A Space–Time Model for Mobile Radio Channel With Hyperbolically Distributed Scatterers

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***Abstract*—In this letter, we present a geometrical and time-variant wireless vector channel model with hyperbolically distributed scatterers for a macrocell mobile environment. This model is based on the assumption that the scatterers are arranged circularly around the mobile station, whereby the distance between the mobile and the local scatterers and the distance between the local scatterers and the dominant scatterers are distributed hyperbolically. The proposed model allows investigation of beamforming aspects as well as space–time processing techniques. Simulation results for this model are presented and compared with the exponential model results.**

***Index Terms*—Beamforming, channel, macrocell, mobile, multipath, scatterer, space–time.**

I. Introduction

OVER the past few decades, radio communication systems have undergone extensive developments. The demands that a radio system must fulfill are greater by the day. The propagation of radio signals on both forward (base station to mobile) and reverse (mobile to base station) links is affected by the physical channel in several ways. The objects and structures in these physical channels are buildings, hills, and trees. The collection of objects in any given physical region describes the propagation environment [1].

A signal propagating through a wireless channel usually arrives at the destination along a number of different paths, referred to as multipaths. These paths arise from scattering, reflection, refraction or diffraction of the radiated energy off objects that lie in the environment. The received signal is much weaker than the transmitted signal due to phenomena such as mean propagation loss, slow fading, and fast fading [1]. A stochastic-geometrical channel model was proposed by Stege [2]. Liberti and Rappaport [3] developed a geometrical-based single-bounce model (GBSB) for microcells.

Lohse [4] developed a geometrical exponential model for macrocells. This model assumes a circular distribution of scatterers around the mobile station (MS), and the distances between the MS and the scatterers are distributed exponentially.

A spatial Rayleigh-fading correlation model for multiple-input multiple-output (MIMO) has been proposed in [5]. This model assumes uniform distribution of the scatterers with respect to θ, the angle-of-arrival at the subscriber unit. The model also assumes that all received rays are equal in power, which is not a realistic assumption for multipath environments. An extension of this model for Rician-fading channels appeared in [6], which assumes von Mises angular distribution for scatterers.

In [7], we developed a geometrical based hyperbolic channel model for macrocells, which provides the directional information of the multipath components. This model assumes a circular distribution of dominant scatterers around mobile station (MS), and the distances between the MS and scatterers are subject statistically to a hyperbolic distribution.

In this paper, we present a combination of scalar stochastic fading model for local scatterers with the geometrical hyperbolic model for the distribution of dominant scatterers. Parts of this work appeared in [7]. Slow fading effects as well as mobile movement including the appearance and disappearance of remote scatterers are also taken into account.

II. Proposed Channel Model

The proposed space–time geometrical-based hyperbolically distributed scatterers (GBHDS) model is shown in Fig. 1. Although the model is applicable for downlink as well, we will analyze the uplink environment only. We assume that the mobile antenna radiates uniformly in azimuth. The propagation environment is densely populated with both natural and man-made structures. These structures act as wave scatterers and are classified into two categories: local scatterers, which are distributed around the mobile, and dominant scatterers, which are far away from the mobile. Scalar channel models do not explicitly include the effects of these dominant reflectors and don't provide any directional information [1]. In this model, we assume that for the kth path, the distance between the mobile and the local scatterer r_k and the distance between the local and dominant scatterer R_k are distributed hyperbolically [7]. This assumption is more realistic and flexible than other existing probability density functions (pdfs), like the exponential-decaying pdf [4] and the angular distribution [5], [6]. This is so because the hyperbolic distribution allows scatterers to be more likely in a flexible vicinity of the mobile rather than of a decaying likelihood that drops immediately after the MS (the exponential distribution) or a uniform likelihood along a specific angle (the angular distribution).

The maximum length of r_k is a few meters (vicinity of the mobile). Unlike other models that consider local scatterers (e.g., [1] and [2]), the propagation distance spread for the local scatterers in our model, $\Delta r_k = \max_k(r_k) - \min_k(r_k)$, is not considered negligible with respect to r_k.

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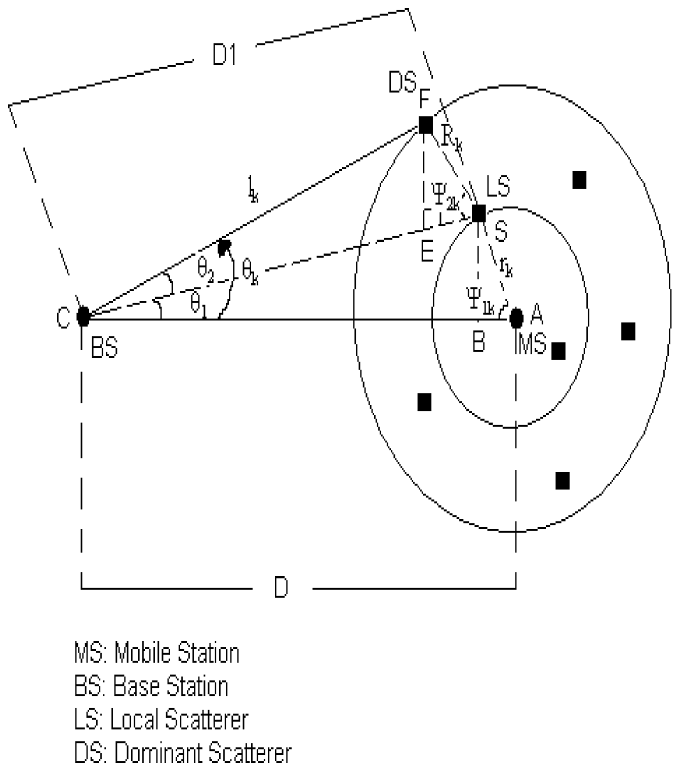

Fig. 1. Geometry of the proposed model.

A. Signal Model

A narrowband space–time channel with one transmit antenna and M receiving antennas is considered. Let the transmitted signal be

$$x(t) = s(t)e^{j2\pi f_c t} \tag{1}$$

where $s(t)$ is the complex baseband signal with bandwidth B and carrier frequency f_c. If we assume that the mobile is moving at speed v and there is no direct line-of-sight (LOS) component for macrocell, the corresponding received signal $y_m(t)$ at the m^{th} antenna element is given by [2]

$$y_m(t) = \sum_{k=1}^{L} \mathbf{a}_m(\theta_k)\sqrt{P(\tau_k)}\alpha_k(t) \times x(t-\tau_k)e^{j\phi_k(t)} + n_m(t) \tag{2}$$

where L is the number of multipaths, $m = 0, \ldots, M-1$, $\mathbf{a}_m(\theta_k)$ is the steering vector of a uniform linear array (ULA), $\sqrt{P(\tau_k)}$ describes the path attenuation, $\alpha_k(t)$ characterizes the fast fading of the space–time channel, $x(t)$ represents the transmitted signal, $n_m(t)$ is the receiver additive white Gaussian noise (AWGN), τ_k is the path delay, and $\phi_k(t) = 2\pi[f_d\cos(\psi_k t) - f_c\tau_k]$, where the expression $f_d\cos(\psi_k t)$ represents the Doppler shift, $f_d = v/\lambda$ is the maximum Doppler shift (v being the mobile velocity, λ is the wavelength), and ψ_k is the direction of the k^{th} scatterer with respect to the mobile velocity vector. The Doppler spread f_m is given by $2f_d$. These parameters vary with time. The DOA and the path delay are determined geometrically from the model. The time-variant channel impulse response is described by

$$h(t,\tau) = \sum_{k=1}^{L} \sqrt{P(\tau_k)}\alpha_k(t)e^{j\phi_k(t)}. \tag{3}$$

As the mobile station moves in the environment, the number and position of scatterers affecting the received signal will change with time, causing the time delay, the fast-fading amplitude, the phase, the pathloss, and the number of resolvable paths to be time-varying [2].

B. Parameters of the Proposed Model

The proposed model provides the power of each path, the TOA, and the DOA of the multipath component as well as the fading effect. The probability density functions of the distances r_k and R_k are assumed in this model to have the forms

$$f_{r_k}(r_k) = \frac{a_1}{\tanh(a_1 R)\cosh^2(a_1 r_k)};\ 0 \le r_k \le R \tag{4}$$

$$f_{R_k}(R_k) = \frac{a_2}{\cosh^2(a_2 R_k)};\ 0 \le R_k < \infty \tag{5}$$

where R is the radius of the circle enclosing the local scatterers under consideration (scatterers beyond R are considered dominant). The applicable values of a_1 and a_2 lie in the interval (0,1). The values of the parameters a_1 and a_2 control the spread (standard deviation) of the local and the dominant scatterers, respectively, around the mobile station. Increasing a_1 and a_2 reduces the spread of the pdf's of r_k and R_k. Accordingly, a_2 should be smaller than a_1. It will be clarified after (18) in this subsection that the value of a_2 is controlled by the choice of the maximum excess delay $\tau_{e_{\max}}$.

The random variable R_k with pdf given in (5) can be obtained from a random variable x_k uniformly distributed over the interval [0,1] using the following transformation:

$$R_k = \frac{1}{a_2}\tanh^{-1}(x_k). \tag{6}$$

From the triangles ABS, ACS, EFS, and CFS in Fig. 1, the path delay (τ_k) of the multipath component is given by

$$\tau_k = \frac{(r_k + R_k + l_k)}{c} \tag{7}$$

where $l_k = \sqrt{R_k^2 + D_1^2 - 2R_k D_1 \cos(\psi_{2k})}$ and

$$D_1 = \sqrt{r_k^2 + D^2 - 2r_k D\cos(\psi_{1k})} \tag{8}$$

ψ_{1k} and ψ_{2k} are uniformly distributed on the interval [0,2π]. The DOA for the k^{th} path is given by

$$\theta_k = \theta_1 + \theta_2 \tag{9}$$

where

$$\theta_1 = \tan^{-1}\left(\frac{r_k\sin(\psi_{1k})}{D - r_k\cos(\psi_{1k})}\right) \tag{10}$$

and

$$\theta_2 = \begin{cases} \tan^{-1}\left(\frac{R_k\sin(\psi_{2k})}{D_1 - R_k\cos(\psi_{2k})}\right) \\ \text{for } R_k\cos(\psi_{2k}) \le D_1;\ \text{and} \\ \tan^{-1}\left(\frac{R_k\sin(\psi_{2k})}{D_1 - R_k\cos(\psi_{2k})}\right) + \pi \\ \text{for } R_k\cos(\psi_{2k}) > D_1. \end{cases} \tag{11}$$

The steering vector to an incoming signal $x_k(t)$ from a DOA θ_k has the form

$$\mathbf{a}_m(\theta_k) = [1,\ a_2(\theta_k),\ \ldots,\ a_M(\theta_k)] \tag{12}$$

where $a_m(\theta_k)$ is a complex number denoting the amplitude and the phase shift of the signal at the m$^{\text{th}}$ antenna relative to that at the first antenna. For a uniform linear array, $a_m(\theta_k) = e^{[j2\pi(m-1)\ d\ \sin(\theta_k)/\lambda]}$, where d is the space between adjacent antennas and λ is the wavelength of the carrier.

The mean power of each multipath component depends on the propagation delay τ_k and is usually defined by a characteristic power delay profile (PDP) P(τ_k) which is given by [8]

$$P(\tau_k) = P_{\text{ref}} - 10\, n \log\left(\frac{\tau_k}{\tau_{\text{ref}}}\right). \tag{13}$$

The pathloss exponent n depends on the propagation scenario to be simulated [2]. P_{ref} is a reference power that is measured at a distance d_{ref} from the transmitting antenna when omnidirectional antennas are used at both the transmitter and the receiver. The reference power is given by [8]

$$P_{\text{ref}} = P_T - 20\ \log\left(\frac{4\pi d_{\text{ref}} f_c}{c}\right) \tag{14}$$

where P_T is the transmitted power in decibels and f_c is the carrier frequency. The fast fading $\alpha_k(t)$ is modeled as a Rayleigh-distributed random process.

The maximum path delay $\tau_{\max}$ for the model simulation is controlled by the value of a_1 and a_2. This delay occurs when $\psi_{2k} = 180°$. Then from (6) we have

$$\tau_{\max} = \frac{1}{c}(r_{k_{\max}} + D_{1_{\max}} + 2R_{\max}) \tag{15}$$

where $r_{k_{\max}}$ is the maximum distance between the MS and the local scatterer, $D_{1_{\max}}$ is the maximum distance between the local scatterer and the base station, and $R_{\max}$ is the maximum distance between the local scatterer and the dominant scatterer. The probability of that the dominant scatterers are beyond $R_{\max}$ is

$$p = \Pr(\mathrm{R_k} > \mathrm{R_{max}}) = 1 - \mathrm{F_R}(\mathrm{R_{max}}) \tag{16}$$

where the cumulative probability distribution of $R_{\max}$ is given by

$$F_R(R_{\max}) = \int_0^{R_{\max}} \frac{a_2}{\cosh^2(a_2 R)} dR = \tanh(a_2 R_{\max}). \tag{17}$$

By inserting (17) and (15) into (16) we get

$$a_2 = \frac{2\tanh^{-1}(1-p)}{(r_{\max} D - D_{1_{\max}} - r_{k_{\max}})} \tag{18}$$

where $p \in [0,1]$ and $D_{1_{\max}} = r_{k_{\max}} + D$. The value of $r_{\max}$ is given by $(\tau_0 + \tau_{e_{\max}})/\tau_0$, where τ_0 is the LOS path delay and $\tau_{e_{\max}}$ is the maximum excess delay. Varying the maximum excess delay produces different values of a_2.

C. Delay Spread and Coherence Bandwidth

In this subsection we characterize briefly the root mean square (rms) delay spread and the coherence bandwidth of the channel in terms of the parameters of the proposed model. The rms delay spread is the square root of the second central moment of the power delay profile and is defined by [8]

$$\sigma_r = \sqrt{\frac{\sum_{k=1}^{L} P(\tau_k)\tau_k^2}{\sum_{k=1}^{L} P(\tau_k)} - \left(\frac{\sum_{k=1}^{L} P(\tau_k)\tau_k}{\sum_{k=1}^{L} P(\tau_k)}\right)^2} \tag{19}$$

where $P(\tau_k)$ is the power as in (9) and τ_k is the delay of the kth multipath component arriving at the base station [see (7)]. The coherence bandwidth for the frequency correlation function above 0.5 is defined as [8]

$$B_c \approx \frac{1}{5\sigma_\tau}. \tag{20}$$

The above formulas will be utilized in the simulation of the proposed model in the next section.

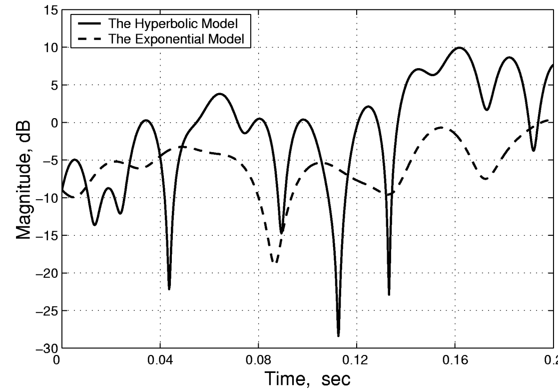

Fig. 2. Comparison between Rayleigh-fading magnitudes of the hyperbolic channel model (with $f_d = 200$ Hz, $T_s = 200\ \mu$s, $a_1 = 0.2$, $a_2 = 0.007$, $\tau_{e_{\max}} = 400$ ns, and $D = 1$ km) and the exponential channel model (with $D = 1$ km and $\tau_{e_{\max}} = 400$ ns).

III. Simulation Results and Discussion

The space–time vector channel is designed to model a wide variety of possible mobile channel scenarios. It produces directional information about the mobile and the fading effects of the environment. This information includes the path delay (τ_k), the DOA (θ_k), the fast fading (α_k), and the power (P_k) for multipath components. In simulating the multipath component parameters using this model, it is necessary to generate samples of the random variable x_k distributed uniformly on [0,1]. The second step is to set the value of a_1, which is chosen in this simulation to be 0.2, then determine the value of a_2 from (18), which is controlled by the maximum excess delay, $\tau_{e_{\max}}$, taken here as 400 ns, which gives $a_2 = 0.007$. Note that the applicable values of a_1 and a_2 are in the interval (0,1). When a_2 is determined, the distances $\{R_k | k = 1, \ldots, L\}$ between the local and the dominant scatterers can be evaluated from (5). The next step is to generate the time delay τ_k of each path, and its DOA θ_k by using (6) and (7), respectively.

The proposed and the exponential models have been simulated for an urban area. In this simulation, the pathloss exponent n, the number of simulated multipath components L, and the distance between the mobile and the base station, D, were set

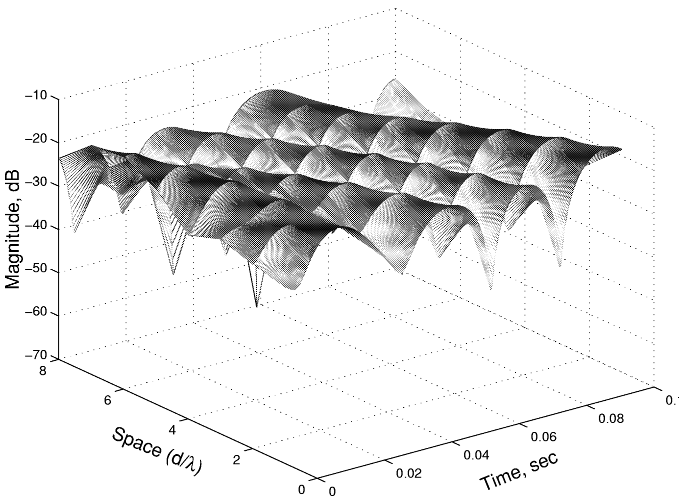

Fig. 3. Space–time spatial fading of the hyperbolic channel model with $f_d = 200$ Hz, $T_s = 200\,\mu$s, $a_1 = 0.2$, $a_2 = 0.007$, $\tau_{e_{\max}} = 400$ ns, and $D = 1$ km.

respectively to 4, 100, and 1 km. For the proposed model we chose $R = 50$ m and $R_{\max} = 700$ m, hence, $0 \leq r_k \leq 50$, and $0 \leq R_k \leq 700$. Fig. 2 shows the signal magnitudes under Rayleigh fading for the hyperbolic and the exponential channel models using the above-mentioned parameters (1000 symbols were considered). The mobile speed v results in a maximum Doppler shift of $f_d = v/\lambda = vf_c/c$ Hz, λ is the wavelength, c being the speed of light, and f_c is the carrier frequency. In this simulation we considered $f_d = 200$ Hz. Fig. 3 shows the received signal level across the array (space–time fading) over a period of 500 symbols for the hyperbolic model with specific values of the model parameters. The angular spread is $\triangle\theta \approx 49°$. The corresponding angular spread for the exponential model is $\triangle\theta \approx 23°$. The rms delay spread for the proposed hyperbolic model and its coherence bandwidth are 417.27 ns and 479.30 kHz, respectively. The corresponding rms delay spread and coherence bandwidth for the exponential model are 654.13 ns and 305.74 kHz, respectively.

IV. Conclusion

In this letter, a space–time GBHDS model for a macrocell mobile environment is proposed. The combination of stochastic and geometrical assumptions results in a mathematically tractable and computationally efficient channel model. This model provides the power of each path, the TOA, and the DOA of the multipath component as well as the fading effect. The model enables the simulation of downlink beamforming as well as space diversity concepts and handles both spatially narrowband and wideband signals. We conclude from the simulation results that the hyperbolic model has greater space selective fading than the exponential model.

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