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Mathematical Problem Solving: A Need for Literacy

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Internationally, educators are concerned about effective methods to empower students with skills to equip them to function in a technological, global world. This paper reviews literature that explores the relationship between mathematical problem solving and literacy, through mathematical modelling and top-level structuring of text. The mathematical modelling problem solving process offers students an opportunity for learning mathematical skills through what is very much a literary process. Top-level structure is an organizational strategy where readers structure texts enabling identification and recall of a text's main idea/s. An extensive search of recent literature reveals strong relationships between reading comprehension and mathematical word problems because text comprehension must interact with mathematical processes to create mathematical literacy. This paper will argue that mathematical modelling and top-level structuring have a strong relationship due to the literary structure of modelling tasks. Secondly, it will investigate implications of skills attainment from such tasks as an empowering tool for the future.

To function effectively in today's global society, all students need to be taught to think mathematically. Mathematical modelling encourages these thinking skills through what is very much a literacy-based process. Mathematical modelling is a problem-solving process that requires students to interpret information from a variety of narrative, expository and graphic texts that reflect authentic real-life situations. As such, mathematical modelling requires students to elicit main ideas from texts and organise the information meaningfully so they can describe, explain, predict, manipulate and share ideas (Lesh & Doerr, 2003). Consequently, students participate in a literary problem-solving process through which they can learn vital mathematical skills and ultimately produce a tool or model which can be transported and reused in like situations.

The mathematical modelling process mirrors real-life situations in that students engage in a literary process, which takes place in a social context. Therefore, students collaborate to discuss and exchange ideas and form possible solutions prior to presenting their cases to peers who in turn may question and comment on the product. Students are required to read and interpret text, and participate in oral and written communication.

Consequently, it is evident that literacy is very much a part of this mathematical problem-solving process. To fully participate in the process, students must firstly comprehend the textual information presented. Students are required to interpret meaningfully a wide variety of text types. As a result, it is envisaged that equipping students with a strategy that could aid competent extraction and structural organisation of the textual main ideas could positively enhance students' ability to learn through the mathematical modelling process.

Mathematical problem solving

Problem solving in mathematics has been topical for discussion since Polya's work in the 1940s (Polya, 1945). Numerous definitions have been proposed on problem solving over the years, but problem solving is an evolving sphere. As the world has progressed and transformed into the global form of today, so too has the concept of problem solving for today's society. Rather than the traditional 'question' and distinct answer phenomenon of the past, modern definitions of problem solving offer more open definitions. Problem solving for today's information age of complexities needs to be dynamic. Lesh and Doerr (2003, p.3) translate problem solving as mathematical thinking and define the process as directly connected to "constructing, describing, explaining, manipulating, predicting, or controlling mathematically significant systems." Lesh and Doerr (2003, p.31) elaborate on the nature of problem solving. They state that "real-life problem solving involves developing useful ways to interpret the nature of givens, goals, possible solution paths, and patterns and regularities beneath the surface of things. Solutions typically involve several "modelling cycles" in which descriptions, explanations, and predictions are gradually refined and elaborated.

Through mathematical modelling, students are given opportunities to engage in tasks, together with other team members, to develop a reusable model or tool that they can apply to other like circumstances. In turn, engaging in these tasks requires students to efficiently draw on their knowledge: a complex process requiring mathematical literacy. The language of mathematics can involve the genres of fields such as scientific, geographical or other societal-based settings, as well as the symbolic and graphic representations of mathematical information. To be mathematically literate necessitates the relevant selection, understanding and organization of this information so that one can acquire the relevant knowledge. It is only when students have knowledge that they can draw upon it, and build upon it in the process of developing new mathematical understandings. So the process becomes a journey where skills are learned by participation in the journey. The more students can understand information, the greater chance they have of participating and developing their mathematical skills.

Mathematical modelling: a literary journey

Demonstrated below is an example of a mathematical modelling problem focus. This example is an extract from a problem given to year four students. It does not contain further background information that would be provided for students in a classroom setting, for example, scientific information on plants and favourable growing conditions. For the purpose here, only the central idea of the problem is posed.

The problem: Farmer Bill's flower farm

Bill has a flower farm and sells flowers to the markets in most months of the year. Use the information on the table to determine what seems to be the best season for Bill to make the most money in 2005?

Investigation 1
Provide an information report to Farmer Bill so that he can determine when he will make the most money during the year. Explain how you came to your decision. Remember to look carefully at the information on the table and use it to explain why you made your decision.

Bill’s flower totals for 2004

Month	Total of Flowers Sold
January	1600
February	536
March	815
April	612
May	588
June	350
July	0
August	0
September	4150
October	6400
November	2285
December	2260

Investigation 2
What about other crops? What if Farmer Bill decided to grow strawberries, tomatoes or corn? Will the pattern or model you found for the flowers work for these too? Investigate these other crops. Explain your model. Explain why the model will or will not work.

In this example, the need for interpretation of texts is evident. Firstly, the students must determine the purpose of the problem in investigation one, that is, what is the problem? Students need to list the questions so that as their team works through each part, they can mark it accordingly. Secondly, the students must interpret the table. Throughout these stages, the students are engaging in reading and oral communication about the texts. As they work through the stages, the students communicate, describe, explain, justify and make decisions about their ideas (Doerr & English, 2003). These literacy components continue throughout investigation two that would also require further research. Finally, the students share their decisions in presentations to their peers. It is at this stage that they must be able to explain fully their decisions and their reasoning. They must communicate their mathematical thinking and be able to competently justify their ideas.

Mathematical modelling and top-level structure

Learning occurs within a context of meaning, that is, a context that is personally relevant and with which students have had prior experience or want to be involved. Meaningful situations enhance understanding of new concepts. When text of any genre is encountered, the goal is to derive meaning, in other words to comprehend text. Reading comprehension has been described by Mayer (2004, p.723) as "the process of making sense out of a text passage, that is building a meaningful mental representation of the text." Mayer confirms the fact that a reader must select relevant information from text, organize the information coherently as a mental representation and integrate new information with existing information for active learning to occur.

There are several differing definitions of comprehension (Pardo, 2004) however; comprehension remains a complex, interactive, cognitive process where the reader is intimately involved with the text. To elicit meaning from text, the reader needs an efficient strategy to facilitate: (1) the integration of prior knowledge, (2) the selection and organization of applicable knowledge, (3) making inferences and (4) using metacognitive knowledge.

The extent to which readers extract meaningful content from text and are able to retrieve and recall information from memory depends largely on their ability to organize the content strategically. Top-level structure equates to the key structuring of the written symbolic language in a logical and systematic manner (Bartlett, 2003). Bartlett, Liyange, Jones, Penridge & McKay (2001) describe four main textual structures: 1. listing/description, 2. comparison, 3. problem/solution and 4. cause/effect. Basically, a reader determines the 'author's plan as he/she reads a text and applies this plan when organizing the information attained from the text. Top-level structuring of text potentially strengthens a reader's ability with text organizational skills and recall, therefore empowering the reader with a tool to organize their thinking during and after reading. It is described by Bartlett et al, (2001, p.67) as:

> A procedure through which a strategist applies what is known about the hierarchal organization of content in order to achieve memory, comprehension and expression outcomes. It allows the strategic reader, listener or reviewer to form an opinion on what a writer, speaker or performer considers as essential content and if necessary, then to move onto critical or inferential analysis. Conversely, it allows a strategist as writer, speaker or performer to produce coherent text and to signal what he/she wants to be seen as essential content.

It would appear that this tool could be most advantageous when used in conjunction with mathematical modelling. The ideas generated by this definition offer evidence demonstrating how top-level structuring could collaborate with mathematical modelling to form a firm foundation for meaningful skills learning through the problem-solving process. As is demonstrated in the 'Farmer Bill' problem above, if students can organize the information they need to work with, it seems they would be better able to memorize, comprehend and express the outcomes. Therefore, it would appear that students would have more chance of being able to list exactly what they were being asked to do, analyse the table and compare the results for each season, think about the causes and effects for these results and express opinions on the results. This, in turn would enable them to form definite opinions on their contributions to the problem-solving process. Having a definite opinion lends itself to clearer thinking and ability to explain and justify these opinions.

Investigation 2 requires the students to critically analyse their results in order to make further inferences on reapplying their model. It would appear that having organised thinking structures should equip students with a superior means of constructing their ideas and applying their knowledge in order to make educated inferences. Mathematical modelling problems require students to share their models through written and oral language in presentations to their peers. Equipping students with an organizational strategy like top-level structuring should provide students with a method of organizing their thinking coherently and communicating their models resourcefully and competently.

Mathematical modelling requires students to search content that includes narrative and expository texts, together with mathematical representations such as graphs and tables. Students must search for the main ideas in these texts. The student must "make sense of the situation so that he or she can mathematize it in ways that are meaningful to her or him" (Doerr & English, 2003: ¶ 9). Top-level structuring offers students a strategic method of abstracting ideas and information from text, that is: making sense of a situation in an organized and planned way (Bartlett et al., 2001). Solving mathematical modelling tasks "involves multiple simultaneous interpretations" (English, 2003, p.7). Top-level structuring provides a strategic approach by which to contemplate, discuss and act upon the information provided. Students are given the ability to decide whether the information is providing them with a list of criteria, a comparison, a cause and effect or a problem and a solution. Using this strategy allows the student to elicit the main idea and use this language of top-level structure to discuss, argue and communicate their position in a planful, strategic way (Bartlett, 2003).

Literacy and mathematical problem solving

There seems no question that to be a competent mathematical problem-solver requires competent literacy skills. There is significant literature emphasizing the crucial contribution of literacy comprehension to successful mathematical problem solving such as Helwig, Almond, Rozek-Tedesco, Tindal and Heath (1999), LeBlanc and Weber-Russell (1996), Littlefield and Rieser (2005), Lucangeli, Tressoldi & Cendron (1998), Mayer (2004) and Passolunghi, Cornoldi & De Liberto (1999). Although these authors refer to the more traditional written word problems, their claims are applicable to mathematical modelling texts because of the considerable reliance on students understanding of textual information. Mathematical modelling problems are presented with a greater volume and combination of text types so literacy skills are vital.

Students who have poor literacy skills inevitably have poor problem-solving skills when problems require reading and interpreting texts because these students cannot gain sufficient meaning from text. This does not necessarily mean that these students are disadvantaged with all mathematical problem solving. These students may be competent with problems that do not require significant text interpretation such as, numerical or spatial problems. Therefore, these students are not necessarily poor at mathematics, but, their mathematical proficiency can be hampered by their lower literacy skills.

Textually-based mathematical problems require students to discriminate between relevant and irrelevant information to efficiently solve the problem. Mayer (2004) identified four cognitive processes that students engage as they attempt solving written problems: 1. the need to translate each sentence into a mental representation, 2. the need to integrate the information to form a mental representation of the whole problem not just parts of it, 3. planning a solution and monitoring or tracking its progress during the problem-solving process, and 4. carrying out the solution procedure. These cognitive processes are linguistically correlated in that they are transforming information into an operable language. Linked with the linguistic cognitive processes is working memory which is required to maintain and process information efficiently (Le Blanc & Weber-Russell, 1996; Passolunghi et al, 1999). Passolunghi et al found that poor problem solvers used what they remembered less efficiently than good problem solvers because they could not filter irrelevant information.

As a result of this apparently strong connection between literacy and textually-based mathematical problem solving, it is envisaged that teaching students the top-level structuring strategy will enhance their ability to actively participate and learn mathematical skills during the problem-solving process. Good strategy users know how to resourcefully implement their strategies to develop their thinking efficiently (Pressley, Borkowski & Schneider, 1989). As has been demonstrated in the sample mathematical modelling problem, 'Farmer Bill', there is an implied perception that to maximize learning opportunities through such textually-based mathematical problems, students could greatly benefit from a strategy like top-level structure.

Conclusion

Mathematical modelling problems reflect real-life situations in their content, method of investigation and communication of resulting models. Top-level structure is a strategy that can be applied in real-life scenarios. Because of the high literary content of mathematical modelling problems, it would seem that mathematical modelling and top-level structuring could form a most amiable partnership. To think and to learn requires literacy skills. To think and to learn requires organization of thoughts and ideas. Top-level structuring offers a method by which to organize thoughts and ideas easily and efficiently. Once the organization process is in place, it becomes straightforward to communicate these thoughts and ideas in written and/or oral form. These skills are required in the mathematical modelling process because it is so based in literacy. As a result, the link between literacy and mathematics has been established here.

The argument here that mathematical modelling is very much a literacy-based process gives rise to the need for further investigation on the issues raised here. Mathematics is not an entity divorced from literacy and language, but is consumed by its own literacy and language. Consequently, a marriage between the two disciplines is proposed here. Current research is being undertaken by the author to investigate the literacy components of mathematical modelling and their effects on students' abilities to participate in the mathematical modelling process. In particular, the extent to which mathematical modelling can be enhanced by top-level structuring is being investigated. Early results are indicating positive implications for the employment of top-level structuring skills with mathematical modelling. Students who had been taught top-level structuring demonstrated enhanced reasoning skills and employed the strategy in their written and oral communication, but further research is to be undertaken to confirm these early indications.

Continued investigation and research into the partnership of mathematics and the role of literacy in textually-based mathematical problems will ultimately benefit student outcomes in learning to think mathematically. The questions remain: to what extent does literacy effect textually-based mathematics like mathematical modelling? to what extent can strategies such as top-level structuring enhance mathematical problem-solving skills? and to what extent can strategies such as top-level structuring facilitate students' learning of mathematics through textually-based mathematics problems like mathematical modelling?

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