MOVING FORCE IDENTIFICATION: PRACTICE AND REVIEW

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Abstract

Moving force identification from dynamic responses of a bridge is an important inverse problem in the civil and structural engineering field. Current knowledge on factors affecting performance of moving force identification methods is reviewed in this paper under main headings below: background of moving force identification, experimental verification both in laboratory and in field. Although there are still many challenges and obstacles to be overcome before these methods can be implemented in practice, some results serve as a good indicator to steer the direction of further work in the field.

Keywords: WIM, Moving Force Identification, Dynamic Responses, Practice, Review.

Résumé

L’identification des forces mobiles à partir de la réponse dynamique d’un pont est un problème inverse important en génie civil et des structures. Les connaissances actuelles sur les facteurs qui influent sur la performance des méthodes d’identification des forces mobiles sont passées en revue selon les principales rubriques : bases de l’identification des forces mobiles, vérification expérimentale en laboratoire et sur site. Bien qu’il restent de nombreux enjeux et obstacles à franchir avant que ces méthodes puissent être mises en œuvre en pratique, certains résultats donnent de bons indicateurs pour orienter les futurs travaux dans le domaine.

Mots-clés: Pesage en Marche, Identification des Forces Mobiles, Réponses Dynamiques, Révision.

移動載重辨識之回顧及應用

摘要：

利用橋梁之動態反應辨識移動荷重爲土木及結構工程學領域中一個重要的問題。本文主要回顧影響移動載重辨識系統績效之相關因素，包括：移動載重辨識之基礎發展以及在實驗室及現場之驗證工作。雖然在上述方法實際應用前仍有許多挑戰及困難，但部分已獲知之研究結果亦可做爲未來相關工作之參考。

關鍵字：動態地磅、移動載重辨識別、動態反應、實際應用、回顧

1. Introduction

Accurate and reliable data on the nature and extent of heavy vehicles use of both the road and bridge network, especially dynamic moving vehicle loads, is extremely important for pavement and bridge design (Stevens, 1987; Fryba, 1999). Most of the traditional ways mentioned acquire data could only measure static axle loads, and they are expensive and subject to bias, e.g. the Weigh-in-Motion (WIM) systems can only acquire static equivalent axle loads of a vehicle (Moses, 1978; Peters 1984&1986; Koniditsiotis et al., 1995). The dynamic load data are valuable not only to the design of bridges and pavements but also to their monitoring and retrofitting since the dynamic wheel loads might increase road surface damage by a factor of 2-4 over that due to static wheel loads (Cebon, 1987). Whittemore et al (1970) and Cantieni (1992) have separately described systems that use instrumented vehicles to measure dynamic loads on bridge deck. Unfortunately, the acquired data are also subject to bias. Over the past years, some great effort has been made for obtaining an advanced WIM technique, a series of moving force identification methods have been put forward, which can compute dynamic wheel loads with an acceptable accuracy (O'Connor and Chan, 1988; Chan et al., 1999; Law et al., 1997; Law et al., 1999), further these methods have been enhanced and merged into a moving force identification system (MFIS) (Yu, 2002). However, there still exist some limitations if these methods could actually be operated in practice (Chan et al., 2000). This paper tries to provide a review on current knowledge of the factors affecting the performance of moving force identification methods, as well as to find the right way to steer the direction of further work.

2. Background of Moving Force Identification

2.1 Models of Bridge-Vehicle System

The bridge-vehicle system is a very complicated system. The interaction between bridge and vehicle is a complex phenomenon governed by a large number of different parameters. The use of simplified models is more effective to establish a clear connection between the governing parameters and the bridge response than a complex model. Normally, the bridge decks are modelled as beams using either the *Beam-element model* (O'Connor and Chan, 1988) or the *Continuous Beam Model* (Bernoulli-Euler beam or Timoshenko beam) or modelled as plates (isotropic plates or orthotropic plates). The vehicles are modelled as a moving force, a moving mass (Lin and Trethewey 1990) or a moving oscillator (Pesterev and Bergman, 1997; Yang et al., 2000). Usually, a quarter-truck model, a half-single-unit two-axle truck model, and a half five-axle semi-trailer truck model developed by Todd et al (1989), or 3-D, 2-D, and single sprung mass (1-D) system (Chatterjee et al., 1994) are adopted.

2.2 Identification Methods

Four identification methods have been put forward, which can compute dynamic wheel loads with an acceptable accuracy. The interpretive method I (IMI, O'Connor and Chan, 1988) developed a system to measure the dynamic vehicle-bridge interaction forces from the bridge total responses caused by the inertial or D'Alembert's forces and the damping forces, in which the bridge deck is modeled as an assembly of lumped masses interconnected by massless elastic beam elements. The interpretive method II (IMII, Chan et al., 1999) was similar to IMI but used Euler's equation for beams to model the bridge deck in the interpretation of dynamic loads crossing the deck. The Euler beam theory together with modal analysis was used to identify moving loads from the bridge responses. The time domain method (TDM, Law et al., 1997) modeled the bridge deck as a simply supported Euler beam with viscous damping and the vehicle/bridge interaction forces was modeled as one-point or two-point loads with fixed axle spacing, moving at constant speed. The moving forces were then identified using the modal superposition principle in time domain. Further, the frequency-time domain method (FTDM, Law et al., 1999) performed Fourier transformation on the equations of motion which are expressed in modal co-ordinates. The relation between the responses and the moving forces was established first and the force spectrums were calculated by the least-square method in the frequency. The time histories of moving forces can then be obtained by performed the inverse Fourier transformation. These methods above have been enhanced and merged into a moving force identification system (MFIS) (Yu, 2002). The first method is developed based on the *beam-element model*, the others based on *continuous beam model*.

It is easy to find that most of the identification methods are eventually converted to a linear algebraic equation, such as, $Ax = b$. Where, x is the time series vector of the unknown time-varying moving force $P(t)$, b is the time series vector of the measured response of bridge deck. The system matrix A is associated with the bridge-vehicle system. In principle, the equation will have a solution given by the least-squares method, which be written as $x = A^{+} b = [(A^{T} A)^{-1} A^{T}]\, b$. Where A^{+} denotes the *pseudo-inverse* (PI) of matrix A. The solution vector x is called *PI solution*. This definition requires matrix A to have full rank. However, the matrix A is often rank deficient or close to rank deficient then A^{+} is best calculated from singular value decomposition (SVD) of matrix A (Lindfield and Penny, 1995). If matrix A is real, the SVD of A is USV^{T}, the solution can easily be calculated by $x = A^{+} b = [VS^{-1}U^{T}]\, b$, the solution vector x here is called *SVD solution*.

3. Experimental Verification in Laboratory

3.1 Experimental Setup

In order to evaluate the moving force identification methods, a series of experiments have been carried out in laboratory. As an example, a simply supported beam with a span of 3.678*m* long and 101*mm* by 25*mm* uniform cross-section is constructed, which made from a kind of mild steel with a density of $7335\, kg/m^3$ and a flexural stiffness $EI = 29.92\, kN/m^2$. Two kinds of model cars are also made to simulate two-axle vehicle and multi-axle vehicle respectively. One has two axles at a spacing of 0.55*m* and is mounted on four rubber wheels. The static mass of the whole car is 12.1*kg* with a rear axle of 3.825*kg*, its axle spacing to span ratio (ASSR) is 0.15. The other has three axles based on the AASHTO (1996) loading code. Two types of ASSRs, 0.15:0.15 and 0.15:0.20, are set, their corresponding axle load ratios are 2W:8W:8W and 3W:7W:7W respectively, the total weight of car is 18kg for the former case and 17kg for the latter respectively as W=1 *kg*. The model car was pulled by a string wound on the drive wheel of an electric motor in the front of the beam. Its speed can be adjusted to control and determine the car speed exactly. When the car traversed the beam bridge, the induced bridge responses were recorded simultaneously using strain gauges and accelerometers installed on the lower surface of the beam, and then used as input data for moving force identification.

3.2 Parameter Studies

Many parameters play an important role in the moving force identification; therefore it is necessary to study the effects of main parameters on the four identification methods. They are bridge-vehicle parameters, measurement parameters and algorithm parameters.

Effect of Bridge-Vehicle Parameters

Usually, a sufficient number of vibration modes of bridge must be included in the identification calculation. But, what is the sufficient number of modes? The answer depends not only on the characteristics of the bridge-vehicle system but also on the solution to the over-determined equation used in the moving force identification method. The IMI is independent of the mode number. The IMII needs at least the first three modes or more to correctly identify the two moving forces. For both the TDM and FTDM, the minimal necessary mode number required is 4. If the first five modes are used to identify the two moving forces, the identification accuracy is the highest in the cases studied (Chan, Yu et al., 2000). Generally, the mode number involved should be bigger than or at least equal to one more than axle number of vehicle.

Vehicle speed plays an important role for the dynamic behavior of a bridge subjected to loads moving across the bridge (Fryba, 1999). It is interesting to notice that the identification accuracy first increase and then decrease with increase in car speed for the IMI. But it is not so significant for both IMII and TDM. It may be concluded that both IMII and TDM are independent of car speeds. Anyway, the IMII is more suitable for the higher vehicle speed. The TDM can effectively identify the forces in all the speed cases. The faster vehicle speed is also of benefit to both the TDM and FTDM, and unfortunately, the FTDM fails in the lower speed cases if PI solution is used to solve the equations (Chan et al., 2001b). However, when using the SVD solution the situation is completely changed, it makes the FTDM from original ineffective to effective and it shows the FTDM has a better identification accuracy in the fast speed as well (Yu and Chan, 2003).

In reality, heavy vehicles mainly comprise articulated and nonarticulated vehicle frames. Previous research using analytical model has shown that nonarticulated vehicles generate much higher vibration on the bridge compared to articulated vehicles (Chan and Yung, 2000). The three-axle vehicle models are allowed to switch from articulated to rigid connection, i.e. nonarticulated connection between tractor and the semi-trailer by two fixed studs in the laboratory. The results show that the identified multi-axle vehicle loads are reasonable and acceptable for both the articulated and nonarticulated vehicles. The moving force identification system can correctly identify the multi-axle vehicle loads even if the middle axle of the nonarticulated vehicles is hanging in the air (Yu and Chan, 2004).

Three types of suspension systems are incorporated in the vehicle models. They are rigid connection, sprung connection, and pre-compressed sprung connection between vehicle frame and axle respectively to simulate different suspension systems. The results show that the suspension systems make an obvious impact on both dynamic characteristics of vehicles and identification accuracy. The fundamental frequency of vehicles is significantly changed with different suspension systems. It is evidently beneficial to the improvement of identification accuracy when the nonarticulated vehicles are suspended and provided with more suspension systems (Yu and Chan, 2004).

Effect of measurement parameters

The effects of sampling frequency using the IMI and IMII are not too obvious within 333 Hz, but after this range, the effects become more significant even make the two methods failure at sampling frequency 1000 Hz The TDM is suitable for the higher sampling frequency. The effect of sampling frequency on the FTDM increases as the sampling frequency, and the FTDM fails if the sampling frequency is higher than 333 Hz and PI solution is used (Chan, Yu et al., 2000). However, the use of the SVD not only makes the FTDM method effective but also results in good identified results with higher accuracy, whereas direct calculation of the PI solution causes the identification method to fail (Yu and Chan, 2003). Moreover, both the TDM and FTDM have higher identification accuracy than both the IMI and IMII.

Measurement stations affect the identification accuracy. For the IMI and IMII, the required numbers of stations can be determined referring to the references (O'Connor and Chan, 1988; Chan et al., 1999). For the TDM, it requires at least three stations to obtain the two moving forces correctly. The FTDM should have at least one more measurement station than using the TDM, i.e., 4. In addition, the FTDM is sensitive to the locations of measuring station, which should be selected carefully when the PI solution is adopted (Chan, Yu et al., 2000). Once the SVD is used for the FTDM, the identified results are acceptable and achieve a very high accuracy even they exceed the acceptable accuracy range when using the PI solution in all the study cases. It is predicted that the identification method is independent of the measurement stations if the SVD method is adopted (Yu and Chan, 2003). In general, the identification accuracy is better if more measuring stations are adopted for both the TDM and FTDM, but it will take longer computational time.

Effect of algorithm parameters

Usually, the SVD technique applies a variant of the QR algorithm to reduce the super-diagonal elements to a negligible size and to result in a diagonal form through an iteration procedure. Here, a given tolerance parameter, i.e. *epsilon* ε, should be set as a criterion related to rejecting or accepting of zero singular values. The criterion may depend on the accuracy of the expected results and, in practice, may be difficult to establish. Results show that a smaller tolerance parameter ε is beneficial to moving force identification. However, if ε is too small the computation cost (CPU) is higher because it need more iteration times for convergence. In addition, the SVD technique increases the CPU time by 60% as compared to that for the PI solution. It is too expensive and not beneficial to the real-time analysis in situ. To take account of all the above aspects at the same time, ε value set to be 1.0e-6 for study cases is appropriate (Yu and Chan, 2003).

The identified results are noise sensitive and they exhibit fluctuations at the beginning and end of the time histories. These moments correspond to the change in the vibration state, vice versa, and the solutions are ill-conditioned. A regularization method developed by Tikhonov and Arsenin (1977) can be introduced to provide bounds to the solution (Law et al., 2001). The results obtained are greatly improved over those without regularization. The TDM is found better than the FTDM in solving for the ill-posed problem. Both simulation and laboratory test results indicate that the total weight of a vehicle can be estimated indirectly using moving force identification methods with some accuracy at least with FTDM (Law et al., 2001).

3.3 Evaluation of identification methods

When a model car traversed the model bridge constructed in laboratory, the induced bridge responses at stations were recorded first and then input into the moving force identification system. Any methods in the system can be chosen to identify the moving axle loads, which are used to reconstruct the so called “rebuilt” responses of bridge. The relative percentage error (RPE) between the measured and rebuilt responses is adopted to assess the efficiency and robustness of identification methods. Table 1 gives a comparison between RPE values when different identification methods are used to estimate the two moving axle loads from measured bending moments at seven measurement stations, here, *L* denotes the beam span length. It shows that all the RPE data are lower than 10% except at the first station (*L*/8) and 7th station (7*L*/8) for IMII method. This illustrates that all the four identification methods involved in the moving force identification system are correct and effective. Moreover, the results from both TDM and FTDM are clearly better than ones from both IMI and IMII. Further, the results provided by SVD solution are better than ones by PI solution, particularly for FTDM. It shows SVD technique can improve the identification accuracy.

Table 1 – Comparison of identified results

Method	**RPE (%)**						
	***L*/8**	**2*L*/8**	**3*L*/8**	**4*L*/8**	**5*L*/8**	**6*L*/8**	**7*L*/8**
IMI	9.37	9.51	9.73	9.62	9.55	9.14	8.36
IMII	12.2	6.00	6.98	4.40	6.05	5.75	13.2
TDM	5.42 5.43	3.08 3.09	1.80 1.80	2.58 2.56	1.95 1.94	3.44 3.43	4.83 4.83
FTDM	5.74 4.53	2.80 2.52	2.15 1.87	2.08 1.95	2.14 1.82	2.41 2.24	4.74 3.17

Notes: The undervalued values are from SVD solution, others from PI solution.

4. Experimental Verification in Field

Before the field measurements, a moving force identification method taking into account the effects of prestressed concrete bridge is presented (Chan and Yung, 2000a) based on IMI method. A 2-axle medium lorry is modeled as a set of forces, with their magnitudes either constant or time-varying, moving across a bridge model with a span length of 28 m. Results show that identified forces are identical to input forces only when no noise is added to the simulated bridge responses. The identified forces from responses which include noise are poor, even with a noise level as low as 1%. However, the identification can be greatly improved using a low-pass filter. Results also show that the identified forces are over- and underestimated, respectively, when the prestressing effects are neglected in the identification and calibration processes. Large errors are also obtained when the prestressing effects are neglected in both processes. Additionally, the errors decrease with decreasing prestressing forces, and do not change with the magnitude of axle forces and mass per unit length.

As an extension work of above theory (Chan and Yung, 2000a), field measurements were carried out to verify the proposed method in October 1995 on an existing prestressed concrete bridge of Ma Tau Wai Flyover, Hunghom, Kowloon, Hong Kong (Chan, Law et al., 2000). A two-axle heavy vehicle of 15 tons was hired for the calibration test of the field measurements. The dynamic bending moments of the test bridge deck brought about by both hired one and in-service 77 vehicles were acquired respectively. Dynamic axle forces were identified by means of the TDM method. The equivalent static axle loads of the two cases are tabulated in Table 2, which shows that the gross weights of both Test 2 and Test 3 are acceptable with percentage differences of 3.69% and 0.85% respectively (COST323, 1999). After obtaining the identified two axle loads of the control vehicle, the rebuilt bending moment responses can be calculated based on the forward problem, and then the RPE data between the measured and the rebuilt responses at each channel can be estimated as list in Table 3 for the control vehicle. Since no information was available for the axle loads of the in-service vehicle, the accuracy of identified dynamic axle loads was also studied using only RPE between the measured and rebuilt responses. Results show that the axle loads can be identified with acceptable results for both hired and in-service vehicles. Therefore the proposed method is valid for identifying dynamic axle forces. Gross weights can be obtained by summing up the equivalent axle load of each axle.

Table 2 - Summary of equivalent static loads identified with considering prestressing

Test case	Axle 1		Axle 2		Total	
	Equivalent static load (kN)	Difference (%)	Equivalent static load (kN)	Difference (%)	Gross weight (kN)	Difference (%)
2	65.22	1.97	91.22	4.95	156.44	3.69
3	62.48	-2.31	89.69	3.19	152.17	0.85

Table 3 - Summary of RPE between responses for control vehicle

Test case	RPE (%)				
	Ch.2	Ch.3	Ch.4	Ch.5	Ch.6
2	2.12	3.13	2.74	9.82	7.31
3	5.87	9.96	3.96	2.38	6.40

5. Conclusions and Recommendations

Recent advances on identification methods of moving loads identified from bridge responses are reviewed in this paper. The background of moving force identification is introduced. Numerical simulations, illustrative examples and comparative studies on the effects of different parameters on the system have been carried out and critically investigated. Bridge-vehicle system models have also been fabricated in the laboratory to validate the correctness and robustness of the proposed methods involved in the moving force identification system (MFIS). Field measurements have also conducted to assess the applicability of the methods in practice. The results show that all four identification methods involved in the MFIS can effectively identify moving axle loads on bridges and can be accepted as practical methods with higher identification accuracy. However, there are still many challenges and obstacles to be overcome before these methods can be implemented in practice, further studies on the moving force identification are necessary and recommended based on the previous experiences and results:

- Although the moving force identification methods are developed and have been proved to be successful, the work mainly focused on laboratory studies. Further field work is necessary and recommended to validate the methods and to accommodate them to practical bridge-vehicle system in engineering.
- Present research is mainly based on the measured bending moment responses induced by the passage of vehicles on bridges; however, other types of bridge responses, especially for acceleration responses easy to measure and operate, should be used to identify the moving loads on bridges. In the cases, a higher sampling frequency, more measurement stations and mode numbers are recommended to adopt in the calculation of identification.
- It is always recommended to use the SVD solution instead of the PI solution, especially for the frequency-time domain method. However, performing a full-SVD is too expensive in practical application. An algorithm to compute a partial SVD may be used instead (Vogel et al., 1994), or replacing the full-SVD by a RRQR factorization (Bazan et al., 1996). The fast computing method and powerful computer is necessary for real time computation and some techniques of splitting the larger coefficient matrix into smaller sub-matrix are recommended in order to make the computation more cost effective.
- Moving force identification from bridge responses is a typical inverse problem tends to be ill-conditioned, in which the effects of errors are of major concern. Numerical scaling and regularization techniques are needed to improve ill-conditioned effects (Stevens, 1987). However, finding the optimal regularization parameter λ is the main difficulty of applying the regularization technique, S-curve (Busby and Trujillo, 1999), L-curve (Hansen, 1992), and GCV methods (Zhu et al., 2002) can be used but subjected to researcher’s experiences and prior information.
- The central difference, Newmark-β and Wilson-θ methods are often used in the moving force identification process, the measurement errors are easy to be amplified and spread abroad, the precise time-step integration technique (Zhong et al., 1994) may be used to establish a calculation format between responses of bridges and moving loads on bridges for more accurate results, especial for both IMI and IMII methods.

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