Robust decentralised load-frequency control using an iterative linear matrix inequalities algorithm

H. Bevrani, Y. Mitani and K. Tsuji

Abstract: The load-frequency control (LFC) problem has been one of the major subjects in electric power system design/operation and is becoming much more significant today in accordance with increasing size, changing structure and complexity of interconnected power systems. In practice LFC systems use simple proportional-integral (PI) controllers. However, since the PI controller parameters are usually tuned based on classical or trial-and-error approaches, they are incapable of obtaining good dynamical performance for a wide range of operating conditions and various load changes scenarios in a multi-area power system. For this problem, the decentralised LFC synthesis is formulated as an $H\infty$-control problem and is solved using an iterative linear matrix inequalities algorithm to design of robust PI controllers in the multi-area power systems. A three-area power system example with a wide range of load changes is given to illustrate the proposed approach. The resulting controllers are shown to minimise the effect of disturbances and maintain the robust performance.

List of symbols

Δf_i	frequency deviation
ΔP_{gt}	governor valve position
ΔP_{ci}	governor load setpoint
ΔP_{ti}	turbine power
ΔP_{tie-i}	net tie-line power flow
ΔP_{di}	area load disturbance
M_i	equivalent inertia constant
D_i	equivalent damping coefficient
T_{gi}	governor time constant
T_{ti}	turbine time constant
T_{ij}	tie-line synchronising coefficient between areas i and j
B_i	frequency bias
α_i	area control error participation factor
R_i	drooping characteristic

1 Introduction

Currently, the electric power industry is in transition from vertically integrated utilities (VIU) providing power at regulated rates to an industry that will incorporate competitive companies selling unbundled power at lower rates. In the new power system structure, load-frequency control (LFC) acquires a fundamental role to enable power exchanges and to provide better conditions for electricity trading.

The common LFC objectives in the restructured power system, i.e. restoring the frequency and the net interchanges to their desired values for each control area remain [1]. That is why during the past decade several proposed LFC scenarios attempted to adapt well tested traditional LFC schemes to the changing environment of power system operation under deregulation [2–5]. In the new environment the overall power system can also be considered as a collection of control areas interconnected through high voltage transmission lines or tie-lines. Each control area consists of a number of generating companies (Gencos) and it is responsible for tracking its own load and performing the LFC task.

There has been continuing interest in designing load-frequency controllers with better performance to maintain the frequency and to keep tie-line power flows within prespecified values, using various decentralised robust and optimal control methods during the last two decades [6–13]. But most of them suggest complex state-feedback or high-order dynamic controllers, which are not practical for industrial practices. Furthermore, some authors have used the new and untested LFC frameworks, which may have some difficulties in being implemented in real-world power systems. Usually, the existing LFC systems in the practical power systems use the proportional-integral (PI) type controllers that are tuned online based on classical and trial-and-error approaches. Recently, some control methods have been applied to design the decentralised robust PI or low-order controllers to solve the LFC problem [14–17]. A PI control design method has been reported [14], which used a combination of $H\infty$ control and genetic algorithm techniques for tuning the PI parameters. The sequential decentralised method based on μ-synthesis and analysis has been used to obtain a set of low-order robust controllers [15]. The decentralised LFC method has been used with structured singular values [16]. The Kharitonov theorem and its results have been used to solve the same problem [17].

In this paper, the decentralised LFC problem is formulated as a standard $H\infty$ control problem to obtain the PI controller via a static output feedback design. An iterative linear matrix inequalities (ILMI) algorithm is used to compute the PI parameters. The proposed strategy is applied to a three-control area example. The obtained

IEE Proceedings online no. 20040493

doi:10.1049/ip-gtd:20040493

Paper first received 21st August 2003 and in revised form 26th February 2004

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robust PI controllers, which are ideally practical for industry, are compared with the H∞-based output dynamic feedback controllers (using the standard ILMI-based H∞ technique). Results show the controllers guarantee the robust performance for a wide range of operating conditions as well as full-dynamic H∞ controllers.

2 H∞-static output feedback using ILMI

This Section gives a brief overview of H∞-static output feedback controller design based on an ILMI approach. Consider a linear time invariant system $G(s)$ with the following state-space realisation.

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{12} u \\ y &= C_2 x \end{aligned} \tag{1}$$

where x is the state variable vector, w is the disturbance and other external input vector, z is the controlled output vector and y is the measured output vector.

The static output H∞ controller problem is to find a static output feedback $u = Ky$, as shown in Fig. 1, such that the resulting closed-loop system is internally stable, and the H∞ norm from w to z is smaller than γ, a specified positive number, i.e.

$$||T_{zw}(s)||_\infty < \gamma \tag{2}$$

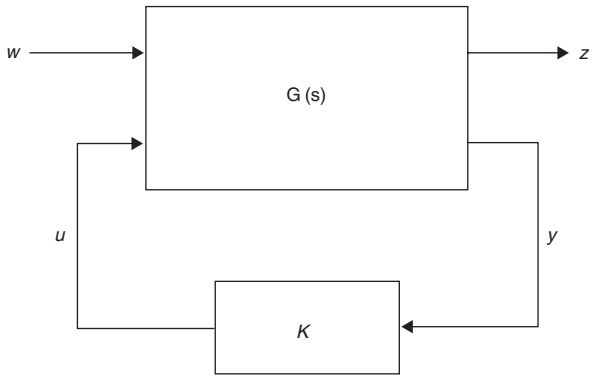

Fig. 1 *Closed-loop system via H∞ control*

Theorem 1 It is assumed (A, B_2, C_2) is stabilisable and detectable. The matrix K is an H∞ controller, if and only if there exists a symmetric matrix $X > 0$ such that

$$\begin{bmatrix} A_{cl}^T X + X A_{cl} & X B_{cl} & C_{cl}^T \\ B_{cl}^T X & -\gamma I & D_{cl}^T \\ C_{cl} & D_{cl} & -\gamma I \end{bmatrix} < 0 \tag{3}$$

where

$$A_{cl} = A + B_2 K C_2, \quad B_{cl} = B_1$$

$$C_{cl} = C_1 + D_{12} K C_2, \quad D_{cl} = 0$$

The proof is given in [18, 19]. We can rewrite (3) as the following matrix inequality [20]:

$$\overline{XBK}\bar{C} + (\overline{XBK}\bar{C})^T + \bar{A}^T \bar{X} + \overline{XA} < 0 \tag{4}$$

where

$$\bar{A} = \begin{bmatrix} A & B_1 & 0 \\ 0 & -\gamma I/2 & 0 \\ C_1 & 0 & -\gamma I/2 \end{bmatrix}, \bar{B} = \begin{bmatrix} B_2 \\ 0 \\ D_{12} \end{bmatrix}, \bar{C} = [C_2 \quad 0 \quad 0] \tag{5}$$

$$\bar{X} = \begin{bmatrix} X & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$$

Hence, the static output feedback H∞ control problem is reduced to find $X > 0$ and K such that matrix inequality (4) holds. It is a generalised static output feedback stabilisation problem of the system ($\bar{A}$, $\bar{B}$, $\bar{C}$) which can be solved via theorem 2, given in the Appendix (Section 9).

A solution of the consequent nonconvex optimisation problem, introduced in theorem 2, cannot be directly achieved by using general LMI technique. On the other hand, the matrix inequality (22) points to an iterative approach to solve the matrix K and X, namely, if P is fixed, then it reduces to an LMI problem in the unknowns K and X. For this purpose, we introduce the following ILMI algorithm that is mainly based on the approach given in [21]. The key point is to formulate the H∞ problem via a generalised static output stabilisation feedback such that all eigenvalues of (A-BKC) shift towards the left half-plane through the reduction of a, a real number, to close to feasibility of (22).

In summary, the H∞-static output feedback controller design based on the ILMI approach for a given system consists of the following steps:

Step 1 Compute the new system ($\bar{A}$, $\bar{B}$, $\bar{C}$), according to (5). Set $i = 1$ and $\Delta\gamma = \Delta\gamma_0$. Let $\gamma_i = \gamma_0$ a positive real number.

Step 2 Select $Q > 0$, and solve $\bar{X}$ from the following algebraic Riccati equation:

$$\bar{A}^T \bar{X} + \bar{X}\bar{A} - \overline{XB}\bar{B}^T \bar{X} + Q = 0 \tag{6}$$

Set $P_1 = \bar{X}$.

Step 3 Solve the following optimisation problem for $\bar{X}_i$, K_i and a_i.

Minimise a_i subject to the LMI constraints

$$\begin{bmatrix} \bar{A}^T \bar{X}_i + \bar{X}_i \bar{A} - P_i \bar{B}\bar{B}^T \bar{X}_i - \bar{X}_i \bar{B}\bar{B}^T P_i \\ \quad + P_i \bar{B}\bar{B}^T P_i - a_i \bar{X}_i \\ \bar{B}^T \bar{X}_i + K_i \bar{C} \\ (\bar{B}^T \bar{X}_i + K_i \bar{C})^T \\ -I \end{bmatrix} < 0 \tag{7}$$

$$\bar{X}_i = \bar{X}_i^T > 0 \tag{8}$$

Denote a_i^* as the minimised value of a_i.

Step 4 If $a_i^* \leq 0$, go to step 8.

Step 5 For $i > 1$ if $a_{i-1}^* \leq 0$, K_{i-1} is the desired H∞ controller and $\gamma^* = \gamma_i + \Delta\gamma$ indicates a lower bound such that the above system is H∞ stabilisable via static output feedback.

Step 6 Solve the following optimisation problem for $\bar{X}_i$ and K_i. Minimise trace ($\bar{X}_i$) subject to the above LMI constraints

(7) and (8) with $a_i = a_i^*$. Denote $\bar{X}_i^*$ as the $\bar{X}_i$ that minimised trace ($\bar{X}_i$).

Step 7 Set $i = i+1$ and $P_i = \bar{X}_{i-1}^*$, then go to step 3.

Step 8 Set $\gamma_i = \gamma_i - \Delta\gamma$, $i = i+1$. Then do steps 2–4.

The matrix inequalities (7) and (8) give a sufficient condition for the existence of the static output feedback controller.

3 Problem formulation and dynamical model

A large-scale power system consists of a number of interconnected control areas. Figure 2 shows the block diagram of control area-*i*, which includes *n* Gencos, from an *N*-control area power system. As is usual in the LFC design literature, three first-order transfer functions are used to model generator, turbine and power system (rotating mass and load) units. w_{1i} and w_{2i} show local load disturbance and area interface, respectively. The other parameters are described in the list of symbols at the front of this paper. Following a load disturbance within a control area, the frequency of that area experiences a transient change, the feedback mechanism comes into play and generates appropriate rise/lower signal to the participating Gencos according to their participation factors (α_{ji}) to make generation follow the load. In the steady state, the generation is matched with the load, driving the tie-line power and frequency deviations to zero.

The balance between connected control areas is achieved by detecting the frequency and tie-line power deviations to generate the area control error (ACE) signal which is, inturn, utilised in the PI control strategy as shown in Fig. 2. The ACE for each control area can be expressed as a linear combination of tie-line power change and frequency deviation.

$$ACE_i = B_i \Delta f_i + \Delta P_{tie-i} \tag{9}$$

It can be shown that considering w_{1i} and w_{2i} as two input disturbance channels is useful to decentralised LFC design [22]. These signals can be defined as follows:

$$w_{1i} = \Delta P_{di} \tag{10}$$

$$w_{2i} = \sum_{\substack{j=1 \\ j\neq i}}^{N} T_{ij} \Delta f_j \tag{11}$$

According to Fig. 2, in each control area the ACE acts as the input signal of the PI controller which is used by the LFC system. Therefore we have

$$u_i = \Delta Pci = k_{Pi} ACE_i + k_{Ii} \int ACE_i \tag{12}$$

In the next step, as shown in Fig. 3, the PI-based LFC design can be reduced to a static output feedback control problem. To change (12) to a simple static feedback control as

$$u_i = K_i y_i \tag{13}$$

we can rewrite (12) as follows [14]:

$$u_i = [k_{Pi}\ k_{Ii}] \begin{bmatrix} ACE_i \\ \int ACE_i \end{bmatrix} \tag{14}$$

Finally, the described ILMI-based H∞ algorithm will be used to obtain the desired PI parameters. The main control framework to formulate the PI-based LFC via a static

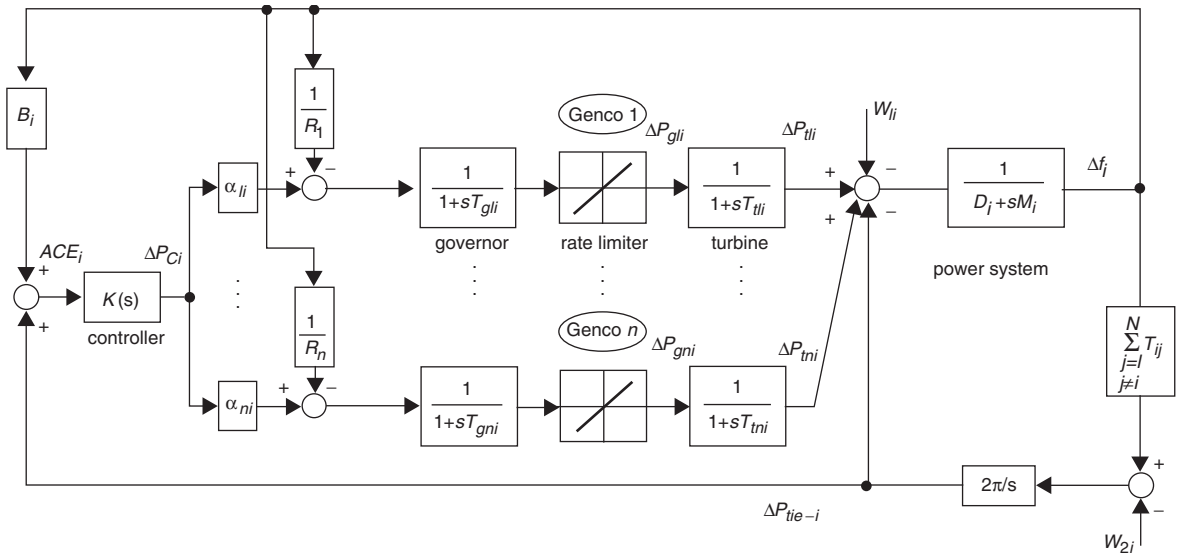

Fig. 2 *General control area*

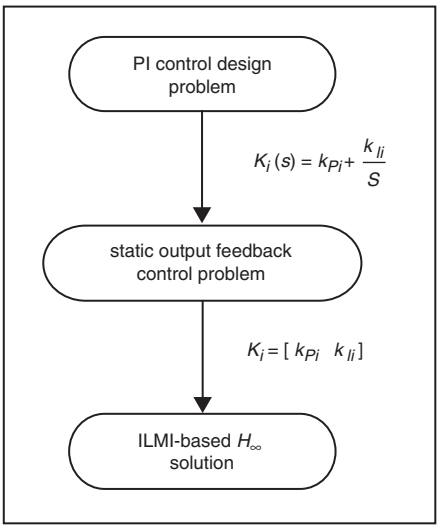

Fig. 3 *Problem formulation*

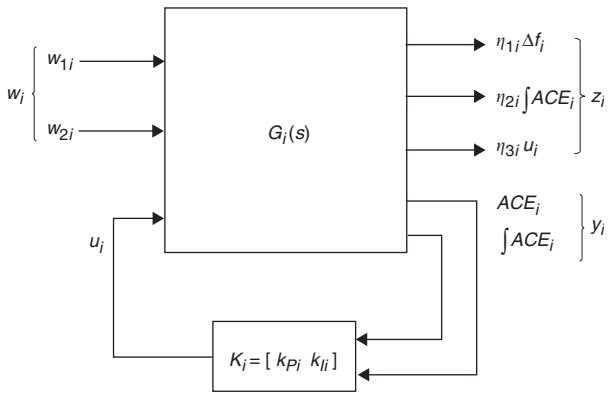

Fig. 4 *Proposed control framework*

output H∞ controller design problem, for a given control area, is shown in Fig. 4.

$G_i(s)$ denotes the dynamical model corresponds to control area i shown in Fig. 2. According to (1), the state space model for each control area i can be obtained as

$$\begin{aligned}\dot{x}_i &= A_i x_i + B_{1i} w_i + B_{2i} u_i \\ z_i &= C_{1i} x_i + D_{12i} u_i \\ y_i &= C_{2i} x_i\end{aligned} \tag{15}$$

where

$$x_i^T = [\,\Delta f_i \quad \Delta P_{tie-i} \quad \int ACE_i \quad x_{ti} \quad x_{gi}\,] \tag{16}$$

$$x_{ti} = [\,\Delta P_{t1i} \quad \Delta P_{t2i} \quad \cdots \quad \Delta P_{tni}\,]$$

$$x_{gi} = [\,\Delta P_{g1i} \quad \Delta P_{g2i} \quad \cdots \quad \Delta P_{gni}\,]$$

$$y_i^T = [\,ACE_i \quad \int ACE_i\,], \quad u_i = \Delta P_{Ci} \tag{17}$$

$$z_i^T = [\,\eta_{1i}\Delta f_i \quad \eta_{2i}\int ACE_i \quad \eta_{3i} u_i\,] \tag{18}$$

$$w_i^T = [\,w_{1i} \quad w_{2i}\,] \tag{19}$$

and

$$A_i = \begin{bmatrix} A_{i11} & A_{i12} & A_{i13} \\ A_{i21} & A_{i22} & A_{i23} \\ A_{i31} & A_{i32} & A_{i33} \end{bmatrix}, \; B_{1i} = \begin{bmatrix} B_{1i1} \\ B_{1i2} \\ B_{1i3} \end{bmatrix}$$

$$B_{2i} = \begin{bmatrix} B_{2i1} \\ B_{2i2} \\ B_{2i3} \end{bmatrix}$$

$$C_{1i} = [\,c_{1i} \quad 0_{3\times n} \quad 0_{3\times n}\,], \; c_{1i} = \begin{bmatrix} \eta_{1i} & 0 & 0 \\ 0 & 0 & \eta_{2i} \\ 0 & 0 & 0 \end{bmatrix}$$

$$D_{12i} = \begin{bmatrix} 0 \\ 0 \\ \eta_{3i} \end{bmatrix}$$

$$C_{2i} = [\,c_{2i} \quad 0_{2\times n} \quad 0_{2\times n}\,], \; c_{2i} = \begin{bmatrix} B_i & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_{i11} = \begin{bmatrix} -D_i/M_i & -1/M_i & 0 \\ 2\pi \sum_{\substack{j=1 \\ j\neq i}}^{N} T_{ij} & 0 & 0 \\ B_i & 1 & 0 \end{bmatrix}$$

$$A_{i12} = \begin{bmatrix} 1/M_i & \cdots & 1/M_i \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}_{3\times n}$$

$$A_{i22} = -A_{i23} = diag[\,-1/T_{t1i} \quad -1/T_{t2i} \quad \cdots \quad -1/T_{tni}\,]$$

$$A_{i33} = diag[\,-1/T_{g1i} \quad -1/T_{g2i} \quad \cdots \quad -1/T_{gni}\,]$$

$$A_{i31} = \begin{bmatrix} -1/(T_{g1i}R_{1i}) & 0 & 0 \\ \vdots & \vdots & \vdots \\ -1/(T_{gni}R_{ni}) & 0 & 0 \end{bmatrix},$$

$$A_{i13} = A_{i21}^T = 0_{3\times n}, \; A_{i32} = 0_{n\times n}$$

$$B_{1i1} = \begin{bmatrix} -1/M_i & 0 \\ 0 & -2\pi \\ 0 & 0 \end{bmatrix}, \; B_{1i2} = B_{1i3} = 0_{n\times 2}$$

$$B_{2i1} = 0_{3\times 1}, \; B_{2i2} = 0_{n\times 1}$$

$$B_{2i3}^T = [\,\alpha_{1i}/T_{g1i} \quad \alpha_{2i}/T_{g2i} \quad \cdots \quad \alpha_{ni}/T_{gni}\,]$$

Similar to [14], three constant weighting coefficients are considered for controlled output signals. η_{1i}, η_{2i} and η_{3i} must be chosen by the designer to obtain the desired performance.

4 Application to a 3-control area power system

To illustrate the effectiveness of proposed control strategy, a three-control area power system, shown in Fig. 5, is considered as a test system. It is assumed that each control area includes three Gencos. The total generation of each Genco in MW is given in Table 1. The power system parameters are considered to be the same as in [14].

For the sake of comparison, in addition to the proposed control strategy to obtain the robust PI controller, a robust H∞ dynamic output feedback controller using the LMI control toolbox is designed for each control area. Specifically, based on a general LMI, first the control design is reduced to a LMI formulation [14], and then the H∞ control problem is solved using the function hinflmi, provided by the MATLAB LMI control toolbox [23]. This function gives an optimal H∞ controller through the minimising the guaranteed robust performance index (2) subject to the constraint given by the matrix inequality (3) and returns the controller $K(s)$ with optimal robust performance index.

The resulted controllers using the hinflmi function are of dynamic type and have the following state-space form, whose orders are the same as size of plant model (9th order in the present paper):

$$\begin{aligned}\dot{x}_{ki} &= A_{ki} x_{ki} + B_{ki} y_i \\ u_i &= C_{ki} x_{ki} + D_{ki} y_i\end{aligned} \tag{20}$$

At the next step, according to the synthesis methodology described in Section 2 and summarised in Fig. 6, a set of three decentralised robust PI controllers are designed. As has already been mentioned, this control strategy is fully suitable for LFC applications which usually employ the PI

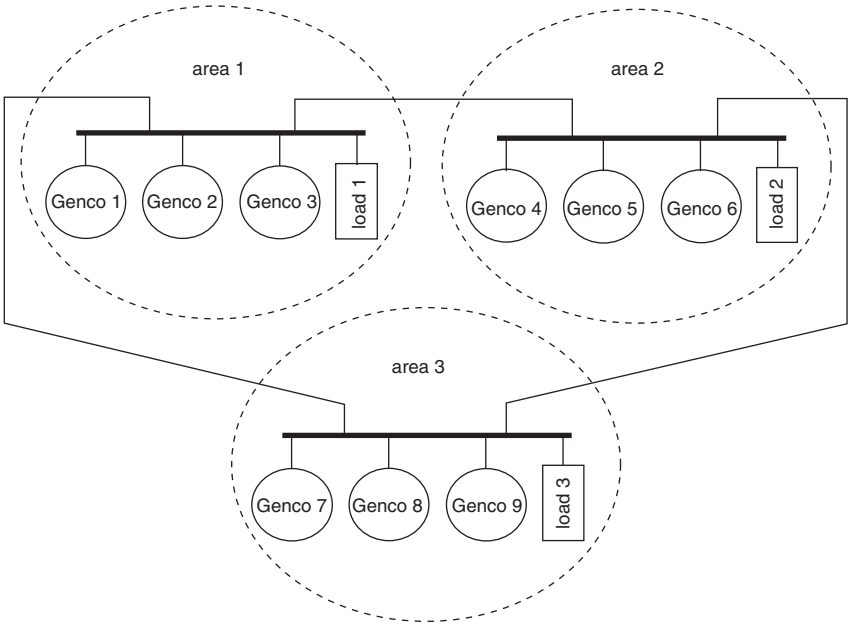

Fig. 5 *Three-control area power system*

Table 1: Total generation of Gencos

Genco	1	2	3	4	5	6	7	8	9
Rate, MW (MVA_{base} : 1000 MW)	1000	1200	1000	1100	900	1200	900	1000	1100

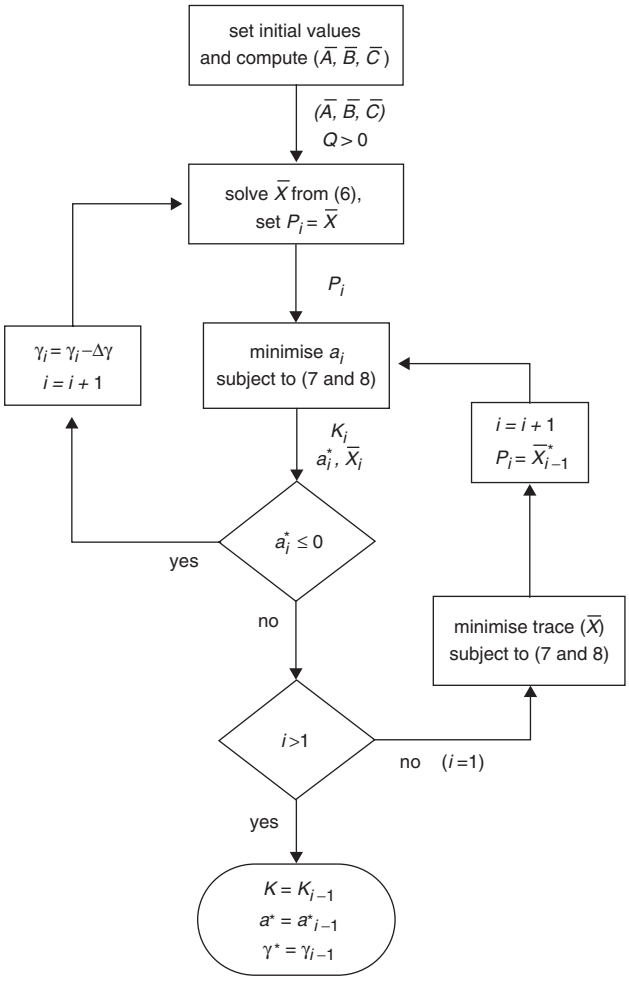

Fig. 6 *LFC design algorithm using ILMI*

Table 2: Control parameters (ILMI design)

Parameter	Area 1	Area 2	Area 3
a^*	−0.3285	−0.2472	−0.3864
k_{Pi}	0.0371	0.0465	0.0380
k_{Ii}	−0.2339	−0.2672	−0.3092
η_{ji}	$\eta_{1i}=0.5$	$\eta_{2i}=1$	$\eta_{3i}=500$

control, while most other robust and optimal control designs (such as the LMI approach) yield complex controllers whose size can be larger than real-world LFC systems. Using the ILMI approach, the controllers are obtained following several iterations. The control parameters are shown in Table 2.

A set of suitable values for constant weights [η_{1i}, η_{2i}, η_{3i}] can be chosen as [0.5, 1, 500], respectively. An important issue with regard to selection of these weights is the degree to which they can guarantee the satisfaction of design performance objectives. The selection of these weights entails a trade-off among several performance requirements. The coefficients η_{1i} and η_{2i} at controlled outputs set the performance goals, e.g. tracking the load variation and disturbance attenuation. η_{3i} sets a limit on the allowed control signal to penalise fast change and large overshoot in the governor load set-point signal. The recent objective is very important to realise the designed controller in the real-world power systems. The large coefficient '500' for η_{3i} results in a smooth control action signal with reasonable changes in amplitude.

It is notable that the robust performance index given by the standard H∞ control design (2) can be used as a valid

Table 3: Robust performance index

Control design	Control structure	γ_1^* (Area 1)	γ_2^* (Area 2)	γ_3^* (Area 3)
H∞	9th order	500.0103	500.0045	500.0065
ILMI	PI	500.0183	500.0140	500.0105

tool to analyse robustness of the closed-loop system for the proposed control design. The resulting robust performance indices (γ^*) of both synthesis methods are too close to each other and are shown in Table 3. It shows that although the proposed ILMI approach gives a set of much simpler controllers (PI) than the robust H∞ design, they also give a robust performance like the dynamic H∞ controllers.

5 Simulation results

To demonstrate the effectiveness of the proposed control design, some simulations were carried out. In these simulations, the proposed controllers were applied to the three-control area power system described in Fig. 5. In this Section, the performance of the closed-loop system using the robust PI controllers compared to the designed dynamic H∞ controllers will be tested for the various possible load disturbances.

Case 1: As the first test case, the following large load disturbances (step increase in demand) are applied to three areas:

$$\Delta P_{d1} = 100\,\text{MW},\ \Delta P_{d2} = 80\,\text{MW},\ \Delta P_{d3} = 50\,\text{MW}$$

The frequency deviation (Δf), area control error (ACE) and control action (ΔPc) signals of the closed-loop system are shown in Fig. 7. Using the proposed method (ILMI), the area control error and frequency deviation of all areas are quickly driven back to zero as well as dynamic H∞ control (LMI).

Case 2: Consider larger demands by areas 2 and 3, i.e. $\Delta P_{d1} = 100\,\text{MW},\ \Delta P_{d2} = 100\,\text{MW},\ \Delta Pd_3 = 100\,\text{MW}$. The closed-loop responses for each control area are shown in Fig. 8.

Case 3: As another severe condition, assume a bounded random load change, shown in Fig. 9, is applied to each control area, where $-50\,\text{MW} \le \Delta P_d \le 50\,\text{MW}$.

The purpose of this scenario is to test the robustness of the proposed controllers against random large load disturbances. The control area responses are shown in Fig. 10. This figure demonstrates that the designed controllers track the load fluctuations effectively. The simulation results show that the proposed PI controllers perform as robustly as the robust dynamic H∞ controllers (with complex structures) for a wide range of load disturbances.

6 Conclusions

A new method for robust decentralised LFC design using an iterative LMI approach has been proposed for a large-scale power system. The design strategy includes enough flexibility to set the desired level of performance and gives a set of simple PI controllers via the H∞ static output control design, which is commonly used in real-world power systems.

The proposed method was applied to a three-control area power system and was tested with different load change scenarios. The results were compared with the results of applied dynamic H∞ output controllers. Simulation results demonstrated the effectiveness of the methodology. It was shown that the designed controllers can guarantee the robust performance, such as precise reference frequency tracking and disturbance attenuation under a wide range of area-load disturbances.

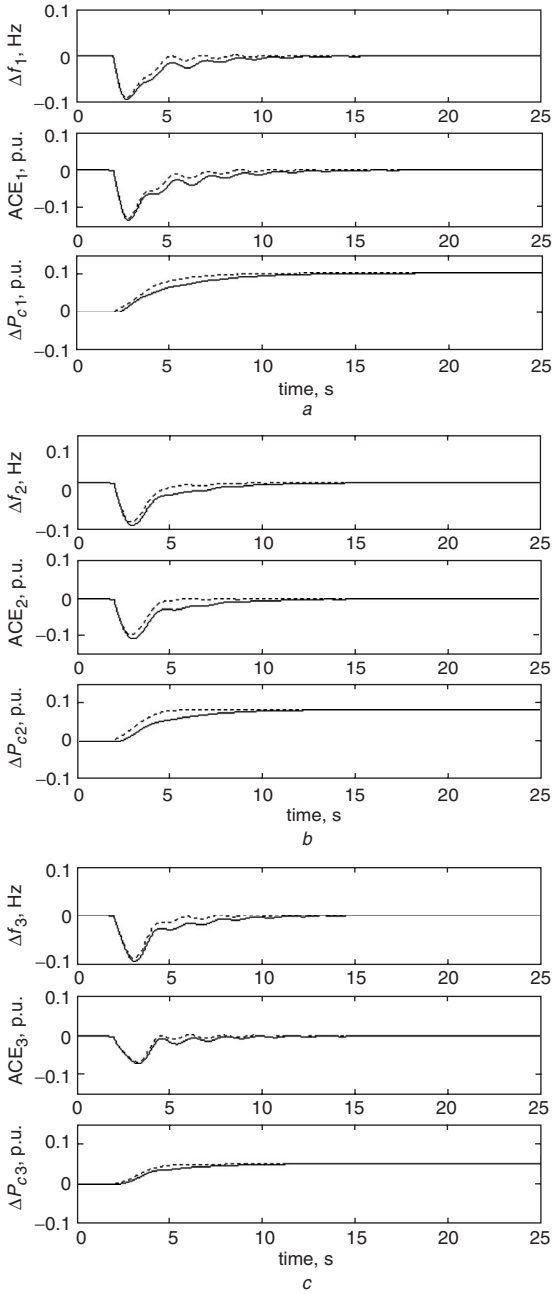

Fig. 7 *System response in case 1*
— ILMI
---- H∞
a Area 1
b Area 2
c Area 3

7 Acknowledgments

The authors are grateful to Dr. Y. Y. Cao from the University of Virginia, USA, Prof. M. Ikeda from Osaka University, Japan and Dr. D. Rerkpreedapong from Kasetsart University, Thailand for their kind discussions.

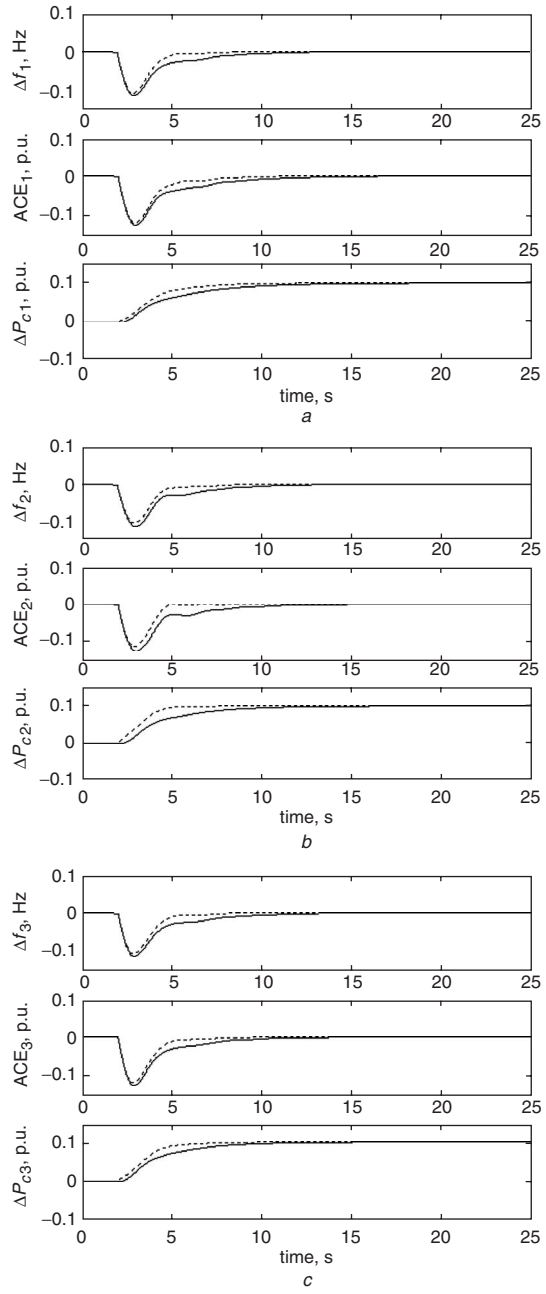

Fig. 8 *System response in case 2*
a Area 1
b Area 2
c Area 3
— ILMI
---- H∞

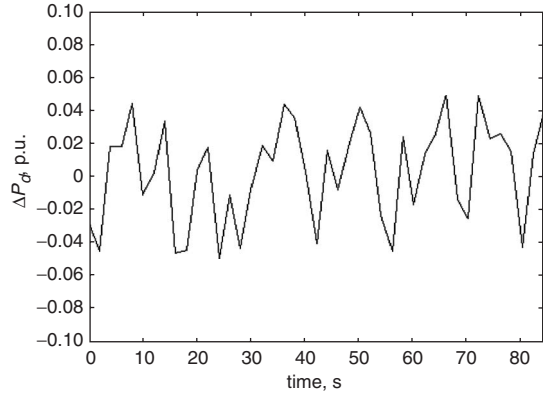

Fig. 9 *Random load demand signal*

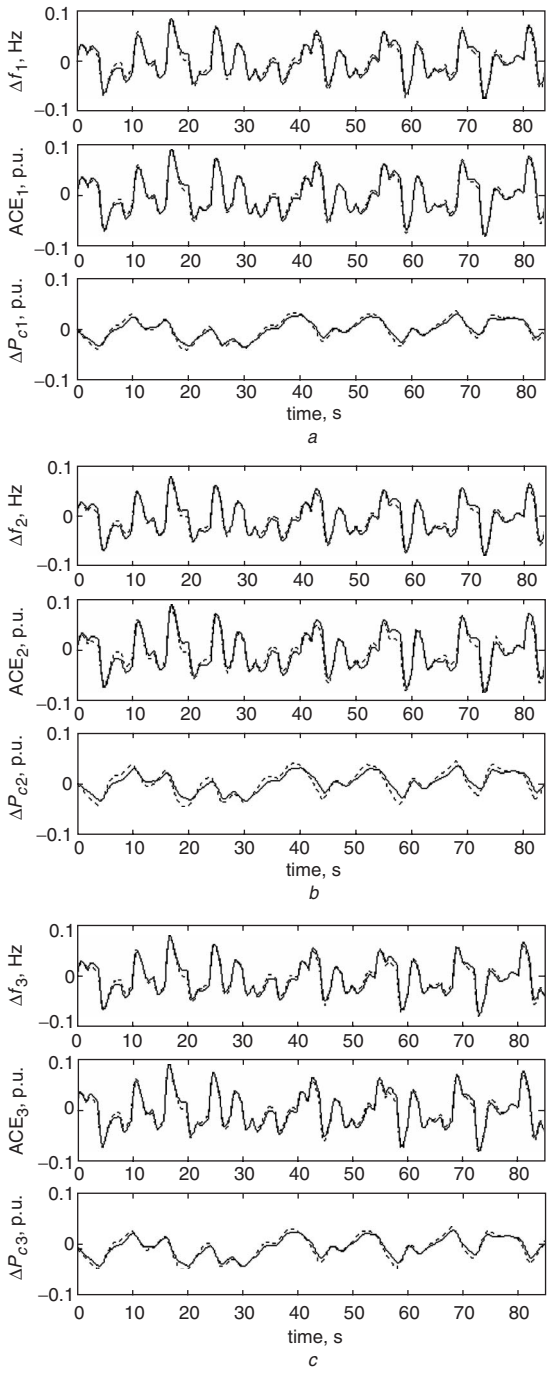

Fig. 10 *System response in case 3*
a Area 1
b Area 2
c Area 3
— ILMI
---- H∞

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9 Appendix

Theorem 2 The system (A, B, C) that may also be identified by the following representation:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{21}$$

is stabilisable via static output feedback if and only if there exist $P>0$, $X>0$ and K satisfying the following quadratic matrix inequality:

$$\begin{bmatrix} A^TX + XA - PBB^TX & (B^TX + KC)^T \\ -XBB^TP + PBB^TP & \\ B^TX + KC & -I \end{bmatrix} < 0 \tag{22}$$

Proof According to the Schur complement, the quadratic matrix inequality (22) is equivalent to the following matrix inequality:

$$\begin{aligned} &A^TX + XA - PBB^TX - XBB^TP + PBB^TP \\ &+ (B^TX + KC)^T(B^TX + KC) < 0 \end{aligned} \tag{23}$$

For this new inequality notation (23), the sufficiency and necessity of the theorem are already proven [24].