Modelling the Cutting Process and Cutting Performance in Abrasive Waterjet Machining with Controlled Nozzle Oscillation

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By

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Keywords

Abrasive waterjet cutting, nozzle oscillation, experimental design and data analysis, cutting performance, kerf characteristics, mathematical modelling, dimensional analysis, model verification.
Abstract

Abrasive waterjet (AWJ) cutting is one of the most recently developed manufacturing technologies. It is superior to many other cutting techniques in processing various materials, particularly in processing difficult-to-cut materials. This technology is being increasingly used in various industries. However, its cutting capability in terms of the depth of jet penetration and kerf quality is the major obstruction limiting its further applications. More work is required to fully understand the cutting process and cutting mechanism, and to optimise cutting performance.

This thesis presents a comprehensive study on the controlled nozzle oscillation technique aiming at increasing the cutting performance in AWJ machining. In order to understand the current state and development in AWJ cutting, an extensive literature review is carried out. It has found that the reported studies on controlled nozzle oscillation cutting are primarily about the use of large oscillation angles of 10 degrees or more. Nozzle oscillation in the cutting plane with such large oscillation angles results in theoretical geometrical errors on the component profile in contouring. No published attempt has been found on the study of oscillation cutting under small angles although it is a common application in practice. Particularly, there is no reported research on the integration of nozzle oscillation technique into AWJ multipass cutting, which is expected to significantly enhance the cutting performance.

An experimental investigation is first undertaken to study the major cutting performance measures in AWJ single pass cutting of an 87% alumina ceramic with controlled nozzle oscillation at small angles. The trends and characteristics of cutting performance quantities with respect to the process parameters as well as the science behind which nozzle oscillation affects the cutting performance have been analysed. It has been shown that as with oscillation cutting at large angles, oscillation at small angles can have an equally significant impact on the cutting performance. When the optimum cutting parameters are used for both nozzle oscillation and normal cutting, the former can statistically increase the depth of cut by 23% and smooth depth of cut by 30.8%, and reduce kerf surface roughness by 11.7% and kerf taper by 54%. It has
also been found that if the cutting parameters are not selected properly, nozzle oscillation can reduce some major cutting performance measures.

In order to correctly select the process parameters and to optimise the cutting process, the mathematical models for major cutting performance measures have then been developed. The predictive models for the depth of cut in both normal cutting and oscillation cutting are developed by using a dimensional analysis technique. Mathematical models for other major cutting performance measures are also developed with the aid of empirical approach. These mathematical models are verified both qualitatively and quantitatively based on the experimental data. The assessment reveals that the developed models conform well to the experimental results and can provide an effective means for the optimum selection of process variables in AWJ cutting with nozzle oscillation.

A further experimental investigation of AWJ cutting of alumina ceramics is carried out in order to study the application of AWJ oscillation technique in multipass cutting. While high nozzle traverse speed with multipass can achieve overall better cutting performance than low traverse speed with single pass in the same elapsed time, it has been found that the different combination of nozzle traverse speed with the number of passes significantly affects cutting process. Optimum combination of nozzle traverse speed with the number of passes is determined to achieve maximum depth of cut. It has also demonstrated that the multipass cutting with low nozzle traverse speed in the first pass and a comparatively high traverse speed for the following passes is a sensible choice for a small kerf taper requirement. When nozzle oscillation is incorporated into multipass cutting, it can greatly increase the depth of cut and reduce kerf taper. The predictive models for the depth of cut in both multipass normal cutting and multipass oscillation cutting are finally developed. With the help of dimensional analysis, the models of the incremental cutting depth for individual pass are derived based on the developed depth of cut models for single pass cutting. The models of depth of cut for a multipass cutting operation are then established by the sum of the incremental cutting depth from each pass. A numerical analysis has verified the models and demonstrated the adequacy of the models’ predictions. The models provide an essential basis for the development of
optimization strategies for the effective use of the AWJ cutting technology when the multipass cutting technique is used with controlled nozzle oscillation.
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Nomenclature

\( C_f \)  coefficient of friction on kerf wall

\( C_k \)  characteristic velocity that combines the particle and material characteristics

\( C_v \)  orifice efficiency

\( C_y \)  compressibility coefficient

\( d_j \)  jet diameter

\( d_n \)  mixing tube diameter

\( D \)  abrasive particle diameter

\( E \)  elastic modulus of target material

\( f_w \)  fraction of effective stress wave energy

\( F \)  oscillation frequency

\( h \)  depth of cut

\( h_1 \)  depth of cut in normal cutting

\( h_2 \)  depth of cut in oscillation cutting

\( h_c \)  depth of cutting wear zone

\( h_d \)  depth of deformation wear zone

\( h_n \)  incremental cutting depth for the \( n \)th pass

\( h_p \)  depth of cut made before the \( n \)th pass

\( h_s \)  smooth depth of cut

\( H \)  standoff distance

\( H_m \)  hardness of target material

\( H_n \)  standoff distance for the \( n \)th pass

\( k_d \)  function of the drag coefficient and the insert diameter

\( K_c \)  fracture toughness of target material

\( m \)  average mass of an individual particle
$m_a$  abrasive mass flow rate
$m_w$  water mass flow rate
$P$  water pressure
$q$  function of the abrasive velocity and the threshold velocity
$r$  particle radius
$R$  average material removed by an individual particle
$R_a$  surface roughness assessed by the centre-line average
$R_f$  particle roundness factor
$R_w$  surface waviness
$t_{rw}$  ratio of abrasive to water mass flow rate
$T_K$  kerf taper
$u$  nozzle traverse speed
$u_n$  nozzle traverse speed for the $n$th pass
$v$  particle velocity
$v_e$  the threshold particle velocity
$v_j$  water velocity
$v_0$  initial particle velocity
$w$  kerf width
$w_n$  average kerf width for the $n$th pass
$W_b$  bottom kerf width
$W_m$  minimum kerf width
$W_t$  top kerf width
$\alpha$  particle attack angle
$\alpha_1$  particle attack angle in normal cutting
$\alpha_2$  particle attack angle in oscillation cutting
$\alpha_e$  attack angle of particle at kerf exit
\( \alpha_n \)  \hspace{1cm} \text{particle attack angle for the } n\text{th pass} \\
\( \alpha_0 \)  \hspace{1cm} \text{angle of impingement at the top of cutting surface} \\
\( \theta \)  \hspace{1cm} \text{oscillation angle} \\
\( \gamma \)  \hspace{1cm} \text{fracture energy per unit area} \\
\( \varepsilon \)  \hspace{1cm} \text{the energy required to remove unit volume in deformation wear} \\
\( \eta \)  \hspace{1cm} \text{momentum transfer efficiency} \\
\( \sigma_f \)  \hspace{1cm} \text{material flow stress} \\
\( \rho_p \)  \hspace{1cm} \text{particle density} \\
\( \rho_w \)  \hspace{1cm} \text{water density}
Statement of Original Authorship

The work contained in this thesis has not been previously submitted for a degree or diploma at any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.

Signature: ___________________________

Date: ____________________________
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Chapter 1

Introduction

Abrasive waterjet (AWJ) machining technology was first commercialised in the late 1980’s as a pioneering breakthrough in the area of non-traditional processing technologies. It is used to cut the target materials with a fine high pressure water-abrasive slurry jet. AWJ machining is superior to many other cutting techniques in processing various materials, such as no thermal distortion on the workpiece, omnidirectional cutting capability, high machining versatility to cut virtually any material and small cutting forces. This technology has found extensive applications in industry [1], particularly in contouring or profile cutting and in processing difficult-to-cut materials such as ceramics and marbles [2-5], and layered composites [6-10].

Since the introduction of this new technology, a great deal of research and development has been conducted towards exploring its applications and the associated science [1]. However, its cutting capabilities in terms of the depth of cut or depth of the jet penetration and kerf quality are the major obstructions limiting its further application. More work is required to fully understand the cutting process and cutting mechanism, and to model and then optimise cutting performance. The cutting performance in AWJ machining is often assessed by such measures or quantities as the depth of cut, smooth depth of cut in the upper cutting zone, kerf profile and geometrical accuracy and surface quality. These measures are normally referred to as cutting performance measures.

In recent years, a significant contribution to the AWJ cutting technology is the introduction of a controlled nozzle oscillation technique aiming at enhancing the cutting performance [2, 11-13]. With this cutting technique, a pendulum-like, forward and backward motion of the cutting head (nozzle) in the cutting plane at predetermined frequency and angular amplitude is superimposed to the normal nozzle traverse motion. By correctly selecting the oscillation parameters and combining cutting conditions, this technique can significantly increase the cutting performance. Nevertheless, there has not been sufficient study on how the nozzle oscillation affects
the cutting performance, the resultant trends and strategies for the selection and optimization of the oscillation parameters based on the other optimized cutting parameters and work materials. In addition, the cutting mechanisms when AWJ machining with controlled nozzle oscillation have never been studied in any detail. Such a study will explain the science and fundamental reason why nozzle oscillation affects the cutting performance, and form the fundamental basis for optimizing the oscillation parameters. Particularly, it is well known that most of the AWJ applications are related to profile cutting or contouring operation, which requires AWJ oscillation operation under small angles to avoid geometrical errors on the components. However, the previous studies all focus on the use of large nozzle oscillation angles of more than 10°. A pressing research study at present is required to investigate if nozzle oscillation under small angles can equally improve cutting performance. Furthermore, no reported study has been found on the development of predictive mathematical models for the cutting performance in AWJ machining with this novel cutting technique, although such models are essential for the selection of the operating parameters and optimisation of the cutting performance in process planning and control.

Therefore, experimental and theoretical studies have been undertaken in this project to investigate the cutting performance under AWJ cutting of 87% alumina ceramics with nozzle oscillation with the research interest on the use of small oscillation angles. An experimental investigation was first conducted to evaluate the kerf characteristics obtained by AWJ single pass cutting with nozzle oscillation under small oscillation angle. The trends and characteristics of cutting performance quantities with respect to the process parameters as well as the science behind which nozzle oscillation affects the cutting performance were analysed. The trends together with the analyses and the large quantity of experimental data can be used in the selection of optimum oscillation parameters under different process conditions. It followed that the mathematical models for the major cutting performance measures were developed based on the experimental data and the theoretical findings achieved in the study. Moreover, a further experimental study was carried out to investigate AWJ oscillation cutting of alumina ceramics under multipass cutting environment. The purpose of this study is to increase the application domain of nozzle oscillation to multipass cutting, obtain an in-depth understanding of this novel cutting mode and
explore a solution for the optimum selection of the number of passes and the corresponding cutting condition. The mathematical models were finally developed to predict the depths of cut under both normal cutting (cutting without nozzle oscillation) and oscillation cutting.

Consequently, this study was carried out with an attempt to attain the following objectives:

- Conduct an experimental investigation to examine the quantitative effect of the nozzle oscillation parameters on the major cutting performance measures, study the trends of this effect with respect to the other process parameters and develop a strategy for selecting the optimum cutting and nozzle oscillation parameters for AWJ machining.

- Develop predictive mathematical models for the major cutting performance measures in single pass AWJ cutting with nozzle oscillation for process control and optimisation, run numerical studies to assess the plausibility of the predictive models, verify the predictive models and estimate the models’ predictive capabilities.

- Conduct an experimental investigation to study the kerf characteristics of AWJ multipass cutting with controlled nozzle oscillation, analyse the effect of process parameters on the kerf characteristics and optimize the selection of the number of passes with the corresponding cutting condition.

- Develop mathematical models to predict the depth of cut produced by multipass cutting with and without nozzle oscillation and assess their predictive ability.

This project, for the first time, systematically studied the cutting phenomenon and cutting performance with respect to the various operation variables in AWJ cutting with nozzle oscillation at small angles. It has been found that similar to oscillation cutting at large oscillation angles, oscillation at small angles can equally have a significant impact on the cutting performance. This research also experimentally demonstrated that if the cutting parameters are not selected properly, nozzle oscillation cutting can reduce the major cutting performance measures rather than increase the cutting performance. As an instance, it was found that there exists a
threshold value of the product of oscillation angle and oscillation frequency for the increase of the depth of jet penetration. If the employed product value is lower than the threshold value, oscillation cutting will produce a small depth of cut compared to the corresponding cutting without nozzle oscillation. More generally, if the oscillation parameters are not selected properly, this technique may result in adverse effects on the cutting performance. This finding corrected the well held intuition that the use of oscillation technique can constantly increase this cutting quantity. This study has obtained fundamental understanding of the kerf formation process and mechanisms in AWJ oscillation cutting with small angles.

This project also for the first time conducted a detailed study on AWJ multipass cutting incorporated with nozzle oscillation technique. The advantages of multipass cutting over single pass cutting have been demonstrated by experiment taking into account the same operation time. The findings for the best combination of nozzle traverse speed and the number of passes to achieve maximum depth of cut together with the guidelines for the selection of cutting variables provide a basis for the efficient use of this advanced technology in industry.

Mathematical models developed for the major cutting performance measures in both oscillation single pass cutting and oscillation multipass cutting provide the quantitative prediction for the cutting performance, which has not been found for these cutting modes. These models are the essential steps towards the correct selection and combination of operation variables as well as the optimization of the cutting process.

In accordance with the motivation, objectives and the research scope of this project, this thesis is organized into 6 chapters with brief outlines of each chapter being as follows:

Chapter 2 gives a detailed review of the current state of the research and development in AWJ machining. The major achievements and problems related to AWJ cutting performance, modelling work and new techniques to enhance AWJ cutting capabilities are examined. Chapter 3 presents a study of AWJ oscillation cutting of an alumina ceramic. This includes the experimental study and theoretical investigations of kerf characteristics (depth of cut, smooth depth of cut, kerf surface
roughness, kerf taper, top kerf width and bottom kerf width) using different process parameters (oscillation angle, oscillation frequency, water pressure, nozzle traverse speed, standoff distance and abrasive mass flow rate). Based on the findings and the experimental data, mathematical models for major cutting performance measures are developed in chapter 4. In this chapter, the models of depth of cut for both normal cutting and oscillation cutting are developed by using dimensional analysis technique. The development of other models, such as smooth depth of cut, kerf surface roughness, kerf taper, top kerf width and bottom kerf width, are also presented based on empirical approach. In chapter 5, a detailed experimental study of AWJ cutting of alumina ceramics is performed to investigate the cutting performance by multipass cutting with nozzle oscillation. The major cutting performance measures as represented by the depth of cut, kerf taper, top kerf width and bottom kerf width are analysed in relation to the effects of major process parameters such as nozzle traverse speed, the number of passes, oscillation parameters and water pressure. The development of mathematical models for the depth of cut suitable for normal cutting and oscillation cutting is also presented. Finally, chapter 6 gives the general conclusions and the perspective for future study.
Chapter 2

A Review on the Current State of AWJ Machining

2.1 Introduction

Abrasive waterjet (AWJ) machining is one of the most recently developed non-traditional cutting processes. It uses a fine jet of ultrahigh pressure water and abrasive slurry to cut the target material by means of erosion [14, 15]. In AWJ cutting process, water serves primarily as an accelerating medium while the abrasive particles take over the role of material removal. This novel technology was first initiated by Franz to cut laminated paper tubes in 1968 [16] and was first introduced as a commercial system in 1983 [17] for the cutting of glass. Nowadays, this process is being widely used for machining of difficult-to-machine materials such as ceramics, concrete, ceramic composites, fibre-reinforced composites, laminates, and titanium alloys where conventional machining is often not technically or economically feasible. Its various distinct advantages over the other machining technologies such as no thermal distortion on work material, high speed and multidirectional cutting capability, high cutting efficiency, ability to cut complicated shapes of even nonflat surfaces very effectively at close tolerances and low deformation stresses within the work material make it ideal for automation [17, 18].

In the last two decades, a great deal of research effort has sought to understand this machining process, its cutting mechanisms and the associated science [2, 19-25]. In this chapter, an extensive review of the current state of research and development in AWJ machining will be conducted.

2.1.1 General description of AWJ machining

AWJ machining uses a beam of water to accelerate abrasive particles to speeds fast enough to cut through much harder workpiece materials. Figure 2.1 is a schematic of AWJ machining process illustrating its cutting head assembly and the basic cutting principle. Water is pumped to high pressure (typically up to 380 MPa) and exits a tiny orifice which has a diameter of commonly about 0.1mm. The high velocity
water creates a vacuum which sucks abrasives from the abrasive line and mixes them with the water in the mixing chamber. Part of the waterjet's momentum is transferred to the abrasives, whose velocities rapidly increase. As a result, a focused, high-velocity stream of abrasive waterjet with a speed normally from 430m/s to 660m/s exits the nozzle and performs the cutting action of the workpiece surface.

Figure 2.1. Schematic of AWJ machining process [26].

A typical abrasive waterjet machining system is mainly composed of the following components as shown in Figure 2.2 [27]: water treatment unit, pump, high pressure intensifier, waterjet, abrasive feed system, abrasive-jet nozzle, and supporting accessories such as hoses and control valves. This system is often manipulated by a cutting head positioning device, typically a robot handling system, which can execute a nozzle traverse motion along the predetermined path. In addition, a water catcher is installed at the bottom of the work material to collect the water and the abrasive particles.

According to their fluid medium, waterjet can generally be distinguished by plain waterjets, waterjets with additives (soluble), and abrasive (non-soluble) waterjets [1]. A plain waterjet uses only a pressured stream of water to cut through materials. This type of cutting is limited to material with naturally occurring small cracks or softer material.
Abrasive waterjet systems are further divided into two categories according to their formation and the ways of adding abrasive particles [28]:

- Direct pumping system: in a direct pumping system, the abrasive particles mix with the water at an early stage before the water is pressured. The slurry is pumped and expelled through a nozzle to form an abrasive slurry jet (ASJ). In this system, the abrasive particles can be mixed thoroughly with the water. However, this process causes more wear on the internal parts of the system than entrainment system. Further, due to the high density of the ASJ, the operating pressures are relatively lower than AWJ (<190 MPa), which has limited its application to relatively low pressures [29, 30].

- Entrainment system: in this system, the high pressure water passes through an orifice and a waterjet is formed. The abrasive particles are entrained into a mixing chamber just next to the orifice through a separate supply line. The waterjet is then mixed with the abrasive particles in the mixing chamber. The abrasive particles are accelerated into the stream and exit the nozzle with the stream to perform the cutting. In this system, the water can reach very high
pressure of up to 380MPa. But, the abrasive particles may not mix uniformly with the water, which will affect the cutting performance. Most of the abrasive waterjet cutting systems currently used in industry are entrainment system. As this project will be conducted based on entrainment AWJ system, the major components will be examined below.

A typical entrainment AWJ machine includes a high pressure pump system, a water supply system, an abrasive feed system, AWJ cutting head, a position control system and a catcher unit (Figure 2.2).

The high pressure system pumps the water to a high pressure and delivers it through the system. For this purpose, a hydraulic radial displacement pump is first used to pressurise the hydraulic oil. The pressured hydraulic oil is then fed into and drives an intensifier pump, providing a carrier medium for the abrasive material. An attenuator is often included on the water outlet of the intensifier to absorb pressure fluctuation.

The water supply system is used to transport high pressure water from the pump to the cutting head. High pressure pipes, joints and various valves are the major parts of the water supply system.

The abrasive feed system consists of an abrasive hopper, an abrasive metering valve and a delivery hose. The abrasive hopper acts as a container to store the abrasives. The metering valve turns on/off the abrasive flow and control the abrasive mass flow rate. The delivery hose transports the abrasive particles from the metering valve to the abrasive inlet on the nozzle assembly. It is a requirement that the distance between the abrasive hopper and the nozzle assembly be short to obtain the high cutting efficiency.

The cutting head (or nozzle assembly) consists of an orifice, a mixing chamber and an abrasive waterjet nozzle. It is used to provide and control high pressure abrasive waterjet. With diameter typically from 0.1mm to 0.5mm, the orifice transfers the high pressure water into high velocity waterjet. The common materials of an orifice are sapphire or diamond. A sapphire orifice could work for up to 200 hours while a diamond orifice usually lasts ten times longer [31]. Located between the orifice and the nozzle, the mixing chamber serves as a space to mix the waterjet with the
abrasive particles which are entrained by vacuum suction. After mixing with the waterjet in the mixing chamber, the abrasive particles are accelerated through the nozzle. The nozzle is typically made of tungsten carbide, which gives it a working life of four hours. In order to achieve an optimum cutting performance, there are strict requirements for the diameter and length of the nozzle. The determination of the nozzle diameter must be associated with a consideration of orifice diameter, which requires that the nozzle diameter be usually 2.5 to 5 times of orifice diameter [32, 33]. The nozzle length must be proper as too short a nozzle cannot ensure sufficient acceleration of the abrasive particle while too long a nozzle could result in high wear caused by passing abrasive waterjet, which would also result in momentum loss. In order to reduce the nozzle wear, a great deal of effort has been made [1, 32, 34-42] including improved design of nozzle structure, proper selection of nozzle material and good alignment of the nozzle with the orifice.

The position control system moves the cutting head according to the required path. The common device is a mechanical manipulator or a high precision multi-axis robot. An essential requirement for a positioning system is that it could cut intricate shapes. The control procedure of a cutting operation is usually programmable. A computer design or manufacturing software coupled with a CNC controller can be used to position the cutting head.

The catcher serves as the functions of dissipating the residual energy of the jet, as well as collecting the water, abrasive particles and the debris from the cut material. Depending on the motion configuration of a machine, a catcher could be a tank catcher or a point catcher. A moving cutting head combined with a fixed workpiece requires a tank catcher while a point catcher is used when cutting head is fixed and the workpiece moves.

### 2.1.2 Characteristics

When compared with other traditional and non-traditional machining processes, abrasive waterjet technology offers the following advantages [43]:

- High machining versatility. An abrasive waterjet can cut virtually any material, particularly for machining many difficult-to-machine materials such as pre-
hardened steel, titanium alloys, copper, brass, aluminium, as well as brittle materials like glass, ceramic, quartz and laminates. It can cut wide range of thickness for workpiece.

- Ability to produce contours. Abrasive waterjets are exceptionally good at two dimensional machining. It is possible to cut very complicated shapes or bevels of any angle in addition to three-dimensional profiling because the process is unidirectional.

- Minimal or no heat affected zone (HAZ). During AWJ cutting process, the heat generated is instantaneously carried away by the water. As a result, no temperature rise occurs in the workpiece. This is especially useful for cutting thermal sensitive material like tool steel and other metals where excessive heat may change the properties of the material.

- Small cutting forces and fast setup. The cutting forces are very small and so for most parts very little fixture is needed. Flat material can be positioned by laying it on the table and putting a couple of weights on it. No tool changes required, so there is no need to program tool changes or physically qualify multiple tools.

- Environmentally friendly. Because there is no heat build-up with abrasive waterjet cutting, there is no fire hazard. Unlike other machining or grinding, AWJ cutting does not produce any dust or particles that are harmful if inhaled. It provides the most environmentally friendly machining around. Some of the waterjet systems are used in the food industry and many of the pumps even use vegetable oil for assembly lubrication. The spent abrasive and waste material become suitable for landfill. The red colour of garnet abrasive also looks nice in the garden.

- Easy integration with mechanical manipulator. It is possible to integrate AWJ cutting into computer-controlled system, optical tracers and full scale six-axis robots. So AWJ machining can be easily automated for production use.

- Availability. As water is the working fluid and the most commonly used abrasive materials, garnet and silica, are at low cost, the process is widely available.
AWJ machining is a very useful machining process that can be readily substituted for many other cutting methods; however, it has some limitations as to what it can cut. Listed below are these limitations and brief descriptions:

- **High initial capital cost and hourly cost.** The major operating cost is the cost of equipment. It is estimated [27] that primary investment of an AWJ system will require US$50,000~500,000. Operating and maintenance cost will be at least US$10~30 per hour. So AWJ cutting is more expensive than other forms of non-traditional machining like plasma or lasers, etc.

- **Limited machining capability.** The cutting capability of the AWJ cutting is limited in terms of the thickness of the target materials that it can penetrate. Very thick parts cannot be cut through with AWJ cutting and still hold dimensional accuracy. If the part is too thick, the jet may dissipate some and a large pocket is often formed at the bottom of the kerf. In addition, a rough waviness pattern is often left at the lower part of the cut surface.

- **Kerf taper.** Taper is also a problem with waterjet cutting in very thick materials. Taper is formed when the jet exits the part at a different angle than it enters the parts, and can cause dimensional inaccuracy.

- **Noise associated with AWJ cutting.** The cutting process involves both mechanical and aerodynamic noises. The mechanical noise is primarily from the engine or electric motor while the aerodynamic noise is from the free jet travelling at high velocities. This occurs especially when large standoff distances are used. In addition, substantial noise is also generated when the abrasive waterjet hits the catcher.

- **The wear of nozzle.** AWJ nozzle is one of the most critical parts that influences the technical and economical performance of the AWJ machining system. It is subjected to constant wear as machining progresses. Typical nozzles made of tungsten carbide/cobalt wear out in about 4 hours at an operating pressure of 240MPa [44]. The increased wear of the AWJ nozzle makes the clearance between the abrasive mixture and the nozzle larger. This causes incomplete mixing of the abrasive particles with the high velocity waterjet which results in deterioration in cutting ability and poor surface quality, affects the precision of
machining and causes undesirable changes in workpiece geometry. It is known that many process variables affect nozzle wear rate. Hashish [32] reported that for tungsten carbide nozzles, the size of the abrasive particles affects the rate of nozzle wear.

2.1.3 Applications

The potential applications of AWJ are numerous [14, 45]. Because of the technical and economic performance of abrasive waterjets, many industries could immediately benefit from this new technology. Over the last decades, AWJ has been found to be widely used in various industries, including manufacturing industry, civil and construction industry, coal mining industry, food processing industry, oil and gas industry, electronic industry and cleaning industry.

- In general factory applications as metal manufacturing, automotive, glass and aerospace industries, AWJ machining has been particularly used in cutting “difficult-to-cut” materials [10, 46-48] as well as in pattern cutting on various materials [49]. These materials were cut by a variety of other technologies before the application of AWJ cutting including diamond saws, plasmas and lasers. However, the thermal devices may produce undesirable changes in material characteristics and, in some cases, may not be able to cut as effectively as an abrasive waterjet.

- In the construction industry, in addition to cutting and scarifying for reinforced concretes, AWJ could perform several other useful functions. For example, it could be used to sandblast and cut corroded rebar, or it can drill holes for bolting posts. It is also often used in road and bridge repair, underground work and pile cutting.

- In the oil and gas industry, a ship can be equipped with an AWJ cutting system for offshore work. Some examples of what an AWJ cutting system can offer are casing cutting for decommissioning of oil wells, rescue operations, platform cutting and repair, underwater construction and pipe cutting.
The coal mining industry can benefit from AWJ cutting by being able to safely cut metal structures in the potentially explosive environment underground. Coal picks can be replaced with abrasive jets for high productivity.

In electronic industry, waterjet cutting is mostly used to cut out smaller circuit boards from a large piece of stock. With waterjet cutting, a very small kerf, or cutting width, can be generated, and there is no much waste of materials. Because the jet is so concentrated, it can also cut very close to the given tolerances for parts mounted on the circuit board without damaging them.

In food industry, waterjet cutting can be used for food preparation. The cutting of certain foods such as bread and trimming fat from meats can also be easily done with it. Since the waterjet exerts such a small force on the food, it does not crush it, and with a small kerf width, very little is wasted.

Cleaning with high pressure waterjet is one of the recent applications of AWJ cutting [17]. Previous cleaning technologies are based on the use of special chemical compounds. Tightening environmental regulations make this approach to surface processing illegal and too expensive due to the usage of chlorofluorocarbons and ozone depleting solvents. Waterjet cleaning is a viable technique for coating removal. It eliminates debris contamination, workpiece surface damage and has the potential for recycling water and off-products.

Only a few examples of the potential applications of the abrasive waterjet cutting technology have been presented in this section. However, they do demonstrate the broad range of applications of abrasive waterjet technology. The possibilities for the industrial use of abrasive waterjets in the immediate future seem unlimited.

2.2 Material removal mechanisms

As a relatively new manufacturing technology, AWJ machining has been extensively explored in many aspects including [17] the understanding of the physics of the process, research oriented toward operations of the process, research focused on the system, sensing, monitoring, and controlling, and the development of new applications to broaden the scope of this technology. Among these research efforts, the investigation on the material removal mechanisms is considered an essential step
to study the cutting performance and model the cutting process. In this section, the basic material removal mechanisms and the associated research effort are examined.

In AWJ cutting, the material removal mechanisms can be broadly classified into micro-mechanism and macro-mechanism, which the former refers to the mechanism in relation to the material removal by individual particles, while the latter is concerned with the kerf formation process. Before discussing these two categories, an examination of the kerf geometry produced in AWJ cutting is necessary.

2.2.1 Kerf geometry of AWJ machining

In AWJ machining, it is found that there are two categories of kerfs produced, “through” cuts and “non-through” cuts. When the jet is provided with sufficient energy, a through cut is formed as shown in Figure 2.3. It is characterized by a wider entry and then a gradual reduction towards the exit so a kerf taper is formed. At the top edge of the kerf two small rounded corners are generated due to the plastic deformation of material caused by jet bombardment. The longer the cut is exposed to the abrasive waterjet, the more evident is the rounding at the top of the cut. When cutting ductile materials, burr may form at the exit of the cut because the plastically deformed material rolls over at the bottom of the kerf.

Figure 2.3 (a) shows the kerf surface cut by an AWJ. In the upper portion of the kerf (named steady cutting zone), the primary surface irregularity is roughness and it is free of waviness. The depth of this zone is named smooth depth of cut. For the lower portion of the kerf (named unsteady deformation wear zone), the waviness “striations” are the dominant characteristic feature. In addition, the waviness marks on the workpiece surface have backward drag angles corresponding to the particle traces, and the drag angles increase as the jet cuts into the workpiece. It was reported [50] that this drag angle is the result of the waterjet that changes the direction as the depth of jet penetration increases. The height penetrated by an AWJ is defined as the depth of cut.
When the waterjet is unable to penetrate the specimen, a non-through cut is formed (Figure 2.4). In an experimental study of kerf geometry for AWJ cutting of ceramic materials, Chen et al found [51] that the typical kerf geometry for ceramics can be divided into three zones while the two upper zones resemble that of a through cut, but a later cut at the bottom causes the kerf to widen forming an enlarged “pocket” with an irregular shape. The upper zone has a smooth surface and no visible striations and pits. In the lower zone which is characterized by numerous pits, the penetration process occurs due to jet upward deflection. The middle zone, which has obvious striations but no pits, marks the transition from the upper smooth zone to the lower enlarged zone. The width of kerf tapers in the upper zone and the width at the end of this zone is equal to the least width of the cut. In the middle zone the width of kerf remains unchanged and is also equal to the least width. In the lower zone the kerf curvature changes greatly and a ballooning formation is observed. As the depth of penetration increases, such formation becomes enlarged. It was reported [10] that this is attributed to the reduced jet stability and effectiveness, jet side deflection and the particles rebounding from the bottom of the kerf.
2.2.2 Micro mechanisms of AWJ machining

AWJ cutting is a micro-machining process in which material removal takes place due to the erosive action of abrasive particles that strike on the cut surface with high impact velocity. The impact of single solid particle is the basic action in the material removal by abrasive waterjets. The literature about micro mechanisms of material removal is extensive. Ludema and Meng [53] defined four sub-mechanisms by which solid particles separate material from a target surface. These mechanisms are cutting, fatigue, melting, and brittle fracture and they generally do not act separately but in combination. Their importance for particular erosion process depends on factors like the impact angle of jet, target material properties and environmental conditions. As the micro erosion mechanisms is greatly varied dependent on the characteristics of target materials, the published literature can broadly be classified into micro mechanisms associated to ductile materials and brittle materials. In general, traditional wear theories for ductile materials [54-56] and brittle materials [57-59] apply to erosive material removal process in AWJ machining.

2.2.2.1 Erosion mechanisms for ductile materials

A vast published literature focused on the investigations of erosion mechanisms for ductile materials [11, 55, 56, 59-68]. Most of the material erosion models derived based on an early Finnie’s [54] “micro-cutting model”. He discussed the erosion process by assuming a plastic response character of the material determined by its
flow stress. The trajectory of a particle cutting a material was calculated and the
eroded volume was determined for shallow impact angles and large impact angles
which denote the angles of inclination of the abrasive particles with respect to the
local kerf geometry. Finnie and McFadden [67] refined the theory by making more
realistic assumptions about the forces presented during particle and surface
interaction. The new modification supports Finnie’s theory, especially for shallow
angles of impact.

Bitter [55, 56] conducted a deeper study on the collision of an elastic sphere with a
plane that deforms both elastically and plastically. He divided the entire material
removal process into two modes including “cutting wear” that happens at low impact
angles and “deformation wear” that occurs at high impact angles and developed
“cutting–deformation model”. Cutting wear is caused by the cutting action of the free
moving particles while deformation wear is a result of repeated deformation by
particles during collisions. He proposed that these two modes occur simultaneously.
Two wear formulas resulted depending on the angle of incidence. In the first, the
particle will exit the material with a velocity component tangential to the surface; in
the second, all the tangential kinetic energy will be dissipated. The total wear at any
angle will be due to the combined effect of cutting and deformation wear. Based on
his finding, Bitter developed two equations to calculate the material removal under
each mechanism. Then, the total material removal is the sum of the two equations.

Hutchings [68] introduced an alternative discussion of the micro-cutting processes
during the solid particle erosion. He defined two modes of material removal due to
micro-cutting, that is, cutting deformation and ploughing deformation as shown in
Figure 2.5, so developed “ploughing-deformation model”. The ploughing
deformation mode occurs when particle is spherical or a round part of an irregular
particle strikes the material so the particle has a large negative rake angle; whereas
cutting deformation is significant for sharp-edged, angular particles that have more
positive rake angles. Hutchings further divided the cutting deformation into type I
cutting deformation and typelcutting deformation depending on the direction of the
particle rotation. It was found that for forward rotating particles, typelis dominant
and therefore a pronounced crater lip is formed, while for particle backward rotation,
type II is valid and this results in chip removal. Hutchings concluded that the cutting deformation and ploughing deformation occur simultaneously during the erosion process.

Figure 2. 5. Cutting deformation and ploughing deformation [1].

2.2.2.2 Erosion mechanisms for brittle materials

There are many models to describe material removal mechanisms for brittle materials [4, 44, 57, 58, 63, 69-72]. Most of them are applied to specific materials. In spite of their diversities, most of them agree that the erosion of brittle materials is a cracking process [73] in which material is removed by the propagation and intersection of cracks ahead of and around the abrasive particle. In fact, AWJ cutting of any material takes place as a combination of brittle and ductile erosion wear mechanisms, but one or the other may dominate the cutting process.

Zeng and Kim [63] stated that material removal is a process through the formation, propagation and interaction of micro-cracks produced by particle impacts. Based on this observation, the brittle material erosion mechanisms are roughly classified into three types: conical crack model, lateral crack model and intergranular crack model.

Conical crack model was proposed by Sheldon and Finnie [57] with the assumption that erosion occurs entirely by crack propagation and chipping as a result of contact stresses during impact. These stresses cause cracks to grow from pre-existing cracks
in the material surface. The load at which crack propagation occurs is related to the distribution of surface flaws through Weibull statistics. The volume removed per impact is proportional to particle penetration depth and cracked material area.

Lateral crack mechanism summarised by Evans et al [58] considered the lateral crack formation during a single particle impact as the major reason in establishing the material volume loss or removal model. This theory assumes that the material erosion rate is proportional to the amount of material removed by each particle impact. The material removed by a single particle impact is determined by the depth of the lateral fracture and the impression radius. Accordingly, the volume loss can be calculated by the depth of particle penetration and the maximum size of the lateral cracks that form during impact as shown in Figure 2.6.

The intergranular crack model developed by Ritter [59] directly assumes that the damage produced by an impacting particle is in the form of grain boundary cracking. Under this mechanism, a fraction of the solid particle kinetic energy is used in grain-boundary cracking and the crater size is proportional to the particle kinetic energy. It further confirms that the energy associated with the formation of the crater on the target material surface is the grain boundary fracture energy multiplied by the product of the number of grains per crater in the surface area.
In their study of erosion phenomenon associated with AWJ cutting of polycrystalline ceramics, Zeng and Kim [63] further summarized erosion mechanisms for brittle materials according to the published brittle material erosion literature and classified them into six categories: (1) conical, radial and lateral crack system; (2) intergranular and transgranular cracking; (3) ring fracture; (4) micro-chipping; (5) plastic deformation and melting; and (6) mixed mode damage. Among these mechanisms, the conical, radial and lateral crack system is the commonly observed impact damage mechanism in brittle materials and the general appearance of the impact damage is a central plastic impression surrounded by a combination of conical, radial and/or lateral cracks. The conical and radial cracks are formed during the loading period of the particle and target interaction while the lateral cracks are formed during the unloading period. Material removal is usually due to the lateral cracking. Intergranular and transgranular cracks have been observed on some coarse grain alumina ceramics. Ring fracture damage includes a main ring crack and many short circumferential fractures. This type of damage is associated with impacts by soft erodents. Plastic deformation has also been observed in many impacts events of brittle materials, involving both angular and rounded erodents.

In an effort to better understand erosion mechanisms for brittle materials, Ness and Zibbell [44] conducted a series of tests on hard materials. They analysed the erosion and abrasion modes and recognized two modes of material removal: a ductile ploughing type of material removal and a fracture induced removal process. In the case of observed hard materials, both processes may occur depending on the specific stress state of the wear environment. A more brittle material may erode in a ductile fashion at low angles of impact, but exhibit fracture induced material removal at high angles of impact (such as 90° angle). If an external load is applied, a ductile type of material removal may be observed at low loads and then transfer to brittle fracture at higher loads. Both types of material removal may occur simultaneously in one test since the local stress state may also vary; e.g. at initial contact with fresh, undeformed abrasive compared to later stages of contact after the abrasive has been deformed.
Choi et al [4] conducted studies on the erosion of brittle materials from the point of view of acting force. They noticed that there are two different modes of material removal depending on the force that the abrasive particle exerts on the surface. Their experimental results indicate that for crack initiation, there exists a threshold force. If the used force is smaller than the required threshold, the plastic flow is the dominant material removal mechanism, as in ductile material. However, in cases where the force is larger than the required threshold, material removal is mainly due to the micro-crack propagation.

Many other researchers also made observations of micro-cracking due to quasi-static point indentation under various experimental conditions. For example, Lawn and Swain [74] summarized the behaviour of brittle materials under point indentation as shown in Figure 2.7. First, the sharp indenter induces a plastic deformation zone (Figure 2.7 (a)). The size of deformation zone increases with the exerted load. At a certain critical indenter load, a crack is suddenly initiated below the contact point where the stress concentration is maximum, as shown in Figure 2.7 (b) (median crack). As the indenter load is increased, the median crack is further extended (Figure 2.7 (c)). During the unloading of the indenter the median crack closes, and the residual stress generated as a result of incompatibility between the plastic deformation zone and the surrounding elastic material initiates cracks. These cracks extend mainly parallel to the material surface, as shown in Figure 2.7 (d) (lateral crack). The lateral cracks usually divert up toward the surface as shown in Figures 2.7 (e) and (f) and are mainly responsible for material removal in the indentation of brittle materials. It is known that there exists a critical load over which the lateral cracks begin to form.
Paul et al [71] went further in the research of micro-mechanism of material removal for polycrystalline brittle material. They compared the mechanisms proposed by Hashish [21] with that proposed by Zeng and Kim [63]. Based on his experimental observations, Hashish suggested that the mechanism at or near the top of the kerf is predominantly micro-cutting at shallow angles, followed by plastic deformation at near-orthogonal angles of impact. Such deformation occurs to remove the step formation. However, Zeng pointed out that all of the mechanisms (micro-cutting or intergranular cracking) take place at shallow angles and the step is removed by secondary or tertiary erosion at shallow angles and not at near-orthogonal angles of impact. After studying the two models, Paul et al [71] summarized that the micro-mechanism of material removal of polycrystalline brittle materials is a mixed mode of predominantly inter-granular cracking, micro-cutting by free flowing abrasive particles and plastic deformation along with ploughing. They divided the global mechanism into two zones. In the first zone (near the top of the kerf) the mechanism is predominantly inter-granular cracking because of stress wave energy associated with shallow angles of impact along with micro-cutting at shallow angles. But in this
region plastic deformation and ploughing do take place although they are insignificant and can be neglected. As the abrasive particles penetrate the material, their velocity vectors become parallel to the generated kerf and thus the local impact angle becomes very near to zero. Based on the assumption that at zero local impact angle the material removal rate will be zero, at such a location a step would be formed. Paul et al [71] finally concluded that the step is removed by near-orthogonal impact of abrasive particles leading to gross plastic deformation and intergranular cracking.

2.2.3 Macro mechanisms of AWJ machining

The widely accepted and used theoretical basis for the macro mechanism of the material removal process or the kerf formation process is the work done by Hashish [19, 75, 76]. He conducted an investigation of the AWJ machining process using a high speed camera to record the material removal process on a plexiglass sample. He found that the material removal process is a cyclic penetration process that consists of two cutting regimes or cutting zones which he called “cutting wear zone” and “deformation wear zone” following Bitter’s erosive theory [55, 56]. Hashish also proposed that the cutting process consists of three stages, which are the entry stage, the cyclic cutting stage and the exit stage in the jet traverse direction.

2.2.3.1 Two cutting zones

In order to investigate the interaction between the AWJ and the workpiece, Hashish [76] conducted an experimental study to record the jet/material interface in transparent plexiglass specimens using high-speed movies. It was observed from his study that a steady state interface exists from the top of the kerf to a depth $h_c$ as shown in Figure 2.8. Below this depth, a step(s) forms in the work material and appears to move under the impact of the jet until it reaches the final depth $h$. The kerf curvature at depth $h_c$ changes suddenly, marking a transition from one material removal mode to another.
Figure 2.8. The wear cutting and deformation cutting mode [19].

The zone up to the depth $h_c$ is referred to as the cutting wear zone, as defined by Bitter [55] for shallow angle impact where the abrasive particles are at a shallow angle of inclination with respect to the local kerf geometry. In this zone, the material removal occurs primarily by micro-cutting with particle impacts at shallow angles. This process is in a steady cutting state with the characteristic that the material removal rate is equal to the jet material displacement rate by traversal. The kerf wall surface produced by cutting wear is characterized by the roughness at the upper portion of the kerf. When this condition terminates at the depth $h_c$, the jet penetrates the material by removing a step at a decreasing rate as the depth increases. The step formation zone below $h_c$ is termed the deformation wear zone, where the material removal is caused by particle impacts at large angles. In this zone, the material is removed by a different erosion mode that is associated with multiple particle bombardment, surface hardening by plastic deformation and crack formation. The geometrical expression of this deformation wear zone is jet-induced waviness “striation” which is regularly distributed on the lower portion of the kerf. It was found that the height of the waviness increases as the depth of cut increases.

The step formation mechanism is explained by Paul et al [71]. They suggested that as the material removal progresses along the depth of kerf, the inclination angle between the abrasive particles and the local kerf geometry becomes zero. At a zero impact angle, no material removal takes place. However, due to the continuous traverse of the AWJ, small steps are formed, such step formation leading to a sudden
change in the curvature of the kerf and the impact angle. At the step, the particles impact the site orthogonally and thus the material removal at such sites can be modelled as plastic deformation.

Based on the investigation conducted by Hashish, Momber and Kovacevic [1] also explained the step formation mechanism from the point of view of jet body-force. At the beginning of the material removal process, the jet hits the work material surface parallel to its axis. With the increase of the depth of cut, the local impact angle also increases. Because of their high body-force, the abrasive particles cannot follow the abrupt change in the jet flow direction, and a de-mixing process occurs. This process leads to a local accumulation of particle impacts. The result of this process is the formation of a local step. The shape of the step changes as the material removal progresses until it reaches a critical shape. This critical shape is characterized by a 90° impact angle. Then the material removal process changes into a drilling process. The abrasive particles cannot remove more material and the final stage of the cutting process is reached.

Nevertheless, many other researchers were also devoted to studying the existence of two different material removal regimes and exploring the contributing factors in the formation of the physical phenomena. In contrast to the two material removal modes assumed by Hashish, Zeng and Kim [72] developed another two-stage model illustrated in Figure 2.9. They questioned the Hashish’s conclusion that the formation of steps leads to large angle impacts. Instead of the underlying concept that the particle impact angle changes with an increase in the depth of cut, they proposed that direct impact of the jet only covers the top portion of the cutting front, below which secondary and tertiary impacts of the deflected jet occurs. In the first stage, called the “direct impact zone”, the abrasive particles directly impact the entire cutting front. As the jet moves further into the material, the area behind the back side of the abrasive waterjet is exposed to a “secondary impact” by deflected abrasive particles that results in a sudden change of the cutting front curvature. The entire cutting front consists of several cycles of step formation. The key feature of Zeng and Kim’s model is that the entire cutting process is associated with abrasive particle impacts at glancing angles, regardless of the type of target material.
In an effort to explore the cause of the step formation, Zeng and Kim explained with Figure 2.10 that as the jet advanced into the workpiece from the right angle edge, a smooth cutting front is developed and continuously grows in size until the rear side of the jet arrives on the edge of workpiece (Figure 2.10 (a)). During this stage, the entire cutting front is directly impacted by the original jet. This stage corresponds to the “steady state” interface observed by Hashish [15]. As the jet moves inward further, the material behind the rear side of the jet is exposed to the secondary impact by the deflected jet, resulting in a sudden change in the material removal rate. As a result, the curvature of the cutting front also exhibits a sudden change (Figure 2.10 (b)). This is the initiation of the step. Then, the step tends to grow because the most inner layer of the jet is deflected away by the front side of the step and becomes a protecting layer for the rest of the step. When the step grows into a certain size, it is exposed to a great flux of jet and its top side begins to erode. At this time, a stable step is formed. As the material on the front side of the step is removed, the step appears to move downstream (Figure 2.10 (c)). After that, another cycle of curvature change will start trailing the jet (Figure 2.10 (d)) and as the step moves downstream for a certain distance, this curvature change starts and leads to the formation of another step (Figure 2.10 (e)).
(a) Smooth cutting front impacted by the original jet; (b) Sudden change of curvature due to different erosion rate—initiation of step; (c) The step grows and moves downstream; (d) Next cycle of curvature change starts; (e) Next step forms and moves downstream again.

Figure 2. 10. The step formation in AWJ cutting processes [72].

### 2.2.3.2 Stages of AWJ cutting process

Many investigations have been carried out to observe the cutting process along the jet traverse direction [5, 38, 43, 77-81]. Most of the researchers originate their results from Hashish’s finding [19, 76]. Hashish conducted a visualization study for transparent materials such as glass, Lexan and Plexiglas and reported that the cutting process in the traverse direction proceeds in three stages which are called an entry stage, a cutting stage and an exit stage as shown in Figure 2.11.

In the entry stage, the different cutting mechanisms develop until the maximum depth of cut is reached. During this stage, the jet enters the workpiece and results in the increase of depth. At the beginning of this stage, cutting is accomplished by erosion at shallow angles of impact. With the increase of the depth of cut, penetration occurs gradually due to erosion at large angles of impact. When the depth of cut reaches its maximum, the penetration process is fully developed and controlled by erosion wear at large angles of impact associated with jet upward deflection.
Following the entry stage, the cutting stage starts. This stage is a cyclic cutting process that continues until the jet reaches the end of the work material. In this stage, a steady state interface to a depth $h_c$ exists at the top portion of the kerf. Below $h_c$, there exists a step(s) and it will move under the impact of the jet until it reaches the final depth. The kerf curvature at depth $h_c$ changes suddenly and this marks a transition from one material removal mode to another.

The exit stage is the one in which the cutting process comes to an end. As the jet approaches the exit edge of the workpiece, an uncut triangle is observed which is associated with a jet sideward deflection. This phenomenon shows that over a certain depth $h_c$ the cutting process is steady, which approximately marks the location of the top of the triangle. This important observation indicates that for effective separation of the two cut sides without triangle it requires $h_c$ be greater than or equal to the material thickness.

![This figure is not available online. Please consult the hardcopy thesis available from the QUT Library](image)

Figure 2.11. Three stages of AWJ cutting process [19].

### 2.3 Jet generation and characteristics

In an abrasive waterjet cutting system, three phases [1], i.e. abrasive particles, a high velocity waterjet and air, enter a cutting head from different entries. They are mixed in the cutting head. During this process, the momentum is transferred from the high velocity waterjet to the abrasive particles that are injected at comparatively low
velocities and the air is sucked with the abrasive materials. As a result, the abrasive particles and the air are accelerated and an abrasive waterjet is formed when the mixture exits the cutting head. Thus, in order to completely mix these three phases, the cutting head must possess three features, i.e. possessing inlets for the entry of high pressure water and the abrasive particles, permitting a proper pressure to suck in the abrasive particles and air, as well as accelerating the particles and focusing the three-phase jet.

2.3.1 Jet generation

Yanaida and Ohashi [82, 83] conducted a series of experiments to investigate the geometrical structure of a high-speed waterjet escaping from an orifice in the air. They divided the jet into three zones in the axial direction including a core zone, a transition zone and a final zone. In the conical-shaped core zone, the flow properties are constant along the jet axis. The length of this zone is related to the orifice diameter, the pump pressure and jet velocity. According to an investigation by Hashish and Plessis [84], the length of this zone is also dependent on the nozzle shape and the water pressure. In the transition zone, the water velocity is a function of the jet radius. The radial profile of the water velocity has a typical bell shape which means that jet will spread out in the radial direction with an increase in the distance travelled. The influence of the ambient air that enters the waterjet becomes more significant in the transition zone (Figure 2.12).

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Figure 2. 12. The structure of a waterjet [1].
The photographs obtained by Geskin et al. [85] show that a jet is surrounded by an array of droplets and solid particles. The jet oscillates in axial as well as in radial directions. These oscillations grow into large disturbances along the axis and finally make the jet discontinuous.

Jet diameter is significantly influenced by the abrasive mass flow rate. Under very low abrasive mass flow rate, the core structure of the waterjet is still visible and several single particle trajectories may be distinguished at the focus exit. At higher abrasive flow rate, the jet diameter significantly increases and no particle trajectories are visible at the focus exit. This is because the coherence of AWJ is fine when the distance from the focus is several millimetres. An increase in the abrasive mass flow rate can also improve the jet structure after the focus exit.

### 2.3.2 Jet characteristics and distribution

A number of studies have been conducted to analyse the structure and the characteristics of mixed air-water-abrasive jets as they exit the mixing tube [83, 84, 86-89]. The main objective of these researches is to arrive at a better understanding of the variations of water pressure and jet velocity, particle distribution and velocity distribution.

The experimental studies by Yanaida and Hashish [83, 84] show that the dynamic water pressure in the core zone is constant along the axis of the jet. Between the core zone and the final zone, the water pressure on the jet axis diminishes at a small rate. By contrast, in the final zone the dynamic pressure along the jet axis decreases abruptly. An investigation conducted by Davies and Jackson [88] has also resulted in the similar finding stating that under certain experimental conditions, the dynamic pressure has a minimum decay with a jet travelling distance of 100 times the nozzle diameter. However, beyond this distance, the pressure decays with the axial distance at a higher rate.

The variation of waterjet velocity along the axial direction is another feature evaluated for the characteristics of abrasive waterjet. Neusen et al. [87] found that the waterjet velocity along the axial direction is almost constant within the distance of 48-125 times of the orifice diameter. But, Davies and Jackson [88] pointed out that this distance could be extended to 300 times of the orifice diameter. Beyond this
distance, the water velocity decreases rapidly along the flow direction, which can be expressed by a function of the standoff distance. The reason for this decrease is believed to be due to the waterjet energy dispersion as the jet flows downstream [86]. Orifice/nozzle diameter ratio also has a big influence on the waterjet velocity. A large orifice/nozzle diameter ratio shows higher velocity levels along the axial direction than the case of smaller ratio.

In his study of particle distribution, Simpson [90] distinguished three main zones in the radial direction within an abrasive waterjet: a core zone, an inner zone, and an outer zone which are gradually outward from the centre of the abrasive waterjet. He reported that the distribution of abrasive mass in the core zone is small while the abrasive mass in the inner zone increases as the distance from the centre of the jet increases. Towards the outer limits of the inner zone, a maximum distribution of the abrasive mass is reached, with no abrasives detected in the outer zone. As for the mass flow distribution in the axial direction, within a certain distance from the focus exit the proportion of abrasives in the core zone does not vary much. Beyond this point, the abrasive mass flow distribution drops. With the increase of jet length the abrasive number in the inner zone is sharply reduced and more abrasive particles spread into the outer zone.

Neusen et al [91] also systematically observed the distribution of abrasive mass with a scanning X-ray densitometer and found the similar result. They suggested that there is no pure “core” remaining as the waterjet leaves the mixing tube. Between $\frac{1}{2}$ to $\frac{3}{4}$ of the mixing tube radius, rather than at the central axis, the water and the particle concentration reach their maximum value. The general distribution of waterjet is saddle-shaped along the radial direction. The similar saddle-shaped distribution for water and abrasive was also observed for the air-water-abrasive jets.

### 2.4 AWJ cutting performance

Considerable research has been conducted to investigate the effect of major process variables on cutting performance, predict the major cutting performance measures and optimise the cutting parameters. It was found from the published literature that abrasive waterjet cutting involves a large number of process variables and virtually
all these variables will affect cutting performance [92]. Before discussing the cutting performance in AWJ machining, a summary of these parameters is necessary.

### 2.4.1 Major AWJ process parameters

Generally, all the involved parameters can be classified into two categories: the input parameters or independent parameters and output parameters or dependent parameters [79]. Mombre and Kovacevic [1] further summarized these parameters into different types in each category as follows:

**Input parameters:**
- Hydraulic parameters: water pressure and waterjet diameter.
- Cutting parameters: nozzle traverse speed, number of passes, standoff distance and impact angle;
- Mixing and acceleration parameters: nozzle diameter and nozzle length;
- Abrasive parameters: abrasive mass flow rate, abrasive particle diameter, abrasive particle size distribution, abrasive particle shape and abrasive particle hardness.
- Target material parameters: fracture strength, flow stress, hardness and ductility.

**Output parameters:**
- Depth of cut and smooth depth of cut.
- Kerf geometry: top kerf width, bottom kerf width and kerf taper.
- Kerf topography: surface roughness and surface waviness.
- Material removal rate.

**2.4.2 Depth of cut**

A vast literature published focuses their interest on the modelling of depth of cut including the exploration of the influence of main AWJ process parameters on the depth of cut and prediction of the cutting performance.
2.4.2.1 The effect of water pressure

The general trend of the depth of cut with reference to water pressure is investigated by a number of studies [15, 19, 30, 36, 41, 43, 46, 52, 72, 93-98]. Their studies show that the depth of cut increases with the water pressure. In his experimental investigation on the cutting of ductile materials and gray cast iron, Hashish [19, 41] noticed that there exists a critical threshold pressure ($P_c$) below which no cutting occurs. Beyond the critical pressure the depth of cut increases linearly with an increase on the water pressure for a certain range. This threshold pressure is independent of the other parameters in his test range. El-Domiaty et al and Hocheng and Chang [43, 46] obtained the similar conclusion with their research for AWJ steel cutting.

Kovacevic [36] designed a system to monitor the depth of AWJ cutting. In his point of view, increasing the water pressure is a most effective method of increasing the cutting ability. The main reason for this is that the transfer rate of momentum and the velocity from the jet to the particles at the nozzle exit is increased in accordance with the waterjet pressure, thus resulting in increased impact energy and accordingly the depth of cut.

Contrary to the finding that the depth of cut linearly increases with the water pressure [52], Momber and Kovacevic’s study of AWJ cutting of rocklike materials [94] indicates that the relationship between the applied water pressure and the depth of cut is non-linear with a decreasing progress at higher water pressure, indicating that the material removal process is less efficient at higher pressure levels. They pointed out that a critical pressure exists which is corresponding to the minimum pressure to cut the material and this critical pressure increases with an increase in the material strength properties.

Chen et al [52] also conducted a research of AWJ cutting of alumina-based ceramics to investigate the effect of water pressure on both depth of cut and smooth depth of cut. They reported that the effect of varying water pressure on the smooth depth of cut is similar to that on the depth of cut. However, it was found that the ratio of smooth depth of cut to the depth of cut decreases as the water pressure increases.
This means that the influence of water pressure on the lower and middle zones is greater than on the upper zone.

2.4.2.2 The effect of nozzle traverse speed

The exploration of the relation between the nozzle traverse speed and the depth of cut has been reported by many authors [1, 27, 36, 43, 52, 94, 99-107]. It was found that the typical relationship is a non-linear curve in which the depth of cut decreases with an increase in the nozzle traverse speed.

Momber and Kovacevic [1] found that the dependence is very significant for low traverse speed but at high traverse speed, this curve may cross the abscissa, which means that the depth of cut approximates a saturation value. They gave an explanation for this decrease in depth of cut with the increase of traverse speed. It was found that the time of erosion process decreases as the traverse speed increases or the increase of the traverse speed causes the reduction in the density of particles impacting on the target materials. In their other investigation, Momber and Kovacevic [94] classified the range of nozzle traverse speeds into three different areas depending on their influence. They reported that if the nozzle traverse speed is very low, the cutting process has some similarities with a piercing process. In the range of small traverse speeds, the cutting process is very sensitive to the variation in the speeds. Even a small variation in the traverse speed may yield a significant change in the depth of cut. For very high traverse speeds, there exists a critical traverse speed. Above this critical value, almost no material removal will occur because the number of impacting abrasive particles becomes very small and the water may not be able to penetrate the material.

Chen et al [27, 52] also investigated the influences of main variables on the depth of cut with an objective to find the optimum nozzle traverse speed. They reported that there is indeed an optimum traverse speed for maximum depth of cut when AWJ cutting of ceramics. However, this optimum traverse speed may deteriorate other cutting performance measures such as the kerf surface finish. It is possible to reach a point where a traverse speed may produce maximum depth of cut and at the same time does not sacrifice surface quality.
2.4.2.3 The effect of standoff distance

A vast published literature explored the influence of the standoff distance on the depth of cut [3, 24, 27, 36, 52, 84, 96, 108-115]. Their results show that with an increase in the standoff distance the depth almost linearly decreases. Chen et al [27] explained that this is because the jet power reaching the workpiece decreases when standoff distance increases and therefore the lower part of the kerf cannot be machined efficiently. But if the standoff distance is too low it may cause damage to the cutting head. Ramulu and Arola [108] give another explanation for this phenomenon after their research on the cutting performance of graphite/epoxy laminates. They pointed out that increasing standoff distance results in an increased jet diameter as cutting is initiated and in turn reduces the energy density of the jet at impingement, which consequently generates a lower jet penetration depth. Hashish and Guo [84, 109] believe the existence of an optimum standoff distance for maximum depth of cut. After consideration of both cutting performance and cutting efficiency, Guo [109] suggests an optimum standoff distance which is equal to 2.0mm. It should be noted that compared to other cutting parameters, the influence of standoff distance is not significant in many cases in which the standoff distance varies with a small range.

2.4.2.4 The effect of abrasive mass flow rate

The effects of abrasive mass flow rate on the depth of cut were systematically explored in many investigations [1, 5, 19, 27, 36, 43, 51, 86, 93, 94, 102, 112, 116-118]. From these literature, the effect of abrasive mass flow rate on the depth of cut is relatively small compared to that of water pressure and nozzle traverse speed [94]. The relationship between the depth of cut and the abrasive mass flow rate is initially linear in which every increase in the abrasive mass flow rate leads to a proportional increase in the depth of cut [19]. However, this linearity terminates at higher abrasive mass flow rate. Thus, a critical mass flow rate exists beyond which the depth of cut starts to decline.

Some authors explained this phenomenon according to their investigations. Wang and Guo [93] explained that at the higher abrasive mass flow rate, not all the abrasive particles in the jet will strike the target material or at least not remove the material
with the same efficiency because higher abrasive mass flow rate will increase the
chance of particle interference between particles. This will reduce the particle energy
and the effectiveness of individual particle in cutting the material. Thus the overall
cutting performance in terms of the depth of penetration does not increase linearly
with abrasive mass flow rate. This explanation is in agreement with the observations
of other researchers, such as Hashish [19], Chen [27] and Momber [94].

Momber and Kovacevic [1] also gave a more detailed explanation for this
phenomenon. They reported that because the abrasive mass flow rate determines the
number of impacting abrasive particles as well as their kinetic energies, the higher
the abrasive mass flow rate, the higher the number of particles involved in the mixing
and cutting processes. This increase in the abrasive mass flow rate leads to a
proportional increase in the depth of cut under the assumption of no contact between
the single abrasive grains in the course of mixing and cutting. But if the abrasive
mass flow rate is higher, some damping mechanisms like particle collision occur in
the mixing chamber, in the accelerating focus and in the cut. On the other hand, the
limited kinetic energy of the waterjet has to distribute over the very high number of
particles. This results in a decrease in the kinetic energy of the single particle. This
effect counteracts the positive effect of the higher impact frequency.

From the viewpoint of the variation in particle velocity when using different abrasive
mass flow rate, Chen et al [27] also analysed the influence of abrasive mass flow rate
on the depth of cut. It was reported that the decrease of the penetration depth with the
increase of abrasive mass flow rate beyond a critical value is because an increase in
abrasive particles makes particle velocity decrease more rapidly than the increase in
the number of impacts. However, Neusen et al [119] did not find any influence of the
abrasive mass flow rate on the abrasive velocity, stating that the decrease in the depth
of cut with an increase in abrasive mass flow rate above a certain value is due to the
factors other than the drop in the particle velocity.

Based on the previous research, Momber [5] furthered his research in an effort to
find an optimum abrasive mass flow rate to achieve a maximum depth of cut. He
found that the optimum abrasive mass flow rate is greatly dependent on the property
of target materials. For ductile-behaviour materials they reach the optimum point at
relatively high abrasive mass flow rate, while very brittle materials reach their
optimum region at low abrasive mass flow rate. The reason for this is due to the higher sensitivity of brittle materials to the impact energy of the abrasive particles, which makes them easier to be cracked under the impact of abrasive particles. But a material that reacts with plastic deformation is more sensitive to the number of the impacting particles.

The above review of published literature shows that water pressure and abrasive mass flow rate positively affect the depth of cut in that an increase in these variables results in an increase in the depth of cut, while standoff distance and nozzle traverse speed adversely influence the depth of cut, i.e. the increase in these two parameters is associated with a decrease in the depth of cut. However, it must be noted that a reduced progress for the increase in the depth of cut is observed when water pressure is high, which means that the effectiveness of increasing water pressure to achieve high depth of cut is reduced at high water pressure values. In addition, it has been found that there exists a critical value for the effect of abrasive mass flow rate. A further increase in abrasive mass flow rate above this value will lead to a decrease in the depth of cut.

2.4.3 Kerf surface characteristics

Abrasive waterjet generated surfaces can be divided into two zones along the depth of cut, characterized by two texture types [79]. At the top of the cut, a smooth, uniform surface texture occurs when waterjet particles impact the kerf wall at shallow angles, which can be assessed by a surface roughness measure. At the lower part of the kerf, the abrasive waterjet exiting the workpiece forms large striations where the surface is characterized by waviness. Striation or waviness is a surface characteristic defined by larger irregularities rather than surface roughness. It is a special feature of cuts with beam cutting technology like waterjet, laser or plasma [120]. Waviness is formed when the ratio between the available energy of the beam and the required energy of the destruction becomes comparatively small. A number of investigations [6, 9, 10, 20, 25, 46, 79, 108, 121-128] have been conducted to experimentally determine the formation process and the correlation between the AWJ cutting conditions and the two major kerf surface characteristics - surface roughness and waviness.
2.4.3.1 Surface roughness

- The effect of water pressure

Kovacevic systematically observed the influence of water pressure on the surface roughness in conjunction with the thickness of the cut [79]. His study shows that at the upper portion of the kerf, the effect of water pressure on surface roughness is insignificant. However, with the increase in the depth of cut, the influence of water pressure increases. Generally, an increase in water pressure causes a decrease in surface roughness. This is because the increase of water pressure results in an increase in particle velocity and particle fragmentation inside the abrasive nozzle, which causes positive effect on surface quality. But when water pressure is higher, it will generate negative effect. High water pressure makes the abrasive particles lose cutting ability when they become too fragmented.

Ramulu and Arola had the similar finding when AWJ cutting of graphite/epoxy laminate [108]. They explained the phenomenon from the point of view of kinetic energy. High water pressure increases the kinetic energy of the abrasive particles and prolongs their capacity for material removal through the depth of cut. As a result, the surface roughness can be decreased by increasing water pressure.

Wang and Wong [121] found a different result from their study of AWJ cutting of metallic coated sheet steels. They noticed that surface roughness initially decreases with an increase in the water pressure. However, with a further increase in the water pressure, the surface roughness dramatically increases. It is because that high water pressure leads to larger effective jet diameter and as a result, the increased overlapping of the waterjet produces a wider kerf as well as a smoother surface than a smaller jet does. With the further increase of water pressure, the outer rim of diverged jet will gain enough energy for cutting the material, which tends to increase the irregularity and roughness of the surface.

- The effect of nozzle traverse speed

An increase in the nozzle traverse speed causes a constant increase in the surface roughness. The reason for this relation is that the increase in traverse speed causes less overlap cutting action and fewer abrasive particles to impinge the surface, which
increases the roughness of the surface. This result was confirmed by the studies of AWJ cutting stainless steel [79], ceramics [46], metallic coated sheet steels [121] and polymer matrix composites [9, 10]. But if the nozzle traverse speed is too low, the change in the surface roughness is undiscernible [79]. In addition, the traverse speed has little influence on the surface roughness at the top of the cut.

- The effect of standoff distance

It has been found by Wang and Wong [121] that when AWJ cutting metallic coated sheet steels, the increase of standoff distance results in a constant increase in the surface roughness. This is attributed to the divergence of the abrasive waterjet when spreading out from the mixing tube, which generates rougher surface. This effect is most predominant near the jet entrance within the initial damage zone, as reported by Ramulu and Arola [108] when AWJ cutting graphite/epoxy laminates. This is due to its effect on the coherency of the abrasive waterjet prior to impingement on the workpiece. Generally, increasing standoff distance results in an increase of jet diameter and in turn reduces the energy density of the jet at impingement. Densities of abrasive particles in the outer perimeter of the expanding jet are low due to this expansion. This decrease of density in the external rim generates more random peaks and valleys on the machined surface created by singular particles and thus results in a higher surface roughness. The influence of standoff distance decreases with the depth of cut to the deformation wear zone of material removal.

- The effect of abrasive mass flow rate

Several studies obtained a similar result with regard to the influence of abrasive mass flow rate on the surface roughness, i.e., an increase in abrasive mass flow rate helps to reduce roughness [46, 79, 108, 121]. This is attributed to the increased number of abrasive particles impinging the surface which are beneficial to cutting the peaks and valleys on the surface. In addition, a high number of abrasive particles involved in mixing increases the probability of particle collision that decreases the average diameter of the impacting particles. Furthermore, the influence of abrasive mass flow rate on surface roughness increases as the depth of cut increases.
2.4.3.2 Surface waviness

The waviness, or striation marks, corresponds to the rough cutting zone or deformation wear zone of an abrasive waterjet generated surface. Waviness patterns on the cut surface result from the properties of abrasive waterjet and the work materials. Chen and Siores [73] investigated in detail the formation mechanisms of waviness in the lower part of the depth of cut. They classified the causes of waviness formation into two categories, i.e., internal effect and external effects. As described by Chen and Siores, the internal effect is due to the kinetic energy distribution of the wavy abrasive particle. Since the particle kinetic energy distribution in the jet is not uniform and has a wavy profile in the jet cross-section such a wavy distribution leads to the wavy striation formation.

The external effects refer to the fluctuation or unsteadiness of the AWJ process parameters including nozzle traverse speed, water pressure and abrasive mass flow rate as well as the vibration of the workpiece and cutting head during the operation. When AWJ cutting the upper part of the kerf, most of the particles have higher kinetic energy than the required destruction energy of material and the whole cut surface can be cut through and a smooth surface can be generated. In the lower part of the kerf, the number of the particles with higher kinetic energy decreases, which means that the effective jet diameter decreases and the wavy particle kinetic energy distribution becomes sharp. Thus, the strong forefront particles cluster continues to cut through the surface while the weak trailing particles cluster is unable to do so by their own energy and just follows the strong particle trace. This leads to leave peak and rough trace marks on the surface.

Water pressure has a significant influence on surface waviness. Waviness is minimized under high water pressure since high pressure provides sufficient energy to the abrasive enabling cutting without severe jet deflection [108]. In addition, high water pressure also increases the size of the cutting wear zone corresponding to good surface quality owing to the increase of kinetic energy. Surface waviness is also critically dependent on the nozzle traverse speed [9, 22, 128, 129]. Wang [9] notices that an increase in the traverse speed results in an increase in the amplitude of the waviness, since increasing traverse speed allows less overlapping machining action on the surface and fewer abrasive particles impinging on the surface. Increasing
standoff distance generally results in a lower jet penetration depth, which means small size of deformation wear zone, thus increasing the surface waviness [108]. An increase in the abrasive mass flow rate substantially reduces the surface waviness [1]. But the effect of abrasive mass flow rate is associated with the variations of traverse speed. With high abrasive mass flow rates, the waviness is less sensitive to the changes in the traverse speed.

2.4.4 Kerf geometry

When discussing the kerf geometry of AWJ cutting, it often refers to the top kerf width, the bottom kerf width and the kerf taper [20] (Figure 2.3). Kerf geometry is another major criterion to assess the kerf characteristics of AWJ machining. The slot generated by abrasive waterjet is generally tapered with the top kerf width $W_t$ being wider than the bottom kerf width $W_b$. The taper of the cut is usually defined as the ratio between the top kerf width and the bottom kerf width [1].

$$T_R = \frac{W_t}{W_b}$$

(2.1)

Or an alternative form:

$$T_R = \frac{W_t - W_b}{2t}$$

(2.2)

where $t$ is the depth at which the minimum kerf width is measured.

2.4.4.1 Kerf taper

The relationship between kerf taper and the major process parameters for both ductile behaving materials and brittle behaving materials has been extensively examined by many researchers [6, 9, 24, 46, 52, 114, 121, 130-134].

In an investigation of ductile materials conducted by Chung et al [24], the taper of the cut is inversely proportional to nozzle traverse speed. They also noticed that kerf taper increases with an increase in standoff distance because the bottom kerf width has no correlation with standoff distance but the top kerf width of the kerf is directly
proportional to it. In addition, they found no clear relation between the kerf taper and water pressure as well as between the kerf taper and abrasive mass flow rate.

Wang and Wong [121] reported that the kerf taper shows somewhat increase with the water pressure but a decrease for higher water pressure where the top kerf width shows flattening (or even decreasing) when AWJ cutting metallic coated sheet steels. The standoff distance has a lesser effect on the bottom kerf width than the top kerf width. As a consequence of this effect, kerf taper increases with standoff distance. They have also found that the kerf taper appears to increase with an increase in nozzle traverse speed but there is no clear trend of the kerf taper with the abrasive mass flow rate.

Matsui et al [114] further described the influence of nozzle traverse speed on kerf taper by their observation of AWJ cutting metallic materials. For low traverse speed, the kerf taper non-linearly increases with an increase in nozzle traverse speed. But the progress of the increase drops in the range of high traverse speed. The authors also noticed an interesting phenomenon in relation to the effect of the nozzle traverse speed on kerf taper in their experiment. It was shown that the kerf taper changed from convergent for high nozzle traverse speed to divergent for low traverse speed. A transition traverse speed exists where the taper is zero.

For the brittle materials, Hocheng and Chang [46] observed that for high nozzle traverse speed, the kerf taper is large since through-cutting of the material is obtained with incomplete widening of the bottom kerf width. At high water pressure, the abrasive possesses abundant kinetic energy and therefore the bottom part of the kerf can be machined more effectively, resulting in a small kerf taper. The abrasive mass flow rate causes no evident change to the kerf taper since it increases or decreases the kerf width in a parallel manner.

Arola and Ramulu [131] investigated the taper formation and influential factors using AWJ cutting graphite/epoxy composite. They noticed that kerf taper has close relation with material thickness. When material thickness is less than 5mm, kerf taper is primarily controlled by the standoff distance. Water pressure and nozzle traverse speed do not show significant influence until approaching cutting depth is greater than 10mm. In a relatively large depth of cut, kerf taper increases with higher traverse speed. An increase in nozzle traverse speed has a big adverse effect on the
required jet energy for material removal due to the reduction in jet exposure time. In addition, increasing traverse speed when cutting with low water pressure has considerably more effect on kerf taper than when cutting with high water pressure. The use of higher water pressure increases the energy of the abrasive particles and this facilitates more efficient cutting and reduces the localized effects of a higher traverse speed. At cutting depths greater than 15mm, no single parameter clearly dominates the kerf taper as it is affected by a combination of all the independent variables.

2.4.4.2 Top kerf width

The investigations on the effect of major process variables on top kerf width are extensive [9, 24, 52, 75, 121, 131, 134-136]. These research efforts can be divided into based on ductile materials and based on brittle materials.

For ductile materials, it is shown that the nozzle traverse speed significantly influences the top kerf width in that a decreased top kerf width occurs with an increase in the nozzle traverse speed [25, 75, 121]. The reason for this trend, Wang and Wong [121] explained, is due to the fact that a faster passing of abrasive waterjet allows fewer particles to strike on the target material and hence generates a narrower slot.

Top kerf width is also strongly influenced by the standoff distance, as confirmed by Chung et al [24], Wang and Wong [121], and Duflu et al [130] with their studies showing that the top kerf width increases with standoff distance. This is due to the result of jet divergence which increases the jet diameter when the standoff distance increases.

It was also found that the top kerf width linearly increases with an increase in abrasive mass flow rate [24], although the influence of abrasive mass flow rate is less important than that of nozzle traverse speed [75]. However, Wang and Wong [121] reported from their study of AWJ cutting of metallic coated sheet steels that there is no clear trend of the kerf width with respect to the abrasive mass flow rate.

No effect of the water pressure on top kerf width was found by Chung et al [24]. They explained that probably the accelerated particles are concentrated in the inner part of the jet. In contrast, Wang and Wong [121] found that the top kerf width
appears to increase with the water pressure. The reason is simply that high water pressure should result in great jet kinetic energy and open a wide slot on the workpiece. But when water pressure reaches a higher value, it exhibits a reduced effect on the top kerf width.

Ramulu and Arola [108], Arola and Ramulu [131], Hocheng and Chang [46], and Wang [10] conducted the systematic studies on the top kerf width in brittle behaving materials. Arola and Ramulu [131] noticed that standoff distance accounts for the largest variation in top kerf width and water pressure, nozzle traverse speed and abrasive particle diameter assume secondary roles. With an increase in the depth of cut, the effect of nozzle traverse speed on the kerf width increases. Once the depth of cut is greater than 8mm, the kerf width is equally affected by water pressure, traverse speed, and abrasive particle diameter, whereas the influence of standoff distance is insignificant.

The relationship between top kerf width and water pressure, according to Wang’s research on polymer matrix composites [10], is an initially increased top kerf width with the water pressure, but then this trend becomes flattened. This trend can be explained that an increase in the water pressure leads to the increase of effective jet width, and in turn increases the top kerf width. However, when the whole jet width becomes effective, any further increase in the water pressure has little effect on the top kerf width.

Hocheng and Chang [46] also presented their results to examine the effect of nozzle traverse speed the top kerf width. They observed that the top kerf width is inversely proportional to the nozzle traverse speed. A small traverse speed allows more particles to strike the target materials and accordingly a wide slot is generated.

### 2.4.4.3 Bottom kerf width

A number of published literature focused the studies on the bottom kerf width by AWJ cutting ductile materials [5, 9, 24, 46, 121, 130, 131]. A research study carried out by Duflou et al [130] indicates that bottom kerf width is strongly affected by nozzle traverse speed. Higher traverse speed results in a narrow bottom kerf width. This observation is supported by Chung et al [24] according to their investigation of
steel aluminium and titanium. This is due to the energy dissipation in the jet with the nozzle traverse speed, which affects mostly peripheral particles and, thus, reduces the width of jet when penetrating into the target material.

The water pressure has more effect on the bottom kerf width rather than the top kerf width [130]. The higher the pressure, the wider the bottom kerf width. It is expected that a higher water pressure results in greater jet kinetic energy at the lower cutting part, which opens a wider slot at the kerf bottom.

Standoff distance appears to have little effect on the bottom kerf width [24]. This result was changed by Wang’s study on polymer matrix composites [9] stating that an increase in standoff distance is associated with an increase in the bottom kerf width although the rate of increase is much smaller than that of the top kerf width.

Abrasive mass flow rate has not much influence on bottom kerf width [24]. However, other researchers reported a different result showing that an increase in abrasive mass flow rate causes an increase in bottom kerf width [5, 46].

The similar trends were reported from the study of brittle behaving materials. the nozzle traverse speed has a more significant influence on the bottom kerf width than other process parameters. When nozzle traverse speed increases bottom kerf width decreases [131]. This is because a faster passing of abrasive waterjet allows fewer abrasive to strike on the material and thus generates a narrower slot.

Wang [9] carried out detailed research on the effect of water pressure on kerf geometry using polymer matrix composites. He suggested that bottom width increases approximately linearly with the water pressure as greater water pressure leads to higher jet kinetic energy impinging onto the material.

The influence of standoff distance is not significant although the bottom kerf width will increase with an increase of standoff distance. This is a result of jet divergence as high velocity waterjet spreads out. When the jet penetrates into the lower part of the material, it loses the kinetic energy and hence the outer rim of the diverged jet does not take effect. As such, the standoff distance has lesser effect on the bottom kerf width than on the top kerf width.
2.5 Modelling study of AWJ cutting performance

With the more applications of AWJ cutting technology, there is a need to quantitatively describe this cutting process in order to select and optimize the process parameters and predict the cutting performance. The mathematical models for the major cutting performance measures are therefore required. Over the last decades, a great deal of research effort has been directed to develop such models. While most of the developed models are concentrated to estimate the depth of cut for different materials [10, 19, 21, 51, 52, 60, 70-72, 93, 137-140], some other models are also developed to express other cutting performance measures such as material removal rate (MRR) [47, 63, 140], the shape of kerf profile [141], kerf surface roughness [10, 22], kerf taper and kerf widths [10] under certain cutting conditions. Both semi-empirical and empirical approaches were utilised in the development of those models, together with the aid of different methodologies which included the use of erosion mechanism in AWJ cutting, an energy conservation theory and regression analysis technique.

2.5.1 Depth of cut models

2.5.1.1 Models based on AWJ erosion mechanism

According to his theories of cutting wear and deformation wear for ductile materials, Hashish [19] developed mathematical models for the depth of cut due to cutting wear, \( h_c \), and deformation wear, \( h_d \), respectively.

\[
h_c = \frac{cm_a v^2 \alpha_0}{4 \sigma_f d_j u}
\]

\[
h_d = \frac{2(1-c)m_a v^2}{\pi d_j \varepsilon u}
\]

Where \( c \) is a constant, \( m_a \) is abrasive mass flow rate, \( v \) is particle velocity, \( \alpha_0 \) is angle of impingement at the top of cutting surface \( (h=0) \), \( \sigma_f \) is material flow stress, \( d_j \) is jet diameter, \( u \) is nozzle traverse speed, and \( \varepsilon \) is the energy required to remove unit
volume in deformation wear. The depth of cut in an AWJ operation can be found by adding $h_c$ and $h_d$.

Later, Hashish improved his models [21] by including the effect of particle size and more accurately expressing the effect of other process parameters. The refined models are expressed as

$$h_c = \frac{\left(\frac{v_0}{C_k}\right) d_j}{\left(\frac{\pi \rho_p u d_j}{14 m_u}\right)^{2/5}} + \left(\frac{v_e}{C_k}\right)$$

(2.5)

$$h_d = \frac{1}{2 C_1 m_u (v_0 - v_e)^2} + \frac{C_f}{d_j} \left(\frac{v_0}{C_k}\right) - \frac{C_f}{C_k}$$

(2.6)

where $v_0$ is the initial particle velocity on the material surface, $\rho_p$ is the particle density, $v_e$ is the threshold particle velocity to removal material, $\sigma_f$ is the material flow stress, $C_f$ is the coefficient of friction on kerf wall, $C_j$ is a constant, and $C_k$ is a characteristic velocity that combines the particle and material characteristics:

$$C_k = \sqrt{\frac{3 \sigma_f R_f^{3/5}}{\rho_p}}$$

(2.7)

where $R_f$ is the particle roundness factor, i.e., the average diameter of particle corner to the diameter of maximum inscribed circle.

It has been found that the improved model is in a simplified form and correlates well with the experimental results. However, additional refinements need to be made to include particle size and velocity distribution, consider the kerf width variation along the kerf wall and determine the patterns of particle-material interaction.

By considering the variation in the kerf width along the kerf wall and choosing the specific energy as the material parameter, Paul et al. [60] developed another set of depth of cut model for AWJ cutting steel and aluminium:

$$h_c = \frac{28 m_u C}{2.5 \pi p u w_t (1 + w_m / w_t)} \left(\frac{v_0}{C_k}\right)^{2.5} \alpha_0^{1.5}$$

(2.8)

$$h_d = \frac{Q - h_c (Qk_d q - 1)}{1 + Qk_d q - k_d^2 q^2} Qh_c - h_c$$

(2.9)
where \( C \) is the part of the jet involved in the micro-cutting process, \( W_t \) is the top kerf width, \( W_m \) is the minimum kerf width, \( k_d \) is a function of the drag coefficient and the insert diameter, \( q \) is a function of the abrasive velocity and the threshold velocity, and \( Q \) is a parameter expressed by

\[
Q = \frac{(1-C)m_a(v_a - v_c)}{2\sigma f_w m_f v} \tag{2.10}
\]

It was shown that this set of analytical models matches nicely with the experimental data. However, in addition to its complexity, these models can not be used before other information in the equation is determined.

Zeng and Kim [72] developed their model for brittle materials after the study of material removal of ceramics. By relating the macro material removal rate on the cutting front to the accumulated micro material removal rate by individual abrasive particles they derived the equation of depth of cut (\( h \)):

\[
h = \left( \frac{\eta C_v C_f}{1 + t_r} \right)^2 \left( \frac{C_m P}{\rho_w d_x u} \right) \left( \frac{2f_w \beta D \sigma_f \gamma^2}{3\gamma E} + \frac{\alpha}{\sigma_f} \right) \tag{2.11}
\]

Where \( \eta \) is the momentum transfer efficiency; \( C_v \) is the orifice efficiency; \( C_f \) is the compressibility coefficient; \( t_r \) is the ratio of abrasive to water mass flow rate, \( C \) is the coefficient of impact efficiency, \( P \) is water pressure, \( \rho_w \) is water density, \( d_x \) is mixing tube diameter, \( f_w \) is the fraction of effective stress wave energy, \( \beta \) is the function of Poisson’s Ratio of the target material, \( D \) is abrasive particle size, \( \sigma_f \) is flow stress of target material, \( \gamma \) is the fracture energy per unit area, and \( E \) is the elastic modulus of target material. The form of this model is too complex. Furthermore, the stress wave energy as a percentage of the total energy must be determined to predict the depth of cut using this model.

2.5.1.2 Models based on fracture mechanics

It was found that the erosion of brittle materials is generally viewed as a brittle fracture process, which is controlled by the formation and propagation of cracks [58]. Since fracture toughness is usually considered as a measure of resistance to crack propagation, El-Domiaty and Abdel-Rahman [70] developed two depth of cut models
based on two available erosion models containing the property of fracture toughness of the target material. They suggested that an erosion model based on fracture mechanics is suitable for brittle materials.

The two models take the following forms:

\[ h = C_1 g_1(\alpha_e) \left( \frac{1}{ud} \right) \left( u m_a v^{19/6} r^{2/3} \rho_p^{7/12} K_c^{-4/3} H_m^{-1/4} \right) \]  

(2.12)

and

\[ h = C_2 g_2(\alpha_e) \left( \frac{1}{ud} \right) \left( u m_a v^{22/9} r^{2/3} \rho_p^{-2/9} K_c^{-4/3} H_m^{1/9} \right) \]  

(2.13)

where \( C_1 \) and \( C_2 \) are constants, \( r \) is the particle radius, \( \rho_p \) is particle density, \( K_c \) is the fracture toughness of target material, \( H_m \) is the hardness of target material, and \( g_1(\alpha_e) \) and \( g_2(\alpha_e) \) are the function of \( \alpha_e \), where \( \alpha_e \) is the attack angle of particle at kerf exit.

These two models offer relatively simple and closed form equations. However, experimental verification showed that they only gave accuracy prediction when water pressure is around 200MPa. For pressures higher or lower than 200MPa, the predictions of the both models start to deviate from the experimental data.

### 2.5.1.3 Models based on energy approach

Energy conservation approach has been used to develop mathematical models in AWJ cutting. The fundamentals of this approach are to assume that the rate of material removed from the workpiece is proportional to the kinetic energy of the abrasive particles.

Using this approach, Chen et al [52] developed a mathematical model to predict the smooth depth of cut (the depth of cut due to cutting wear) and the depth of cut produced by AWJ cutting alumina ceramics:

\[ h_e = 0.88 \left( \frac{m_a \rho^{0.51}}{u^{0.27}} \right) \]  

(1.14)
In their study of AWJ cutting of polymer matrix composites, Wang and Guo [10, 93] found that the erosive process possesses a unique mechanism which is a combination of the erosive mechanisms for ductile and brittle materials. It does not seem to be appropriate to use either the erosion theory or the fracture mechanics approach to model the cutting process. Based on energy conservation approach, they developed a semi-empirical model to predict the depth of cut. This model takes the following form:

\[ h = 0.9 \left( \frac{m_a P}{u^{0.78}} \right) \]  

(2.15)

Where \( k, \alpha_1, \alpha_2, \alpha_3 \) are constants respectively.

The simplicity of those models is obvious. However, a neglect of the energy loss may occur during the process in order to simplify the derivation and therefore the prediction accuracy of the models needs to be improved.

### 2.5.2 Surface characteristic models

Because of the lack of full understanding regarding the mechanisms of kerf surface roughness in AWJ cutting and the complexity involved in the development of surface roughness model, most of the models are based on empirical approaches [10, 11, 79, 95, 108, 121]. While these models can give good prediction for surface roughness, they can only be suitable for particular material under particular cutting conditions.

After his investigation of waviness phenomenon, Hashish [22] suggested a simplified physical prediction model for surface waviness. Based on the hypotheses that the interface between the jet and target material is not steady in the striation zone and all the material in this zone is removed through deformation wear at 90° impact angles, this model was developed by means of an idealized geometry for waviness formation. This model shows a good quantitative agreement with the experimental data according to Hashish’s verification.
\[ 2R_w = 1 - \sqrt{1 - \left( \frac{\pi}{4} \right)^2 \left[ \frac{d_n (h - h_j) u}{0.5m_v v^2 / \epsilon} \right]^2} \] (2.17)

Where \( R_w \) is surface waviness (striation), \( d_n \) is the mixing tube diameter, \( \epsilon \) is the specific energy for deformation wear mode.

However, quantitative discrepancies were observed if the effect of kerf taper was not considered. This suggests that to accurately predict surface waviness using this model the variation of kerf width must be included.

### 2.5.3 Models of kerf geometrical features

Mathematical models for kerf geometrical features such as kerf widths and kerf taper were found through the search of the published literature, of which the most were based on pure empirical approach [9, 10, 108, 121, 131]. These models provide a practical way to predict the kerf geometrical features produced in AWJ machining and they usually give reasonably accurate prediction for the particular materials and cutting conditions. While these models only give the general formats of the equations with the coefficients required to be determined from experiment, their applications are just limited to the discussed problems.

One of the examples is a kerf taper model developed by Ramulu and Arola [108] using regression technique. This equation takes the form of second order polynomial by relating the kerf taper ratio \( (T_R) \) to the independent variables:

\[ T_R = c_0 + \sum_{i=1}^{4} c_i x_i + \sum_{i=1}^{4} c_{ii} x_i^2 + \sum_{i=1}^{3} \sum_{j=i+1}^{4} c_{ij} x_i x_j \] (2.18)

where \( c_0 \) is a constant, \( c_i \), \( c_{ii} \) and \( c_{ij} \) are the first order, second order and interaction coefficients, \( x_1 \) is the water pressure, \( x_2 \) is the abrasive grit size, \( x_3 \) is the nozzle traverse speed, and \( x_4 \) is the standoff distance.

Another example is a model developed by Wang [10] for predicting the top kerf width \( (W_t) \). Using backward elimination procedure in the regression technique, this equation is expressed by water pressure \( (P) \), nozzle traverse speed \( (u) \) and abrasive mass flow rate \( (m_a) \)
2.6 Techniques for enhancing the cutting performance

As a newly developed non-traditional machining method, AWJ machining is evolving and undergoing sophistication by finding new techniques to enhance the cutting performance and to improve the machining qualities. The exploration of the new techniques with the purpose to improve the AWJ cutting performance has never been ceased. The past several years have seen a significant development in several research branches in this regard. Various techniques have been developed to increase the depth of jet penetration and smooth depth of cut as well as to reduce kerf surface roughness and kerf taper angle. Among these attempts, cutting with forward angling the jet, controlled nozzle oscillation technique and multipass cutting are found to have great prospects in improving the cutting performance without any additional costs to the cutting process.

2.6.1 Forward angling the jet

The angle of impact is the angle between the initial jet flow direction and the plane of the workpiece material (Figure 2.13). During the AWJ cutting, as the abrasive particles cut into the lower part of the material, the cutting direction changes due to the jet deflection and instability near the bottom of the kerf. This changed angle is indicated by drag angle which reduces the component of energy for impacting the material and results in irregularity near the bottom of the kerf. It is proved that by introducing a jet forward angle in the cutting plane the drag angle in the kerf profile can be effectively compensated so as to improve the kerf qualities [10].
Experimental results show that there exist optimum impact angles for obtaining the maximum depth of cut and improved kerf surface finish. The research conducted by Hashish on glass, Lexan and plexiglass [142] proves that when the jet angle is at 80° (a jet forward angle of 10°) the depth of cut peaks. Particularly, at high nozzle traverse speeds, where the depth of cut is shallow and the kerf has larger curvature, the influence of angle is more pronounced. The material removal rate can be increased by a factor of 3 or 4 when the jet is angled. The surface waviness can also be improved when the jet is angled in the direction of the jet traverse. The reason for these effects is that jet deflection occurs after more energy is absorbed in cutting.

Chen et al’s result [51] shows that the optimal jet impact angle for the maximum depth of cut is between 80°-85° when AWJ cutting ceramics. In addition, at 70°-75° of the jet impact angle, the surface drag angle in the lower part of the kerf could be reduced to zero, which means an improved surface quality.

Wang [10] conducted a study of polymer matrix composites to investigate the influence of jet impact angle on the major kerf characteristics. It is demonstrated that the depth of cut increases as the jet impact angle increases from 50° upward and the peak value occurs at about 80°. This is due to the different distribution of jet energy when jet angle changes. Reducing the jet angle will reduce the tangential component of the particle velocity along the cutting wall. In the lower part of the cut the angled jet compensates for the jet drag angle so that the tangential component of the particle energy is increased. It was also found that the kerf width and kerf taper are
independent of the jet impact angle, which may be explained that the kerf geometry only depends on the properties of the material and the jet structure. In addition, jet angle can significantly improve the surface finish when it increases from 50° to 70°. When jet angle is small, the reduction in the tangential component of particle energy results in a significant change in the cutting wear mode erosion and this increases the surface roughness.

2.6.2 Controlled nozzle oscillation

The idea of controlled nozzle oscillation or cutting head oscillation was introduced by Veltrup [143] and was then developed as a most effective cutting technique by many other researchers [2, 11-13, 51, 118, 120, 139, 144, 145]. In this technique, in addition to its normal traverse motion during the cutting, the nozzle is still moved in an angular direction parallel to the cutting direction (in the cutting plane) at a given angle and frequency. Thus, the main parameters using oscillation technique are oscillation angle and oscillation frequency. Nozzle oscillation has been found to be an effective way in the improvement of the depth of cut, smooth depth of cut and surface finish compared to normal cutting (cutting without nozzle oscillation).

The study of forward jet angling has confirmed that it can effectively reduce striation drag angles and accordingly improve surface qualities. It is then believed that cutting head vibration, by which the jet can scan the surface of the cut, affects the formation and pattern of surface striation [127]. It was proven that the depth of upper smooth zone with oscillation increases by more than 30% compared with that of normal cutting [2]. Both the striation drag angle and striation frequency (the number of striation along the cutting path) in the lower part of the kerf decrease. This result was supported by AWJ cutting mild steels, aluminium alloys and fibre-reinforced composites [11, 145]. These better cutting results are attributed to the fact that the cutting head oscillation technique drives abrasive waterjets to scan through the cut surface and more uniformly attacks the cut surface by the particle kinetic energy.

The use of nozzle oscillation can also significantly improve the surface roughness [11]. It was found that when AWJ cutting without nozzle oscillation, the kerf surface is only momentarily exposed to a stationary jet. If the nozzle traverse speeds are higher, the residence time of the jet is lower, resulting in a rough and striated kerf
surface. However, by superimposing nozzle oscillation at these higher speeds, the kerf surface under oscillation cutting is repeatedly scanned by fresh abrasive waterjets, which increases waterjet residence time at a particular spot. This may produce better kerf surface quality.

The nozzle oscillation cutting can also increase the depth of cut, although no quantitative data have been found through the search of the published literature. It was found [11] that the successive traces of particles on the cut surface with nozzle oscillation are steeper than those without oscillation, which results in more particle energy in the cutting direction and deeper cuts. The particle scanning action in oscillation cutting also helps to reduce particle interference and clear the target surface for subsequent particles to cut.

2.6.2.1 The effect of oscillation angle

Siores et al and Chen et al [2, 51] observed the effect of oscillation angle on smooth depth of cut by their study of AWJ oscillation cutting ceramics. It is indicated that as the oscillation angle increases the smooth depth of cut increases and the smooth depth difference between the two side surfaces decreases. When oscillation angle is between 15° and 20° the striation drag angle in the cutting surface becomes zero and the maximum smooth depth of cut can be obtained. It is claimed that when the oscillation angle increases within 20°, the scanning abrasive wear becomes more effective and the smooth depth of cut increases. But when the oscillation angle increases to about 20°, the smooth depth difference increases sharply. This is because once the oscillation angle exceeds 20°, the nozzle unbalancing and vibration becomes obvious and this causes one of the two cut surfaces to wear more effectively than the other. Thus, the optimum oscillation angle was found between 15° to 20° when AWJ oscillation cutting ceramics.

The investigation of the relationship between oscillation angle and the surface roughness has found that an increase in oscillation angle leads to significant improvement in surface quality [2, 11]. The results show that the improvements in surface quality are directly proportional to the oscillation angles used. When the oscillation angle increases within 30°, the average striation frequency of the two side surfaces decreases. It is so because more effective scanning abrasive wear takes place
and this results in the decrease in the number of striation along the cutting path. But it is also observed that when the oscillation angle increases from 20° to 30°, the striation on one side of the cut surface increases slightly and the other side is more effectively worn.

It can be seen from the previous studies of the effect of oscillation angle on the cutting performance that all the effort was concentrated on the investigation of relatively large oscillation angles, i.e. more than 10°. While this way of oscillation cutting is not appropriate in practice for contouring operation due to the generation of geometrical errors on the component, it is limited to the application of straight cutting. Therefore, it remains an issue to investigate oscillation cutting under small oscillation angles.

2.6.2.2 The effect of oscillation frequency

As the oscillation frequency increases, the average smooth depth of cut of the two side cut surfaces increases and the smooth depth difference between the two cut surfaces decreases [51]. But if the oscillation frequency increases to a particular given value, the average smooth depth of cut will decrease. This trend is believed to be a result of an unbalanced cutting head at high oscillation frequency which reduces the effectiveness of the scanning abrasive wear mechanism. Furthermore, too high frequency oscillation is harmful for the life and balancing of the waterjet equipment. It was also found [2] that the optimum oscillation frequency corresponding to the turning point of smooth depth of cut is associated with the nozzle traverse speed. When the oscillation frequency is about six times of the nozzle traverse speed, the maximum smooth depth of cut can be achieved.

The influence of oscillation frequency on surface quality is similar to that of oscillation angle on the surface quality. When the oscillation frequency increases within a certain range, the average striation frequency of the two side surface will decrease [2].

2.6.3 Multipass cutting

In AWJ cutting, there are cases that the material thickness is beyond the jet cutting capacity in a single pass or cases where for a given depth of cut, reduced total cutting
time is required. It was found that with a proper combination of cutting parameters such as nozzle traverse speed, multipass cutting over the same kerf a number of times demonstrates distinct superiority over the single pass cutting [146]. The research on this technique has been reported in some investigations [1, 19, 84, 106, 146-149]. Experimental results indicate that multipass cutting at high nozzle traverse speed may generate deeper total depth of cut and smooth depth of cut than single pass cutting carried out over the same total elapsed time [84]. Multipass cutting with high nozzle traverse speed can also result in kerfs of small taper angle and better surface finish [147].

Multipass cutting has a profound influence on the depth of cut and smooth depth of cut compared with single pass AWJ cutting. The influence is almost linear in the beginning, but the progress drops at a certain critical number of passes [1]. An optimum combination between the number of passes and the nozzle traverse speed exists that yields a maximum depth of cut and smooth depth of cut. This is due to a balance between the impact damping and wall friction at a certain depth of cut.

Multipass can also significantly affect surface finish of machined material. It is observed [146] that the surface roughness decreases constantly with an increase in the number of passes. This is due to the smoothening action on the kerf wall by the subsequent passes to remove the peaks left in the precedent pass.

The research on the influence of multipass cutting on the kerf geometry indicates [146] that if the first pass can cut through the specimen, the subsequent passes have been found to be able to reduce kerf taper so that the two side walls tend to be parallel to each other. In the case where the first pass is unable to penetrate the workpiece, multipass cutting is necessary to cut through the material. In this case the subsequent passes are able to cut through the workpiece and reduce the taper angle of the kerf wall in the upper and lower portion. But there is still a widened portion in the final kerf resulting from the large pocket generated in the first pass. The number of passes and the nozzle traverse speed do not generate significant variation in the top kerf width. The kerf taper angle is also greatly affected by the nozzle traverse speed in the subsequent passes. Lower speed is always favourable in achieving smaller kerf taper angle. As a result, a good combination between the number of passes and the traverse speed is important for different application situation. A large
nozzle traverse speed may be used for high cutting rate and low cost while lower speed is needed to generate good surface finish and small kerf taper.

All the studies published in multipass cutting show that while high traverse speed coupled with multipass cutting can achieve better overall cutting performance over low traverse speed with single pass, the research on the strategy of optimum combination of the nozzle traverse speed and the number of passes associated with other cutting parameters has not been conducted in detail.

2.7 Conclusions

An extensive review of the state of the art of AWJ cutting technologies has been conducted in this chapter. It was shown that as a newly developed machining technology, AWJ machining technology is receiving more and more attention in the machining areas especially for the machining of difficult-to-cut materials. Its unique advantages over other traditional methods make it a new choice in the machining industry. As summarized in the foregoing review, a great deal of effort has been made to explore the applications and the associated science. The wide range of research and development in this area includes the research on the material removal mechanisms in the AWJ cutting, abrasive waterjet characteristics, the analysis of the cutting performance with respect to the major cutting parameters, and development of practical mathematical models to predict the cutting performance and optimise the selection of cutting parameters, and the exploration of new techniques to enhance the cutting performance.

A brief review of the composition, characteristics and applications of AWJ machining was first conducted in the first part of this chapter. It was followed by a summary of the research on the material removal mechanisms. According to the previous investigations, the material removal mechanisms can be classified into micro mechanism which is related to the material removal by individual particles and macro mechanism which is referred to the kerf formation process. It has been found that the solid particle erosion models of the micro-cutting process for ductile materials include cutting wear model, deformation wear model and ploughing-deformation model. For brittle materials, conical crack, lateral crack and intergranular crack are the dominant mechanisms in micro-material removal.
Depending on the wear environment and the material properties, brittle materials may exhibit a ductile ploughing type of material removal.

The extensive studies on the macro mechanisms of AWJ cutting has suggested that the cutting process consists of an entry stage, a cutting stage and an exit stage along the nozzle traverse direction, while there exist a cutting wear zone and a deformation wear zone in the cutting front along jet penetration direction. Other models were also suggested to explain the kerf formation such as two stage model. The step formation along the kerf wall was also studied and the mechanism of this phenomenon was investigated by a number of researchers. Consequently, the kerf geometry machined by AWJ is characterised by a wide entry on the top surface and a gradual reduction towards the exit so there is a taper on the kerf. For a non-through cut, there is an enlarged pocket at the lower part of the kerf which is due to the jet upward deflection.

Jet generation and characteristics have briefly reviewed in this chapter. It was shown that the jet can be divided into three zones which include a core zone, a transition zone and a final zone along the jet flow direction. Water pressure in the core zone is constant and beyond this zone, water pressure reduces. However, the variation of jet velocity is not discernable along the flow direction in a distance of 48 to 125 times of the orifice diameter. The particle distribution along the radial direction can be distinguished into three zones including a core zone, an inner zone and an outer zone with the maximum distribution of abrasives to be in the inner zone.

The experimental studies of the effects of process variables on the cutting performance consist of a large part of the published literature. While these investigations show a good understanding of the cutting performance and the associated science, most of the results are for particular cutting conditions and materials. The review of the depth of cut, kerf surface characteristics and kerf geometry indicates that the right choice of the process parameters is very important for good cutting performance. The survey of the depth of cut shows that this performance quantity is significantly affected by water pressure and nozzle traverse speed, while abrasive mass flow rate has relatively small effect on it. An increase in water pressure and abrasive mass flow rate or a decrease in the nozzle traverse speed and standoff distance is associated with an increase in the depth of cut. Towards the
requirement of the maximum depth of cut, high water pressure, slow nozzle traverse speed, short standoff distance and large abrasive mass flow rate should be selected. Kerf surface roughness increases with nozzle traverse speed and standoff distance, but decreases with the water pressure and abrasive mass flow rate. Corresponding selection strategy can therefore be made to reduce surface roughness. The review on the effect of process parameters on the kerf taper shows that standoff distance has the most effect on it but there is no clear trend between the kerf taper and the abrasive mass flow rate. An increase in standoff distance and nozzle traverse speed or a decrease in water pressure results in the increase in kerf taper. Top kerf width is greatly affected by standoff distance and nozzle traverse speed and has no discernable relationship with water pressure and abrasive mass flow rate. Top kerf width increases with an increase in standoff distance or a decrease in traverse speed. Nozzle traverse speed has strong effect on the bottom kerf width and the higher the traverse speed, the narrower the bottom kerf width. High water pressure increases the bottom kerf width but standoff distance and abrasive mass flow rate have little effect on this cutting performance measure.

The mathematical models to predict the cutting performance have been extensively reviewed in this chapter. It was noted that most of the developed models are in an effort to predict the depth of cut. Those models were developed using various approaches including AWJ erosion mechanism, fracture mechanics, energy conservation approach and regression analysis technique, which can be broadly classified into semi-empirical and pure empirical categories. While these models provide an effective way to predict the cutting performance, most of them are limited to particular cutting conditions and target materials. Moreover, most of the developed models have a complex mathematical expression which is difficult for practical use. Some of them include unknown factors needed to be determined by other research. Comparatively, semi-empirical method offers a more reasonable way for the modelling of engineering problem. With resort to this approach, the first part of the model development begins with the theoretical analysis and derivation and it then followed by experimental verification to determine the constants in the model.

A number of new techniques have been explored to enhance the cutting performance of AWJ cutting. These include forward angling the jet, controlled nozzle oscillation
and multipass cutting. By introducing a jet forward angle in the cutting plane, the surface drag angle in the kerf profile can be compensated and the cutting performance can be improved. The review of the controlled nozzle oscillation has shown that by superimposing an angular motion of the cutting head to the normal traverse motion, controlled nozzle oscillation technique can significantly increase the overall cutting performance without additional costs to the process. The relationship between the major cutting performance measures and the oscillation parameters are extensively reviewed. The published literature focused on the investigations of the effect of oscillation parameters on the smooth depth of cut and the surface roughness under large oscillation angles. It has been found that when using a relatively large oscillation angle of more than 10°, smooth depth of cut increases with both oscillation angle and oscillation frequency, while surface roughness decreases with those two oscillation parameters. To cope with the low capabilities, multipass cutting technique has been studied in recent years. It was indicated that by proper combinations of nozzle traverse speed and the number of passes, multipass cutting can significantly increase the depth of cut and smooth depth of cut as well as reduce surface roughness and kerf taper.

It can be concluded from the above literature review that while the cutting performance obtained by AWJ machining has been extensively studied with some of the literature concentrating on the research of oscillation and multipass cutting techniques, the cutting performance under the cutting environment of oscillation cutting and multipass cutting requires more detailed investigations. Particularly, the cutting performance at small oscillation angles has never been studied from the search of published literature, although a large number of applications which are related to AWJ profile cutting or contouring require the use of small oscillation angles. It is therefore necessary to explore if oscillation cutting under small angles can still increase cutting performance. From the survey of the literature, it can also be noted that in multipass cutting the research on the selection strategy for the optimum combination of nozzle traverse speed and the number of passes remains a research topic. Furthermore, the study on cutting performance using multipass cutting with controlled nozzle oscillation has not been found from the literature, although it is expected that this new cutting mode will produce an overall better cutting
performance. It is therefore on this ground that this project is conducted to study the cutting performance under AWJ cutting with controlled nozzle oscillation.

More importantly, it is well understood that the accurate and efficient use of oscillation and multipass cutting technology depends, to a great extent, on the right choice of the process parameters. Therefore, it is essential to develop reliable mathematical models correlating all the parameters involved. However, the mathematical model to predict the cutting performance under oscillation cutting suitable for small angles and multipass cutting has not been found yet from the search of the literature. Thus, another attempt of this project is to develop mathematical models to predict the major cutting performance measures under oscillation cutting and multipass cutting.
Chapter 3

A Study on the Cutting Performance of AWJ Machining with Controlled Nozzle Oscillation

3.1 Introduction

As an advanced manufacturing technology, abrasive waterjet (AWJ) machining is being increasingly used in various industries. However, the cutting capacity of this technology in terms of depth of cut (or depth of jet penetration) and kerf quality has been the major obstruction that limits its applications. In the last decades, considerable research and development effort have been made to develop new techniques to enhance the cutting performance and cutting capacity of this technology such as the depth of cut and surface finish. As reviewed in chapter 2, some newly developed techniques include cutting with forward angling the jet in the cutting plane, multipass cutting and controlled nozzle oscillation [10, 92, 150]. Among these new techniques, controlled nozzle oscillation or cutting head oscillation has been found to be one of the most effective ways in improving the cutting performance without additional costs to the cutting process. With this cutting technique, a pendulum-like nozzle forward and backward motion in the cutting plane at predetermined frequency and angular amplitude is superimposed to the normal nozzle traverse motion, as shown in Figure 3.1. Therefore, when using the nozzle oscillation technique, oscillation angle and oscillation frequency are two of the major cutting parameters. It has been found that the nozzle oscillation cutting technique can significantly improve some major cutting performance measures such as the depth of cut and surface finish. It has been reported [11, 92, 151] that the depth of the upper smooth zone in nozzle oscillation cutting can be increased by more than 30% as compared with that without oscillation, while kerf surface finish as measured by the centre line average $R_a$ can be improved by as much as 30%.
Figure 3.1. Schematic of controlled nozzle oscillation.

However, it appears that all the reported studies in controlled nozzle oscillation are primarily on the use of large nozzle oscillation angles (or angular amplitudes) of 10 degrees or more. Nozzle oscillation in the cutting plane (in the direction tangential to the curved profile in contouring) with such large oscillation angles results in theoretical geometrical errors on the component profile in contouring and is therefore not preferred in practice. As a result, it is necessary to investigate if nozzle oscillation at small angles can be employed to enhance the cutting performance. Furthermore, it has been noticed that if the oscillation parameters were not correctly selected, nozzle oscillation could have an adverse effect on the cutting performance. However, there has been no adequate study to thoroughly understand this phenomenon, examine how the cutting parameters affect the major cutting performance measures and under what conditions nozzle oscillation can have adverse effect on the cutting performance. In order to yield quantitative summaries...
on the cutting performance, regression analysis [152] and analysis of variance (ANOVA) [153-155] are performed to comparatively study the data acquired under the normal cutting (cutting without nozzle oscillation at a 90° jet impact angle) and nozzle oscillation cutting, assess the effect of each variable and accordingly distinguish the importance of these variables to the cutting performance, and identify the best combinations of cutting conditions for the optimum cutting performance. Graphical studies using scatter plots [156, 157] are then conducted to qualitatively analyse the effect trends of the cutting parameters on major cutting performance measures. Finally, a general guide and suggestion are given for the selection and combination of cutting parameters to achieve optimum cutting performance.

3.2 Experimental procedure

3.2.1 Material and AWJ cutting system

In this set of experiments, the specimens used were 87% alumina ceramics in the form of plates with the thickness of 12.7mm to represent brittle materials. The specimens were cut by a Flow International Waterjet Cutter driven by a “Model 20X” dual intensifier pumping system with the operating pressure of up to 380MPa. The motion of the nozzle is numerically controlled by a computer and a five-axis robot positioning system. The robot positioning system was programmed to be able to simultaneously execute the linear (the nozzle traverse speed) and oscillation motion of the nozzle. The oscillation movement consists of a forward angular movement at a given angle and angular speed and a backward or return movement to the original position of nozzle with the same speed. The original position of the nozzle was perpendicular to the workpiece surface. The other basic components in this system consist of “M-263” abrasive delivery system, “Paser II” Abrasive jet cutting head, “ASI CNC controller”, a water catcher and a remote terminal to program the machine.

3.2.2 Design of experiment

Depending on the characteristics and physical limitations of experiments, there are various designs of experiments (DOE). Taguchi’s approach provides a systematic and efficient method for conducting the engineering experiments [153]. The Taguchi
method of experimental design uses a special set of arrays called orthogonal arrays to construct an experimental plan. By choosing proper level combinations of various independent variables, these standard arrays stipulate the way of conducting limited number of runs but give the full information of all the factors that affect the performance parameters. In this way, an extremely large number of runs in an experiment could be avoided. The AWJ cutting experiment of this study employs Taguchi method as the theoretical basis of experimental design.

In AWJ cutting, a large number of variables have an impact on cutting performance [12, 19]. To simplify the analysis of oscillation cutting, four major process variables in normal AWJ cutting as identified in earlier studies [92] and two oscillation variables were chosen for investigation. These six variables include water pressure, nozzle (jet) traverse speed, abrasive mass flow rate, standoff distance between nozzle and workpiece surface, nozzle oscillation angle and oscillation frequency.

The selection of the feasible operation ranges for these process variables was made based on their ranges of practical applications and the machine system limitations. According to the major objective in conducting this research, i.e. investigating the cutting performance produced by AWJ oscillation cutting at small angles, small oscillation angles of less than 10º were selected with the purpose that the oscillation does not result in significant kerf geometrical errors. The selection of other process variables was then based on the determination of oscillation angle together with an anticipation of generating both through cuts and non-through cuts from this experiment for analysis. To achieve a through cut it was required that the combinations of the process variables give the jet enough energy to penetrate through the specimens. Therefore, the four variables in AWJ normal cutting were varied as follows: water pressure from 275MPa to 380MPa, nozzle traverse speed from 0.67mm/s to 1.67mm/s, standoff distance from 2mm to 5mm, and abrasive mass flow rate from 6.8g/s to 13.6g/s. Based on the previous finding from AWJ oscillation cutting ceramics [51], the oscillation frequencies were determined as approximate 6 times of nozzle traverse speeds to achieve overall improved cutting performance, giving 4~10 Hz. Therefore, the oscillation frequencies were determined from 2Hz to 14Hz.
The determination of corresponding levels for each variable was based on the effect nature of different level settings on the performance parameters, i.e. to consider the relationship between the process variables and performance parameters (linear or other forms). In the absence of exact nature of relationship between the process variables and the performance parameters and in order to derive more general form of mathematical models in the later stage, four levels for each variable were selected. The six process variables and their level settings are shown in Table 3.1. The other parameters that were kept constant during the tests include the nominal jet impact angle (the angle between the initial jet flow direction and the workpiece surface, 90º), orifice diameter (0.33mm), mixing tube or nozzle diameter (1.02mm) and abrasive material (80 mesh garnet sand).

Table 3.1. Experimental design in oscillation cutting.

<table>
<thead>
<tr>
<th>Process variables</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abrasive flow rate $m_a$ (g/s)</td>
<td>6.8</td>
<td>9.1</td>
<td>11.3</td>
<td>13.6</td>
</tr>
<tr>
<td>Standoff distance $H$ (mm)</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Water pressure $P$ (MPa)</td>
<td>275</td>
<td>310</td>
<td>345</td>
<td>380</td>
</tr>
<tr>
<td>Nozzle traverse speed $u$ (mm/s)</td>
<td>0.67</td>
<td>1.00</td>
<td>1.33</td>
<td>1.67</td>
</tr>
<tr>
<td>Oscillation angle $\theta$ (degree)</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Oscillation frequency $F$ (Hz)</td>
<td>2</td>
<td>6</td>
<td>10</td>
<td>14</td>
</tr>
</tbody>
</table>

Three groups of cutting tests were chosen in the experimental design. The first group used the four-level, six-factor design scheme in Taguchi orthogonal arrays [153, 155] with all six selected variables in order to study the influence of oscillation cutting on the cutting performance. This design scheme required 64 experimental runs ($L_{64}$). For comparatively studying the difference of cutting performance between oscillation cutting and normal cutting, another four-level, four-factor design scheme ($L_{16}$) was used; the four process variables were waterjet pressure, nozzle traverse speed, standoff distance and abrasive mass flow rate. This resulted in 16 more runs. Furthermore, in order to facilitate the analysis using the as-measured data, a third
group was designed using some typical cutting conditions from Table 3.1. This group of design included 30 tests. Thus, a total of 110 runs were undertaken in this four-level, six-factor experiment by combining the above designs. All specimens were machined into slots of 30mm to ensure steady state cutting conditions. The distance between each two cuts was 10mm and 5 specimens were used for the experiment.

3.2.3 Data acquisition

Measurements with the assistance of metrological instruments were conducted for the major cutting performance measures that included the depth of cut, smooth depth of cut, top and bottom kerf widths, kerf taper, and surface roughness. Of these quantities, kerf taper was calculated by recording the top kerf width, the minimum kerf width and the depth where the minimum kerf width was measured, while the other five quantities were directly measured from each cut. At least three measures for each quantity were made on each cut and the average was taken as the final reading.

The top kerf width, bottom kerf width, minimum kerf width and depth of cut were measured from the end view of the kerf profile by using a “SigmaScope 500” profile projector prior to separating the specimens. Kerf surface roughness and smooth depth of cut were measured after the specimens were separated to expose the kerf wall and examine the cutting front. Surface roughness values as assessed by the centre-line average $R_a$ were taken using a “Surtronic 3+” surface stylus profile meter. A sample length of 12.5mm was chosen with a cut-off length of 2.5mm for all specimens. $R_a$ values were measured at three different cutting locations, i.e. 1mm, 3mm and 5mm, from the top kerf edge. The measurements of smooth depth of cut were conducted by using a digital vernier micrometer under an “Olympus” optical microscope.

3.3 Results and analysis

In this section, kerf profiles and micro-characteristic features of machined specimens will first be examined and discussed. Various statistical analysis methods will then be used to reveal the relationships between oscillation angle, oscillation frequency, water pressure, nozzle traverse speed, standoff distance as well as abrasive flow rate and the major cutting performance measures, i.e. depth of cut, smooth depth of cut,
surface roughness, top kerf width, bottom kerf width and kerf taper as well as the
trends of their effects. When using ANOVA to quantitatively analyse the data
acquired from the tests, mean value, maximum value and minimum value of each
major cutting performance measure in oscillation cutting are examined to compare
with those obtained in normal cutting under corresponding cutting conditions and
consequently used to highlight the advantages of oscillation cutting over normal
cutting. Significant variables to affect each performance measure are distinguished
by using the mean squares for each variable. Based on the level means in ANOVA,
opimum level for each variable, optimum cutting condition for each cutting
performance measure, and therefore the consequent optimum cutting performance
measure are determined. The trends of each cutting performance measure with
respect to the process variables are systematically analysed. The mechanisms behind
these trends are discussed according to the understanding of the AWJ cutting
process. All the figures presented in this section to assist in portraying the trends in
the cutting are generated with the use of the regression equations with “backwards
elimination” technique on the experimental data. Those regression models are
performed with the aid of SPSS software [158]. The results of the analysis will form
the basis to suggest a strategy for the selection of cutting parameters for AWJ cutting
with small oscillation angles and to develop mathematical models in the later part of
this thesis.

3.3.1 Kerf profiles

A visualization study was conducted to evaluate the cuts from this experiment to gain
a grasp of the general kerf characteristics. It was clear that the two types of kerfs
produced in this experiment depending on the process variables, i.e. through cut and
non-through cut, have the similar characteristics with those reported in the literature.

Figure 3.2 shows an end view of kerf characteristics produced in a normal cutting
(left) and an oscillation cutting (right). These two cuts were machined using the same
normal cutting variables, i.e. an abrasive mass flow rate of 9.1g/s, a standoff distance
of 2mm, a water pressure of 310MPa, and a nozzle traverse speed of 1.33mm/s in
addition to an oscillation angle of 8° and an oscillation frequency of 10Hz for
oscillation cutting. This figure indicates the presence of three distinct zones, namely,
cutting wear zone, deformation wear zone and jet upward deflection zone in the
order in which they occur along the penetration depth. The former two zones are formed by shallow angles of attack and large angles of attack respectively [19] which are defined as the angle between the material surface and particle moving direction at the point of attack. The two kerfs have the similar geometry that is characterised by a wide entry at the top and a narrow width towards the exit so that a taper is formed. When the jet has no sufficient energy to penetrate the specimen, a non-through cut is produced as shown in Figure 3.2 (left) in normal cutting. The non-through cut has a wide top width but eventually reduces towards the lower part of the kerf, and at the bottom an enlarged pocket is left due to the jet upward deflection. Figure 3.2 (right) shows a kerf generated using the equivalent cutting condition but incorporating into oscillation operation. Obviously, oscillation cutting generated a higher depth of jet penetration than normal cutting. As the jet was just able to cut through the specimen, there was no large pocket at the bottom but the exit edge was widened as a result of the target material cracking and the deformation wear in this portion. It then can be deduced that if the jet possesses sufficient energy to easily penetrate the specimen there will be no this enlarged pocket at the exit of the kerf, which is one of the goals pursued in an AWJ cutting.

Two typical surface profiles produced from this experiment are as shown in Figure 3.3 for normal cutting (left) and oscillation cutting (right) under the corresponding combinations of process variables. The specimen in normal cutting was machined with a mass flow rate of 9.1g/s, standoff distance of 4mm, water pressure of 380MPa, and a nozzle traverse speed of 0.67mm/s, while oscillation cutting used the above process variables with the incorporation of oscillation angle of 4° and oscillation frequency of 14Hz. In spite of their equivalent cutting conditions in these two cuts, evident uneven marks were observed at the lower region of the surface for normal cutting (lift). However, oscillation cutting reached a relatively uniform surface.
The right surface profile is characterised by an upper smooth zone matching cutting wear zone and a lower rough zone corresponding to deformation wear zone, each exhibiting unique surface features. Upper smooth zone is observed from the top surface down to the distance where clear striations are visible. It is distinguished by shallow abrasive wear tracks and exhibits the surface roughness which is free of striations. The lower rough zone exists from the end of the cutting wear zone to the jet exit edge. Surface irregularity within this region is primarily dominated by waviness patterns (striations) due to deflection of penetrating jet. The waviness on the machined surface increases with depth of cut and the highest degree of waviness appears nearest to the jet exit. The normal cutting produced surface (left) has the similar features on the upper surface with that in oscillation cutting. But the machined surface has an additional uneven region near the jet exit edge corresponding to the widened part from the end view, indicating the jet upward deflection action caused by the insufficient jet energy in penetrating the specimen. This observation indicates that by just superimposing oscillation operation on the normal way of AWJ cutting, the cutting capacity of an AWJ was greatly enhanced and accordingly, smoother
surface finish was produced. These observations demonstrate the superiorities of AWJ oscillation cutting over normal cutting.

Figure 3.3. Surface profiles in oscillation cutting (Left: normal cutting, $m_a=9.1\, \text{g/s}$, $H=4\, \text{mm}$, $P=380\, \text{MPa}$, $u=0.67\, \text{mm/s}$. Right: oscillation cutting, $m_a=9.1\, \text{g/s}$, $H=4\, \text{mm}$, $P=380\, \text{MPa}$, $u=0.67\, \text{mm/s}$, $\theta=4^\circ$, $F=14\, \text{Hz}$).

### 3.3.2 Surface roughness

#### 3.3.2.1 Comprehensive analysis

This section gives a general view and quantitative analysis of the surface roughness from the experimental results. The roughness values machined grouped by oscillation cutting and normal cutting is compared using statistical techniques. With the aid of ANOVA, the importance of each process variable, the best combination of these variables to achieve optimum surface finish and the corresponding minimum surface roughness are then discussed by using the general linear model available in SPSS software [158]. In addition to the influence of cutting parameters, surface roughness is also dependent on the depth of cut. As we can see from Figure 3.3, machined surface becomes rough along the cutting depth. Therefore, the discussion of surface roughness should make to the same depth. The following discussion uses the experimental data measured at the depth of 1mm from the top edge and the surface roughness is assessed based on the centre-line average $R_a$. 
An inspection of all the cuts in oscillation cutting revealed that most of the machined cuts had the surface features as described in Figure 3.3 except that 5 cuts (test No. 28, 52, 53, 67 and 74) were found to be extremely rough so that striations actually were the predominant feature. The examination of cutting conditions for these cuts was found that these were the cases when high water pressures, large oscillation angles and high oscillation frequencies were used. The roughness values for 94 oscillation cuts varied between 3.61μm and 13.69μm with the average value being 6.43μm, while for 16 normal cuts $R_a$ was found to be in the range of 3.6μm to 8.5μm and the average value was 5.23μm. Comparison of the $R_a$ values under the corresponding cutting conditions showed that in the majority cases where nozzle oscillation was used, the surface was rougher than when cutting without nozzle oscillation. The worst result was observed to be machined using high water pressure, high nozzle traverse speed, large oscillation angle and large frequency. The increased $R_a$ values in oscillation cutting may be attributed to the jet instability and the system vibration caused by nozzle oscillation under some cutting conditions. These conditions therefore generated cuts with extremely high $R_a$ values and consequently increased the total average of the roughness value.

It is interesting to note that under some conditions, nozzle oscillation can produce the surface roughness values which are significantly lower than those in normal cutting under corresponding cutting conditions (such as Test No. 19, 30, 31, 51 and 100). It appears that the proper combinations of process variables are crucial to surface roughness. The study has found that when high water pressures and high nozzle traverse speeds were used, oscillation cutting increased surface roughness irrespective of the oscillation frequencies or angles used. It was noted that when water pressure was high, low nozzle traverse speed with small oscillation angle and low to medium oscillation frequency improved surface roughness. It was also found that when nozzle traverse speed was high while water pressure was selected at small to medium level together with small oscillation angle and low oscillation frequency, better surface finish was achieved. When these good combinations are used, the improvement in surface finish by oscillation cutting with respect to the corresponding normal cutting is more pronounced at high nozzle traverse speed than at low speed. Similar to the normal way of AWJ cutting, oscillation cutting at low
nozzle traverse speed was found to be able to reduce surface roughness. These observations are in line with the results of other reported work [11].

ANOVA was performed to distinguish the most important variables to affect surface roughness as shown in Table 3.2. In this table, the F test values can be obtained by calculating the ratio of the variance for each process variable to that of all the variables included in the error terms. F test is used to measure the significance of the process variable under investigation with respect to all the process variables considered. The significance level chosen here for the assessment is 5%. The null hypothesis is to assume that for a particular process variable, the group means of surface roughness are all equal with respect to different levels of this variable, while the alternative hypothesis is that they are not. The higher this computed T value is, the more likely the process variable contributes to the changes of the mean surface roughness within the confidence level. The computed significance value is computed with the selected significance level. If this value is less than the selected significance level (0.05), the null hypothesis is discredited and it can be asserted that the changes in the value of this process variable result in a different surface roughness value. Therefore, if a process variable has a significance value of less than the selected significance level it is considered to be a primary variable to affect the surface roughness.

In Table 3.2, a test of the null hypothesis that the mean surface roughness is equal when changing oscillation frequency levels reports an F test value of 4.715. The significance value is 0.000 which is less than 0.05 and so it can be concluded that oscillation frequency is a primary process variable in affecting surface roughness. According to the results of mean square, F test and computed significance value, among the six independent variables tested, water pressure and oscillation frequency have important influence on surface roughness which are considered as primary process variables. Of these two primary variables, water pressure applies the most effect on the surface roughness due to its higher F test value. The other four variables are less important based on their significance values. The less effect of oscillation angle may be owing to the small range and increment used in the study, between 2° to 8°, which has no substantial effect on the variation of surface roughness.

Best cutting conditions for both oscillation cutting and normal cutting to achieve minimum surface roughness and accordingly their minimum roughness values are
determined based on the calculation of level mean values for each process variable, as shown in Table 3.2. It is indicated that the optimum cutting conditions for oscillation cutting are: oscillation frequency at 2Hz, oscillation angle at 2°, nozzle traverse speed at 1mm/s, water pressure at 275MPa, standoff distance at 5mm and abrasive mass flow rate at 6.8g/s. Using this set of optimum cutting variables can produce a surface roughness $R_a$ of 2.05μm. A similar analysis has found that the optimum conditions for cutting without nozzle oscillation are nozzle traverse speed at 1mm/s, water pressure at 275MPa, standoff distance at 2mm and abrasive mass flow rate at 11.3g/s, and the resulted surface roughness $R_a$ is 2.29μm.

This analysis demonstrates that cutting with nozzle oscillation under optimized combination of cutting parameters can improve surface finish. However, if the combination of cutting conditions is incorrectly selected, cutting with nozzle oscillation can increase the surface roughness. This finding is somehow different from the previous investigations [2, 11, 51, 120] in which the use of nozzle oscillation technique is shown to constantly improve the cutting performance.

<table>
<thead>
<tr>
<th>Opti. cutting cond.</th>
<th>Mean square</th>
<th>F test</th>
<th>Sig.</th>
<th>Oscillation</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oscillation frequency (Hz)</td>
<td>17.339</td>
<td>4.715</td>
<td>0.000</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Oscillation angle (degree)</td>
<td>1.351</td>
<td>0.367</td>
<td>0.777</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Water pressure (MPa)</td>
<td>24.773</td>
<td>6.737</td>
<td>0.000</td>
<td>275</td>
<td>275</td>
</tr>
<tr>
<td>Nozzle traverse speed (mm/s)</td>
<td>1.730</td>
<td>0.470</td>
<td>0.704</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Standoff distance (mm)</td>
<td>9.563</td>
<td>2.601</td>
<td>0.064</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Abrasive flow rate (g/s)</td>
<td>1.014</td>
<td>0.274</td>
<td>0.842</td>
<td>6.8</td>
<td>11.3</td>
</tr>
<tr>
<td>Minimum roughness (μm)</td>
<td>2.05</td>
<td>2.29</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Many trends have been established to qualitatively assess surface roughness with variations in oscillation frequency, oscillation angle, water pressure, nozzle traverse
speed, standoff distance and abrasive mass flow rate with the emphasis on oscillation variables. The results of graphical representation for this study are reported below.

### 3.3.2.2 The effect of oscillation frequency

The effect of oscillation frequency on surface roughness has been studied graphically as illustrated in Figure 3.4. It is apparent from Figure 3.4 (a) that when oscillation angle varies surface roughness increases with the oscillation frequency monotonically and linearly. This linear trend also applies to other cutting conditions when varying water pressure or abrasive mass flow rate as shown in Figures 3.4 (b) and (e). However, the nozzle traverse speed and standoff distance appear to affect the slope of the linear relationship between oscillation frequency and surface roughness, as shown in Figures 3.4 (c) and (d). From Figure 3.4 (c), an increase in nozzle traverse speed causes a steep rise in the slope, indicating that increasing nozzle traverse speed aggravates the effect of oscillation frequency. This is because under high nozzle traverse speed less overlapping action causes fewer abrasive particles impacting on the specimen under the same oscillation angle and oscillation frequency and cannot effectively cut off the “peaks” left on the surface. The similar observation is noted from Figure 3.4 (d) from which the increase in the standoff distance results in a decrease in the slope. The standoff distance at 5 mm is associated with a negative slope of the linear relationship, i.e. the surface roughness decreases slightly with the oscillation frequency. This phenomenon is attributed to the fact that at high standoff distances, the scanning scope of waterjet is widened under the same oscillation angle, which leads to an increased overlapping cutting action on the cutting front and hence reduces the surface roughness. This observation is well in agreement with the previous finding [11]. Thus it can be seen that under the most of the combinations of process variables the increase of oscillation frequency results in an increase of surface roughness.
Figure 3.4. The effect of frequency on surface roughness: (a) $P=345\text{MPa}, u=1\text{mm/s}, H=2\text{mm}, m_a=9.1\text{g/s}$; (b) $\theta=4^\circ, u=1\text{mm/s}, H=2\text{mm}, m_a=9.1\text{g/s}$; (c) $\theta=4^\circ, P=345\text{MPa}, H=2\text{mm}, m_a=9.1\text{g/s}$; (d) $\theta=4^\circ, P=345\text{MPa}, u=1\text{mm/s}, m_a=9.1\text{g/s}$; (e) $\theta=4^\circ, P=345\text{MPa}, u=1\text{mm/s}, H=2\text{mm}$. 
3.3.2.3 The effect of oscillation angle

Figure 3.5 shows the influence of oscillation angle on surface roughness. It can be found from the figures that in most cases initially surface roughness increases slightly with an increase in oscillation angle and reaches a maximum turning point. As the oscillation angle further increases, surface roughness starts to decrease. The angle value corresponding to maximum roughness value is in the vicinity of 5° in most of the cases. This may be a result of the scanning action of the jet on the cutting front and there appears to be an optimum scanning scope corresponding to a set of cutting parameters, similar to the discussion above about the effect of standoff distance. At smaller oscillation angles, the jet scanning action cannot effectively cut off the “peaks” left on the cut surface and causes jet turbulence or instability and system vibration that increase the surface roughness. Larger oscillation angles increase the overlap cutting action and the number of scanning actions on a given part of surface, so that the scanning action is dominant and reduces the surface roughness. This indicates that from the point of view of reducing surface roughness medium angle in the test range should be avoided.

![Graphs showing the effect of oscillation angle on surface roughness](image-url)
Figure 3.5. The effect of oscillation angle on surface roughness: (a) $P=345\text{MPa}, u=1\text{mm/s}, H=2\text{mm}, m_a=9.1\text{g/s}$; (b) $F=6\text{Hz}, u=1\text{mm/s}, H=2\text{mm}, m_a=9.1\text{g/s}$; (c) $F=6\text{Hz}, P=345\text{MPa}, H=2\text{mm}, m_a=9.1\text{g/s}$; (d) $F=6\text{Hz}, P=345\text{MPa}, u=1\text{mm/s}, m_a=9.1\text{g/s}$; (e) $F=6\text{Hz}, P=345\text{MPa}, u=1\text{mm/s}, H=2\text{mm}$.

3.3.2.4 The effect of other process variables

The typical effect of water pressure, nozzle traverse speed, abrasive mass flow rate and standoff distance on surface roughness is plotted in Figure 3.6. The first three variables have the same effects on surface roughness as reported in chapter 2 for AWJ normal cutting. An increase in the water pressure, nozzle traverse speed or abrasive mass flow rate is associated with an increase in surface roughness although the degrees of their effects are different. In fact, surface roughness just shows a slight linear increase with the abrasive mass flow rate, which can be assumed to be negligible. Therefore, the preferred choice in considering surface roughness requirement in practice should be small abrasive flow rate from the point of view of reducing cutting cost.
Figure 3.6. The effect of other process variables on surface roughness: (a) $\theta=4^\circ$, $F=6\,\text{Hz}$, $H=2\,\text{mm}$, $m_a=9.1\,\text{g/s}$; (b) $\theta=4^\circ$, $F=10\,\text{Hz}$, $H=3\,\text{mm}$, $m_a=9.1\,\text{g/s}$; (c) $\theta=4^\circ$, $P=345\,\text{MPa}$, $u=1\,\text{mm/s}$, $H=3\,\text{mm}$; (d) $\theta=4^\circ$, $F=6\,\text{Hz}$, $P=345\,\text{MPa}$, $u=1\,\text{mm/s}$.

The standoff distance affects the surface roughness in a different way with that in AWJ normal cutting. An increase of standoff distance causes a constant increase in surface roughness when normal cutting mode applies [108, 121]. However, the observation from Figure 3.6 (d) shows that surface roughness exhibits an initial increase with an increase in standoff distance and then a slow decrease with the further increase in standoff distance.

When using oscillation technique, while an increase in the standoff distance may reduce the density of abrasive particles in the outer perimeter of the expanding jet and so the particle energy at the attack point as the case in AWJ normal cutting, standoff distance may take an effect jointly with the oscillation angle and frequency. The increase in standoff distance results in a broader scanning scope of the abrasive
waterjet. Therefore, more overlapping action occurs, which is beneficial to correct the irregularities of previous cutting. When standoff distance is relatively small the effect of the divergent jet plays a dominant role with the increase of standoff distance so that the surface becomes rough. After standoff distance reaches a certain value the effect of scanning scope may be more significant thus results in a decreased $R_a$ value.

### 3.3.2.5 Overall considerations on surface roughness

In summary, an investigation into the surface roughness generated by AWJ oscillation cutting has quantitatively and qualitatively been carried out. The statistical analysis shows that although the surface became rougher in most cutting cases with oscillation cutting from this experiment, under proper combinations of cutting conditions nozzle oscillation cutting can produce much lower surface roughness than normal cutting under the corresponding cutting conditions. The results of ANOVA suggest that water pressure has the most influence on surface roughness, followed by oscillation frequency. The least important variable is abrasive flow rate with the next being oscillation angle so that the effect of abrasive flow rate can in fact be neglected. Under their respective optimum cutting conditions, oscillation cutting can reduce surface roughness by 11.7% compared to normal cutting in this study.

From the qualitative analysis using graphical study, surface roughness increases with oscillation frequency, water pressure and nozzle traverse speed, but initially slightly increases and then slowly decreases with an increase in oscillation angle or standoff distance. Therefore, the selection of low oscillation frequency and water pressure, small oscillation angle and nozzle traverse speed coupled with short or high standoff distance could produce improved surface finish compared to normal cutting. Practically, for a given surface quality requirement, it is recommended to use slow oscillation frequency, low water pressure, short standoff distance, small oscillation angle and abrasive flow rate combined with suitably low nozzle traverse speed in order to save the cutting costs. Under any circumstances, high water pressure and large nozzle traverse speed must not be used at the same time when using oscillation cutting as such combination could cause cutting system vibration and jet instability regardless of oscillation parameters incorporated.

Therefore, proper combination of process variables is exceedingly important to achieve improved surface finish compared to normal cutting. This result is a
correction to the previous reports showing that the use of oscillation technique can constantly improve surface quality.

3.3.3 Depth of cut

3.3.3.1 Comprehensive analysis

Experimental data from this set of experiments revealed that nozzle oscillation significantly affects the depth of cut. The depth of cut for all the tests with nozzle oscillation shows an average increase of 27.7% relative to that obtained without nozzle oscillation under the corresponding cutting conditions. In some extreme cases, nozzle oscillation cutting increased the depth of cut by as much as 82%. Nevertheless, there are still cases where the depth of cut in oscillation cutting is less than that of the corresponding normal cutting. This may be different from the usual intuition that oscillation technique can constantly increase this cutting quantity. The analysis shows that whether or not nozzle oscillation can increase the depth of cut is dependent on both oscillation angle and oscillation frequency. Depending on the other operating parameters, there appears to exist a critical or threshold value of the product of oscillation frequency and oscillation angle which is defined as oscillation strength (Figure 3.7). Only above this critical value can nozzle oscillation increase the depth of cut. This indicates that the variation of oscillation angle per unit time (or angular velocity of oscillation) determines the effect of oscillation cutting. This also means that a small angle with a high frequency has a same effect as a large angle with a low frequency. In the case of cutting condition shown on Figure 3.7, the critical oscillation strength is found to be approximately 17 deg/s).

![Figure 3.7. The influence of oscillation strength on depth of cut: $m_o=6.8g/s$, $H=3mm$, $P=310MPa$, $u=1mm/s.$](image)

Figure 3. 7. The influence of oscillation strength on depth of cut: $m_o=6.8g/s$, $H=3mm$, $P=310MPa$, $u=1mm/s.$
In looking into the relations between the critical oscillation strength and major process variables, it has been found that nozzle traverse speed and standoff distance positively affect critical oscillation strength, while water pressure and abrasive flow rate have adverse effect on this value. The increase in nozzle traverse speed causes a dramatic increase in critical oscillation strength, which may be due to the fact that higher nozzle traverse speed reduces the time of erosion process and more powerful oscillation action needs to be superimposed to overcome the resistance of target material. The critical oscillation strength decreases with the increase in water pressure. This fact may be explained by the understanding that higher water pressure provide high energy to overcome the resistance of target material. Standoff distance has a relatively minor influence on the critical value in that the critical oscillation strength slightly increases when standoff distance increases. This may be caused by the reduced jet energy in the lower part of the kerf with the increased standoff distance and stronger oscillation action is required in order to compensate for the energy reduction of the jet. An increase in abrasive mass flow rate leads to a decrease in critical oscillation strength. High abrasive mass flow rate means more impacting abrasive particles involving in the cutting process and therefore high kinetic energy of the jet. The corresponding result of high abrasive mass flow rate is that even a small oscillation strength could generate an improved depth of cut.

AVONA was performed to distinguish the most important process variables on depth of cut. It has been found that the significance values of oscillation frequency, nozzle traverse speed, water pressure and abrasive mass flow rate are less than the selected significance level (0.05) and these four variables are considered to be primary process variables to affect depth of cut. Therefore, the most dominant process variables for depth of cut are oscillation frequency, nozzle traverse speed, water pressure and abrasive mass flow rate in order of their importance based on their respective F test values, whereas standoff distance and oscillation angle have a little effect. The determinations of optimum cutting conditions for both oscillation cutting and normal cutting suggest that if a 14Hz oscillation frequency and a 6° oscillation angle are used under 380MPa water pressure, 0.67mm/s nozzle traverse speed, 3mm standoff distance, and 11.3g/s abrasive flow rate, oscillation cutting can produce the maximum depth of cut of 16.3mm, as shown in Table 3.3. However, with the normal way of cutting the optimum combination of the cutting parameters were found to be
water pressure at 345MPa, nozzle traverse speed at 0.67mm/s, standoff distance at 2mm, and abrasive mass flow rate at 11.3g/s, which yields the maximum depth of cut of 13.3mm. Thus, statistically nozzle oscillation cutting can increase the depth of cut by 23% with respect to the normal cutting technique under the respective optimum combinations of cutting parameters.

<table>
<thead>
<tr>
<th></th>
<th>Mean square</th>
<th>F test</th>
<th>Sig.</th>
<th>Opti. cutting cond.</th>
<th>Oscillation</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oscillation frequency (Hz)</td>
<td>11.939</td>
<td>18.280</td>
<td>0.000</td>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oscillation angle (degree)</td>
<td>0.022</td>
<td>0.034</td>
<td>0.991</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water pressure (MPa)</td>
<td>6.152</td>
<td>9.420</td>
<td>0.000</td>
<td>380</td>
<td>345</td>
<td></td>
</tr>
<tr>
<td>Nozzle traverse speed (mm/s)</td>
<td>11.555</td>
<td>17.692</td>
<td>0.000</td>
<td>0.67</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>Standoff distance (mm)</td>
<td>0.114</td>
<td>0.175</td>
<td>0.912</td>
<td>3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Abrasive flow rate (g/s)</td>
<td>5.084</td>
<td>7.785</td>
<td>0.000</td>
<td>11.3</td>
<td>11.3</td>
<td></td>
</tr>
<tr>
<td>Maxi. depth of cut (mm)</td>
<td></td>
<td>16.3</td>
<td></td>
<td>13.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In order to qualitatively study the various trends of depth of cut with respect to major process variables, following sections present the detailed analysis of the effect of oscillation frequency, oscillation angle, water pressure, nozzle traverse speed, standoff distance and abrasive mass flow rate.

### 3.3.3.2 The effect of oscillation frequency

The relationship between oscillation frequency and the depth of cut is plotted in Figure 3.8. In most of the cases, the depth of cut increases approximately linearly with the oscillation frequency, as illustrated by Figures 3.8 (b), (d) and (e) when water pressure, standoff distance and abrasive mass flow rate vary. It is believed that a higher oscillation frequency increases the number of repeated scanning actions and reduces the particle interference, which in turn increases the overall abrasive cutting capacity and the depth of cut. Under some conditions, an increase in oscillation
frequency may result in a slight decrease in the depth of cut, as shown in Figures 3.8 (a) and (c). These two figures also show that the slope of the linear relationship between oscillation frequency and depth of cut is affected by oscillation angle and nozzle traverse speed. Under large oscillation angle, the scanning effectiveness of the jet reduces with an increase in oscillation frequency due to the jet instability caused by nozzle vibration, which leads to a decreased depth of cut. A higher nozzle traverse speed is associated with a more rapid increase of depth of cut as the oscillation frequency increases. When a low nozzle traverse speed of 0.67mm/s is used, an increase in oscillation frequency in fact results in a decrease in the depth of cut. This may be explained that at low nozzle traverse speed, the jet scanning action cannot take effect and may cause increased particle interference and a reduction in the jet cutting capability.
Figure 3.8. The effect of oscillation frequency on depth of cut: (a) $P=345\text{MPa}$, $u=1\text{mm/s}$, $H=2\text{mm}$, $m_a=9.1\text{g/s}$; (b) $\theta=4^\circ$, $u=1\text{mm/s}$, $H=2\text{mm}$, $m_a=9.1\text{g/s}$; (c) $\theta=4^\circ$, $P=310\text{MPa}$, $H=2\text{mm}$, $m_a=9.1\text{g/s}$; (d) $\theta=4^\circ$, $P=345\text{MPa}$, $u=1\text{mm/s}$, $m_a=9.1\text{g/s}$; (e) $\theta=4^\circ$, $P=345\text{MPa}$, $u=1\text{mm/s}$, $H=2\text{mm}$.

3.3.3.3 The effect of oscillation angle

Figure 3.9 shows the effect of oscillation angle on the depth of cut from the experimental data. Under most combinations of cutting variables, an increase in oscillation angle results in an initially slight increase in the depth of cut, while the increasing rate decreases with the oscillation angle. As the oscillation angle further increase to beyond $6^\circ$, the depth of cut exhibits a decreased trend. A maximum turning point for the depth of cut occurs at about $4^\circ$ to $6^\circ$ of oscillation angles.
Figure 3.9. The effect of oscillation angle on depth of cut: (a) $P=310$ MPa, $u=1$ mm/s, $H=3$ mm, $m_{a}=9.1$ g/s; (b) $F=6$ Hz, $u=1$ mm/s, $H=3$ mm, $m_{a}=9.1$ g/s; (c) $F=6$ Hz, $P=310$ MPa, $H=3$ mm, $m_{a}=9.1$ g/s; (d) $F=6$ Hz, $P=310$ MPa, $u=1$ mm/s, $m_{a}=9.1$ g/s; (e) $F=6$ Hz, $P=310$ MPa, $u=1$ mm/s, $H=2$ mm.

This effect is also dependent on the oscillation frequency as shown in Figure 3.9 (a). At relatively large oscillation frequencies (e.g. 10Hz and 14Hz), an increase in oscillation angle is associated with a steady decrease of the depth of cut while the decreasing rate slightly increases with the oscillation angle. This may be due to the fact that at high oscillation frequencies, an increase in oscillation angle increases the jet instability which decreases the jet cutting capability and hence decreases this cutting performance measure. To this end, large oscillation frequencies with small oscillation angles are preferred to increase the depth of cut.
3.3.3.4 The effect of other process variables

The effects of water pressure, nozzle traverse speed, abrasive mass flow rate and standoff distance on the depth of cut are presented in Figure 3.10. An increase in water pressure or abrasive mass flow rate produces an increased depth of cut, while the reverse trend applies when the nozzle traverse speed is increased. These trends are the same as previously reported in normal cutting [15, 52, 94].

Standoff distance plays a limited role in affecting the depth of cut in oscillation cutting, as shown in Figure 3.10 (c). An increase in standoff distance results in a slight increase in the depth of cut, which is different from the intuition and previous findings in normal AWJ cutting indicating that with an increase in standoff distance
the depth of cut almost linearly decreases [20, 52, 84, 108]. The increase in the standoff distance causes reduced jet energy on the point where it attacks the target material, as is the case in normal cutting. However, when oscillation operation applies, its effect may be controlled by the oscillation parameters used. With a larger standoff distance, the jet scanning scope on the cutting front is increased, which may increase or decrease the depth of cut depending on the other parameters used, in a similar way to the oscillation angle.

### 3.3.3.5 Overall considerations on depth of cut

A detailed investigation has been performed to statistically and qualitatively study the depth of cut obtained with AWJ nozzle oscillation cutting. The comparisons of the corresponding depth of cut values between normal cutting and oscillation cutting have been made with the purpose of understanding how oscillation cutting can improve the AWJ penetrating depth. Quantitative analysis indicates that overall increase in the depth of cut using oscillation cutting in this experiment is 27.7% compared to the corresponding normal cutting.

However, the increase in the depth of cut is greatly dependent on the oscillation strength used (the product of oscillation frequency and oscillation angle), which means the use of oscillation cutting does not really always improve the depth of cut. It has been found that there exists a critical value of oscillation strength to achieving an improved depth of cut. If the used oscillation strength is lower than the critical value, oscillation cutting will reduce the depth of cut compared to normal cutting. The critical oscillation strength varies according to other cutting parameters used. The analysis has shown that an increase in nozzle traverse speed or standoff distance causes an increased critical oscillation strength, while high water pressure and abrasive mass flow rate reduce this value.

ANOVA performed on the experimental data suggests that oscillation frequency, nozzle traverse speed, water pressure and abrasive mass flow rate are the primary process variables in affecting the depth of cut. Standoff distance and oscillation angle have less effect on the depth of cut than other parameters. The optimum cutting conditions for both oscillation cutting and normal cutting has been identified and if
under respective optimum condition, oscillation cutting could increase the depth of cut by 23%.

Qualitative study shows that oscillation frequency, water pressure, abrasive mass flow rate and standoff distance positively affect the depth of cut, of which standoff distance only has limited effect, and nozzle traverse speed has adverse effect on the depth of cut. In addition, the effect of oscillation angle is varied in the selected range, i.e. initially increases and then decreases the depth of cut with the increase in oscillation angle. Consequently, in order to achieve maximum depth of cut, the strategy for the selection of process variables should be large oscillation frequency, medium oscillation angle, high water pressure, large abrasive mass flow rate and relatively small nozzle traverse speed. However, it should be noted that in AWJ contouring operation, the smallest possible oscillation angle is a priority. The combination of the cutting variables should consider the cutting mode and work requirements.

### 3.3.4 Smooth depth of cut

#### 3.3.4.1 Comprehensive analysis

Smooth depth of cut is the depth from the top edge of the specimen down to where clear striations on the cut surface are visible. The major feature on this surface of the kerf front is roughness, as described in Chapter 2. It is always expected that the smooth depth of cut is equal to the required depth of jet penetration.

An inspection of all the cuts shows that most of the cuts have apparent upper smooth zone and lower striation zone apart from two kerfs on which the striation is actually the predominant feature (test No. 28 and 33). These two cuts were produced when cutting with high water pressure, fast nozzle traverse speed, large oscillation angle and high oscillation frequency. The measurements of smooth depth of cut show a similar result with that for depth of cut in that most of the kerfs in oscillation cutting have a higher smooth depth of cut than their corresponding normal cutting. Comparison of the average smooth depths of cut under their corresponding cutting conditions indicates that oscillation cutting increased the average smooth depth of cut by 18.7%. The higher smooth depths of cut are found to be produced using high oscillation frequency and high water pressure.
An investigation into the influence of process variables performed by means of ANOVA has found that oscillation frequency, water pressure, nozzle traverse speed, abrasive mass flow rate and standoff distance play primary roles in affecting smooth depth of cut based on their significance values, while the change in the oscillation angle has little effect as indicated by Table 3.4. Among these primary variables, oscillation frequency has the most effect, followed by nozzle traverse speed, water pressure, abrasive mass flow rate and standoff distance according to their F test values. In fact, the influence of frequency is so significant that the influence of other process variables is diminished. In the case of best combination of process variables for oscillation cutting, i.e. oscillation frequency of 14Hz, oscillation angle of 8°, nozzle traverse speed of 0.67mm/s, water pressure of 380MPa, standoff distance of 5mm, and abrasive mass flow rate of 13.6g/s, the maximum smooth depth of cut would be 5.98mm. Accordingly, if oscillation technique is not used the best combination of process variables is water pressure at 380MPa, nozzle traverse speed at 0.67mm/s, standoff distance at 5mm and abrasive mass flow rate at 13.6g/s. Such combination would produce a maximum smooth depth of cut of 4.57mm, which means that oscillation technique could increase smooth depth of cut by 30.8% compared to normal cutting under their respective optimum cutting conditions.

By means of graphical study, the general trends of smooth depth of cut with respect to the process variables such as oscillation frequency, oscillation angle, water pressure, nozzle traverse speed, standoff distance and abrasive mass flow rate, will be discussed below.
### Table 3. 4. ANOVA of smooth depth of cut.

<table>
<thead>
<tr>
<th></th>
<th>Mean square</th>
<th>F test</th>
<th>Sig.</th>
<th>Opti. cutting cond. Oscillation</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oscillation frequency (Hz)</td>
<td>8.872</td>
<td>77.733</td>
<td>0.000</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Oscillation angle (degree)</td>
<td>0.117</td>
<td>1.023</td>
<td>0.392</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Water pressure (MPa)</td>
<td>1.726</td>
<td>15.123</td>
<td>0.000</td>
<td>380</td>
<td>380</td>
</tr>
<tr>
<td>Nozzle traverse speed (mm/s)</td>
<td>1.754</td>
<td>15.368</td>
<td>0.000</td>
<td>0.67</td>
<td>0.67</td>
</tr>
<tr>
<td>Standoff distance (mm)</td>
<td>0.552</td>
<td>4.833</td>
<td>0.006</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Abrasive flow rate (g/s)</td>
<td>0.798</td>
<td>6.993</td>
<td>0.000</td>
<td>13.6</td>
<td>13.6</td>
</tr>
<tr>
<td>Maxi. smooth depth of cut (mm)</td>
<td>5.98</td>
<td>4.57</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### 3.3.4.2 The effect of oscillation frequency

Figure 3.11 illustrates the influence of oscillation frequency on smooth depth of cut under various cutting conditions. It can be seen from the Figure that increasing oscillation frequency results in a linear increase in the smooth depth of cut. These linear trends unanimously apply for all the cutting conditions tested in the range of this experiment, irrespective of the variations in oscillation angle, water pressure, nozzle traverse speed, standoff distance and abrasive mass flow rate (Figures 3.11 (a), (b), (c), (d), and (e)). It is believed that within per unit time high oscillation frequency increases the number of repeated scanning action on kerf wall by fresh abrasive waterjet, which causes the increase of smooth depth of cut.

![Figure 3.11](image-url)
Figure 3.11. The effect of oscillation frequency on smooth depth of cut: (a) $P=345\text{MPa}$, $u=1\text{mm/s}$, $H=2\text{mm}$, $m_a=9.1\text{g/s}$; (b) $\theta=4^\circ$, $u=1\text{mm/s}$, $H=2\text{mm}$, $m_a=9.1\text{g/s}$; (c) $\theta=4^\circ$, $P=345\text{MPa}$, $H=2\text{mm}$, $m_a=9.1\text{g/s}$; (d) $\theta=4^\circ$, $P=345\text{MPa}$, $u=1\text{mm/s}$, $m_a=9.1\text{g/s}$; (e) $\theta=4^\circ$, $P=345\text{MPa}$, $u=1\text{mm/s}$, $H=2\text{mm}$.

### 3.3.4.3 The effect of oscillation angle

The influence of oscillation angle on smooth depth is presented in Figure 3.12. Although the trend shows a generally increased smooth depth of cut with the increase of oscillation angle in the test range, at first stage smooth depth of cut slightly decreases when increasing oscillation angle to approximately $5^\circ$, and then with an additional increase in oscillation angle smooth depth of cut begins to increase. This trend applies when changing oscillation frequency, water pressure, standoff distance and abrasive flow rate (Figure 3.12 (a), (b), (d), and (e)). However, nozzle traverse speed affects the trend of smooth depth of cut with the oscillation angle, as indicated by Figure 3.12 (c). Under low nozzle traverse speed, smooth depth of cut presents a
similar trend with those when other process variables vary. However, when high nozzle traverse speed is used, smooth depth of cut decreases slightly and this trend remains unchanged with the increase of oscillation angle although the rate of decrease declines. Thus, if high smooth depth of cut is the only requirement for cutting performance, small or large oscillation angle coupled with appropriate low nozzle traverse speed should be preferred consideration.
Smooth depth of cut is found to increase approximately linearly with water pressure (Figure 3.13 (a)) and to reduce following a non-linear curve with the nozzle traverse speed (Figure 3.13(b)), which is well in line with their effect on smooth depth of cut in normal cutting [92]. Compared to the effect of water pressure on depth of cut showing an initial increase and then gradually reduced rate for this increase at high pressures, water pressure affects smooth depth of cut with a more significant rate than depth of cut. The effect of abrasive mass flow rate and standoff distance is not so much significant than that of water pressure and nozzle traverse speed as described in the forgoing ANOVA Table 3.4.
Figure 3.13. The effect of other process variables on smooth depth of cut: (a) $\theta=4^\circ$, $u=1\text{mm/s}$, $H=2\text{mm}$, $m_a=9.1\text{g/s}$; (b) $\theta=4^\circ$, $F=6\text{Hz}$, $P=345\text{MPa}$, $H=3\text{mm}$; (c) $\theta=4^\circ$, $F=6\text{Hz}$, $u=1\text{mm/s}$, $H=3\text{mm}$; (d) $\theta=4^\circ$, $F=6\text{Hz}$, $u=1\text{mm/s}$, $m_a=9.1\text{g/s}$.

When using oscillation cutting, abrasive mass flow rate affects smooth depth of cut in different way with that using normal cutting, as illustrated by Figure 3.13 (c). Smooth depth of cut shows a slight decrease with abrasive mass flow rate despite not significant, rather than an increase as in the case of normal cutting [52]. This indicates that when using oscillation cutting mode, an increase in abrasive mass flow rate may cause increased particle interference to be dominant in the jet scanning action and hence a reduction in the smooth depth of cut.

Smooth depth of cut is found to decrease with an increase in standoff distance at first. When standoff distance is at around 3mm, smooth depth of cut begins to increase (Figure 3.13 (d)) although in the whole test range, the effect of standoff distance is diminished as described in the ANOVA of smooth depth of cut (Table 3.4). This may be explained that in the first stage, increasing standoff distance results in the jet spreading and accordingly reduces the energy density of the jet at the impacting site as is the case in normal cutting. But with the further increase in standoff distance, the effect of increased scanning scope of the abrasive waterjet is more prominent and this increased overlapping action may achieve higher smooth depth of cut.
3.3.4.5 Overall considerations on smooth depth of cut

The above study of smooth depth of cut machined by AWJ oscillation cutting has shown that oscillation cutting technique greatly increased smooth depth of cut compared to normal cutting. Quantitative analysis revealed that oscillation cutting statistically increased smooth depth of cut by 18.7% compared to the corresponding normal cutting in this experiment. In terms of the effect of process variables on smooth depth of cut, the result of ANOVA indicates that oscillation frequency ranks the first and then flowed by nozzle traverse speed, water pressure, abrasive mass flow rate and standoff distance according to their importance, whereas oscillation angle has diminished effect. If using their respective optimum cutting condition, the percentage increase with oscillation cutting could be 30.8% compared to normal cutting.

The analysis of the trend with respect to the process variables has exhibited that an increase in oscillation frequency and water pressure or a decrease in nozzle traverse speed and abrasive flow rate leads to an increase in smooth depth of cut. Oscillation angle and standoff distance present a similar effect on smooth depth of cut. Smooth depth of cut first slightly decreases and then increases with the oscillation angle or standoff distance.

Consequently, in consideration of a proper combination of process variable for practical use, the recommendation is large oscillation frequency, high water pressure, reasonably low nozzle traverse speed, small abrasive mass flow rate with small oscillation angle and low standoff distance.

3.3.5 Kerf taper

3.3.5.1 Comprehensive analysis

Kerf taper is a quantity which is often used to reflect the inclination of the kerf wall from the top surface to the bottom of the kerf. As stated in the foregoing section, the value of kerf taper, $T_R$, for each cut was obtained from the calculation using other measured quantities, i.e. the top kerf width $W_t$, the minimum kerf width along the kerf profile $W_b$, and the depth $t$ at which the minimum kerf width was measured, based on the following equation
\[ T_R = \frac{(W_f - W_b)}{2t} \]  

(3.1)

It is noted from an observation to all the cuts that most of the cuts generated by oscillation operation have a smaller kerf taper value than that machined by corresponding normal cutting. Those kerfs possessing larger kerf taper in oscillation cutting are found to be machined using small oscillation frequency, high nozzle traverse speed and low water pressure. Statistically, based on the average kerf taper values, oscillation cutting decreased taper angle by 18.1% compared to normal cutting under the corresponding cutting conditions in this experiment.

ANOVA analysis of the experimental data suggests that according to the criterion that the calculated significance value is less than 0.05, the primary variables to have significant effect on kerf taper include oscillation frequency, oscillation angle, nozzle traverse speed, water pressure and abrasive mass flow rate (Table 3.5). By contrast, standoff distance has the least effect on kerf taper. Of these primary process variables, oscillation frequency accounts for most of the kerf taper formation, while water pressure, nozzle traverse speed, abrasive mass flow rate and oscillation angle assume the second role in order of their importance for kerf taper variation. Table 3.5 also shows that the optimum combination for the smallest kerf taper has been found to be oscillation frequency of 14Hz, oscillation angle of 8º, water pressure of 380MPa, nozzle traverse speed of 0.67mm/s, standoff distance of 2mm, and abrasive mass flow rate of 11.3g/s. This optimum combination can produce a kerf taper of 0.028. Similarly, the analysis of the experimental data has found that statistically, the minimum kerf taper for the normal way of AWJ cutting is 0.061 under the optimum combination of process variables, i.e. water pressure is at 380MPa, nozzle traverse speed at 0.67mm/s, standoff distance at 2mm, and abrasive mass flow rate at 11.3g/s. Thus, under the optimum parameter combinations, cutting with nozzle oscillation can reduce the kerf taper by 54%. 

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Chapter 3  
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Table 3.5. ANOVA of kerf taper.

<table>
<thead>
<tr>
<th></th>
<th>Mean square</th>
<th>F test</th>
<th>Sig.</th>
<th>Opti. cutting cond.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Oscillation</td>
</tr>
<tr>
<td>Oscillation frequency (Hz)</td>
<td>5.509</td>
<td>18.668</td>
<td>0.000</td>
<td>14</td>
</tr>
<tr>
<td>Oscillation angle (degree)</td>
<td>1.169</td>
<td>3.961</td>
<td>0.011</td>
<td>8</td>
</tr>
<tr>
<td>Water pressure (MPa)</td>
<td>4.692</td>
<td>15.901</td>
<td>0.000</td>
<td>380</td>
</tr>
<tr>
<td>Nozzle traverse speed (mm/s)</td>
<td>2.404</td>
<td>8.147</td>
<td>0.000</td>
<td>0.67</td>
</tr>
<tr>
<td>Standoff distance (mm)</td>
<td>0.185</td>
<td>0.627</td>
<td>0.600</td>
<td>2</td>
</tr>
<tr>
<td>Abrasive flow rate (g/s)</td>
<td>1.758</td>
<td>5.956</td>
<td>0.000</td>
<td>11.3</td>
</tr>
<tr>
<td>Minimum kerf taper</td>
<td></td>
<td></td>
<td>0.028</td>
<td>0.061</td>
</tr>
</tbody>
</table>

3.3.5.2 The effect of oscillation frequency and oscillation angle

The effect of oscillation frequency on kerf taper is plotted in Figure 3.14. Since oscillation frequency has little effect on kerf width (to be analysed in the following section), but has a significant effect on the jet penetration depth as discussed in section 3.3.3.2, an increase in oscillation frequency is associated with a reduced kerf taper from Equation (3.1). This trend is consistent for cutting under various conditions. The effect of oscillation frequency on the kerf taper becomes interesting when nozzle traverse speed is varied, as shown in Figure 3.14(c). The slope of the kerf taper plots with respect to oscillation frequency becomes steeper as the nozzle traverse speed increases. It is believed that under high nozzle traverse speed, oscillation cutting affects the depth of cut in a higher rate with an increase in oscillation frequency as described in Figure 3.8 (c).
Figure 3.14. The effect of oscillation frequency on kerf taper: (a) $P=310\text{MPa}$, $u=1\text{mm/s}$, $H=2\text{mm}$, $m_a=9.1\text{g/s}$; (b) $\theta=4^\circ$, $u=1\text{mm/s}$, $H=2\text{mm}$, $m_a=9.1\text{g/s}$; (c) $\theta=4^\circ$, $P=310\text{MPa}$, $H=2\text{mm}$, $m_a=9.1\text{g/s}$; (d) $\theta=4^\circ$, $P=310\text{MPa}$, $u=1\text{mm/s}$, $m_a=9.1\text{g/s}$; (e) $\theta=4^\circ$, $P=310\text{MPa}$, $u=1\text{mm/s}$, $H=3\text{mm}$. 
Figure 3.15. The effect of oscillation angle on kerf taper: (a) $P=310\text{MPa}$, $u=1\text{mm/s}$, $H=2\text{mm}$, $m_a=9.1\text{g/s}$; (b) $F=6\text{Hz}$, $u=1\text{mm/s}$, $H=2\text{mm}$, $m_a=9.1\text{g/s}$; (c) $F=6\text{Hz}$, $P=310\text{MPa}$, $H=2\text{mm}$, $m_a=9.1\text{g/s}$; (d) $F=6\text{Hz}$, $P=310\text{MPa}$, $u=1\text{mm/s}$, $m_a=9.1\text{g/s}$; (e) $F=6\text{Hz}$, $P=310\text{MPa}$, $u=1\text{mm/s}$, $H=2\text{mm}$.
Oscillation angle has a similar effect on kerf taper with oscillation frequency, although the rate of the decrease in kerf taper is not so fast with the oscillation angle as can be seen from the comparison of Figure 3.14 with Figure 3.15. Thus, it can be deduced that using large oscillation strength is advantageous to reduce kerf taper noticing that oscillation frequency and oscillation angle affect kerf taper with the similar trend.

3.3.5.3 The effect of other process variables

![Graphs showing the effect of different process variables on kerf taper.](image)

Figure 3.16. The effect of other process variables on kerf taper: (a) $\theta=4^\circ$, $F=6$Hz, $u=1$mm/s, $H=2$mm; (b) $\theta=4^\circ$, $F=6$Hz, $H=3$mm, $m_a=9.1$g/s; (c) $\theta=4^\circ$, $F=6$Hz, $P=345$MPa, $m_a=9.1$g/s; (d) $\theta=4^\circ$, $F=6$Hz, $P=345$MPa, $H=3$mm.

Figure 3.16 presents the effects of water pressure, nozzle traverse speed, standoff distance and abrasive mass flow rate on kerf taper. It can be seen from the figure that kerf taper linearly decrease with water pressure, proportionally increase with nozzle traverse speed, increase with standoff distance and decrease with abrasive flow rate.
traverse speed and not apparently affected by abrasive mass flow rate, which has been demonstrated to be in line with the previous study in normal cutting [46, 114, 121, 131].

Figure 3.16 (c) suggests that kerf taper initially shows somewhat increase with the standoff distance but a slight decrease for large standoff distance when nozzle traverse speed is relatively high. This means that small standoff distance does not necessarily produce a minimum taper angle, which manifests different trend with the result obtained in normal cutting [24, 121]. The reason may be that at first stage with the increase in standoff distance the lower part of kerf is not machined effectively due to the decrease of jet energy. At the same time, the scanning scope broadens with the standoff distance which is beneficial to the rise in the depth of jet penetration. The influence of the latter may play a predominant role at a high standoff distance. However, the use of low nozzle traverse speed consistently decreases kerf taper with an increase in standoff distance owing to the increased exposure time of jet on the target material and reduced particle interference, which may significantly increase the depth of cut.

3.3.5.4 Overall considerations on kerf taper

In summary, the investigation of kerf taper angle machined by AWJ oscillation cutting using statistical and graphical studies has indicated that oscillation cutting could noticeably reduce kerf taper angle as compared to normal cutting. This experiment achieved a decrease of 18.1% in taper angle by using oscillation cutting compared to the corresponding normal cutting.

Statistical analysis has found that oscillation frequency, water pressure, nozzle traverse speed, abrasive mass flow rate and oscillation angle are primary process variables and affect kerf taper in the listed order, while standoff distance accounts for less effect on kerf taper. In terms of the best combinations to achieve minimum kerf taper, the result of ANOVA shows that when using their respective optimum cutting conditions, oscillation cutting could reduce kerf taper by 54% compared to normal cutting.

The graphical study of the effect of the process variables reveals that kerf taper presents a decreased trend with an increase in oscillation frequency, oscillation angle
and water pressure, but with a decrease in nozzle traverse speed. Standoff distance first increases and then decreases kerf taper throughout the test range. Kerf taper is independent of abrasive mass flow rate. If minimizing kerf taper is the only goal in practice, high oscillation frequency, large oscillation angle, high water pressure, short standoff distance and corresponding small value of nozzle traverse speed are necessary choice together with the use of small abrasive mass flow rate from the point of view of reducing cutting cost.

3.3.6 Top kerf width and bottom kerf width

3.3.6.1 Comprehensive analysis

Top kerf width and bottom kerf width are often assessed in practice with the objective of achieving minimum widths to control kerf taper. The top kerf width and bottom kerf width measurements obtained from machined specimens in both oscillation cutting and normal cutting disclose that the top width value and the bottom width value machined by oscillation cutting group and normal cutting group have no noteworthy difference. To substantiate this observation, a statistical analysis has been conducted to compare the respective top kerf width values grouped by oscillation cutting and normal cutting using their corresponding cutting condition. Then, the same analytical technique is applied with regard to the bottom kerf width.

It has been found that in this study oscillation cutting only increased the top kerf width and bottom kerf width by 3.76% and 2.27% respectively compared to normal cutting, which can be reasonably believed that oscillation cutting as a whole has no significant influence on both top kerf width and bottom kerf width. A further observation confirms that generally, large top kerf width was produced using high oscillation frequency and long standoff distance. In the case of bottom kerf width, the wide width is found to be produced under the impingement with the low oscillation frequency and small nozzle traverse speed.

According to the result of ANOVA performed on top kerf width (Table 3.6), top kerf width is primarily controlled by standoff distance, oscillation frequency, nozzle traverse speed and abrasive mass flow rate as indicated by their significance values of less than 0.05. Water pressure and oscillation angle are considered to have little effect on the top kerf width. Of these having primary effects, standoff distance is the
most influential variable, while oscillation frequency, nozzle traverse speed and abrasive mass flow rate assume to account for the second and the third roles based on their F test values. Table 3.6 also suggests that to generate minimum top kerf width of 1.172mm in oscillation cutting, the process variables should be combined as standoff distance at 2mm, oscillation frequency at 2Hz, nozzle traverse speed at 1.67mm/s, abrasive mass flow rate at 6.8g/s, water pressure at 380MPa and oscillation angle at 8°. Accordingly, if optimum cutting condition for normal cutting is employed, i.e. water pressure of 345MPa, nozzle traverse speed of 1.33mm/s, standoff distance of 2mm and abrasive mass flow rate of 9.1g/s, the narrowest top kerf width could be 1.108mm. Comparatively, the use of oscillation cutting only changes the top kerf width by 5.7% if both cutting modes use their best cutting conditions.

Table 3.6. ANOVA of top kerf width.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean square</th>
<th>F test</th>
<th>Sig.</th>
<th>Oscillation</th>
<th>Opti. cutting cond.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oscillation frequency (Hz)</td>
<td>4.537</td>
<td>10.210</td>
<td>0.000</td>
<td>2</td>
<td>Normal</td>
</tr>
<tr>
<td>Oscillation angle (degree)</td>
<td>0.507</td>
<td>1.141</td>
<td>0.343</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Water pressure (MPa)</td>
<td>0.938</td>
<td>2.111</td>
<td>0.112</td>
<td>380</td>
<td>345</td>
</tr>
<tr>
<td>Nozzle traverse speed (mm/s)</td>
<td>3.899</td>
<td>8.775</td>
<td>0.000</td>
<td>1.67</td>
<td>1.33</td>
</tr>
<tr>
<td>Standoff distance (mm)</td>
<td>8.899</td>
<td>20.001</td>
<td>0.000</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Abrasive flow rate (g/s)</td>
<td>1.367</td>
<td>3.076</td>
<td>0.037</td>
<td>6.8</td>
<td>9.1</td>
</tr>
<tr>
<td>Minimum top width (mm)</td>
<td></td>
<td></td>
<td></td>
<td>1.172</td>
<td>1.108</td>
</tr>
</tbody>
</table>

The results from ANOVA conducted on bottom kerf width are presented in Table 3.7. An analysis of process variable influence on bottom kerf width using mean square, F test and significance value divulges that oscillation frequency and nozzle traverse speed primarily affect the formation of bottom kerf width. Other four variables are considered to have little effect on bottom kerf width as their significance values are great than 0.05, of which standoff distance is the least influential. Under their respective optimum combination of process variables
presented in Table 3.7, the minimum bottom kerf widths are 0.407mm for oscillation cutting and 0.372mm for normal cutting, which only has a difference of 8.6%.

Table 3.7. ANOVA of bottom kerf width.

<table>
<thead>
<tr>
<th></th>
<th>Mean square</th>
<th>F test</th>
<th>Sig.</th>
<th>Opti. cutting cond.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oscillation frequency (Hz)</td>
<td>2.972</td>
<td>10.638</td>
<td>0.000</td>
<td>Oscillation</td>
</tr>
<tr>
<td>Oscillation angle (degree)</td>
<td>0.386</td>
<td>1.382</td>
<td>0.260</td>
<td>Normal</td>
</tr>
<tr>
<td>Water pressure (MPa)</td>
<td>0.222</td>
<td>0.797</td>
<td>0.502</td>
<td></td>
</tr>
<tr>
<td>Nozzle traverse speed (mm/s)</td>
<td>0.807</td>
<td>2.889</td>
<td>0.046</td>
<td>1.67</td>
</tr>
<tr>
<td>Standoff distance (mm)</td>
<td>0.070</td>
<td>0.252</td>
<td>0.860</td>
<td>3</td>
</tr>
<tr>
<td>Abrasive flow rate (g/s)</td>
<td>0.745</td>
<td>0.668</td>
<td>0.059</td>
<td>6.8</td>
</tr>
<tr>
<td>Minimum bottom width (mm)</td>
<td>0.407</td>
<td></td>
<td></td>
<td>0.372</td>
</tr>
</tbody>
</table>

As the use of oscillation cutting has no significant effect on top kerf width and bottom kerf width, a deep analysis of the influence of process variables on kerf width is neither important nor necessary. The following section involves brief discussions from the point of view of qualitative analysis to substantiate the findings in the quantitative analysis.

3.3.6.2 The effect of oscillation frequency and oscillation angle

The typical effect of oscillation frequency and oscillation angle on top kerf width and bottom kerf width is plotted in Figure 3.17. It can be seen from Figures 3.17 (a) and (b) that no noticeable trend is observed on the effect of oscillation frequency and oscillation angle on top kerf width. Figures 3.17 (c) and (d) indicate that although a marginal decrease is observed in bottom kerf width with an increase in oscillation frequency or oscillation angle, the degree of their influence is still considered to be negligible. This observation is applied for other cutting conditions when varying other process parameters.
3.3.6.3 Overall considerations on top kerf width and bottom kerf width

The top kerf width and bottom kerf width machined by using oscillation technique have been statistically analysed and also qualitatively discussed in brief in this section. Statistical analysis has indicated that no significant difference can be found in top kerf width by using oscillation cutting and normal cutting. The same result is observed for bottom kerf width. Thus, it can be inferred that AWJ cutting by overlapping oscillation operation on normal cutting has no remarkable effect on both top kerf width and bottom kerf width.

When using AWJ oscillation cutting, the primary variables to affect top kerf width are standoff distance, oscillation frequency, nozzle traverse speed and abrasive mass.
flow rate based on the ANOVA of the top kerf width, while the other two variables have no significant effect. As for the bottom kerf width, oscillation frequency and nozzle traverse speed are the variables to primarily affect it. The analysis of optimum cutting conditions using ANOVA suggests that the use of oscillation cutting only increases the top kerf width and the bottom kerf width by 5.7% and 8.6% respectively if both cutting modes use their best cutting combinations. This again confirms the small effect of oscillation cutting on kerf widths.

Qualitative analysis has shown that no apparent trends are observed for the effect of oscillation parameters (oscillation frequency and oscillation angle) on top kerf width and bottom kerf width.

### 3.4 Conclusions

A detailed and extensive experimental investigation has been planned and carried out in order to study the characteristics of major cutting performance measures in AWJ machining of 87% alumina ceramics with controlled nozzle oscillation under small oscillation angles. This involves running a comprehensive set of tests covering a wide range of process variables including oscillation frequency, oscillation angle, water pressure, nozzle traverse speed, abrasive mass flow rate and standoff distance. The Taguchi experimental design technique using orthogonal arrays was used to construct the experimental scheme. In planning this experiment, AWJ normal cutting was also included using the corresponding cutting conditions with those in oscillation cutting for the purpose of comparatively studying the performance characteristics under the two different cutting modes. Both statistical analysis and graphical study were carried out to investigate the kerf characteristics and assess the effects of process variables on major cutting performance measures in terms of kerf surface roughness, depth of cut, smooth depth of cut, kerf taper, top kerf width and bottom kerf width. Based on the study, the following conclusions are made:

A visualization study made to the two types of kerfs, through cut and non-through cut, in AWJ oscillation cutting has shown that the kerfs produced in oscillation cutting have the similar characteristics with those in normal cutting, i.e. presence of cutting wear zone and deformation wear zone as well as jet upward deflection zone if non-through cut is formed. The inspection of kerf surface indicates that the kerfs
possess upper smooth zone assessed by surface roughness and lower rough zone characterised by striations, which is much like those in normal cutting. Under the corresponding cutting conditions with normal cutting, the higher depth of jet penetration and smooth zone are observed when oscillation cutting is used.

The effect of process variables on performance characteristics has been extensively investigated by statistical and qualitative studies. ANOVA was performed to statistically distinguish the importance of each process variable. The best cutting conditions to achieve the optimum cutting performance were identified and the corresponding optimum cutting performance measures were determined. Qualitative study using graphical analysis has also been carried out to analyse the trends of major cutting performance measures with regard to the process variables.

The trends of major cutting performance measures were affected by the process variables in the different ways. It has been found that oscillation frequency is the most important in affecting the overall cutting performance. Of the six discussed performance measures, oscillation frequency applied the most effect on the depth of cut, smooth depth of cut, kerf taper and bottom kerf width. Oscillation frequency was also the second important variable among the six process variables in controlling kerf surface roughness and top kerf width. An increase in the oscillation frequency increased depth of cut, smooth depth of cut and kerf surface roughness, and reduced kerf taper. Top and bottom kerf width were not significantly affected by oscillation frequency. Therefore, selection of large oscillation frequency in practice can effectively increase overall cutting performance in oscillation cutting.

The effect of oscillation angle on most of the considered cutting performance measures is not significant due to the narrow range of oscillation angles conducted in this experimental study. Except that it applies the third role on bottom kerf width, oscillation angle has the least influence on depth of cut, smooth depth of cut and top kerf width and accounts for the second least effect on surface roughness and kerf taper. An increase in oscillation angle results in the decrease in kerf taper. Smooth depth of cut firstly decreases and then increases with oscillation angle, while the reverse trend is observed on the effect of surface roughness and depth of cut. As oscillation angle is not so significant in affecting cutting performance under oscillation cutting with small oscillation angles, a sensible strategy for the selection
of oscillation angle is to use small values, which can accommodate both AWJ straight and contouring cutting modes and equally improve the overall cutting performance.

The effects of other process variables, i.e. water pressure, nozzle traverse speed, standoff distance and abrasive mass flow rate, on cutting performance were extensively analysed and discussed. Among these variables, water pressure, nozzle traverse speed and abrasive mass flow rate affect the most of the cutting performance in a similar way to that in normal cutting. Water pressure has the most effect on surface roughness. In addition, it is also a second important variable to affect kerf taper and the third important variable to affect depth of cut and smooth depth of cut. When using high water pressure in oscillation cutting, surface roughness, depth of cut and smooth depth of cut all increase but kerf taper decreases. Thus, high water pressure should be selected after the surface roughness requirement is considered.

Nozzle traverse speed was found to be the second most influential variable on depth of cut, smooth depth of cut and bottom kerf width, the third most important variable on kerf taper and top kerf width and the fourth variable to have profound effect on kerf surface roughness. Fast nozzle traverse speed increases kerf surface roughness and kerf taper but reduces depth of cut and smooth depth of cut. Apparently small nozzle traverse speed is beneficial to obtain overall improved cutting performance in oscillation cutting. However, small nozzle traverse speed is associated with low cutting efficiency and accordingly high cutting cost. Thus a compromise must be made in the selection of nozzle traverse speed to consider cutting economics.

Abrasive mass flow rate takes the fourth place in affecting depth of cut, smooth depth of cut, kerf taper and top kerf width. It has less effect on bottom kerf width and the least effect on surface roughness. Depth of cut increases with abrasive mass flow rate but surface roughness, smooth depth of cut and kerf taper have not much dependence on abrasive mass flow rate. Consequently, large abrasive flow rate is recommended in oscillation cutting to achieve overall better cutting performance.

The variation of standoff distance has a different effect on cutting performance with that in normal cutting. In oscillation cutting, an increase in standoff distance reduces the particle energy at the attacking point of target material, as is the case in normal
cutting. However, it takes an effect together with the oscillation parameters in oscillation cutting. With an increase in standoff distance, the jet scanning scope on the cutting front increases which may compensate for the decrease in particle energy. This combined action may have a different effect on cutting performance with that in normal cutting. It was found that standoff distance is the most influential variable on top kerf width but has the least effect on kerf taper and bottom kerf width. It applies the third role to affect surface roughness and has no much effect on depth of cut and smooth depth of cut. When increasing standoff distance, smooth depth of cut shows a trend of first decrease and then increase. A reverse trend is observed for surface roughness and kerf taper. But there is no noticeable relationship between depth of cut and standoff distance. Therefore, using small standoff distance should be considered to obtain an overall improved cutting performance.

It has been found from the forgoing analysis on the cutting performance that the use of oscillation technique in AWJ cutting does not always improve cutting performance, as are the cases of surface roughness and the depth of cut in this experimental study. Surface roughness in oscillation cutting is closely related to the process variables used. When high water pressures and high nozzle traverse speeds are used, surface roughness worsens rather than improves compared to corresponding normal cutting regardless of the oscillation parameters used. Likewise, there exists a threshold value of the oscillation strength (the product of oscillation frequency and oscillation angle) for the increase of depth of cut in oscillation cutting, which is different from the usual expectation that the use of oscillation technique can constantly increase this cutting quantity. Below this threshold oscillation strength, nozzle oscillation will decrease the depth of cut as compared to normal cutting.

The cutting instances for surface roughness and the depth of cut demonstrate that correct selection of oscillation parameters and combination of process variables are crucial to achieve improved cutting performance. If the cutting parameters are not correctly selected nozzle oscillation cutting can cause negative effect on the major cutting performance measures. Thus, for a job requirement of good surface finish, slow oscillation frequency, low water pressure, short standoff distance, small oscillation angle and abrasive mass flow rate are recommended together with a proper low nozzle traverse speed. If high depth of cut is required, it is suggested to
use fast oscillation frequency, medium oscillation angle, high water pressure, large abrasive mass flow rate and relatively low nozzle traverse speed. In order to maximize smooth depth of cut, the sensible choice of the cutting parameters is large oscillation frequency, small oscillation angle, high water pressure, reasonably low nozzle traverse speed, short standoff distance, combined with small abrasive mass flow rate. If minimizing kerf taper is a major concern for the job requirements, a combination of fast oscillation frequency, large oscillation angle, high water pressure, short standoff distance, small abrasive mass flow rate should be selected coupled with a corresponding slow nozzle traverse speed.

Thus, in comparison with oscillation cutting at large angles which was reported in previous findings [2, 51, 120, 145], oscillation cutting at small angles can still greatly improve cutting performance. Under the correct selection and combination of process variables, this technique can significantly increase the depth of cut and smooth depth of cut, noticeably reduce kerf taper angle and effectively improve kerf surface roughness. Consequently, nozzle oscillation technique is a viable technique for increasing the cutting performance in AWJ cutting in both straight and contouring modes if the motion control mechanism can manipulate the cutting head in the desired way.

The comprehensive analysis of cutting performance in AWJ oscillation cutting and extensive discussions for correct selection of process variables in this chapter have formed the basis for suggesting optimum process conditions for AWJ cutting of alumina ceramics. However, in order to develop a strategy to optimize the cutting conditions in process planning in AWJ oscillation cutting, predictive mathematical models for major cutting performance measure are required and those are developed in chapter four.
Chapter 4

Modelling the Cutting Performance in AWJ Machining with Controlled Nozzle Oscillation

4.1 Introduction

The investigation into the major cutting performance of 87% alumina ceramics machined by AWJ cutting with controlled nozzle oscillation in chapter 3 has revealed that whether or not oscillation cutting improves the cutting performance is greatly dependent on the selection and combination of cutting variables in the process. When the selection of cutting variables is not proper, oscillation cutting will worsen the cutting performance rather than improve it. Therefore, the prediction of the major cutting performance measures in AWJ cutting with nozzle oscillation by means of the predictive mathematical models is a fundamental step towards the selection of the optimum process parameters and the control of cutting process.

It was found from the literature review that some mathematical models to express the depth of cut, kerf surface roughness and kerf geometrical features for AWJ cutting without nozzle oscillation have been developed. Those models used a variety of approaches. Taking the depth of cut models developed as an instance, most of the models were developed based on complex micro-cutting mechanisms [71, 72], by considering cutting wear and deformation wear theories [19, 21, 75], using an energy conservation approach [2, 9, 51, 52, 93] or with the aid of fracture mechanics approach [43, 60, 70]. Almost all the mathematical models can essentially be classified into either pure theoretical models or empirical models. While theoretical models usually can give an accurate prediction for the cutting performance but contain many unknown variables and thus are too complex for practical use, empirical models are generally based on the statistical analysis of experimental data and limited for their use.

In this chapter, the mathematical models to predict the major cutting performance measures in AWJ oscillation cutting, i.e. the depth of cut, smooth depth of cut, surface roughness, kerf taper and kerf width, are developed. While some of the
models are resorted to the combination of theoretical derivation and empirical study and thus semi-empirical models are developed, others are simply derived by using empirical analysis of the experimental data due to the insufficient understanding of the mechanisms in generating these performance quantities. The models are assessed by the analysis of predicted results from the models and the corresponding experimental data.

4.2 Predictive depth of jet penetration models

In this section, a mathematical model for depth of cut (or jet penetration) in AWJ cutting without nozzle oscillation is first developed with the parameters covering water pressure, nozzle traverse speed, standoff distance and abrasive mass flow rate. Then, by incorporating oscillation parameters (oscillation angle and oscillation frequency), the model for depth of cut in oscillation cutting is developed.

It is noted from the analysis of cutting process that the development of the depth of cut models for AWJ cutting with and without nozzle oscillation involves the consideration of a host of operating variables. The complex nature of the process makes the modelling process extremely difficult. In addition, there are a number of phenomena associated with AWJ cutting, such as particle interference and fragmentation, that have not yet been well understood and there are no mathematical models to represent these phenomena [93]. As a result, to consider all these variables and theoretically model the depth of jet penetration are either not possible at this stage of development or result in very complicated equations with many unknown parameters, making the model unrealistic for practical use. A good solution to cope with such engineering problems is to resort to the combination of theoretical derivation and empirical study, which leads to a semi-empirical mathematical model.

In this regard, dimensional analysis [159, 160] is a powerful analytical technique in describing the relationship between physical engineering quantities (such as the depth of jet penetration) and independent variables.

4.2.1 Modelling conditions and assumptions

In this study, a dimensional analysis technique is used to establish an operative mathematical equation to describe the depth of cut in terms of the process variables,
while regression analysis of the experimental data is undertaken to determine the empirical constants in the model.

With dimensional analysis, all variables appearing in a problem can be assembled into a smaller number of independent dimensionless products or Pi (\(\pi_i\)) groups. The dimensional homogeneity requires that all terms in a mathematical relationship must have the same dimensions regardless of the choice of units for every variable. This is to ensure that we are dealing with the right dimensions in any given problem. For this purpose, the Pi theorem [159-161] can be used to find the proper dimensionless products. The Pi theorem states that the number of \(\pi_i\) groups needed to correlate the variables is equal to \(n-m\), where \(n\) is the number of variables involved in the problem and \(m\) is the number of selected fundamental dimensions. The relationships connecting the individual variables can be determined by algebraic expressions relating each \(\pi_i\) group, thus reducing the total number of variables. Dimensional analysis is very effective in engineering to make prediction for performance parameter and quantitatively describe the relationship between performance parameter and independent variables as well as the effect trends of these variables.

To simplify the model development process, some assumptions need to be made:

- Abrasive particles are distributed uniformly over any jet cross-sectional area.
- The velocity of an abrasive particle is the same as that of its surrounding water in the jet, and the jet velocity variation along the jet flow direction is ignored.
- Jet side spreading is ignored. Thus, kerf width \((w)\) is considered to be approximately equal to effective jet diameter \((d_j)\).

The underlying principle in the construction of the predictive model for the depth of cut is that the overall material removal rate is equal to the accumulated volume of material removed by individual abrasive particles in the given time span [138]. If assuming that the depth of cut is \(h\), the nozzle traverse speed is \(u\) and the average kerf width is \(w\), the overall material removal rate \((MRR)\) can be expressed as

\[
MRR = uhw
\]  
(4.1)
If the abrasive mass flow rate is \( m_a \) and the average mass of an individual particle is \( m \), then the number of abrasive particles in the jet per unit time is \( \frac{m_a}{m} \). The total accumulated volume of material removed by the abrasive particles can be represented as \( Rm_e/m \), where \( R \) is the average material removed by an individual particle contributing to the material removal process. In abrasive waterjet cutting, not all particles in the jet will impinge the material or have sufficient energy to cut the target material. Some particles may collide with other particles and are not involved in the cutting action. To consider this phenomenon, an efficiency factor, \( K_e \), may be introduced so that the following volumetric relation can be drawn to relate the overall material removal rate to the accumulated volume of material removed by individual particles

\[
MRR = uhw = K_e \frac{m_a}{m} R
\]  

(4.2)

By ignoring jet side spreading, it may be assumed that the average kerf width is equal to the jet diameter \( (d_j) \) which in turn is approximated by the nozzle diameter, i.e. \( w = d_n \). Hence

\[
MRR = uh d_n = K_e \frac{m_a}{m} R
\]  

(4.3)

In Equation (4.3), \( R \) is a parameter yet to be determined and related to its influencing variables in order to derive the formula of depth of cut. Therefore, after correlating the overall material removal with the accumulated volume of material removed by individual abrasive particles, the next step would be to derive the expression of the material removed by an individual particle in terms of its influencing variables. In doing so, a dimensional analysis technique is used as follows.

### 4.2.2 Material removal by individual particles

As pointed out earlier, there are a large number of variables that affect the material removal process in AWJ cutting. Based on Finnie’s models [54, 67] of material removal by a single particle for low attack angles (the angle between the material surface and particle flow direction at the point of attack), four parameters are dominant in controlling the material removal process. These dominant parameters are
related to the properties of work material, the mass of an individual abrasive particle, the impact velocity of the particle and the attack angle at the point of impact. For brittle materials, the material flow stress is used to account for the effect of work material properties. Therefore, the material removed by a single particle is dependent on the particle velocity \(v\), particle attack angle \(\alpha\), the flow stress of the work material \(\sigma_f\), and the average mass of an individual particle \(m\). Mathematically, the relationship between the material removed by a single particle and the above variables can be written in the form

\[ R = f(\sigma_f, v, m, \alpha) \]  

(4.4)

where the mass of an individual particle is taken from the average value based on the average particle size (assuming in spherical shape) and particle material density.

To determine the mathematical expression of Equation (4.4), dimensional analysis technique is used. According to the Pi theorem of dimensional analysis, fundamental dimensions involved in the problem should first be identified in order to determine the number of repeating variables [162, 163] which appear in each Pi group. For this purpose, the dimensions of all the five variables in Equation (4.4) are analysed and listed in Table 4.1.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R)</td>
<td>Material removal by an individual particle</td>
<td>(L^3)</td>
</tr>
<tr>
<td>(\sigma_f)</td>
<td>Flow stress of target material</td>
<td>(MT^{-2}L^{-1})</td>
</tr>
<tr>
<td>(v)</td>
<td>Particle velocity</td>
<td>(LT^{-1})</td>
</tr>
<tr>
<td>(m)</td>
<td>Average mass of an individual particle</td>
<td>(M)</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Average particle attack angle</td>
<td>1 (dimensionless)</td>
</tr>
</tbody>
</table>

From Table 4.1, the set of variables in Equation (4.4) is expressed in terms of three fundamental dimensions, i.e. length L, mass M, and time T, which means that three
repeating variables must be chosen to construct this model. Based on the common rules in the selection of repeating variable \([159, 164, 165]\), \(\sigma\), \(v\) and \(m\) can represent all the three fundamental dimensions and are thus selected as the repeating variables.

The Pi theorem states that the number of independent dimensionless products (Pi group) is equal to the number of variables in the equation minus the number of repeating variables. Therefore, two independent dimensionless products can be formed by the five governing variables in Equation (4.4). Noting that \(\alpha\) is already a dimensionless variable, two independent dimensionless products can be formed from Equation (4.4), with detailed derivations given in Appendix A. Thus, these two dimensional products are deduced as

\[
\pi_1 = \frac{R\sigma_f}{mv^2} \tag{4.5}
\]

\[
\pi_2 = \alpha \tag{4.6}
\]

Based on the dimensional analysis technique, the functional relation between these two dimensionless products in Equations (4.5) and (4.6) is:

\[
\pi_1 = f(\pi_2) \tag{4.7}
\]

or

\[
\frac{R\sigma_f}{mv^2} = f(\alpha) \tag{4.8}
\]

At this stage, \(f()\) is an unknown function to be determined from other mathematical processing. It is noted that a non-dimensional quantity is proportional to the product of other dimensionless groups raised to rational power [166]. Because of the simplicity and wide use of this power law formulation, it is applied in this study. Hence, the complete dimensional equation is given as follows:

\[
\frac{R\sigma_f}{mv^2} = A\alpha^j \tag{4.9}
\]

Equation (4.9) can be rewritten as
\[ R = \frac{Amv^2}{\sigma_f} \alpha^j \]  

(4.10)

where \( A \) is a constant and \( j \) represents the exponent of dimensional group. In determining the particle attack angle \( \alpha \), the cutting processes with and without nozzle oscillation are considered separately.

### 4.2.3 AWJ cutting without nozzle oscillation

It has been reported that the particle attack angle is a variable along the cutting front [47, 63, 72, 138]. This variable depends on the curvature or orientation of surface being impacted and the moving direction of the particles at the impact site. From the study of kerf formation process (or macro mechanism of AWJ cutting) [19, 60, 72], an AWJ forms a complete kerf in a step formation process and the surface curvature of the cutting front changes as the jet cuts into the work material. Likewise, the particle moving direction changes as the particle flows away from the nozzle exit. As a result, it is extremely difficult to model the attack angle of each individual particle. To simplify this analysis, the average particle attack angle from the kerf top to bottom is used and mathematically modelled by using a dimensional analysis technique.

After analysing the effects of various major variables, it was noted that the factors to have a significant influence on both the curvature of the cutting front and the moving direction of the particles are known to have work material, abrasive particle, and waterjet-related natures [19, 60, 72, 167]. The flow stress of the target material, nozzle traverse speed and water pressure greatly affect the slope of the cutting front in that small material flow stress, low traverse speed or high water pressure results in a steep cutting front if other process parameters maintain unchanged during the process. Standoff distance between the nozzle and the work surface has a significant effect on particle moving direction. In addition, different particle size, represented by the average particle diameter, causes a different attack angle at the contact point of the particle and the kerf front. Thus, in this study, we assume that six dimensional variables can be defined to describe the attack angle of an individual particle impact on target material. Consequently, the attack angle can be expressed as a function of
five parameters including nozzle traverse speed \( u \), water pressure \( P \), standoff distance \( H \), average particle diameter \( D \) and the flow stress of target material \( \sigma_f \), i.e.

\[
\alpha_i = \phi(u, P, H, D, \sigma_f)
\]  \hspace{1cm} (4.11)

where \( \alpha_i \) is the average attack angle in AWJ cutting without nozzle oscillation.

Similar to the foregoing dimensional analysis, the six variables in Equation (4.11) can be represented by three fundamental dimensions (length \( L \), mass \( M \), and time \( T \)) as shown in Table 4.2.

Table 4.2. Variables and their dimensions in deriving particle attack angle in normal cutting.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 )</td>
<td>Average particle attack angle in normal cutting</td>
<td>1 (dimensionless)</td>
</tr>
<tr>
<td>( u )</td>
<td>Nozzle traverse speed</td>
<td>( LT^{-1} )</td>
</tr>
<tr>
<td>( P )</td>
<td>Water pressure</td>
<td>( ML^{-1}T^{-2} )</td>
</tr>
<tr>
<td>( H )</td>
<td>Standoff distance</td>
<td>( L )</td>
</tr>
<tr>
<td>( D )</td>
<td>Average particle diameter</td>
<td>( L )</td>
</tr>
<tr>
<td>( \sigma_f )</td>
<td>Flow stress of target material</td>
<td>( ML^{-1}T^{-2} )</td>
</tr>
</tbody>
</table>

It then follows by selecting nozzle travel speed \( u \), standoff distance \( H \), and material flow stress \( \sigma_f \) as repeating variables, so that three Pi groups can be formed from the six variables; namely (The derivation process is shown in Appendix B).

\[
\pi_1 = \alpha_1
\]  \hspace{1cm} (4.12)

\[
\pi_2 = \frac{D}{H}
\]  \hspace{1cm} (4.13)

\[
\pi_3 = \frac{P}{\sigma_f}
\]  \hspace{1cm} (4.14)
Those groups are related by the function of

\[ \pi_1 = \phi(\pi_2, \pi_3) \]  

which leads to the relationship

\[ \alpha_i = \phi\left(\frac{D}{H}, \frac{P}{\sigma_f}\right) \]  

By applying the power law function method [166], the following functional relation is obtained

\[ \alpha_i = B_1 \left(\frac{D}{H}\right)^{x_1} \left(\frac{P}{\sigma_f}\right)^{y_1} \]  

where \( B_1 \) is a dimensionless constant and the exponents of dimensionless products are written as \( x_1 \) and \( y_1 \), respectively. By replacing \( \alpha \) in Equation (4.10) with \( \alpha_i \) from Equation (4.17), the material removal by a particle in AWJ normal cutting is given by

\[ R = \frac{B_2 m v^2}{\sigma_f} \left(\frac{D}{H}\right)^x \left(\frac{P}{\sigma_f}\right)^y \]  

where \( B_2, x \) and \( y \) are used to generalise the constants, such that \( B_2 = A B_1 j, x = x_1 j \), and \( y = y_1 j \).

Substituting Equation (4.18) into Equation (4.3) and replacing the depth of cut \( h \) with \( h_i \) to denote cutting without nozzle oscillation give

\[ h_i = \frac{B_3 m v^2}{\sigma_f d_s u} \left(\frac{D}{H}\right)^x \left(\frac{P}{\sigma_f}\right)^y \]  

where constant \( B_3 \) is the product of \( B_2 \) and \( K_e \).

It is now necessary to determine the particle velocity, \( v \), in the above equation. If assuming that the energy loss in the supply system is negligible and the water is
incompressible, the water velocity in a jet, \(v_j\), before mixing with abrasive particles can be found by using the Bernoulli’s equation, i.e.

\[
v_j = \sqrt{\frac{2P}{\rho_w}} \tag{4.20}
\]

where \(P\) is water pressure, and \(\rho_w\) is water density. If assuming that the particle is entrained by the water to increase its velocity and at the point of particle attacking the material surface, the particle has gained the same velocity as its surrounding water, the particle velocity can be obtained using the momentum transfer equation, i.e.

\[
v = k_m \left( \frac{m_w}{m_w + m_a} \right) v_j \tag{4.21}
\]

where \(m_w\) is the water mass flow rate, \(m_a\) is abrasive mass flow rate, and \(k_m\) is a factor to consider the momentum transfer efficiency.

To work out the mass ratio term in Equation (4.21) will make the model complicated. Therefore, to simplify the derivation, the mass ratio term is approximated as a constant, \(B_4\). For the process conditions used in the experiments of this study, this approximation only results in a less than 2.5% error for the mass ratio and even smaller error for the final depth of cut. Thus, Equation (4.21) can be re-written as

\[
v = k_m B_4 v_j \tag{4.22}
\]

Substituting Equations (4.22) and (4.20) into Equation (4.19) yields the final equation of the depth of cut for AWJ cutting without nozzle oscillation, i.e.

\[
h_1 = \frac{K_1 m_a P}{u \sigma_f \rho_w d_n} \left( \frac{D}{H} \right)^x \left( \frac{P}{\sigma_f} \right)^y \tag{4.23}
\]

where \(K_1=2B_3B_4\sigma_f^2K_m^2\), is a constant generalizing all the constants and can be determined by experiment.
4.2.4 AWJ cutting with nozzle oscillation

In deriving the mathematical expression of the particle attack angle in AWJ cutting with nozzle oscillation, it is noticed that the incorporation of nozzle oscillation technique into AWJ normal cutting can dramatically affect the trace profile on the cutting front. Previous studies have found the underlying theme of why nozzle oscillation cutting can increase the depth of cut [11, 92]. It shows that the successive traces of particles on the cut surface with nozzle oscillation are steeper than those without nozzle oscillation. The changed particle traces in fact affect the particle attack angle. In addition, with the use of nozzle oscillation cutting technique the standoff distance has changed its role in affecting the cutting process. Specifically, the jet scanning or oscillating scope in the cutting front is not only related to the oscillation angle, but also to the standoff distance. With a larger standoff distance, the jet scanning scope on the cutting front is increased. Consequently, in constructing the particle attack angle in AWJ oscillation cutting, the nozzle oscillation parameters, oscillation angle $\theta$ and frequency $F$, must be taken into account in addition to the five parameters considered in the cutting without nozzle oscillation.

Thus, the attack angle for oscillation cutting can be expressed as a function of the seven variables

$$\alpha_2 = \psi(\theta, F, u, P, H, D, \sigma_f)$$

(4.24)

where $\alpha_2$ is used to denote the attack angle in oscillation cutting. Using the same technique in modelling the attack angle for cutting without oscillation, all these eight parameters in Equation (4.24) are quantities with regard to three fundamental dimensions: L, M, and T (Table 4.3).

By identifying $u$, $H$ and $\sigma_f$ as repeating variables and noting that $\alpha$ and $\theta$ are already dimensionless variables, five independent dimensionless groups can be formed from the eight variables in Equation (4.24) (The equation is derived in Appendix C).

$$\pi_1 = \alpha_2$$

(4.25)

$$\pi_2 = \theta$$

(4.26)
\[ \pi_3 = \frac{FH}{u} \]  

(4.27)

\[ \pi_4 = \frac{D}{H} \]  

(4.28)

\[ \pi_5 = \frac{P}{\sigma_f} \]  

(4.29)

Table 4.3. Variables and their dimensions in deriving particle attack angle in oscillation cutting.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_2 )</td>
<td>Average particle attack angle in oscillation cutting</td>
<td>1 (dimensionless)</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Oscillation angle</td>
<td>1 (dimensionless)</td>
</tr>
<tr>
<td>( F )</td>
<td>Oscillation frequency</td>
<td>( T^{-1} )</td>
</tr>
<tr>
<td>( u )</td>
<td>Nozzle traverse speed</td>
<td>( LT^{-1} )</td>
</tr>
<tr>
<td>( P )</td>
<td>Water pressure</td>
<td>( ML^{-1}T^{-2} )</td>
</tr>
<tr>
<td>( H )</td>
<td>Standoff distance</td>
<td>( L )</td>
</tr>
<tr>
<td>( D )</td>
<td>Average particle diameter</td>
<td>( L )</td>
</tr>
<tr>
<td>( \sigma_f )</td>
<td>Flow stress of target material</td>
<td>( ML^{-1}T^{-2} )</td>
</tr>
</tbody>
</table>

Those groups are related by the function of

\[ \pi_1 = \psi(\alpha_2, \pi_3, \pi_4, \pi_5) \]  

(4.30)

Thus, Equation (4.24) becomes

\[ \alpha_2 = \psi\left(\theta, \frac{FH}{u}, \frac{D}{H}, \frac{P}{\sigma_f}\right) \]  

(4.31)

Once again, by using the power law formulation, \( \alpha_2 \) can be expressed using those four groups as
\[ \alpha_2 = C_1 \theta^n \left( \frac{FH}{u} \right)^b \left( \frac{D}{H} \right)^c \left( \frac{P}{\sigma_f} \right)^d \]  

(4.32)

where \( C_1, a, b, c \) and \( d \) are all empirical constants.

Substituting Equation (4.32) into Equation (4.10) and generalizing the constants and exponents by new constants \( C_2, a, b, c \) and \( d \) give:

\[ R = \frac{C_2mv^2}{\sigma_f} \theta^a \left( \frac{FH}{u} \right)^b \left( \frac{D}{H} \right)^c \left( \frac{P}{\sigma_f} \right)^d \]  

(4.33)

Consequently, by substituting Equation (4.33) into Equation (4.3) and making the necessary transformations give

\[ h_2 = \frac{C_3m_v^2}{\sigma_f d_u u} \theta^a \left( \frac{FH}{u} \right)^b \left( \frac{D}{H} \right)^c \left( \frac{P}{\sigma_f} \right)^d \]  

(4.34)

where \( h_2 \) denotes the depth of cut in AWJ oscillation cutting, while \( C_3=2K \) and is a dimensionless constant.

By substituting Equations (4.22) and (4.20) into Equation (4.34), the final form of depth of cut equation for AWJ oscillation cutting is given by

\[ h_2 = \frac{K_2m_v P}{\sigma_f \rho_u d_u u} \theta^a \left( \frac{FH}{u} \right)^b \left( \frac{D}{H} \right)^c \left( \frac{P}{\sigma_f} \right)^d \]  

(4.35)

where \( K_2=2C_3B^2K_m^2 \), is an empirical constant to be determined by experiment.

### 4.2.5 Model assessment

Two mathematical models of the depth of cut for both AWJ normal cutting and oscillation cutting have been established. These models are in their general forms for brittle materials. However, before the models can be of any use, the constants in the models need to be determined first. For this purpose, a nonlinear regression analysis has been performed on the experimental data obtained in chapter 3. The coefficients
and the exponent estimates in Equations (4.23) and (4.35) are given in Table 4.4, with a confidence interval of 95%.

Table 4.4. Coefficients and exponents of the models estimated by nonlinear regression.

<table>
<thead>
<tr>
<th></th>
<th>AWJ normal cutting</th>
<th></th>
<th>AWJ oscillation cutting</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_1$ $x$ $y$</td>
<td></td>
<td>$K_2$ $a$ $b$ $c$ $d$</td>
</tr>
<tr>
<td>$K_1$</td>
<td>0.261</td>
<td>$K_2$</td>
<td>1.223</td>
</tr>
<tr>
<td>$x$</td>
<td>0.156</td>
<td>$a$</td>
<td>0.229</td>
</tr>
<tr>
<td>$y$</td>
<td>0.186</td>
<td>$b$</td>
<td>0.169</td>
</tr>
<tr>
<td></td>
<td>0.289</td>
<td>$c$</td>
<td>0.289</td>
</tr>
<tr>
<td></td>
<td>0.604</td>
<td>$d$</td>
<td></td>
</tr>
</tbody>
</table>

Substituting these constants into the equations gives

$$ h_1 = \frac{0.261 m_a P}{\sigma_f \rho_w d_n u} \left( \frac{D}{H} \right)^{0.156} \left( \frac{P}{\sigma_f} \right)^{0.186} $$

(4.36)

or

$$ h_1 = 0.261 \frac{m_a P^{1.186} D^{0.156}}{\rho_w d_n u \sigma_f^{1.186} H^{0.156}} $$

(4.37)

and

$$ h_2 = \frac{1.223 m_a P}{\sigma_f \rho_w d_n u} \theta^{0.229} \left( \frac{FH}{u} \right)^{0.169} \left( \frac{D}{H} \right)^{0.289} \left( \frac{P}{\sigma_f} \right)^{0.604} $$

(4.38)

or

$$ h_2 = 1.223 \frac{m_a \theta^{0.229} P^{1.604} F^{0.169} D^{0.289}}{\rho_w d_n u^{1.169} \sigma_f^{1.604} H^{0.12}} $$

(4.39)

Equations (4.36) to (4.39) are valid for cutting an 87% alumina ceramic under the ranges of cutting parameters given in chapter 3. An examination of the equations reveals that the forms of the two models are generally feasible and consistent with the experimental trends of the depth of cut with respect to the major process variables in cutting both with and without nozzle oscillation.
For the depth of cut model without nozzle oscillation, Equation (4.37) realistically represents the effects of water pressure, nozzle traverse speed, abrasive mass flow rate and standoff distance as discussed earlier in the literature review. The positive exponents of water pressure and abrasive mass flow rate in the numerator of Equation (4.37) show that an increase in the water pressure and abrasive mass flow rate results in an increase in the depth of cut. Similarly, the positive exponent values for nozzle traverse speed and standoff distance in the denominator indicate the negative effect of these two process variables on the depth of cut. Those results are in good agreement with the reported literature.

When cutting with oscillation, the Equation (4.39) again correctly predicts the trends of the depth of cut with respect to the process parameters. Similar to Equation (4.37), the presence of water pressure, nozzle traverse speed, standoff distance and abrasive mass flow rate is in their rational forms in the model, which reflects the trends that are consistent with the findings of earlier investigations [1, 92] and the experimental results. In addition, the incorporation of oscillation parameters into the model shows that an increase in the oscillation frequency and oscillation angle generally results in an increase in the depth of cut. While there is a discrepancy between the model prediction and the experimental trend described in chapter 3 in terms of the effect of oscillation angle, the model prediction is generally reasonable.

An analysis also finds that the models correctly represent the effect of other process variables. It has been reported in early investigations [92] that larger particles carry more energy and have the potential to remove more material in a cutting action. The particle diameter has a positive exponent in the numerator of the equations and is therefore considered to be correctly incorporated in the models. Likewise, nozzle diameter and the flow stress of target material are in reverse relation with the depth of cut in the equations. It is apparent that a large nozzle diameter will spread the jet energy over a large jet cross-sectional area so that its penetration ability may be reduced. Similarly, materials with a higher flow stress require higher destructive (threshold) energy for particles to take an effective cut and cause a particle to remove less material than cutting a lower flow stress material [168, 169]. These two variables have a positive exponent in the denominator of the models and correctly represent these effects.
It is interesting to note that the corresponding exponents in the models for cutting with and without nozzle oscillation may bear different values. It is understood that nozzle oscillation not only clears the way for particles to effectively cut the material and reduces interferences between particles through a scanning cutting action, but also changes the particle-work interactions and the erosive process (which may involve different erosive mechanisms). Thus, it is not surprising that different exponential values for the same variable were presented in the two models. Consequently, it is considered that the basic forms of the models for cutting with and without nozzle oscillation are correct.

In order to check the adequacy of the models, a qualitative assessment has been made by comparing the predicted depths of cut from the models with the corresponding experimental data. These are carried out by plotting the trends of predicted depth of cut together with experimentally obtained values with respect to water pressure, nozzle traverse speed, standoff distance, abrasive mass flow rate, oscillation angle and oscillation frequency. Some typical and representative samples of the comparisons are given in Figure 4.1 for normal cutting model and Figure 4.2 for oscillation cutting model.

From Figure 4.1, a highly close correlation is observed between the predicted depths of cut and the corresponding experimental data in AWJ normal cutting in terms of water pressure, nozzle traverse speed, standoff distance and abrasive mass flow rate. Shown in Figure 4.2 are the variations of depth of cut with respect to oscillation frequency, oscillation angle, water pressure, nozzle traverse speed, standoff distance and abrasive mass flow rate. While there is a disagreement between the predicted values and the corresponding experimental data for the effect of oscillation angle as shown in Figure 4.2 (b), the influence of the process variables on the depth of cut generally follows the similar trends as analysed in the previous oscillation experimental study. From figure 4.2 (b), there is a gradual increase in the depths of cut with the oscillation angle from the predictions of the model while the experimental depths of cut present a trend of initially slight increase and then decrease. This difference may be due to the narrow range of oscillation angles selected for this study, which is not enough to have substantial effect on depth of cut and therefore to make accurate prediction. From this graphical study it can be seen
that the predicted trends for depth of cut from the two models follow the similar patterns with and are also generally close to the results as described in the previous experimental study. Thus, qualitative verification indicates that the predictions of the two models are considered in good agreement with the experimental results.

![Graphs showing comparisons between predictions and experimental data for depth of cut without oscillation.](image)

Figure 4.1. Comparisons between predictions and experimental data for depth of cut without oscillation: (a) $u=1\text{mm/s}$, $H=4\text{mm}$, $m_a = 9.1\text{g/s}$; (b) $P=310\text{MPa}$, $H=3\text{mm}$, $m_a = 9.1\text{g/s}$; (c) $P=345\text{MPa}$, $u=1\text{mm/s}$, $m_a = 9.1\text{g/s}$; (d) $P=310\text{MPa}$, $u=1.67\text{mm/s}$, $H=3\text{mm}$. 
Figure 4.2. Comparisons between predictions and experimental data for depth of cut in oscillation cutting: (a) $\theta = 4^\circ$, $P = 275$ MPa, $u = 1$ mm/s, $H = 2$ mm, $m_a = 9.1$ g/s; (b) $F = 6$ Hz, $P = 275$ MPa, $u = 1$ mm/s, $H = 4$ mm, $m_a = 9.1$ g/s; (c) $\theta = 2^\circ$, $F = 6$ Hz, $u = 1$ mm/s, $H = 5$ mm, $m_a = 9.1$ g/s; (d) $\theta = 4^\circ$, $F = 6$ Hz, $P = 275$ MPa, $H = 5$ mm, $m_a = 9.1$ g/s; (e) $\theta = 4^\circ$, $F = 6$ Hz, $P = 275$ MPa, $u = 1$ mm/s, $H = 3$ mm; (f) $\theta = 4^\circ$, $F = 6$ Hz, $P = 275$ MPa, $u = 1$ mm/s, $H = 3$ mm.
A quantitative assessment of the models has been carried out based on the percentage deviation of the model predictions with respect to the experimental results under the corresponding cutting conditions. The histograms and the average percentage deviations evaluated using the two models have been established and shown in Figures 4.3 (a) and (b) for cutting without and with nozzle oscillation respectively. In these figures, the percentage deviation is defined as

\[
Std. \text{ Dev} (\%) = \frac{\text{Predicted depth of cut} - \text{Experimental data}}{\text{Experimental data}} \times 100
\]  

(4.40)

The comparisons show that the model’s prediction for the case of normal cutting yields an average percentage deviation of -2.4% with a standard deviation of 19.99%, while for cutting with nozzle oscillation, the average percentage deviation and the standard deviation are -2.3% and 17.20% respectively. The large standard deviation is attributed to the scatter of the experimental data which were used to determine the constants in the models. Consequently, it may be deduced that the depth of jet penetration models developed can give reasonably good predictions both qualitatively and quantitatively.

Figure 4. 3. Percentage deviations between predicted and experimental depth of cut: (a) Without nozzle oscillation; (b) With nozzle oscillation.
The models have been further assessed by comparing the depth of cut values predicted by the two models under the corresponding cutting conditions in order to estimate their predictive capability. These comparisons can also confirm the finding made in the previous chapter revealing that not all nozzle oscillation cuttings can enhance the cutting capability by increasing depth of cut compared to the corresponding normal cutting. As stated in chapter 3 when discussing the trends of depth of cut with respect to different cutting parameters, there exists a threshold value of oscillation strength (the product of oscillation angle and oscillation frequency) for the increase of depth of cut. Oscillation cutting can increase the depth of cut only when the employed oscillation strength is greater than the required threshold value for the considered cutting condition. The threshold strength appears to be variable depending on other parameters used (i.e. water pressure, nozzle traverse speed, standoff distance and abrasive mass flow rate). Table 4.5 shows the comparisons of depth of cut calculated using Equations (4.37) and (4.39) under the corresponding cutting conditions. It is noticed that when the cutting condition is in a combination of abrasive mass flow rate at 9.1g/s, standoff distance at 3mm, water pressure at 310MPa and nozzle traverse speed at 1.33mm/s, the threshold oscillation strength ($C_{\theta,F}$) is 11.23deg/s. Under this cutting condition, when the used oscillation strength is 4deg/s (oscillation angle and oscillation frequency are 2° and 2Hz, respectively) which is smaller than its threshold value, the resultant depth of cut in oscillation cutting is 6.46mm which is smaller than the corresponding normal cutting (7.69mm). But other three oscillation operations produced higher depths of cut than that in corresponding normal cutting due to the use of the higher oscillation strengths than threshold value. A Similar result has been observed when the threshold oscillation strength is 8.26deg/s under the cutting condition of abrasive mass flow rate at 9.1g/s, standoff distance at 4mm, water pressure at 275MPa and nozzle traverse speed at 1mm/s, where two cuts produced lower depth of cut (7.19mm and 8.43mm) than that in corresponding normal cutting (8.49mm).

Thus, the result of the comparison for the values produced by the two models agrees well with the finding in the last chapter stating that whether or not oscillation cutting increases the depth of cut compared to normal cutting depends on both oscillation angle and oscillation frequency used. More specifically, the combination of oscillation angle and oscillation frequency must satisfy the requirement that their
product be equal to or greater than the required threshold value. This comparison therefore further validates the developed models.

Table 4.5. The comparison of depths of cut predicted by the two models.

<table>
<thead>
<tr>
<th>(m_a)</th>
<th>(H)</th>
<th>(P)</th>
<th>(u)</th>
<th>(\theta)</th>
<th>(F)</th>
<th>(\theta \times F)</th>
<th>(C_{\theta \times F})</th>
<th>(h_1)</th>
<th>(h_2)</th>
<th>Increase (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.1</td>
<td>3</td>
<td>310</td>
<td>1.33</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>11.23</td>
<td>7.69</td>
<td>6.46</td>
<td>-16.01</td>
</tr>
<tr>
<td>9.1</td>
<td>3</td>
<td>310</td>
<td>1.33</td>
<td>2</td>
<td>6</td>
<td>12</td>
<td>11.23</td>
<td>7.69</td>
<td>7.78</td>
<td>1.13</td>
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<tr>
<td>9.1</td>
<td>3</td>
<td>310</td>
<td>1.33</td>
<td>2</td>
<td>10</td>
<td>20</td>
<td>11.23</td>
<td>7.69</td>
<td>8.48</td>
<td>10.24</td>
</tr>
<tr>
<td>9.1</td>
<td>3</td>
<td>310</td>
<td>1.33</td>
<td>2</td>
<td>14</td>
<td>28</td>
<td>11.23</td>
<td>7.69</td>
<td>8.98</td>
<td>16.70</td>
</tr>
<tr>
<td>9.1</td>
<td>4</td>
<td>275</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>8.26</td>
<td>8.49</td>
<td>7.19</td>
<td>-15.29</td>
</tr>
<tr>
<td>9.1</td>
<td>4</td>
<td>275</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>8</td>
<td>8.26</td>
<td>8.49</td>
<td>8.43</td>
<td>-0.72</td>
</tr>
<tr>
<td>9.1</td>
<td>4</td>
<td>275</td>
<td>1</td>
<td>6</td>
<td>2</td>
<td>12</td>
<td>8.26</td>
<td>8.49</td>
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<tr>
<td>9.1</td>
<td>4</td>
<td>275</td>
<td>1</td>
<td>8</td>
<td>2</td>
<td>16</td>
<td>8.26</td>
<td>8.49</td>
<td>9.88</td>
<td>16.35</td>
</tr>
</tbody>
</table>

In conclusion, two predictive mathematical models for the depth of cut in AWJ cutting without and with nozzle oscillation have been developed by using a dimensional analysis technique. The general forms of the models are applicable for brittle materials, such as ceramics and marbles, while the final models with the constants determined by employing nonlinear regression technique based on the experimental data have been developed specifically for cutting an 87% alumina ceramic. The qualitative and quantitative comparisons between the predicted and experimental depths of cut have been made to assess the predictive capability of the models. The comparisons between the predictions by the two models under the corresponding cutting conditions are also carried out to confirm the effectiveness of oscillation cutting. All the numerical analyses and assessments have verified the models and demonstrated the reliability of the two models’ predictions. Consequently, those two models have provided an essential basis for the development of optimization strategies for the effective use of the AWJ cutting technology when the nozzle oscillation technique is used.
4.3 Predictive models for other major cutting performance measures

Because of the lack of full understanding for the mechanisms of some phenomena in AWJ machining with nozzle oscillation, to mathematically express some of the major kerf features from theoretical derivation presents a difficult task as mentioned in the development of the depth of cut models. As such, mathematical models for surface roughness, smooth depth of cut, kerf taper, top kerf width and bottom kerf width are empirically developed in this section by using a regression analysis technique on the experimental data. These models relate the required cutting performance measures to the six process variables in conducting this oscillation cutting experiment, namely oscillation angle, oscillation frequency, water pressure, nozzle traverse speed, standoff distance and abrasive mass flow rate. The empirical models have also been assessed both qualitatively and quantitatively.

4.3.1 The determination of the model forms

In the determination of appropriate model forms to describe the discussed major cutting performance measures with respect to the process variables, linear regression model was first tested with the computer software SPSS package [158, 170]. The other four common used types of regression models including logarithmic, power, exponential and quadratic forms were then tested for each individual performance measure. A comparison of the coefficient of determination ($R^2$) was carried out from the preliminary regression analysis in order to select the best form for these models. The $R^2$ values for the five tested regression models are listed in Table 4.6.

From Table 4.6, it can be seen that the quadratic models can best describe the variations in the performance measures with respect to the process variables as it provides the highest $R^2$ values for all the discussed performance quantities. Consequently, all the five performance measure models are developed based on quadratic model.
Table 4. 6. Coefficient of determination of common used regression models.

<table>
<thead>
<tr>
<th></th>
<th>Linear</th>
<th>Logarith.</th>
<th>Power</th>
<th>Exponent.</th>
<th>Quadratic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface roughness</td>
<td>0.646</td>
<td>0.617</td>
<td>0.639</td>
<td>0.664</td>
<td>0.846</td>
</tr>
<tr>
<td>Smooth depth of cut</td>
<td>0.902</td>
<td>0.903</td>
<td>0.914</td>
<td>0.898</td>
<td>0.963</td>
</tr>
<tr>
<td>Kerf taper</td>
<td>0.793</td>
<td>0.750</td>
<td>0.741</td>
<td>0.790</td>
<td>0.892</td>
</tr>
<tr>
<td>Top width</td>
<td>0.809</td>
<td>0.802</td>
<td>0.798</td>
<td>0.804</td>
<td>0.869</td>
</tr>
<tr>
<td>Bottom width</td>
<td>0.590</td>
<td>0.611</td>
<td>0.601</td>
<td>0.586</td>
<td>0.799</td>
</tr>
</tbody>
</table>

4.3.2 The formation of the models

In the construction of quadratic models, as there are six process variables involved in the modelling process, every resultant quadratic equation will be made up of twenty seven estimate items. However, not all the estimate items significantly affect the developed models, thus inclusion of all the twenty seven estimate items in the model may just lead to a too complicated form to practical use. In order to simplify the models, the “backward” elimination procedure in SPSS package was used. In this procedure all estimate items were included at start and some of them were removed at each step in case that the change in $R^2$ was small enough without affecting much of the precision of the prediction. Following the model simplification process, the final forms of predictive models for kerf surface roughness (represented by the centre-line average $R_a$), smooth depth of cut ($h_s$), kerf taper ($T_R$), top kerf width ($W_t$) and bottom kerf width ($W_b$) are given as follows:

Surface roughness:

$$R_a = 1.19 - 0.435H^2 - 2.083u^2 - 0.048\theta^2 + 0.134m_au - 0.0085HP \tag{4.41}$$

$$+ 0.167H\theta - 0.112HF + 0.516uF$$

Smooth depth of cut:
where oscillation angle (θ) is in degree, oscillation frequency (F) is in Hz, water pressure (P) is in MPa, nozzle traverse speed (u) is in mm/s, standoff distance (H) is in mm and abrasive mass flow rate (m_a) is in g/s.

### 4.3.3 Model verification

The predictive capabilities of the five empirical mathematical models are first verified by qualitative analysis in which the predicted trends of the models with respect to the process variables and the comparisons between the predictions and the experimental data are examined. The analysis in the form of plots is shown in Figures 4.4 to 4.8 where the curves represent the predicted trends of models in connection with the respective process variable and the square symbols in the figures are the corresponding experimental data for the purpose of comparison.

From Figure 4.4 to 4.8, the predicted values for surface roughness, smooth depth of cut, kerf taper, top kerf width and bottom kerf width correctly reflect the trends with reference to the six process variables, although the extent of the variation in some of the performance measures has minor difference with that discussed in chapter 3. The
good fits of the predicted performance measures to the experimental data in the various verification cases are apparent from these figures. Figure 4.4 shows that an increase in oscillation frequency, water pressure, nozzle traverse speed or abrasive mass flow rate is associated with an increase in the surface roughness, while surface roughness slightly increases at first stage and then decreases when oscillation angle or standoff distance increases. It also can be seen that the surface roughness is highest when oscillation angle is at about 4° to 6°.

![Graphs showing the relationship between surface roughness and various parameters](image)
Figure 4.4. Comparisons between predictions and experimental data for surface roughness: (a) $\theta=8^\circ$, $P=310$MPa, $u=1.33$mm/s, $H=2$mm, $m_a=9.1$g/s; (b) $F=6$Hz, $P=380$MPa, $u=0.67$mm/s, $H=4$mm, $m_a=9.1$g/s; (c) $\theta=6^\circ$, $F=6$Hz, $u=1.67$mm/s, $H=3$mm, $m_a=9.1$g/s; (d) $\theta=2^\circ$, $F=2$Hz, $P=345$MPa, $H=5$mm, $m_a=9.1$g/s; (e) $\theta=8^\circ$, $F=2$Hz, $u=1.33$mm/s, $P=310$MPa, $m_a=9.1$g/s; (f) $\theta=6^\circ$, $F=6$Hz, $P=380$MPa, $u=1$mm/s, $H=5$mm.

Shown in Figure 4.5 are the relationships between the predicted smooth depths of cut and the six process variables. The results confirm that with an increase in oscillation frequency or water pressure the smooth depth of cut increases, while increasing the nozzle traverse speed and abrasive mass flow rate leads to a decrease in smooth depth of cut. However, smooth depth of cut shows a trend of initial decrease and then increase with the oscillation angle and standoff distance. These trends are well in line with the analysis in chapter 3 except for the prediction based on standoff distance where the predicted smooth depth of cut reduces somewhat quickly than the discussion in chapter 3 when standoff distance increases.
Figure 4.5. Comparisons between predictions and experimental data for smooth depth of cut: (a) $\theta=8^\circ$, $P=275$MPa, $u=1$mm/s, $H=4$mm, $m_a=11.3$g/s; (b) $F=6$Hz, $P=380$MPa, $u=0.67$mm/s, $H=4$mm, $m_a=9.1$g/s; (c) $\theta=6^\circ$, $F=10$Hz, $u=1$mm/s, $H=2$mm, $m_a=13.6$g/s; (d) $\theta=2^\circ$, $F=2$Hz, $P=275$MPa, $H=2$mm, $m_a=6.8$g/s; (e) $\theta=8^\circ$, $F=2$Hz, $u=1.33$mm/s, $P=310$MPa, $m_a=9.1$g/s; (f) $\theta=2^\circ$, $F=6$Hz, $P=310$MPa, $u=1.67$mm/s, $H=4$mm.
Predicted kerf tapers using Equation 4.43 generally follow the analysis in chapter 3, i.e. it linearly increase with nozzle traverse speed, slightly reduces with standoff distance and abrasive mass flow rate but decreases with oscillation frequency and water pressure (Figure 4.6). Nevertheless, a discrepancy exists for the predictions in connection with the oscillation angle (Figure 4.6 (b)) compared to the foregoing analysis in chapter 3. For the effect of oscillation angle, while the result in chapter 3 shows a slowly decreased trend with the oscillation angle in the test range, the prediction by Equation 4.43 presents a trend of first slight increase and then gradual decrease. As stated before, this discrepancy is attributed to the narrow range of
oscillation angle selected in conducting this experiment. Consequently, the kerf taper model works reasonably well in the verification cases.

From Figures 4.7 (a), (b) and 4.8 (a), (b), it was found by comparing the kerf widths predicted by varying oscillation parameters with the previous discussions that insignificantly top kerf width increases with oscillation frequency but decreases with oscillation angle, while the increase in oscillation frequency and oscillation angle slightly decreases bottom kerf width. The variation of kerf widths with respect to water pressure, nozzle traverse speed, standoff distance and abrasive mass flow rate as predicted by the Equations 4.44 and 4.45 is also found to be in good agreement with the reported literature in chapter 2. Thus it can be concluded that the top kerf width and the bottom kerf width models describe well the kerf widths produced by the use of the six process variables from the point of view of qualitative analysis.
Figure 4.7. Comparisons between predictions and experimental data for top kerf width: (a) $\theta=8^\circ$, $P=310\text{MPa}$, $u=1.33\text{mm/s}$, $H=2\text{mm}$, $m_a=9.1\text{g/s}$; (b) $F=6\text{Hz}$, $P=385\text{MPa}$, $u=0.67\text{mm/s}$, $H=4\text{mm}$, $m_a=9.1\text{g/s}$; (c) $\theta=6^\circ$, $F=6\text{Hz}$, $u=1.67\text{mm/s}$, $H=3\text{mm}$, $m_a=9.1\text{g/s}$; (d) $\theta=2^\circ$, $F=2\text{Hz}$, $P=275\text{MPa}$, $H=2\text{mm}$, $m_a=6.8\text{g/s}$; (e) $\theta=8^\circ$, $F=2\text{Hz}$, $u=1.33\text{mm/s}$, $P=310\text{MPa}$, $m_a=9.1\text{g/s}$; (f) $\theta=2^\circ$, $F=6\text{Hz}$, $P=310\text{MPa}$, $u=1.67\text{mm/s}$, $H=4\text{mm}$. 
Figure 4.8. Comparisons between predictions and experimental data for bottom kerf width: (a) $\theta=8^\circ$, $P=310\text{MPa}$, $u=1.33\text{mm/s}$, $H=2\text{mm}$, $m_a=9.1\text{g/s}$; (b) $F=10\text{Hz}$, $P=345\text{MPa}$, $u=1.33\text{mm/s}$, $H=4\text{mm}$, $m_a=6.8\text{g/s}$; (c) $\theta=6^\circ$, $F=6\text{Hz}$, $u=1.67\text{mm/s}$, $H=3\text{mm}$, $m_a=9.1\text{g/s}$; (d) $\theta=2^\circ$, $F=2\text{Hz}$, $P=275\text{MPa}$, $H=2\text{mm}$, $m_a=6.8\text{g/s}$; (e) $\theta=8^\circ$, $F=2\text{Hz}$, $u=1.33\text{mm/s}$, $P=310\text{MPa}$, $m_a=9.1\text{g/s}$; (f) $\theta=6^\circ$, $F=6\text{Hz}$, $P=380\text{MPa}$, $u=1\text{mm/s}$, $H=2\text{mm}$.

The validity of the models is then assessed by means of the quantitative analysis based on the percentage deviations of the model predictions with regard to the corresponding experimental data. The histograms evaluated using Equations 4.41 to 4.45 for surface roughness, smooth depth of cut, kerf taper, top kerf width and bottom kerf width are shown in Figure 4.9. Further, Table 4.7 summaries the average percentage deviations, standard deviations and the coefficient of determinations after “backward” elimination procedure for these five developed models. Thus, it may be stated that the developed empirical models for surface roughness, smooth depth of cut, kerf taper, top kerf width and bottom kerf width can be used for adequate predictions of these performance quantities.
Figure 4.9. Percentage deviations between predicted and experimental data for regression models: (a) Surface roughness; (b) Smooth depth of cut; (c) Kerf taper; (d) Top kerf width; (e) Bottom kerf width.

Table 4.7. Summary of the regression models.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface roughness</td>
<td>2.9</td>
<td>19.42</td>
<td>0.838</td>
</tr>
<tr>
<td>Smooth depth of cut</td>
<td>0.5</td>
<td>6.96</td>
<td>0.958</td>
</tr>
<tr>
<td>Kerf taper</td>
<td>2.2</td>
<td>15.13</td>
<td>0.888</td>
</tr>
<tr>
<td>Top kerf width</td>
<td>0.2</td>
<td>4.4</td>
<td>0.855</td>
</tr>
<tr>
<td>Bottom kerf width</td>
<td>0.8</td>
<td>9.34</td>
<td>0.788</td>
</tr>
</tbody>
</table>

4.4 Conclusions

In this chapter, predictive mathematical models for the major cutting performance measures in AWJ cutting with controlled nozzle oscillation including the depth of cut, kerf surface roughness, smooth depth of cut, kerf taper, top kerf width and bottom kerf width have been developed in order to guide the correct selection of the process variables and optimize the AWJ machining process in process planning.
Both semi-empirical and empirical approaches were involved for the development process. The constants in the models were determined from the AWJ oscillation cutting experiment on the 87% alumina ceramics. Various techniques were utilized to evaluate the predictive capability of the developed models. Qualitative assessment was conducted to examine the trends of the models’ predictions with regard to the process variables and compare their predicted values with the experimental data. Quantitative analysis was also carried out to assess the percentage deviations of the model predicted values in connection with the corresponding experimental results. The verification of the models demonstrated that the developed models have good correlation with the experiment both in terms of their trends and the quantitative values. Hence, they are deemed to be adequate for practical use.

The predictive models for the depth of cut in both normal cutting and nozzle oscillation cutting were developed by first using theoretical derivation taking into account of the micro mechanisms involved in the cutting process and resorting to dimensional analysis technique. Then empirical fit of the experimental data with the equations of the analytical models was carried out to determine the constants in the models. The development is based on the premise that the overall material removal in a given time span is an accumulated action of material removed by individual abrasive particles. In order to obtain the expression of material removal by a single abrasive particle, dimensional analysis was used to relate this volume quantity to its influencing parameters including particle velocity, particle mass, particle attack angle at the point of attack and the flow stress of target material. It followed that the particle attack angles for both normal cutting and oscillation cutting were formulated by using dimensional analysis technique again. According to the micro mechanisms in AWJ normal cutting, the independent variables to affect the particle attack angle were identified which include nozzle traverse speed, water pressure, standoff distance, average particle diameter and the flow stress of target material. By introducing the oscillation parameters to the process, the relationship between the particle attack angle and the influencing variables was derived for oscillation cutting. The general forms of depths of cut for both normal cutting and oscillation cutting take the form of power expression.
The two models of depth of cut have been assessed using different techniques. An examination for each variable in the models was first conducted to verify their feasibility. Qualitative assessment has been made by examining the trends of the predicted depths of cut with the variation of different cutting variable and comparing the predicted values with the corresponding experimental data. A quantitative comparison between the predicted and experimental depths of cut has also been carried out based on their percentage deviations. The depths of cut predicted by the two models have further been examined to confirm the finding made in chapter 3 showing that only when the used oscillation strength is above the threshold value could oscillation cutting increase the depth of cut. All the numerical analyses reveal that the developed models conform well to the experimental results and can provide an effective means for the optimum selection of process variables in AWJ cutting with nozzle oscillation.

The mathematical models for surface roughness, smooth depth of cut, kerf taper, top kerf width and bottom kerf width were established in this chapter by purely empirical approach based on regression analysis technique. Different mathematical forms of the model expression have been compared and the quadratic equation was selected to model these five performance quantities as it gave the highest coefficient of determination. Backward elimination in SPSS software was used for the regression process. Similar assessment processes from qualitative to quantitative analysis were carried out to validate the five developed models. Again, very good agreement between the predicted values and the experimental data was found. Therefore, this set of models provides an essential basis for the development of optimization strategies for the effective use of AWJ cutting technology when nozzle oscillation technique is used.
Chapter 5

Enhancing the AWJ Cutting Performance by Multipass Cutting with Controlled Nozzle Oscillation

5.1 Introduction

In abrasive waterjet machining, it has been troubled for a long time by the cutting capabilities with regard to the depth of jet penetration and the kerf quality. As mentioned in chapter 3, when the jet has no sufficient energy to penetrate the target material there is an enlarged pocket left at the bottom of the kerf. Because of this, AWJ cutting technology has to a great extent been limited in its application to relatively thin materials. In order to process thicker materials, a low nozzle traverse speed together with a high water pressure is normally used [77, 92, 103, 171]. However, such a combination of process parameters is not preferred in practice from an economic point of view. In addition, there are always situations where the material thickness (or the depth of cut requirement) is beyond the AWJ cutting capability. To cope with the capacity requirement in AWJ machining, a large amount of research effort has been directed to explore the new techniques. As a result, several innovative cutting techniques have been developed in recent years to enhance the cutting performance [2, 11, 12, 84, 120, 147, 150, 172]. As mentioned in Chapter 3, multipass cutting and controlled nozzle oscillation have been found to be able to dramatically increase the cutting performance.

In AWJ multipass cutting, the jet or cutting head travels over the same kerf a number of times. It has been found [84, 146, 147] that multipass cutting can not only dramatically increase the depth of cut, and hence increase the application domain of this cutting technology, but also significantly improve the kerf quality such as the kerf profile, kerf geometry and surface quality. By properly selecting the number of passes and the cutting parameters in each pass, multipass cutting has been found to be superior to single pass cutting whereby multipass cutting at high traverse speeds can achieve better overall cutting performance than a single pass cutting at low speed.
within the same cutting time. It was also found that even for the cases where a single pass can cut through the target material, the overall time needed for per unit length of the cut and thus the unit cost of cutting can be minimised by resorting to multipass cutting [19, 107]. Therefore, a multipass cutting is more effective both technologically and economically than a single pass cutting.

As concluded in chapter 3 from the study of AWJ machining alumina ceramic with nozzle oscillation at small angles, AWJ cutting with nozzle oscillation can significantly improve the major cutting performance. However, the research on multipass cutting combined with the controlled nozzle oscillation technique has not been found from the search of literature, let alone the solution strategies for the selection of optimum number of passes and the corresponding cutting conditions under this cutting mode. While it is expected that the integration of nozzle oscillation technique into multipass cutting can notably enhance the cutting performance, it is interesting to study the cutting process and cutting performance produced under such cutting mode.

In this study, an experimental investigation is presented on the AWJ multipass cutting of an 87% alumina ceramic with nozzle oscillation. The cutting capacity in terms of the depth of cut and the major kerf geometrical features as represented by top kerf width, bottom kerf width and kerf taper are analyzed with respect to the major process variables including the number of passes, the nozzle traverse speed, water pressure and oscillation parameters. Optimum combinations of the major process parameters are recommended to maximize the depth of cut, taking into account the economic or productivity aspects. The general guidelines and suggestions are given for the selection of the number of passes and the other major cutting variables. In addition, the mathematical models to predict the depth of cut in AWJ multipass cutting with and without nozzle oscillation are developed. Based on the previously developed depth of cut models for AWJ single pass cutting, the current development extends the same methodology to multipass cutting, by relating the macro material removal rate on the cutting front to the accumulated micro material removal rate by individual abrasive particles and using dimensional analysis technique.
5.2 Experimental tests

5.2.1 Work material and equipment

In this experiment, 87% alumina ceramic plates of 25.4mm thickness were used as the specimens. The selection of high thicknesses of the specimens was in an intention of obtaining more non-through cuts to study the capacity of multipass cutting. The AWJ cutting system used was equipped with a high-pressure water intensifier and a six-degree of freedom robot manipulator. The major components of this cutting system include a Flow International’s 5X intensifier pump, a Motoman robotic traverse system with six degrees of freedom, an abrasive mixing and accelerating head, an abrasive feeding and mass flow monitoring device, a receiver, and a control panel to program the machine. The intensifier is able to supply water up to a maximum pressure of 380MPa. The robot is used to move and position the cutting head to carry out the tests in such a way that it can simultaneously execute the linear (the traverse speed) and oscillation motion of the cutting head.

5.2.2 Experimental plan

From the foregoing description, this experimental study aims at finding the best solution for the major cutting performance in multipass cutting with nozzle oscillation. Particularly, the most concerned is how to combine the number of passes with other cutting parameters so multipass cutting at higher nozzle traverse speeds could achieve superior performance to a single pass cutting at a lower nozzle traverse speed within the same elapsed time. In accordance with the major objectives, three major process variables were chosen for this experiment which included nozzle traverse speed, oscillation parameters (oscillation angle and oscillation frequency) and the number of passes. From the previous ANOVA on single pass oscillation cutting, water pressure is another important variable which can not be ignored when discussing the influence of cutting parameters on cutting performance. Thus, the complete variables in controlling these multipass oscillation tests were selected to include nozzle traverse speed, number of passes, water pressure, oscillation angle and oscillation frequency.
In order to assess the advantages of multipass cutting over single pass cutting on cutting capacity represented by the depth of cut, it is an expectation that most resultant kerfs have non-through cuts after all the passes. For this purpose, the parameters were selected such that the total penetration depth is less than the material thickness under the combinations of these operating variables. In addition, the selection of the cutting conditions should also enable the major cutting performance measures to be acquired and therefore evaluated for each pass. Therefore, the same nozzle traverse speed was used for all the passes in some operations, while in others the nozzle traverse speed in one pass was selected as multiples of that in other passes in the same operation in order to evaluate the cutting performance with respect to the cutting time (or elapsed time). Proper selections of nozzle traverse speeds in different operations were also carefully made for meaningful comparisons of the cutting performance in multipass as well as in single pass cutting. All selected nozzle traverse speeds were used for up to three passes in the tests. Based on the finding in previous study of oscillation single pass cutting on alumina ceramic in chapter 3, the water pressures were 275MPa and 345MPa. Oscillation angle and oscillation frequency were selected at 4º and 6Hz respectively for achieving the best overall cutting performance based on the finding in earlier studies [151]. Cutting without nozzle oscillation was also conducted for a comparison purpose. Table 5.1 gives the variable and constant process parameters used in the experiment. According to this experimental design, a total number of 124 passes were conducted which produced 60 cuts for evaluation.

Measurements with the assistance of metrological instruments were conducted for the major cutting performance measures that included the depth of cut, top kerf widths, bottom kerf width and kerf taper. Of these quantities, kerf taper was calculated using the top kerf width, the minimum kerf width and the depth where the minimum kerf width was measured, while the other three quantities were directly measured from each cut. The top kerf width, bottom kerf width, minimum kerf width and depth of cut were measured from the end view of the kerf profile by using a “SigmaScope 500” profile projector. At least three measures for each quantity on each cut were made and the average was taken as the final reading.
Table 5.1. Operating parameters used in the multipass cutting experiment.

<table>
<thead>
<tr>
<th>Operating parameters</th>
<th>Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nozzle traverse speed (mm/s)</td>
<td>1, 2, 4, 6</td>
</tr>
<tr>
<td>Number of passes</td>
<td>1, 2, 3</td>
</tr>
<tr>
<td>Water pressure (MPa)</td>
<td>275, 345</td>
</tr>
<tr>
<td>Oscillation angle (degree)</td>
<td>4</td>
</tr>
<tr>
<td>Oscillation frequency (Hz)</td>
<td>6</td>
</tr>
<tr>
<td>Initial standoff distance (mm)</td>
<td>3</td>
</tr>
<tr>
<td>Abrasive mass flow rate (g/s)</td>
<td>9.1</td>
</tr>
<tr>
<td>Jet impact angle in normal cutting (degree)</td>
<td>90</td>
</tr>
<tr>
<td>Mixing tub (nozzle) diameter (mm)</td>
<td>0.762</td>
</tr>
<tr>
<td>Orifice diameter (mm)</td>
<td>0.254</td>
</tr>
<tr>
<td>Size of abrasives (mesh number)</td>
<td>80</td>
</tr>
</tbody>
</table>

Because of the relative shallow slots produced in this experiment for most of the cuts, some difficulties are encountered in separating the specimens to expose the kerf surface for measurements. Therefore, the surface roughness and the smooth depth of cut were not measured and these two performance quantities were not investigated in this study.

### 5.3 Results and discussion

#### 5.3.1 Kerf characteristics

A visual evaluation for all the cuts shows that the kerfs grouped by different passes have different characteristics. Figure 5.1 shows some typical kerf profiles. Generally, the kerfs produced by a single pass cutting possess the similar features with the non-through cuts observed in the previous single pass oscillation experiment, i.e. the kerf has wider entry at the top and reduces gradually towards the bottom, so that a taper is formed. There is also an enlarged pocket at the kerf bottom owing to the jet upward deflection. The kerfs after second pass have a swollen portion, indicating the incomplete removal of the pocket produced in the first pass. The marks of the pockets produced in the first and second passes may still be visible on the kerf walls after the third pass cutting. Some of the cuts after three passes have a parent larger
enlarged portion in the lower part of the kerf than those generated by two passes. But, in most cases, there is no clear pocket formed at the kerf bottom by the third pass cutting, possibly because of the deflected particles unable to take effective material removal as reduced energy due to the high standoff distance [107].

Figure 5.1. Kerf characteristics in AWJ multipass cutting with nozzle oscillation (from left to right: single, double and triple pass. \( u=2\text{mm/s}, P=275\text{MPa} \)).

It was also noted that some of the kerfs are without evident pocket at the bottom irrespective of how many passes used. Usually, these kerfs either have shallow depth of cut produced by a single pass or have deep depth of cut after three passes. Of the two cases, the former may be attributed to the fact that the deflected abrasive particles directly rebounded out of the kerf after impinging the target material, while the later can be explained by the insufficient energy of deflected particles. In general, those kerfs have a narrow exit due to the decreased effective diameter of jet with the increase in the standoff distance.

**5.3.2 Depth of cut**

An examination of the experimental data for the 60 cuts has revealed that with the cutting conditions used in this experiment, all the tests produced non-through cuts with the maximum and minimum depths of cut being 18.84mm and 0.84mm respectively. If all the cuts were divided into two groups as multipass normal cutting
and multipass oscillation cutting, nozzle oscillation cutting has significantly increased the depth of cut by an average of 45.43% as compared to that in normal cutting under the corresponding cutting conditions.

Furthermore, it has been found that the depth of cut steadily increases with the number of passes. While this trend is evident as expected, the rate of increase in the depth of cut tends to decrease with the number of passes. In order to assess the rate of increase, the depths of cut produced by individual pass are calculated which is defined as incremental cutting depth. Statistic calculations to all the incremental cutting depths according to different pass revealed that the average incremental cutting depth is 2.63mm, 2.44mm and 2.40mm for the first, second and third pass respectively, which demonstrates a gradually decreased trend in the rate of increase. This is because the actual standoff distance constantly increases with the number of pass, which leads to a decreased jet energy at the lower part of the kerf and accordingly reduces the cutting ability of the jet.

Figure 5.2 illustrates the relationship between the depth of cut and the number of passes. In these cases, the nozzle traverse speed for all the first pass was 2mm/s, but in the subsequent passes, different nozzle traverse speeds were used while the speed for the second and third pass remained unchanged in each operation. From this figure, it can be seen that while the depth of cut increases constantly with the number of passes, the slope of the curves reduces with an increase in nozzle traverse speed at the second and third passes, as a high traverse speed reduces the number of particles impinging the target material in per unit time. Consequently, the incremental cutting depth in a pass as well as the depth of cut after all the passes decrease. Thus, when the first traverse speed is 2mm/s, the depth of cut is affected by the traverse speeds of the following passes in that low traverse speeds are always favourable for generating deeper depth of cut. The general trends apply for both normal cutting and nozzle oscillation cutting as shown in Figures 5.2 (a) and (b), although the quantitative depth of cut values are higher in oscillation cutting than in normal cutting. A solution strategy is needed to determine the number of passes and the nozzle traverse speed in each pass in order to achieve the required depth of cut while maximizing the cutting rate.
Figure 5.2. The effect of number of passes on depth of cut using different traverse speed: (a) $P=345\text{MPa}$, normal cutting; (b) $P=275\text{MPa}$, $\theta=4^\circ$, $F=6\text{Hz}$.

When the same nozzle traverse speed is applied for all the three passes in an operation, the increase in depth of cut is closely dependent on the nozzle traverse speed used, as shown in Figure 5.3. It is obvious that lower traverse speed is always beneficial to the generation of higher depth of cut, which is the same result with that obtained in the study of single pass oscillation cutting. However, it has been found that the difference in the depths of cut after first pass is relatively not significant, but after the second and third pass this disparity becomes progressively great for both normal cutting (Figure 5.3 (a)) and oscillation cutting (Figure 5.3 (b)). This observation results in a deduction that using lower traverse speed is more important after the first pass from the viewpoint of large depth of cut. However, low traverse speed signifies longer machining time and so a high cutting cost, thus the selection of nozzle traverse speed must be at a compromise from the economic point of view.
Figure 5.3. The effect of traverse speed on depth of cut using the same traverse speed: (a) $P=345\text{MPa}$, normal cutting; (b) $P=275\text{MPa}$, $\theta=4^\circ$, $F=6\text{Hz}$.

Figure 5.3 also presents another phenomenon that is unique for multipass cutting. It can be seen that most of the curve slopes from second pass to third pass are smaller than those from first pass to second pass although this trend is hardly noticeable. To further confirm this phenomenon, the values of depth of cut for some traverse speeds are selected and given in Table 5.2. The data show that increasing the nozzle traverse speed from 1mm/s to 2mm/s (for single pass) produced the value of 10.2mm and 5.79mm respectively, which did not certainly reduce the depth of cut by $\frac{1}{2}$. The similar case was observed for the nozzle traverse speed increasing from 2mm/s to 4mm/s (with the depth of cut of 3.17mm). From the above analysis, an optimum cutting strategy exists that yields a maximum depth of cut, i.e. increasing nozzle traverse speed to a certain value and making several cutting passes. However, how to properly combine the nozzle traverse speed and the number of passes remains one of the important issues.

In order to find out the optimum combination of nozzle traverse speed and the number of passes for the given cutting conditions, a dashed line is drawn on the curves as indicated in Figures 5.3 (a) and (b) to represent the points where the depth of cut has been obtained in the same total elapsed time. The elapsed time can be characterised by the ratio of nozzle traverse speed to the number of passes ($u/N$) which is a constant for the dashed line. On this line, the different combinations of nozzle traverse speeds and number of passes also present the same cutting energy. For the cutting conditions shown in Figures 5.3 (a) and (b), the dashed lines have the
same value of 2 for the ratio of nozzle traverse speed to the number of passes. It is observed from the Figure that the slopes of the dashed lines increase in the test range. The positive slope of the dashed line clearly shows the improved depth of cut which results from multipass cutting. A peak point on the dashed line indicates an optimum combination of nozzle traverse speed and the number of passes. Within this experimental data, the depth of cut with three passes and the nozzle traverse speed at 6mm/s is larger than those obtained using single pass at 2mm/s or two passes at 4mm/s for both normal cutting and oscillation cutting, as can be seen from Figure 5.3 (a) and (b).

Table 5.2. The effect of nozzle traverse speed and number of passes on depth of cut ($P=345\text{MPa}, \theta=4^\circ, F=6\text{Hz}$).

<table>
<thead>
<tr>
<th>Traverse speed (mm/s)</th>
<th>Number of passes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>10.2</td>
</tr>
<tr>
<td>2</td>
<td>5.79</td>
</tr>
<tr>
<td>4</td>
<td>3.17</td>
</tr>
<tr>
<td>6</td>
<td>2.16</td>
</tr>
</tbody>
</table>

While multipass cutting demonstrates technical advantage over single pass cutting, another important advantage can also be highlighted from the point of view of economic consideration. Table 5.2 gives the comparisons of depths of cut made by multipass cutting and single pass cutting within the same elapsed time. It can be seen that two passes at 2mm/s penetrated a depth of cut of 11.13mm which was deeper than that generated by a single pass at 1mm/s (10.2mm), although they had the same elapsed time. Likewise, the depth of cut from two passes at 4mm/s (6.36mm) was higher than that from one pass at 2mm/s (5.79mm) and three passes at 6mm/s cut a depth of cut of 6.37mm which was larger than two passes at 4mm/s did. This implies that for a given depth of cut, the employment of multipass cutting can reduce the overall cutting time needed for the job and thus the unit cost of cutting can be minimised.

Figure 5.4 shows the influence of oscillation on the depth of cut. Obviously, multipass cutting incorporating into oscillation technique penetrated larger depth of
cut than that produced by normal cutting under the corresponding cutting conditions. This trend has been found to apply for both the cuttings using different nozzle traverse speeds in the three passes (Figure 5.4 (a)) and the cuttings using the same traverse speed in all the passes (Figure 5.4 (b)). It is also observed that the difference in depth of cut for oscillation cutting and normal cutting gradually becomes large with the number of passes, which indicates the incomparable advantage of oscillation cutting over normal cutting in multipass cutting, particularly in the high number of passes. The efficiency of the jet progressively falls with the number of passes due to the increasingly high standoff distance along with the fact that a part of the jet energy is used to wash the deposited abrasive particles out of the kerf at depth from the last pass. With the incorporation of oscillation operation, the scanning scope of waterjet is widened with the increase of standoff distance which leads to an increased overlapping cutting action on the cutting front. As a result, the overall abrasive cutting capacity increases and this compensates for the loss of jet energy owing to the increase in the standoff distance.

![Diagram](image)

(a) Normal cutting
θ=4°, F=6Hz
(b) Normal cutting
θ=4°, F=6Hz

Figure 5.4. The effect of oscillation on depth of cut: (a) \(u_1=2\text{mm/s}, u_2=u_3=6\text{mm/s}, P=345\text{MPa}\); (b) \(u_1=u_2=u_3=4\text{mm/s}, P=275\text{MPa}\).

The influence of water pressure on the depth of cut is shown in Figure 5.5. It is found that depth of cut steadily increases as the water pressure increases for all the cutting conditions tested irrespective of oscillation cutting (Figure 5.5 (a)) or normal cutting (Figure 5.5 (b)), which is consistent with the result obtained in the previous study of single pass oscillation cutting. The comparison of Figure 5.5 (a) with (b) shows that the slopes of the curves in (b) are flatter than that in (a), further indicating that
multipass cutting incorporating into oscillation may produce larger depth of cut than normal cutting does.

![Graphs showing depth of cut vs number of passes for different water pressures and nozzle traverse speeds](image)

**Figure 5.5** The effect of water pressure on depth of cut: (a) $u_1=2\text{mm/s}$, $u_2=u_3=6\text{mm/s}$, $\theta=4^\circ$, $F=6\text{Hz}$. (b) $u_1=u_2=u_3=6\text{mm/s}$, normal cutting.

In conclusion, the proper combination of cutting parameters, specifically the number of passes and nozzle traverse speed, along with the incorporation of oscillation technique is the key point if depth of cut is the major concern in AWJ multipass cutting. For the cutting condition used in this experiment, three passes combined with nozzle traverse speed of 6mm/s are the best choice to achieve maximum depth of cut.

### 5.3.3 Kerf taper

In this study, kerf taper was calculated using the same formula as Equation (3.1)

$$ T_R = \frac{(W_t - W_b)}{2t} \quad (5.1) $$

where $W_t$, $W_b$ and $t$ denote the top kerf width, the narrowest kerf width along the kerf profile and the depth at which the narrowest kerf width was measured respectively.

All the experimental data were first statistically analysed to get a general view of the influence of multipass cutting on kerf taper. The examination of all the cuts produced in this experiment shows that the taper value averaged 0.16. In terms of different pass, the average tapers were 0.21, 0.15 and 0.13 after first, second and third pass.
respectively. It is apparent that taper value presents a steadily decreased trend with the increase in the number of passes. If the experimental result was considered according to whether the oscillation technique was used, the oscillation group had an average of 0.12 while the normal cutting group averaged out at 0.19 under the corresponding cutting conditions. This result agrees well with that obtained in single pass oscillation cutting experiment in chapter 3, which again demonstrates the effectiveness of oscillation cutting in reducing kerf taper angle.

The effect of number of passes on kerf taper is shown in Figure 5.6. It has been found, as shown in Figure 5.6 (a) for normal cutting and Figure 5.6 (b) for oscillation cutting, that the kerf taper consistently reduces with an increase in the number of passes when first pass uses the same nozzle traverse speed while other passes use the different traverse speeds in one operation. This can be explained by the knowledge that with the increase in the number of passes, the second and third pass unceasingly impact the kerf wall and have the function to widen the lower part of the kerf generated by the first pass, which results in relatively parallel kerf walls. However, the decrease in kerf taper with the number of passes is strongly controlled by the nozzle traverse speed used in the following passes with the trend that lower traverse speeds are beneficial to gain small taper angles. It is also noted that the rate of decrease in kerf taper tends to be slow with the number of passes regardless of the cutting mode (normal cutting or oscillation cutting) or traverse speeds used. This observation can apparently be confirmed by the consistently decreased slopes of the curves from Figure 5.6 (a) and (b). It is believed that because of the increase in standoff distance with the number of passes, the abrasive waterjet tends to become diffused at the lower portion and therefore has no sufficient cutting ability to cut into the kerf walls.
Figure 5.6. The effect of number of passes on kerf taper using different traverse speed: (a) $P=275\text{MPa}$, normal cutting; (b) $P=275\text{MPa}$, $\theta=4^\circ$, $F=6\text{Hz}$.

Figure 5.7 shows the effect of traverse speeds on kerf taper for the cutting conditions where the same traverse speed applied for all the three passes. It is noted that kerf taper unanimously reduces as the nozzle traverse speed decreases whether or not oscillation technique is used during the process. At high speeds, the number of particles impacting on the target material per unit length decreases and so is not enough to effectively broaden the lower part of the kerf wall.

Figure 5.7. The effect of traverse speed on kerf taper using the same traverse speed: (a) $P=275\text{MPa}$, $\theta=4^\circ$, $F=6\text{Hz}$; (b) $P=345\text{MPa}$, normal cutting.

A similar trend was also noted as the cutting cases using different traverse speeds in different passes, i.e. the rate of decrease in kerf taper tends to be slow with the
number of passes. This implies that the nozzle traverse speed of the first pass is more important to obtain a small kerf taper. Consequently, a low nozzle traverse speed should be used where possible for the first pass to significantly reduce kerf taper.

Comparing the kerf taper difference with regard to different passes, it is noticed that this difference becomes smaller as the number of passes increases. A quantitative expression of the kerf taper difference for typical cutting conditions is shown in Table 5.3 for the demonstration of this observation. It is indicated that, under the cutting condition of water pressure at 275MPa and using oscillation operation, the kerf taper differences for nozzle traverse speeds at 2mm/s, 4mm/s and 6mm/s amount to 0.07, 0.06 and 0.04 respectively after the first, the second and the third pass. This constantly reduced difference in kerf taper with the number of passes suggests that the nozzle traverse speed in the latter passes is relatively not critical to gain an improved kerf taper. Thus, a relatively higher nozzle traverse speed may be selected for the high passes for increasing productivity, reducing the cutting costs and equally without deteriorating kerf taper if minimizing kerf taper is the only job requirement.

### Table 5.3. The effect of traverse speed and number of passes on kerf taper

\(P=275\text{MPa}, \theta=4^\circ, F=6\text{Hz}\).

<table>
<thead>
<tr>
<th>Traverse speed (mm/s)</th>
<th>Kerf taper</th>
<th>Traverse speed (mm/s)</th>
<th>Kerf taper</th>
<th>Traverse speed (mm/s)</th>
<th>Kerf taper</th>
<th>Taper difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>First pass</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.14</td>
<td>4</td>
<td>0.20</td>
<td>6</td>
<td>0.21</td>
<td>0.07</td>
</tr>
<tr>
<td>Second pass</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.10</td>
<td>4</td>
<td>0.14</td>
<td>6</td>
<td>0.16</td>
<td>0.06</td>
</tr>
<tr>
<td>Third pass</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.09</td>
<td>4</td>
<td>0.11</td>
<td>6</td>
<td>0.13</td>
<td>0.04</td>
</tr>
</tbody>
</table>

The effect of oscillation operation on kerf taper is illustrated in Figure 5.8. As can be seen from the case of using different nozzle traverse speeds in three passes (Figure 5.8 (a)) and the case of using the same speed in all the three passes (Figure 5.8 (b)), kerf taper is consistently decreased when oscillation operation is applied during multipass cutting. It is also noted that the rate of decrease in the slopes of the curves for normal cutting in the two figures notably slows down after the second pass,
indicating the limited ability of normal cutting in improving the kerf taper with the number of passes. On the contrary, the curves for oscillation cutting present the almost unchanged slopes, which demonstrates the stronger ability of oscillation cutting in the improvement of kerf taper than that of normal cutting, particularly in the later passes. In spite of the fact that normal cutting can progressively reduce kerf taper with the number of passes, the decrease in kerf taper is restrained from the constantly increased standoff distance with the number of passes. In this case, the abrasive waterjet has no enough energy to effectively cut the lower part of the kerf. However, with the employment of oscillation technique, the broadened scanning scope of the jet leads to an increased overlapping cutting action on the lower part of the kerf walls with the number of passes and therefore reduces the kerf taper.

![Graphs showing the effect of oscillation on kerf taper](image)

Figure 5.8. The effect of oscillation on kerf taper: (a) \(u_1=2\text{mm/s}, u_2=u_3=4\text{mm/s}, P=345\text{MPa}\); (b) \(u_1=u_2=u_3=2\text{mm/s}, P=345\text{MPa}\).

The relationship between the water pressure and kerf taper under the cutting conditions of normal cutting and oscillation cutting is shown in Figure 5.9. It is apparent that as the water pressure increases from 275MPa to 345 MPa, the kerf taper steadily decreases regardless of the cutting mode (normal cutting or oscillation cutting) and the nozzle traverse speed. This is quite understandable that when the pressure of 345MPa is used the abrasive particles possess the more abundant kinetic energy than the cutting condition of 275MPa.
Figure 5.9. The effect of water pressure on kerf taper: (a) \( u_1=2\text{mm/s}, u_2=u_3=4\text{mm/s}, \) normal cutting; (b) \( u_1=u_2=u_3=6\text{mm/s}, \theta=4^\circ, F=6\text{Hz}. \)

From the analysis of relationships between kerf taper and the individual cutting variables used in this study it can be concluded that to achieve a small kerf taper in multipass cutting, the use of lower nozzle traverse speed in the first pass is more significant while the selection of the traverse speed for other passes is relatively less important. Consequently, a sensible choice to achieve minimized kerf taper in this experiment should be the lowest nozzle traverse speed of 1mm/s for the first pass and then to apply a relatively high speeds (such as 4mm/s or 6mm/s) to the later passes along with the use of oscillation technique.

### 5.3.4 Top and bottom kerf width

Top kerf width and bottom kerf width are two quantities determining the kerf taper angle. While small kerf taper is always a goal pursued in practice, minimizing both the widths is desirable under any circumstance. A statistic analysis was first conducted to examine the top kerf width and bottom kerf width values machined in this study to get a general knowledge of the result. The average of all the measurements for top kerf width and bottom kerf width revealed that the average top width and the average bottom width were 1.24mm and 0.52mm respectively. Their respective values averaged according to the number of passes are calculated and shown in Table 5.4. As can be seen from the table, both the average top kerf width and bottom kerf width increased with the number of passes. The kerf wall formed during the first pass acts to focus the jet during the following passes with the increase
in standoff distance. In doing so, the jet’s coherence is maintained and kerf width is widened at the same time. But the rate of increase in top kerf width was shown to be lower than that in bottom kerf width. It was found from the statistic calculation that the percentage increases of top kerf width were 6.9% and 4.8% after the second and third pass, whereas the bottom kerf width had the percentage increases of 15.2% and 5.7% respectively when the second pass and third pass completed. The higher rates of increase in bottom kerf width than in top kerf width mean a gradually parallel kerf walls or a decrease in the kerf taper with the number of passes, which is in agreement with the result obtained in the last section.

Table 5.4. Average top and bottom kerf widths with number of passes.

<table>
<thead>
<tr>
<th></th>
<th>First pass</th>
<th>Second pass</th>
<th>Third pass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average top width (mm)</td>
<td>1.16</td>
<td>1.24</td>
<td>1.30</td>
</tr>
<tr>
<td>Average bottom width (mm)</td>
<td>0.46</td>
<td>0.53</td>
<td>0.56</td>
</tr>
</tbody>
</table>

The experimental data of top kerf width and bottom kerf width were also statistically analysed by dividing them into oscillation group and normal group to investigate the effect of oscillation cutting. It was noted that when oscillation operation was applied, the average top kerf width increased from 1.2mm to 1.27mm and the increase in bottom kerf width was from 0.48mm to 0.55mm compared to normal cutting under the corresponding cutting conditions. The percentage increases were 5.8% and 14.6% for top kerf width and bottom kerf width respectively, which is consistent with the finding in the earlier oscillation single pass experimental studies that the use of oscillation technique has a more significant effect on bottom width than on top width. Consequently, the use of oscillation technique can steadily reduce the kerf taper although the top kerf width and the bottom kerf width are both increased.
The effect of the number of passes on kerf width is shown in Figure 5.10. The top kerf width increases with the number of passes (Figure 5.10 (a)) as is the case of bottom kerf width (Figure 5.10 (b)). With the increase in number of passes, abrasive waterjet repeatedly impacts on the same kerf cut and gradually widens the top kerf width and the bottom kerf width. It has been found that standoff distance and therefore jet expansion increase with the number of passes. This expanded jet has small effect on the entrance of the kerf but has great effect on the lower part of the kerf machined by the subsequent passes. Thus the lower part of the kerf is widened more quickly than the top part. Therefore, multipass cutting more significantly increases bottom kerf width.

Figure 5.11 illustrates the effect of nozzle traverse speed on top kerf width and bottom kerf width under the cutting condition of normal cutting with the water pressure at 345MPa. It is apparent that high nozzle traverse speed reduces both top kerf width (Figure 5.11 (a)) and bottom kerf width (Figure 5.11 (b)). As the nozzle traverse speed is high the jet exposure time and therefore the number of abrasive particles attacking the cutting front per unit length decreases. The jet does not possess enough energy to widen top kerf width and bottom kerf width. This influence is more significant to kerf exit width, so the upper portion of the kerf is more easily to broaden than the lower portion.
When oscillation technique is used during multipass cutting, it affects the top kerf width and the bottom kerf width to a different level as shown in Figures 5.12 (a) and (b). It is observed that under the cutting conditions without nozzle oscillation, top kerf width and bottom kerf width almost equally increase. However, when cutting combined with the oscillation operation, the rate of increase in top kerf width considerably decreases with the number of passes while the increase in bottom kerf width keeps unchanged. These can be confirmed by the two gradually approached curves at high passes in Figure 5.12 (a) and two almost parallel curves in Figure 5.12 (b). This phenomenon is most likely due to the overlapping action of oscillation cutting along with the jet expansion with the increase of standoff distance which has more significant effect on lower portion of the kerf than on the entrance of the kerf.

The employment of high water pressure increases the top kerf width and the bottom kerf width, which is evidently observed from Figures 5.13 (a) and (b). High water pressure results in great jet kinetic energy and this in turn increases the effective jet diameter impacting on the target material.
5.4 Modelling the major cutting performance of multipass oscillation cutting

From the above experimental study of multipass cutting with controlled nozzle oscillation, it has been concluded that using multipass cutting technique combined with nozzle oscillation, cutting at high nozzle traverse speed for several passes can obtain better cutting performance than single pass cutting at low traverse speed. More specifically, the cutting capability represented by the depth of cut can be significantly increased by resorting to this novel machining technique. In order to
mathematically describe the capability of multipass cutting with nozzle oscillation, the mathematical models for depth of cut in multipass cutting are developed in this section. Using the similar methodologies with that in the development of the depth of cut models for single pass oscillation cutting, this development also uses dimensional analysis technique by relating the overall material removal rate to the accumulated volume of material removed by individual abrasive particles in a given time span. For comparison study of the cutting abilities in multipass normal cutting and multipass oscillation cutting, both cutting modes are considered in the development and therefore two mathematical models are developed.

In the previous study of single pass cutting of 87% alumina ceramic with controlled nozzle oscillation, the modelling study resulted in two equations to predict the depths of cut

\[ h_1 = \frac{K_1 m_u P}{\sigma_f \rho_w d_n u} \left( \frac{D}{H} \right)^\alpha \left( \frac{P}{\sigma_f} \right)^\gamma \]  

\[ h_2 = \frac{K_2 m_u P}{\sigma_f \rho_w d_n u} \theta^a \left( \frac{FH}{u} \right)^b \left( \frac{D}{H} \right)^c \left( \frac{P}{\sigma_f} \right)^d \]  

where \( h_1 \) is used to predict the depth of cut under normal cutting while \( h_2 \) applies for oscillation cutting. Dimensional analysis technique was used during the development process and nonlinear regression was adopted for the constants estimation of the models. Predicted trends were compared with the experimental data. The numerical analysis was conducted based on the percentage deviations of the model predicted values with respect to the corresponding experimental results. The predicted depths of cut produced by the two models under the corresponding cutting condition were also studied to estimate their predictive capability. The assessment has been shown that the predictions of the two models were in reasonable agreement with the experimental results.

Before developing the models for multipass cutting, it is necessary to define the depths generated by an individual pass as well as by a multipass operation (including a number of passes). While the depth of cut made by an individual pass is defined as the incremental cutting depth, the depth produced by a multipass operation is referred...
to as the depth of cut. In the development process, the models of incremental cutting depth for a given pass are first derived which are equivalent to the depth of cut models in the single pass oscillation cutting. Then, the depth of cut for a multipass operation is established by the sum of the incremental cutting depth from each pass. The prediction ability of the models will be assessed using the similar methods when assessing the models of single pass oscillation cutting.

### 5.4.1. Material removal by individual particles in a given pass

Comparing the cutting conditions between multipass cutting and single pass cutting, it has been noticed that while in the case of single pass cutting, the value of standoff distance is a constant, in multipass cutting each pass has a different standoff distance depending on the initial standoff distance between the nozzle and workpiece for the first pass as well as the depth of cut produced before this pass. The actual standoff distance for a given pass is the sum of initial standoff distance and the accumulation of preceding incremental cutting depths. This constantly increased standoff distance with the number of passes in multipass cutting leads to a series of other variations in the cutting process, such as, average kerf width, the effective number of particles involved in the cutting action and the particle attack angle. Accordingly, the depth of cut varies from pass to pass. In developing the models for multipass cutting, all the above variations must be considered and generalised to accommodate different passes. Therefore, the development process is primarily based on the same fundamentals used in developing single pass oscillation cutting, with the extension to cover the cutting conditions in different passes. For this purpose, the expression of actual standoff distance is first determined to suit different passes.

Assuming that the standoff distance for the \( n \)th pass is \( H_n \), then it can be expressed as

\[
H_n = H + \sum_{i=1}^{i=n-1} H_i
\]

\[
= H + h_p
\]  \hspace{1cm} (5.4)

where
where $H$ is the initial standoff distance; $H_i$ is the incremental cutting depth by the $i^{th}$ pass; and $h_p$ is the depth of cut made before the $n^{th}$ pass. Equation (5.4) must be used instead of $H$ in Equations (5.2) and (5.3) in the derivation of the depth of cut produced by multipass cutting.

By using the same assumptions in developing the depth of cut models for single pass oscillation cutting, the overall material removal rate in $n^{th}$ pass ($MRR$) can be given by

$$MRR_n = u_n h_n w_n$$  \hspace{1cm} (5.6)

where $u_n$, $h_n$ and $w_n$ refer to nozzle traverse speed, incremental cutting depth and the average kerf width for $n^{th}$ pass. The study conducted in the last multipass cutting experiment has found that effective jet diameter impacting on the target material reduces with the number of passes. This in turn reduces the average kerf width in the following passes. Thus

$$MRR_n = A_1 u_n h_n w_1$$  \hspace{1cm} (5.7)

where $w_1$ is the average kerf width produced by the first pass and $A_1$ is a constant introduced to consider the variation of average kerf width for different passes. According to the previous assumptions, the average kerf width for the first pass is approximately equal to the jet diameter which is in turn equal to the nozzle diameter. Thus, Equation (5.7) becomes

$$MRR_n = A_1 u_n h_n d_n$$  \hspace{1cm} (5.8)

According to Equation (4.3) in chapter 4, the overall material removal rate can also be expressed as

$$MRR_1 = K_e \frac{m_a}{m} R$$  \hspace{1cm} (5.9)
where \( MRR_I \) is the overall material removal rate for the first pass; \( m_o/m \) is the number of abrasive particles in the jet per unit time; \( R \) is the average contribution of a particle to the material removal; and \( K_e \) is an efficiency factor considering the fact that not all abrasive particles in the jet impinge or have sufficient energy to cut the material due to the collision among the particles. Generalizing Equation (5.9) to accommodate other passes, the overall material removal rate for the \( n \)th pass can be given as

\[
MRR_n = K_m K_e \frac{m_o}{m} R
\]  

(5.10)

The analysis in the last multipass cutting experiment indicated that the jet energy at the cutting point constantly decreased with the number of passes. Thus, for a given pass, the consequence of this energy decrease is the reduced number of abrasive particles to effectively cut the material. A coefficient, \( K_m \), is therefore introduced to account for the changes in the effective number of abrasive particles in the \( n \)th pass.

Incorporating Equation (5.10) into Equation (5.8), it gives

\[
A_1 u_n h_n d_n = K_m K_e \frac{m_o}{m} R
\]  

(5.11)

Equation (5.11) can be rewritten as

\[
h_n = A_2 \frac{m_o}{mu_o d_n} R
\]  

(5.12)

where \( A_2 \) is a constant generalising the constants of \( A_1 \), \( K_m \) and \( K_e \) (\( A_2 = K_m K_e / A_1 \)).

Similar to the process in deriving the models of single pass oscillation cutting, the average material removed by an individual particle, \( R \), then is determined by relating its influencing variables with the aid of dimensional analysis technique. Based on the analysis in discussing the effect of major cutting variables on removal process in chapter 3, \( R \) is associated with the flow stress of target material, \( \sigma_f \), the impact velocity of the particle, \( v \), the average mass of an individual particle, \( m \), and the particle attack angle, \( \alpha_n \), in the discussed cutting pass. An equation similar to Equation (4.4) can be expressed as
By applying standard dimensional analysis and the power law formation method (refer to chapter 4 and Appendix A for the details of derivation), Equation (5.13) is solved as

\[
R = \frac{A_3 v^2}{\sigma_f} \alpha_n^j
\]

where \( A_3 \) is a constant introduced and \( j \) represents the exponent for \( \alpha_n \). In the next step to determine the expression of the particle attack angle in an individual pass \( (\alpha_n) \), the derivation process is separated into multipass normal cutting and multipass oscillation cutting.

**5.4.2 The incremental cutting depth for multipass normal cutting**

When analysing the dependence of the particle attack angle on various independent variables in chapter 4, it was found that the attack angle could be expressed as a function of nozzle traverse speed \( u \), water pressure \( P \), standoff distance \( H \), average particle diameter \( D \) and the flow stress of target material \( \sigma_f \). Therefore, extending Equation (4.11) to multipass normal cutting by considering the mathematical expression of actual standoff distance in Equation 5.4, the attack angle for the \( n \)th pass, \( \alpha_{n1} \), is

\[
\alpha_{n1} = \phi(u_n, P, H + h_p, D, \sigma_f)
\]

Applying the same approaches to the deriving process, the function relation of the attack angle for the \( n \)th pass is obtained

\[
\alpha_{n1} = B_1 \left( \frac{D}{H + h_p} \right)^{x_1} \left( \frac{P}{\sigma_f} \right)^{y_1}
\]

where \( B_1, x_1 \) and \( y_1 \) are dimensionless constants. By replacing \( \alpha_n \) in Equation (5.14) with \( \alpha_{n1} \) from Equation (5.16), the material removal by an individual particle is given by
\[ R = \frac{B_2 m \nu^2}{\sigma_f} \left( \frac{D}{H + h_p} \right)^x \left( \frac{P}{\sigma_f} \right)^y \]  

(5.17)

where \( B_2, x \) and \( y \) are introduced to generalise the constants, such that \( B_2 = A_3 B_1 \), \( x = x/j \), and \( y = y/j \).

Substituting Equation (5.17) into Equation (5.12) and replacing the incremental cutting depth \( h_n \) with \( h_n^I \) to denote incremental cutting depth without nozzle oscillation give

\[ h_n^I = \frac{B_2 m \nu^2}{\sigma_f d_n u_n} \left( \frac{D}{H + h_p} \right)^x \left( \frac{P}{\sigma_f} \right)^y \]  

(5.18)

In order to determine the particle velocity, \( v \), the Bernoulli’s equation and the momentum transfer equation as expressed in Equations 4.20 and 4.21 respectively in chapter 4 are used. After making the necessary transformations, the final form of incremental cutting depth in multipass normal cutting can be expressed as

\[ h_n^I = \frac{K_1 m \nu P}{\sigma_f \rho \omega d_n u_n} \left( \frac{D}{H + h_p} \right)^x \left( \frac{P}{\sigma_f} \right)^y \]  

(5.19)

where \( K_1 \) is a constant generalizing all the constants designated in the derivation process.

5.4.3 The incremental cutting depth for multipass oscillation cutting

According to the analysis in deriving the particle attack angle in single pass oscillation cutting, this angle is controlled by five parameters considered in cutting without nozzle oscillation, i.e. \( u, P, H, D \) and \( \sigma_f \), and the oscillation cutting, i.e. oscillation angle \( \theta \) and frequency \( F \). Thus, applying this relationship to multipass cutting and substituting the standoff distance with \( H + h_p \), the particle attack angle in the \( n \)th pass, \( \alpha_{n2} \), can be expressed by the following function

\[ \alpha_{n2} = \psi(\theta, F, u_n, P, H + h_p, D, \sigma_f) \]  

(5.20)
Using the same technique in modelling the attack angle for multipass normal cutting, $\alpha_{n2}$ can be given

$$\alpha_{n2} = C_0 \theta^{a_1} \left( \frac{F(H + h_p)}{u_n} \right)^{b_1} \left( \frac{D}{H + h_p} \right)^{c_1} \left( \frac{P}{\sigma_f} \right)^{d_1}$$

(5.21)

where $C_0, a_1, b_1, c_1$ and $d_1$ are constants. Adopting the similar approaches in deriving the model of incremental cutting depth for multipass normal cutting, the form of incremental cutting depth for multipass oscillation cutting is given by

$$h_{n2} = \frac{K_2 m_d P}{\sigma_f \rho_w d_n u_n} \theta^a \left( \frac{F(H + h_p)}{u_n} \right)^{b} \left( \frac{D}{H + h_p} \right)^{c} \left( \frac{P}{\sigma_f} \right)^{d}$$

(5.22)

where $K_2, a, b, c$ and $d$ denote constants needed to be determined by experiment.

The depths of cut in multipass cutting are an accumulative result of incremental cutting depths. Thus, the depths of cut after $n$ passes are given by

$$h_1 = \sum_{i=1}^{i=n} h_{i1}$$

(5.23)

for normal cutting and

$$h_2 = \sum_{i=1}^{i=n} h_{i2}$$

(5.24)

for oscillation cutting.

### 5.4.4 Assessment of the predictive ability of the models

A nonlinear regression analysis technique is used on the experimental multipass cutting of 87% alumina ceramic to estimate the constants in Equation (5.19) and (5.22) and to assess the predictive ability. For this purpose, the incremental cutting depths for an individual pass were calculated and the actual standoff distances for each pass were in turn obtained based on the measured depth of cut values. The
coefficients and the exponents of the regression analysis are obtained as shown in Table 5.5.

Table 5.5. Coefficient and exponent in the models.

<table>
<thead>
<tr>
<th>Multipass normal cutting</th>
<th>Multipass oscillation cutting</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_1$</td>
<td>48.083</td>
</tr>
<tr>
<td>$K_2$</td>
<td>2.079</td>
</tr>
<tr>
<td>$x$</td>
<td>0.056</td>
</tr>
<tr>
<td>$a$</td>
<td>0.415</td>
</tr>
<tr>
<td>$y$</td>
<td>1.676</td>
</tr>
<tr>
<td>$b$</td>
<td>0.083</td>
</tr>
<tr>
<td>$c$</td>
<td>0.176</td>
</tr>
<tr>
<td>$d$</td>
<td>0.975</td>
</tr>
</tbody>
</table>

Accordingly, Equations (5.19) and (5.22) are expressed as

\[
h_{n1} = \frac{48.083m_aP}{\sigma_f \rho_w d_n u_n} \left( \frac{D}{H + h_p} \right)^{0.056} \left( \frac{P}{\sigma_f} \right)^{1.676} \quad (5.25)
\]

and

\[
h_{n2} = \frac{2.079m_aP \theta^{0.415}}{\sigma_f \rho_w d_n u_n} \left( \frac{F(H + h_p)}{u_n} \right)^{0.083} \left( \frac{D}{H + h_p} \right)^{0.176} \left( \frac{P}{\sigma_f} \right)^{0.975} \quad (5.26)
\]

Rearranging Equations (5.25) and (5.26) results in

\[
h_{n1} = 48.083 \frac{m_aP^{2.676}D^{0.056}}{\rho_w d_n u_n \sigma_f^{2.676} (H + h_p)^{0.056}} \quad (5.27)
\]

and

\[
h_{n2} = 2.079 \frac{m_aP^{1.975} \theta^{0.415} F^{0.083} D^{0.176}}{\rho_w d_n \sigma_f^{1.975} u_n^{1.083} (H + h_p)^{0.093}} \quad (5.28)
\]

Two mathematical models of the depth of cut for AWJ multipass normal cutting and multipass oscillation cutting have been established as shown in Equations (5.27) and (5.28) based on the models developed for single pass oscillation cutting. An
inspection was first made to examine the rationality of the items included in the two models. It can be seen that the two models in the form of power function realistically express the incremental cutting depths produced by the use of major process variables in multipass cutting combined with nozzle oscillation operation. The items with the different exponents in numerator or in denominator in the equations correctly reflect the influence of major cutting variables on the depth of cut. As shown by Equation (5.27) and (5.28), the models correctly represent the effects of water pressure $P$, abrasive mass flow rate $m_w$, nozzle traverse speed $u_n$ and the actual standoff distance $(H+h_p)$ with the positive exponents in the numerator or denominator. Of these four variables, water pressure and abrasive mass flow rate are located in the numerator with the positive exponent, which means that the increase of these two variables are associated with an increase in the depth of cut. In addition, included in the denominator with positive exponent, nozzle traverse speed and standoff distance have a negatively effect on the depth of cut. They present the consistent trends with that discussed in the analysis of single pass oscillation cutting experiment and multipass cutting experiment. Particularly, the presence of standoff distance in the denominator with positive exponent reasonably reflects the decreased trends of incremental cutting depth with the increase in the number of passes, which is in agreement with the finding in the study of multipass cutting experiment showing that the rate of increase in the depth of cut tends to decrease with the number of passes.

When incorporating oscillation parameters ($\theta$ and $F$) into the model in Equation (5.28), the oscillation angle and oscillation frequency with the positive exponents in the numerator also correctly predicts the trends of incremental cutting depth. Other process parameters, such as average particle diameter $D$, nozzle diameter $d_n$ and target material flow stress $\sigma_f$, play the similar roles in the models as described in the earlier chapter. The brief inspection has demonstrated that the forms of the models are generally feasible and consistent with the experimental trends of the depth of cut in multipass cutting both with and without nozzle oscillation.
A qualitative assessment of the two models was conducted to compare the predicted depths of cut generated by the models with the experimental data. The trends of predicted values alongside the experimentally measured values for multipass normal cutting are illustrated in Figure 5.14. It is observed that the incremental cutting depth for individual pass and the cumulative depth of cut after all passes all follow the trends describe in the experimental study of multipass cutting performance. When cutting under water pressure of 275MPa combined with nozzle traverse speed of 2mm/s for all the three passes (Figure 5.14 (a) and (b)), the incremental cutting depth slightly reduces and the cumulative depth of cut increases with the number of passes, agreeing with the finding in the previous study that the rate of the increase in depth of cut decreases with the number of passes. The cutting using higher water pressure...
of 345MPa and larger nozzle traverse speed of 6mm/s presents the same trends (Figure 5.14 (c) and (d)). The high correlation between predicted values and the experimental data is observed for the conditions shown in the Figures except for the incremental cutting depth generated at the first pass using water pressure of 275MPa and nozzle traverse speed of 2mm/s.

Similar trends are observed for multipass cutting combined with nozzle oscillation. Some representative comparisons of the predictions from the model with the corresponding measured data are shown in Figure 5.15. It can be seen from the figure that Equation (5.28) again qualitatively expresses the trends of incremental cutting depth and the cumulative depth of cut with the number of passes. It is indicated that while a slightly decreased trend is observed for incremental cutting depth when cutting using water pressure of 275MPa and nozzle traverse speed of 6mm/s (Figure 5.15 (a)), the cumulative depth of cut presents a constantly increased trend with the increase in the number of passes (Figure 5.15 (b)). This observation applies to other cases in the experimental range (Figure 5.15 (c) and (d)). The close correlation between the predictions and the experimental results verifies the feasibility of the model in predicting the depth of cut in multipass cutting with controlled nozzle oscillation.
In order to further verify the two models and to estimate their predictive capability, a quantitative assessment has been conducted in the form of the percentage deviation between predicted and measured incremental cutting depths. The histograms evaluated using Equation (5.27) for multipass normal cutting and Equation (5.28) for multipass oscillation cutting have been established and shown in Figure 5.16. Table 5.6 summaries the means and standard deviations of the percentage deviations of predicted incremental cutting depth. From Table 5.6, it can be noted that for these predictions in normal cutting the mean percentage deviation is 2.3% and the standard deviation is 16.25%. Similarly, the model’s prediction for multipass cutting combined with nozzle oscillation results in a mean percentage deviation of 1.4% with the standard deviation of 13.95%.

Figure 5.15. Comparisons between model predictions and experimental data for multipass oscillation cutting: (a), (b) $P=275\text{MPa}, u_1=u_2=u_3=6\text{mm/s}, \theta=4^\circ, F=6\text{Hz}$. (c), (d) $P=345\text{MPa}, u_1=u_2=u_3=4\text{mm/s}, \theta=4^\circ, F=6\text{Hz}$. 
Figure 5.16. Percentage deviations between predicted and experimental incremental cutting depth in multipass cutting: (a) Multipass normal cutting. (b) Multipass oscillation cutting.

Table 5.6. Mean and standard deviation of measured and predicted depth of cut in multipass cutting.

<table>
<thead>
<tr>
<th></th>
<th>Multipass normal cutting</th>
<th>Multipass oscillation cutting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (%)</td>
<td>2.3</td>
<td>1.4</td>
</tr>
<tr>
<td>Standard deviation (%)</td>
<td>16.25</td>
<td>13.95</td>
</tr>
</tbody>
</table>

The verification of the models has shown that the general forms of the models are applicable for brittle materials. Both the qualitative and quantitative assessments of the models have demonstrated that the depth of cut models can give reasonably good predictions and can be used to adequately predict this cutting performance measure. These developed models provide helpful guideline in the correct selection of the process parameters and the optimization of the AWJ multipass cutting process.

5.5 Conclusions

In this chapter, an experimental study of major cutting performance machined by AWJ multipass cutting with controlled nozzle oscillation has been conducted on 87%
alumina ceramic plates. The cutting capacity in terms of depth of cut, kerf taper, top kerf width and bottom kerf width was evaluated with reference to major process variables including the nozzle traverse speed, number of passes, oscillation parameters and water pressure. Best cutting conditions have been selected to achieve optimum performance and furthermore, the general guidelines of selection of cutting variables have been suggested. The superiority of multipass cutting over the single pass cutting has also been analysed from the viewpoint of technology and economy. The results of this study show that multipass cutting with nozzle oscillation can significantly enhance cutting performance compared to single pass cutting in the same elapsed time or if the job requirement is known the cutting time and thus the cutting cost can be minimized by resorting to this novel cutting technique.

It has been found that the depth of cut dramatically increases with the number of passes. Similar to single pass oscillation cutting, lower nozzle traverse speed, high water pressure and proper use of oscillation parameters are beneficial to obtain high depth of cut. While it is obvious that the use of multipass can increase depth of cut, the rate of increase in the depth of cut decreases with the number of passes. An optimum selection method for the combination of the nozzle traverse speed and the number of passes was then suggested by drawing a group of equal elapsed time lines. The peak point on the line was used to determine the best combination of nozzle traverse speed and the number of passes. It was found by using this technique that to achieve maximized depth of cut in this experiment, the combination of three passes with the nozzle traverse speed at 6mm/s is the best choice.

Kerf taper constantly decreases with an increase in the number of passes and water pressure but a decrease in nozzle traverse speed and the use of oscillation technique. It has been found that the nozzle traverse speed in the first pass is most important to affect kerf taper. Therefore, small nozzle traverse speed should be always used for the first pass and a comparatively high traverse speed could be selected for the following passes along with the use of nozzle oscillation for all the passes. Such a combination does not sacrifice kerf taper but at the same time it can reduce the total cutting time and increase the productivity in practice.

An increase in the number of passes, water pressure, decrease in the nozzle traverse speed or the incorporation of nozzle oscillation increase kerf width. However, the use
of nozzle oscillation in multipass cutting can effectively decrease the rate of the increase in kerf width. It was also found that the extent of effect of the number of passes on top kerf width and bottom kerf width is different. Bottom kerf width is easier to widen than top kerf width with the number of passes, thus kerf taper tends to reduce with the number of passes. As the study of kerf width is often with an intention of minimizing kerf taper in practice, the selection of the cutting parameters should be made together with the consideration of kerf taper.

Mathematical models for the depth of cut for both multipass normal cutting and multipass oscillation cutting were developed in this section. Based on the developed models of depth of cut for single pass oscillation cutting in chapter 4, the fundamental methodologies used in the development of these models are first to develop the incremental cutting depth for each pass and then to sum these depths to obtain the total depth of cut for a multipass operation. For this purpose, the equations of actual standoff distance for each pass were derived. The incremental cutting depth for each individual pass was formulated by using dimensional analysis technique and by relating the overall material removal rate to the accumulated material removal by individual particles. The developed two models were assessed qualitatively and quantitatively using the experimental data of incremental cutting depths for an individual pass and the depth of cut for a multipass operation. The results are satisfactory and it can be claimed that these two models are adequate to predict the depth of cut in multipass cutting.
Chapter 6

Final Conclusions and Future Study

The major objective of this project is to investigate the cutting performance in AWJ cutting of alumina ceramics with controlled nozzle oscillation under small oscillation angles to apply AWJ machining to both straight-slit cutting and contouring. The objective has been achieved through the attempts presented in this thesis. A comprehensive literature survey of the state of the art of AWJ machining was first conducted in order to fully understand the techniques and current development in this area and to develop the plan and methodology in this project. A substantial experimental study was then performed to investigate the kerf characteristics machined by AWJ single pass oscillation cutting on alumina ceramics under small oscillation angles. It was followed by further experimental investigation into the cutting performance produced on alumina ceramics by multipass cutting with controlled nozzle oscillation. The mathematical models for major cutting performance measures in single pass oscillation cutting and the depth of cut in multipass cutting were developed in order to provide effective means of selecting process parameters and optimizing the cutting process. These achievements made during this project are summarized in the following.

The experimental study on the kerf characteristics generated by AWJ single pass oscillation cutting of alumina ceramics has found that the kerf profiles machined by oscillation cutting are like that in normal cutting. The kerf surface of a through cut is characterized by an upper smooth zone and a lower rough zone. A non-through cut has three zones which include an upper smooth zone, a middle striation zone and a lower enlarged pocket. When using the corresponding cutting conditions with normal cutting, oscillation cutting can produce higher depth of cut and smooth zone.

The major cutting performance measures represented by the surface roughness, the depth of cut, smooth depth of cut, kerf taper, top kerf width and bottom kerf width were analysed with regard to the process parameters including oscillation angle, oscillation frequency, water pressure, nozzle traverse speed, standoff distance and abrasive mass flow rate. It has been found that oscillation frequency has the most
effect on the depth of cut, smooth depth of cut, kerf taper and bottom kerf width in addition to its second important effect on kerf surface roughness and top kerf width. An increase in oscillation frequency is associated with an increase in depth of cut, surface roughness and smooth depth of cut but a decrease in kerf taper. These findings indicate that the use of high oscillation frequency is preferred to obtain overall good cutting performance. The study has also found that small oscillation angles as ranged in this experiment have little effect on most of the performance quantities. Kerf surface roughness and the depth of cut initially increase with oscillation angle and then decrease when oscillation angle further increases, while smooth depth of cut shows a reverse trend. Kerf taper constantly decrease as oscillation angle increases. Since oscillation angle has no significant effect on cutting performance when it is small, it is recommended to use small oscillation angle which is beneficial to contouring in practice. Nevertheless, this experimental study has resulted in a new finding related to the correct combination of oscillation angle and oscillation frequency. It was indicated that oscillation cutting can increase the depth of cut only if the used oscillation strength (the product of oscillation angle and oscillation frequency) is above a critical value. This concept clarifies the previous report showing that the superimposing of oscillation operation on the normal AWJ cutting can constantly increase the depth of cut.

Of the other process parameters, water pressure, nozzle traverse speed and abrasive mass flow rate have the similar effect on the major cutting performance with that in normal cutting. High water pressure increases depth of cut, smooth depth of cut and surface roughness but reduces kerf taper, which means that high water pressure should be used if surface roughness requirement is satisfied. As nozzle traverse speed increases, surface roughness and kerf taper increase but depth of cut and smooth depth of cut decrease. Thus, low traverse speed produces overall good cutting performance but is at the cost of sacrificing productivity. Large abrasive mass flow rate helps increase depth of cut and smooth depth of cut but shows no significant effect on surface roughness and kerf taper. Therefore, it is recommended to use large abrasive mass flow rate in oscillation cutting. It was also found from this experiment that standoff distance has changed its way in affecting the cutting process in oscillation cutting. Unlike its role in normal cutting which reduces the jet energy as standoff distance increases, it affects the cutting performance together with the
oscillation angle and oscillation frequency. High standoff distance broadens the jet scanning scope which counteracts the decrease in jet energy and may be beneficial to obtain good cutting performance. An increase in standoff distance initially increases and then reduces surface roughness and kerf taper. Nevertheless, it first decreases and then increases smooth depth of cut but shows no apparent effect on depth of cut. Therefore, to achieve an overall good cutting performance, low standoff distance should be selected.

This experimental study conclusively demonstrated that nozzle oscillation cutting at small angle is still a feasible technology to increase the overall cutting performance compared to oscillation cutting with medium and large angles. More importantly, this machining strategy can broaden the application of AWJ cutting to contouring which is a more common operation in practice. However, this experimental study also reveals that correct selection of the oscillation parameters together with the other corresponding process parameters is critical to achieve overall good cutting performance. Incorrect selection and combination of cutting variables deteriorate rather than improve the cutting performance. Consequently, the trends obtained in the analysis of the cutting performance, the large quantity of experimental data and the understanding of the science behind the cutting process form the useful guidelines for the selection of optimum oscillation parameters under different process conditions.

In order to correctly select the process parameters and to optimise the cutting process, the mathematical models for major cutting performance measures have been developed. The predictive models for the depth of cut in normal cutting and oscillation cutting were developed based on the consideration that AWJ cutting is an accumulated process by individual abrasive erosion of target material. The material removal by a single particle was then formulated by using a dimensional analysis technique and relating to the independent variables including particle velocity, average particle mass, particle attack angle and the flow stress of target material. In order to find required particle attack angle, the dimensional analysis was used again to relate it to its influencing parameters such as nozzle traverse speed, water pressure, standoff distance, average particle diameter and material flow stress in normal cutting. For oscillation cutting, the similar approach was applied to formulate the
particle attack angle in relation to oscillation angle and oscillation frequency in addition to the parameters to affect the particle attack angle in normal cutting. The general forms of depth of cut models for normal cutting and oscillation cutting were then obtained and the empirical coefficients in the equations were determined using regression analysis on the experimental data. Other empirical models for surface roughness, smooth depth of cut, kerf taper, top kerf width and bottom kerf width were performed with the aid of nonlinear regression analysis. The developed mathematical models have been verified both qualitatively and quantitatively. It was concluded that good agreement has been achieved between the predicted values from the models and the experimental data and therefore they are adequate to predict the cutting performance when using nozzle oscillation technique.

A further experimental investigation of AWJ cutting of alumina ceramics was carried out in order to study the application of AWJ oscillation technique in multipass cutting. The cutting capacity assessed by the depth of cut, kerf taper, top kerf width and bottom kerf width was analysed in relation to the nozzle traverse speed, the number of passes, oscillation parameters and water pressure. While high nozzle traverse speed over the same kerf a number of passes can achieve overall better cutting performance than low traverse speed with single pass in the same elapsed time, it was found that the different combination of nozzle traverse speed with the number of passes significantly affects cutting process. Using the ratio of nozzle traverse speed to the number of passes, the optimum combination of traverse speed with the number of passes can be determined for a given cutting condition to achieve maximum depth of cut. It was shown that three passes coupled with the largest traverse speed in this experiment produced maximum depth of cut. It has also been found that the multipass cutting with low nozzle traverse speed in the first pass and a comparatively high traverse speed for the following passes is a sensible choice for a small kerf taper requirement. The increase in the top kerf width and the bottom kerf width is obtained when multipass cutting is applied. However, multipass cutting more significantly affects bottom kerf width than top kerf width. Similar conclusions are reached when oscillation technique and high water pressure are used in multipass cutting as is the case in single pass cutting, i.e., the incorporation of oscillation and the use of high water pressure can greatly increase the depth of cut, increase kerf width but reduce kerf taper.
The predictive models for the depth of cut in multipass normal cutting and multipass oscillation cutting were developed. It was found that in multipass cutting, the average kerf width, the effective number of particles involved in the cutting action and the particle attack angle are different from that in single pass cutting due to the variation of actual standoff distance in different passes. The equation of actual standoff distance for different passes was first formulated. The variations in the average kerf width and the effective number of particles for different passes were considered by introducing a constant in the development process. The models for the incremental cutting depth were then derived by using the same approaches with that in developing the depth of cut models for single pass cutting. The required models for the depth of cut in multipass cutting were performed by the sum of the incremental cutting depth from each pass. The two mathematical models were assessed through analysing the predicted trends and comparing the predicted values with the experimental results. The assessment demonstrated the adequacy and feasibility of employing these models for the prediction of the depth of cut and the selection of cutting parameters in multipass cutting.

Based on the results, findings and the achievements performed from this project, further studies need to be carried out for the better use of nozzle oscillation technique and thorough understanding of the science related to this novel technology:

The mechanisms in AWJ cutting with controlled nozzle oscillation remain certain assumptions. Particularly, the science behind which oscillation operation can improve or deteriorate the cutting performance under different cutting conditions is unknown. To further continue the current study, a visualisation study needs to be conducted by a high speed camera on transparent materials to investigate the kerf formation process and the macro material removal process when AWJ cutting with controlled nozzle oscillation. This study will provide a significant insight into the mechanisms of the effect of nozzle oscillation on kerf formation process. It will also enable to develop a basis for fully modelling the cutting process and a strategy for further increase of the cutting performance.

After an in-depth understanding of the mechanisms in AWJ cutting with controlled nozzle oscillation, the mathematical models for other cutting performance measures need to be refined with the help of theoretical approach. In this project, the
development of mathematical models for the smooth depth of cut, kerf surface roughness, kerf taper, top kerf width and bottom kerf width is completed using empirical approach due to the insufficient understanding of a number of fundamental phenomena in AWJ oscillation cutting. While these models can offer adequate predictions for the required cutting performance measures, their reliance on the use of experimental data limits their application to the tested cutting conditions. More feasible and practical mathematical models for these performance quantities need to be established in the future.

In this project, only the cutting performance of AWJ straight-slit cutting with controlled nozzle oscillation was studied. However, AWJ profile cutting or contouring is a more common cutting process in practice. It is therefore necessary to conduct investigations to study the application of nozzle oscillation in contouring. This study will result in the detailed knowledge of the technological cutting performance and kerf geometrical errors under different kerf curvatures, nozzle oscillation parameters and other cutting conditions. This can be used to develop a strategy for selecting the optimum cutting and nozzle oscillation parameters for AWJ contouring.
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Appendix A

The Derivation of the Material Removed by a Single Particle

Starting with Equation (4.4), the material removed by a particle is expressed according to its influencing variables

\[ R = f(\sigma_f, v, m, \alpha) \]  

(A1)

The units of the variables involved in Equation (A1) are

\[ R = [L^3], \sigma_f = [MT^{-2}L^{-1}], v = [LT^{-1}], m = [M], \text{ and } \alpha = [1] \]  

(A2)

The dimensions of each term on the right hand side of the Equation must equal the dimensions of \( R \) in order for Equation (A1) to be dimensionally consistent. When the dimensions are substituted for the variables in the equation, the dimensional formula of the material removed by individual particles can now be written as:

\[ [L^3]^a [MT^{-2}L^{-1}]^b [LT^{-1}]^c [M]^d = [M^0L^0T^0] \]  

(A3)

As there are three fundamental dimensions and the total numbers of variables are five in this equation, two Pi groups are expected from the dimensional analysis. Selecting \( \sigma_f, v, \) and \( m \) as repeating variables, the first Pi group can be constructed by multiplying \( R \) by the repeating variables.

\[ \pi_1 = R \sigma_f^a v^b m^c \]  

or

\[ [M^0L^0T^0] = [L^3][MT^{-2}L^{-1}]^b [LT^{-1}]^c [M]^d \]  

(A4)

By equating the powers of the three fundamental units in turns on both sides of Equation (A5), a set of simultaneous equations are obtained which can be solved to calculate these constants. The simultaneous equations can be written as

\[ a_1 + c_1 = 0 \]
\[ 3 - a_1 + b_1 = 0 \]
\[ -2a_1 - b_1 = 0 \]  

(A6)
Solving Equation (A6) gives
\[ a_1 = 1, \quad b_1 = -2, \quad \text{and} \quad c_1 = -1 \quad (A7) \]

Thus
\[ \pi_1 = R\sigma_f v^2 m^{-1} \quad (A8) \]

or
\[ \pi_1 = \frac{R\sigma_f}{mv^2} \quad (A9) \]

Because \( \alpha \) is already a dimensionless variable, the second Pi group is
\[ \pi_2 = \alpha \quad (A10) \]

The relation between the two dimensionless products can now be equated as
\[ \pi_1 = f(\pi_2) \quad (A11) \]

By substituting Equations (A9) and (A10) into (A11), the complete dimensional equation is as follows
\[ \frac{R\sigma_f}{mv^2} = A\alpha^j \quad (A12) \]

which can be rewritten as
\[ R = \frac{Amv^2}{\sigma_f} \alpha^j \quad (A13) \]


Appendix B

The Derivation of the Attack Angle in AWJ Normal Cutting

From Equation (4.11), the attack angle in AWJ cutting without oscillation can be expressed as

\[ \alpha_i = \phi(u, P, H, D, \sigma_f) \]  

(B1)

The variables in Equation (B1) have the following dimensions respectively

\[ \alpha_i = [I], \ u = [LT^{-1}], \ P = [MT^{-2}L^{-1}], \ H = [L], \ D = [L], \ \sigma_f = [MT^{-2}L^{-1}] \]  

(B2)

There are three fundamental dimensions (L, T, and M) and six variables involved in this equation. Therefore, three independent dimensionless products can be obtained in which \( \alpha_i \) is the first Pi group. Selecting \( u, H \) and \( \sigma_f \) as repeating variables, the second Pi group can be formulated by multiplying \( D \) by the three repeating variables

\[ \pi_2 = DH^a u^b \sigma_f^{c_i} \]  

or

\[ [L^a T^b M^0] = [LL^a (LT^{-1})^b (MT^{-2}L^{-1})^c] \]  

(B3)

A set of simultaneous equations can be obtained from Equation (B3) by equating the powers of L, T, and M on both sides at a time.

\[
\begin{align*}
1 + a_1 + b_1 - c_1 &= 0 \\
- b_1 - 2c_1 &= 0 \\
c_1 &= 0
\end{align*}
\]  

(B4)

From Equation (B4),

\[ a_1 = -1, \ b_1 = 0 \]  

and \( c_1 = 0 \)  

(B5)

Therefore,

\[ \pi_2 = \frac{D}{H} \]  

(B6)

Similarly, construct the third Pi group using \( P \) and the repeating variables.
\[ \pi_3 = PH^{a_3} u^{b_3} \sigma_f^{c_3} \] \hspace{1cm} (B7)

Its equivalent dimensional form is
\[ [L^0 T^0 M^0] = \left[(MT^{-2} L^1)^{a_3} (LT^{-1})^{b_3} (MT^{-2} L^1)^{c_3}\right] \] \hspace{1cm} (B8)

The corresponding simultaneous equations of Equation (B8) is as follows
\[
\begin{align*}
-1 + a_2 + b_2 - c_2 &= 0 \\
-2 - b_2 - 2c_2 &= 0 \\
1 + c_2 &= 0
\end{align*}
\] \hspace{1cm} (B9)

Solving Equation (B9) yields
\[ a_2 = b_2 = 0 \text{ and } c_2 = -1 \] \hspace{1cm} (B10)

Substituting Equations (B10) into Equation (B7), the third Pi group is solved
\[ \pi_3 = \frac{P}{\sigma_f} \] \hspace{1cm} (B11)

The functional relationship among \( \pi_1, \pi_2, \) and \( \pi_3 \) can be written as
\[ \alpha_1 = \phi \left( \frac{D}{H}, \frac{P}{\sigma_f} \right) \] \hspace{1cm} (B12)

Based on the form of power function, the dimensional equation is rewritten as
\[ \alpha_1 = B_1 \left( \frac{D}{H} \right)^{\kappa_1} \left( \frac{P}{\sigma_f} \right)^{\lambda_1} \] \hspace{1cm} (B13)
Appendix C

The Derivation of the Attack Angle in AWJ Oscillation Cutting

According to Equation (4.24) the attack angle in oscillation cutting is

$$\alpha_2 = \psi(\theta, F, u, P, H, D, \sigma_f)$$ \hspace{1cm} (C1)

The dimensions for each variable in this function are as follows

$$\alpha_2 = [1], \ \theta = [1], \ F = [T^{-1}], \ u = [LT^{-1}], \ P = [MT^{-2}L^{-1}], \ H = [L], \ D = [L], \ \text{and} \ \sigma_f = [MT^{-2}L^{-1}]$$ \hspace{1cm} (C2)

which are also expressed by three fundamental dimensions (L, T and M). Therefore, five Pi groups can be drawn in which \(\alpha_2\) and \(\theta\) are identified as \(\pi_1\) and \(\pi_2\). Selecting \(H, u\) and \(\sigma_f\) as repeating variables, the third Pi group \(\pi_3\) can be calculated using the product of oscillation frequency \(F\) with the repeating variables

$$\pi_3 = FH^a u^b \sigma_f^c$$ \hspace{1cm} (C3)

Equation (C3) is equivalent to

$$\begin{bmatrix} L^a T^0 M^0 \end{bmatrix} = \begin{bmatrix} L^{-1} \left( LT^{-1} \right)^b \left( MT^{-2} L^{-1} \right)^c \end{bmatrix}$$ \hspace{1cm} (C4)

Solving the equation by equating the exponents of L, T, and M in turn, the constants \(a_1, b_1, \) and \(c_1\) are obtained

$$a_1 = 1, \ b_1 = -1, \ \text{and} \ c_1 = 0$$ \hspace{1cm} (C5)

Thus

$$\pi_3 = \frac{FH}{u}$$ \hspace{1cm} (C6)

By multiplying particle diameter \((D)\) and water pressure \((P)\) by the repeating variables respectively, \(\pi_4\) and \(\pi_5\) can be constructed

$$\pi_4 = DH^a u^b \sigma_f^c$$ \hspace{1cm} (C7)

$$\pi_5 = PH^a u^b \sigma_f^c$$ \hspace{1cm} (C8)
Repeating the process when solving Equation (C3), the constants of $a_2$, $b_2$, $c_2$, $a_3$, $b_3$, and $c_3$ are solved

\[
\begin{cases}
  a_2 = -1, b_2 = c_2 = 0 \\
  a_3 = b_3 = 0, c_3 = -1
\end{cases}
\]  

(C9)

So

\[
\begin{cases}
  \pi_4 = \frac{D}{H} \\
  \pi_5 = \frac{P}{\sigma_f}
\end{cases}
\]  

(C10)

The relation between the above dimensionless products can now be equated as

\[
\alpha_2 = \psi(\theta, \frac{FH}{u}, \frac{D}{H}, \frac{P}{\sigma_f})
\]  

(C11)

Thus, the complete dimensional equation is as follows

\[
\alpha_2 = C \theta^{a} \left( \frac{FH}{u} \right)^{b} \left( \frac{D}{H} \right)^{c} \left( \frac{P}{\sigma_f} \right)^{d}
\]  

(C12)
## Appendix D

### AWJ Oscillation Experimental Data

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