Gorjian, Nima and Mittinty, Murthy and Ma, Lin and Yarlagadda, Prasad K. and Sun, Yong (2010) *The Explicit Hazard Model – Part 2 : Applications.* In: PHM 2010 Macau : IEEE - Prognostics & System Health Management Conference, 12-14 January 2010, University of Macau, Macau, China. (In Press)

The Explicit Hazard Model – Part 2: Applications

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Abstract – Hazard and reliability prediction of an engineering asset is one of the significant fields of research in Engineering Asset Health Management (EAHM). In real-life situations where an engineering asset operates under dynamic operational and environmental conditions, the lifetime of an engineering asset can be influenced and/or indicated by different factors that are termed as covariates. The Explicit Hazard Model (EHM) as a covariate-based hazard model is a new approach for hazard prediction which explicitly incorporates both internal and external covariates into one model. EHM is an appropriate model to use in the analysis of lifetime data in presence of both internal and external covariates in the reliability field. This paper presents applications of the methodology which is introduced and illustrated in the *theory* part of this study. In this paper, the semi-parametric EHM is applied to a case study so as to predict the hazard and reliability of resistance elements on a Resistance Corrosion Sensor Board (RCSB).

I. INTRODUCTION

Hazard prediction is imperative for reliability engineers and researches since an engineering asset is aging (or degrading) with time, and its conditional probability of failure is increasing with time. The fundamental notion in hazard prediction is failure times of an engineering asset and its stochastic time-variant covariates. These covariates change stochastically and may influence and/or indicate the lifetime of an engineering asset. There are several significant reasons for the consideration of the hazard and its prediction [1]. One of the major reasons is that it might be physically enlightening to consider the instantaneous risk attaching to an engineering asset known to be survived at age t. Another is that hazard models with covariates are often appropriate to be utilized in the cases of groups of engineering assets, different failure types, and suspended data (i.e. censoring).

The degradation (or aging) process of an engineering asset is influenced by different types of operational and environmental mechanisms that affect it since the moment it is installed [2]. If an engineering asset operates under adverse operational and environmental conditions, its degradation evolves more rapidly than normal operational and environmental conditions. Due to insufficiency of traditional reliability models in individual system reliability under dynamic operational and environmental conditions, statistical hazard models with covariates (also termed as covariate-based hazard models) have been developed. Most of these models are developed based on Cox's proportional hazard model [3]. However, these models have not attracted much attention in the field of reliability due to the prominences of Cox's proportional hazard model.

In industrial applications, multiple failure mechanisms may be recognized through certain diagnostic factors (or internal covariate) [3]. However, multiple failure mechanisms may not be identified by these diagnostic factors alone, since some failures happen due to random shocks (e.g. loads and stress factors) caused by the environment in which the engineering assets operate. Therefore, both diagnostic factors (internal covariates) and operating environment factors (or external covariates) should be incorporated into a covariate-based hazard model to have a more effective hazard prediction for an engineering asset. To this end, in the *theory* part of this study a novel covariate-based hazard model has been developed [3]. This model is named as the Explicit Hazard Model (EHM) that can be presented in two different forms: semi-parametric and non-parametric ones.

EHM is an appropriate model to use in the analysis of lifetime data in the reliability field. EHM proposes a new approach to effectively predict the hazard and reliability of an engineering asset utilizing three different types of data (i.e. historical failure data, internal and external covariates data). This paper presents applications of the methodology which is introduced and illustrated in the *theory* part of this study [3]. In this paper the semi-parametric EHM is chosen to apply for modeling the degradation of resistance elements on a Resistance Corrosion Sensor Board (RCSB). Implementation of the semi-parametric EHM for such a case study can verify this model in the field of reliability. This data are obtained from a laboratory test, which was developed as the standard operating procedure for the measurement of atmospheric corrosion rates using resistance corrosion sensors.

The remainder of the paper is organized as follows. Section II reviews the applications of covariate-based hazard models in the reliability field. Section III illustrates the semi-parametric EHM and its parameter estimation equations. In Section IV a case study to model the degradation of resistance elements on RCSB is conducted. The results of the hazard and reliability prediction are illustrated in the section. Section V provides the conclusions of the paper.

II. OVERVIEW ON APPLICATIONS OF COVARIATE-BASED HAZARD MODELS IN THE RELIABILITY FIELD

A number of covariate-based hazard models have been developed in both the reliability and biomedical fields. Amongst these models, only a few of them have been applied to estimate the hazards of engineering assets. The proportional hazard model is one such model that has been widely applied in the reliability area. Since 1972, this model has generated a great amount of literature in the reliability field [4-33]. Kumar [14] applies the stratified proportional hazard model, which is the simplest and most useful extension of the proportional hazard model, to estimate the hazard and reliability of engineering assets. The proportional intensity model is exercised in different reliability fields [34-38]. Sun et al. [39]

applies the proportional covariate model to predict the hazard of a single stage spur gearbox in a laboratory experiment.

Pijnenburg [40] utilizes the additive hazard model to estimate the hazard and reliability of air conditioning systems of aircrafts, whereas Newby [41] asserts that the model has theoretical limitations which leads to identification problems while estimating parameters of the model. The accelerated failure time model has been used in several applications of the reliability field [42-44]. Shyer et al. [45] applies the extended hazard regression model for accelerated life test with multiple stress loadings. They also extend the model for time-dependent covariates. Liao et al. [31] applies the logistic regression model to predict the residual life of bearings in a laboratory experiment. Kumar and Westberg [46] employs the Aalen's regression model to estimate the hazard and reliability of transmission cables in load-haul-dump machines.

According to literature review, applications of covariate-based hazard models in the reliability field are summarized in the following table. Table 1 illustrates that some of covariate-based hazard models are appropriate to apply with external covariates and some with internal covariates. It also shows that some of these models such as the proportional hazard model are applied for both internal and external covariates but in individual models. With this in mind, none of these covariate-based hazard models explicitly incorporate both external and internal covariates into a model. In fact, all of these approaches neglect the existence of both external and internal covariates in the hazard of an engineering asset. To address this concern and in order to have an effective asset life prediction in presence of these two covariates, the EHM is proposed.

Table 1: Applications of covariate-based hazard models in the reliability field

Model	***External covariates***	***Internal covariates***	***Both external and internal covariates***
Proportional hazard model	[4-24]	[25-33]	Nil
Stratified proportional hazard model	[14]	Nil	Nil
Proportional intensity model	[34-38]	Nil	Nil
Proportional covariate model	Nil	[39]	Nil
Additive hazard model	[34, 40, 41, 47]	Nil	Nil
Accelerated failure time model	[42-44]	Nil	Nil
Extended hazard regression model	[45]	Nil	Nil
Logistic regression model	Nil	[31]	Nil
Aalen's regression model	[46]	Nil	Nil

III. EXPLICIT HAZARD MODEL

Explicit hazard model is a covariate-based hazard model which introduced by Gorjian et al. [3]. Both external covariates (operating environment factors) and internal covariates (diagnostic factors) are explicitly included in the EHM. This model accepts the existence of the two covariates to have more effective prediction results for the hazard and reliability of an engineering asset. EHM allows the external covariate to be considered as a stress factor and the internal covariate as a failure indicator for updating the current status of an engineering asset. This model assumes that internal and external covariates are independent from each other.

If $h\left(t;\bar{z}_1(t),\bar{z}_2(t)\right)$ denotes the hazard of an engineering asset and $h_0\left(\exp(\bar{\gamma}_1\bar{z}_1(t)).t\right)$ is its baseline hazard (or underlying hazard), therefore the generic form of EHM can be expressed as:

$$h(t;\bar{z}_1(t),\bar{z}_2(t)) = h_0\left(\exp(\bar{\gamma}_1\bar{z}_1(t))\cdot t\right)\exp(\bar{\gamma}_2\bar{z}_2(t)) \qquad \textbf{(1)}$$

Where, $\bar{z}_1(t)$ and $\bar{z}_2(t)$ are vectors of internal and external covariates, respectively. $\bar{\gamma}_1$ and $\bar{\gamma}_2$ are vectors of regression coefficients.

Like other covariate-based hazard models, EHM has some merits and drawbacks.

Merits:

- EHM is a covariate-based hazard model to explicitly investigate the influences of both external covariates (operating environment factors) and internal covariates (diagnostic factors) associated with the hazard of an engineering asset.
- EHM utilizes three different sources of data (i.e. historical failure data, internal and external covariates data) to predict effectively the hazard and reliability.
- EHM is presented into two forms: semi-parametric and non-parametric.
- Since EHM incorporates internal covariates into the baseline hazard, the selection of any specific distribution (e.g. Logistic, Log-logistic, and Gamma) in the baseline hazard instead of Weibull distribution is also reasonable.
- Internal covariates in this model can update and reform the baseline hazard in order to show the current status of an engineering asset.
- EHM handles censored and uncensored data.
- EHM can be used in different reliability fields.

Limitations:

- Akin to all other covariate-based hazard models, the estimated values of regression coefficients in EHM are sensitive to omission, misclassification and time dependence of covariates.
- Care must be taken in the selection and formulation of internal and external covariates. Due to complex inter-relationships, in some cases it is very difficult to distinguish between these covariates.

A. MODEL DEVELOPMENT

If the historical failure data follows the Weibull distribution, the semi-parametric EHM is suggested [3]. The semi-parametric EHM involves a specified function (i.e. Weibull distribution) in the form of the baseline hazard. The semi-parametric EHM can be expressed as:

$$h(t;\bar{z}_1(t),\bar{z}_2(t)) = \left[\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}\left(\exp(\bar{\gamma}_1\bar{z}_1(t))\right)^{\beta-1}\right] \times \exp\left(\bar{\gamma}_2\bar{z}_2(t)\right) \qquad \textbf{(2)}$$

If t denotes the lifetime of an engineering asset with $0\le\tau\le t$, the related reliability function of the semi-parametric EHM is given by:

$$R(t;\bar{z}_1(\tau),\bar{z}_2(\tau)) = \exp\left[-\int_0^t \frac{\beta}{\eta}\left(\frac{\tau}{\eta}\right)^{\beta-1}\exp\left[\beta\left(\bar{\gamma}_1\bar{z}_1(\tau)\right)-\bar{\gamma}_1\bar{z}_1(\tau)+\bar{\gamma}_2\bar{z}_2(\tau)\right]d\tau\right] \qquad \textbf{(3)}$$

Suppose the reliability function S has a unit negative exponential distribution, therefore it can be expressed as:

$$S = \int_0^t \exp\left[\beta\left(\bar{\gamma}_1\bar{z}_1(\tau)\right)-\bar{\gamma}_1\bar{z}_1(\tau)+\bar{\gamma}_2\bar{z}_2(\tau)\right]d\left(\frac{\tau}{\eta}\right)^{\beta} \qquad \textbf{(4)}$$

The value of S can be calculated by substituting the estimated values $\hat{\beta},\hat{\eta},\hat{\bar{\gamma}}_1,$ and $\hat{\bar{\gamma}}_2$ into Equation (3), provided that the values of $\bar{z}_1(\tau)$ and $\bar{z}_2(\tau)$ are known for all τ . This assumption may only hold with *continuous time* samples of the covariates. Otherwise, for *discrete time* samples of the covariates where the values of $\bar{z}_1(\tau)$ and $\bar{z}_2(\tau)$ are unknown for all τ , an approximate sample path with the right continuous jump process is required [3].

B. PARAMETER ESTIMATION OF MODEL

In order to estimate the parameters of the semi-parametric EHM, it is required to have the historical failure time data, external and internal covariates data. Suppose that a random sample of r items yields n distinct failure times and $r-n$ censoring (or suspended) times. Therefore, the likelihood function of the semi-parametric EHM is given by Equation (5).

$$\begin{aligned}L(\beta,\eta,\bar{\gamma}_1,\bar{\gamma}_2) &= \prod_{i\in\Theta_F} h(t_i;\bar{z}_1(t_i),\bar{z}_2(t_i)) \times \prod_{j\in\{\Theta_F\cup\Theta_C\}} \mathrm{R}(t_j;\bar{z}_1(\tau),\bar{z}_2(\tau)\big|0\le\tau\le t_j)\\
L(\beta,\eta,\bar{\gamma}_1,\bar{\gamma}_2) &= \prod_{i\in\Theta_F}\left[\frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta-1}\exp\left[\beta\left(\bar{\gamma}_1\bar{z}_1(t_i)\right)-\bar{\gamma}_1\bar{z}_1(t_i)+\bar{\gamma}_2\bar{z}_2(t_i)\right]\right]\\
&\times \prod_{j\in\{\Theta_F\cup\Theta_C\}}\exp\left[-\int_0^{t_j}\frac{\beta}{\eta}\left(\frac{\tau}{\eta}\right)^{\beta-1}\right.\\
&\left.\times\exp\left[\beta\left(\bar{\gamma}_1\bar{z}_1(\tau)\right)-\bar{\gamma}_1\bar{z}_1(\tau)+\bar{\gamma}_2\bar{z}_2(\tau)\right]d\tau\right]\end{aligned} \qquad \textbf{(5)}$$

Where Θ_F indexes the set of failure times and Θ_C indexes the set of censoring (suspended) times; t_i is the failure time of the i^{th} item; and t_j is either the observed failure time or the suspended (censoring) time of the j^{th} item.

If $S_j = \int_0^{t_j}\exp\left[\beta\left(\bar{\gamma}_1\bar{z}_1(\tau)\right)-\bar{\gamma}_1\bar{z}_1(\tau)+\bar{\gamma}_2\bar{z}_2(\tau)\right]d\left(\frac{\tau}{\eta}\right)^{\beta}$ and it has a unit negative exponential distribution, $\Lambda h(t_i;\bar{z}_1(t_i),\bar{z}_2(t_i)) = \ln h(t_i;\bar{z}_1(t_i),\bar{z}_2(t_i))$, and n is the total number of failure times available. Thus the log-likelihood function of the semi-parametric EHM becomes:

$$\begin{aligned}l(\beta,\eta,\bar{\gamma}_1,\bar{\gamma}_2) &= \sum_{i\in\Theta_F}\Lambda h(t_i;\bar{z}_1(t_i),\bar{z}_2(t_i)) - \sum_{j\in\{\Theta_F\cup\Theta_C\}}\mathrm{S_j}\\
l(\beta,\eta,\bar{\gamma}_1,\bar{\gamma}_2) &= n\ln\left(\frac{\beta}{\eta}\right)+\sum_{i\in\Theta_F}\ln\left[\left(\frac{t_i}{\eta}\right)^{\beta-1}\right]\\
&+\sum_{i\in\Theta_F}\exp\left[\beta\left(\bar{\gamma}_1\bar{z}_1(t_i)\right)-\bar{\gamma}_1\bar{z}_1(t_i)+\bar{\gamma}_2\bar{z}_2(t_i)\right]\\
&-\sum_{j\in\{\Theta_F\cup\Theta_C\}}\int_0^{t_j}\exp\left[\beta\left(\bar{\gamma}_1\bar{z}_1(\tau)\right)-\bar{\gamma}_1\bar{z}_1(\tau)+\bar{\gamma}_2\bar{z}_2(\tau)\right]d\left(\frac{\tau}{\eta}\right)^{\beta}\end{aligned} \qquad \textbf{(6)}$$

All parameters can be estimated by maximizing the log-likelihood function using an optimization approach. Equation (6) is applied where the values of $\bar{z}_1(\tau)$ and $\bar{z}_2(\tau)$ are known for all τ . Otherwise the approximate sample path by the right continuous jump process for $\left\{\bar{z}_1(\tau),\bar{z}_1(\tau)\middle|t\ge 0\right\}$ is required. If $S_j^* = S^*$, as a result the log-likelihood function can be expressed by Equation (7).

$$l(\beta,\eta,\bar{\gamma}_1,\bar{\gamma}_2)=\sum_{i\in\Theta_F}\Lambda h(t_i;\bar{z}_1(t_i),\bar{z}_2(t_i))-\sum_{j\in\{\Theta_F\cup\Theta_C\}}\mathrm{S}_j^*$$

$$\begin{aligned} l(\beta,\eta,\bar{\gamma}_1,\bar{\gamma}_2) &= n\ln\left(\frac{\beta}{\eta}\right)+\sum_{i\in\Theta_F}\ln\left[\left(\frac{t_i}{\eta}\right)^{\beta-1}\right] \\ &+\sum_{i\in\Theta_F}\exp\left[\beta(\bar{\gamma}_1\bar{z}_1(t_i))-\bar{\gamma}_1\bar{z}_1(t_i)+\bar{\gamma}_2\bar{z}_2(t_i)\right] \\ &-\left\{\left(\frac{\tau_0}{\eta}\right)^{\beta}\exp\left[\beta(\bar{\gamma}_1\bar{z}_1(0))-\bar{\gamma}_1\bar{z}_1(0)+\bar{\gamma}_2\bar{z}_2(0)\right]\right. \\ &+\sum_{\substack{k=0\\k\in\{\Theta_F\cup\Theta_C\}}}^{n}\exp\left[\beta(\bar{\gamma}_1\bar{z}_1(\tau_k))-\bar{\gamma}_1\bar{z}_1(\tau_k)+\bar{\gamma}_2\bar{z}_2(\tau_k)\right] \\ &\left.\times\left[\left(\frac{\tau_{k+1}}{\eta}\right)^{\beta}-\left(\frac{\tau_k}{\eta}\right)^{\beta}\right]\right\} \end{aligned} \quad \textbf{(7)}$$

IV. CASE STUDY

The semi-parametric EHM was applied to model the degradation of resistance elements on a RCSB which is caused by corrosion. Data in this case study was obtained from a laboratory test, which was developed as the standard operating procedure to use of resistance corrosion sensors for the measurement of atmospheric corrosion rates. This test was carried out for three resistance elements on the RCSB. The typical failure mode of the resistance elements on the RCSB is corrosion. During the degradation of resistance elements on the RCSB, the internal and external covariates were observed and collected for these three samples. These observations were recorded once per month. This laboratory test was conducted over twelve months.

This case study assumed that the failure time occurred when the sectional loss or corrosion rate hit a pre-specified failure threshold. According to the assumed failure threshold, there were three failure times in the samples.

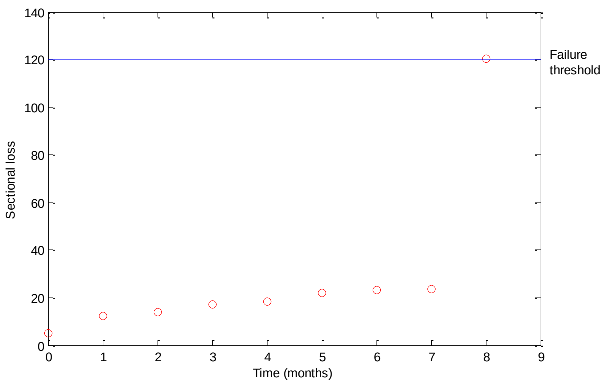

Figure 1: Sectional loss of the resistance element No.1 on the RCSB

The trend of data set shows that the sectional loss was gradually increasing in the three samples. The ranges of changes of the sectional loss were between 1 to 30 μm per year. However, the sectional loss values were increased beyond 100 μm at certain time points in all three samples. Therefore, we assumed that the failure time occurred at in these time points. The failure threshold of one of the sample is illustrated in Figure 1.

In this case study, the temperature in degrees celsius was considered as an external covariate. As a stress factor, it may accelerate or decelerate the failure time of the resistance element. The sectional loss was considered as an internal covariate. It contains information about the current status of the resistance element as a failure indicator. In the case study, we assume that the internal and external covariates are independent from each other in order to test our model. This paper aims to estimate the hazard and reliability of the resistance element No.1 on the RCSB using the semi-parametric EHM. Table 2 shows the partial observations of both the internal and external covariates for this sample of data over eight months. In addition, the estimated values of the shape and scale parameters of the Weibull distribution $(\hat{\beta},\hat{\eta})$, as well as the coefficients of both internal and external covariates $(\hat{\gamma}_1,\hat{\gamma}_2)$ are shown in the table. All of these unknown parameters can be obtained by maximizing the log-likelihood function in Equation (7) using the nonlinear optimization problem.

In order to obtain a better fit as well as to avoid overestimating the coefficients of both the internal and external covariates, these two covariates should be rescaled. For this reason, these covariates are transformed by taking the natural logarithm. If SL indicates as the sectional loss and $Temp$ indicates as the ambient temperature. Therefore, the internal and external covariates are denoted as $z_1(t)=\ln(SL(t))$ and $z_2(t)=\ln(Temp(t))$, respectively. Table 2 shows the values of the temperature and sectional loss after rescaling by above transformed functions.

Table 2: Internal and external covariates observations for the resistance element No.1 on the RCSB

Observation time (month)	***Rescaled temperature***	***Rescaled sectional loss***	***Parameter estimates***
1	3.0910	2.4973	$\hat{\beta}=1.7926$ $\hat{\eta}=0.0843$ $\hat{\gamma}_1=1.7702$ $\hat{\gamma}_2=-2.5865$
2	3.1224	2.6276	
3	3.4177	2.8309	
4	3.2581	2.9167	
5	3.0910	3.0847	
6	3.4045	3.1416	
7	2.9857	3.1667	
8	2.7537	4.7914	

The hazard of the resistance element No.1 on the RCSB can be calculated by substituting the estimated values $\hat{\beta}, \hat{\eta}, \hat{\gamma}_1$, and $\hat{\gamma}_2$ into Equation (2). Figure 2 depicts the hazard estimate of the resistance element No.1 on the RCSB. As it can be seen in the figure, from starting point to 7 months, the hazard rises gradually. From 7 months to 8 months, there is a sharp increase in the amount of hazard and then it peaks at 8 months. According to the historical operational data, the failure time in this sample occurred at 8 months.

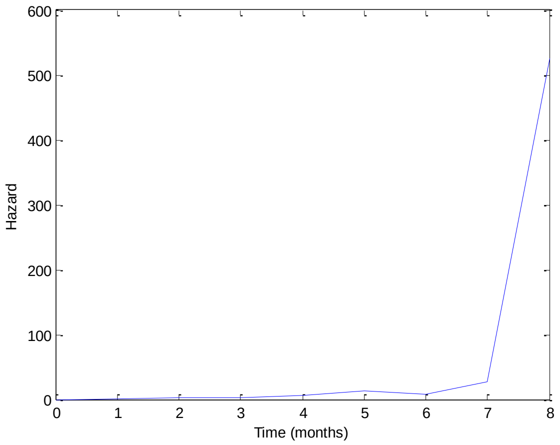

Figure 2: Hazard estimate for the resistance element No.1 on the RCSB

The reliability can be calculated by substituting the estimated values $\hat{\beta}, \hat{\eta}, \hat{\gamma}_1$, and $\hat{\gamma}_2$ into Equation (3) or Equation (4). The integration in the reliability equation can be solved by using the numerically evaluate double integral of the adaptive Simpson quadrature rule. Therefore, the related reliability estimate using EHM is shown in Figure 3.

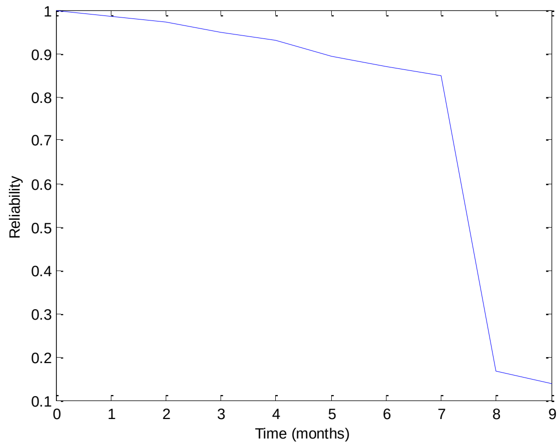

Figure 3: Reliability estimate for the resistance element No.1 on the RCSB

V. CONCLUSIONS

This study presents an application of the new covariate-based hazard model using both internal and external covariates. EHM is an appropriate approach in the analysis of lifetime data and it can be used for prognostics and asset life prediction. This model utilizes three different sources of data (i.e. historical failure data, internal and external covariates data) to effectively predict the hazard and reliability of an engineering asset.

This paper focuses on the application of the semi-parametric EHM. A case study is presented to model the degradation of resistance elements on the RCSB that is caused by corrosion. To test and verify this model, the hazard and reliability of one of the case study's samples are calculated subject to both internal and external covariates. The results of both the hazard and reliability for the sample are illustrated in the paper.

This case study is an initial numerical example to demonstrate EHM; however, it was not specifically designed for the verification of this model. The current case study has some restriction in selection and formulation of these covariates. Therefore, further case studies need to be done as EHM is a new model and its research is still in infancy. Selection and formulation of covariates for EHM will be discussed in more detail in the future work. The Future work is continuing using EHM to hazard and reliability predictions of both an individual and a system in Engineering Asset Health Management (EAHM).

ACKNOWLEDGMENTS

The authors gratefully acknowledge the financial support provided by both the Cooperative Research Centre for Integrated Engineering Asset Management (CIEAM), established and supported under the Australian Government's Cooperative Research Centers Program, and the School of Engineering Systems of Queensland University of Technology (QUT). The authors would like to thank Dr. Avin Mathew and Dr. Eric Kim for their valuable comments and suggestions regarding MATLAB programming during the preparation of the paper.

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