Recently, there has been considerable attention to the development of the mathematical capability of Australasian citizens. It is widely accepted that all citizens should be numerate. That is, they should be able to cope with the everyday mathematical demands of life at school, in the home, at work, and in the community. Additionally, there is a need for adequate numbers of citizens to develop the high-level mathematical capability necessary to support and advance our technologically-oriented society (Howard, 2001; MacGillivray, 2000). Thus, the dual goals of contemporary mathematics education are (1) to develop a numerate citizenry, and (2) to develop a society with sufficient high-level mathematical capability. However, to achieve these goals, we must understand how to adequately cater for exceptional students, such as those who have learning difficulties and those who are gifted in mathematics. While students with learning difficulties and gifted students clearly differ substantively, they are both “at risk” of underachieving in mathematics (Diezmann, Thornton, & Watters, 2003). This chapter provides an overview of the context for the education of students with learning difficulties and mathematically gifted students, reviews the associated research, and suggests avenues for future mathematics education research to support exceptional students.

**Students with Learning Difficulties**

Conscientious teachers of mathematics have always been concerned with those students who find it difficult to learn mathematics. However, for much of our educational history, the aim has been to identify and remove those who would find mathematics difficult. The reason for the development of the IQ test was to allow educators to exclude children for whom education was considered to be a waste of public resources (Richardson, 1991). In more recent times, students facing difficulties have been excluded in other, more subtle ways such as streaming (or “setting”) (Zevenbergen, 2001, 2003), reduced access to the curriculum (such as “core only” mathematics classes) (Faragher, 2001) or through altered teaching approaches (Norton, McRobbie, & Cooper, 2002).
Optimally, all students including those with learning difficulties need opportunities to engage in a mathematics program that encompasses all strands of the mathematics curriculum and includes tasks that challenge their mathematical thinking.

**Philosophical underpinnings**

A review of the research on learning difficulties was not included in the 1996-1999 MERGA review (Owens & Mousley, 2000). However, it is useful to consider the changes that have occurred in the understanding of the field through the 1990s. Writers in the field of gender studies refer to the shift in perspective toward the education of girls to a point of view which acknowledges that girls are not deficient in some way and in need of change to make them fit in (Forgasz, Leder, & Vale, 2000). A similar change has occurred in special education. Stephens (2000) notes that “Traditional approaches to assisting students with learning difficulties in numeracy have tended to focus on remediation and withdrawal” (p. 27). Moving from a medical model of disability focused on identifying and remediating deviance, the sociological model acknowledges the right of people with a disability to be as they are and not to have to change to fit in (Ainscow, 1994; Forman, 2001; Sykes, 1989). Therefore, the emphasis in the last decade has been less of one seeking to define, diagnose and remediate learning difficulties in individual students to one of identifying ways to make the curriculum more accessible to all learners. Despite this shift in philosophy, the diagnostic perspective is still evident in some recent research and practices.

**Context**

The end of the 1990s saw a focus on numeracy, driven at least initially by Australian Commonwealth Government Education Policy. The Department of Education, Training and Youth Affairs (now the Department of Education, Science and Training) funded three significant reports in this period.

The first report, “Mapping the Territory. Primary Students with Learning Difficulties: Literacy and Numeracy” (Louden et al., 2000) provided a national summary of support in regular classrooms for children with learning difficulties in the areas of literacy and numeracy and identified successful strategies. Inconsistency of terms used to describe and classify children across Australia was noted in the report as making research and data gathering difficult. A particular difficulty was evident in the identification of children experiencing difficulties with learning mathematics: “Recognition of what constitutes a learning difficulty in numeracy is not widely shared across the teaching community in Australian primary schools, especially in the early years” (Milton, 2000, p. 111).

In contrast to literacy where early identification of difficulties was widespread, Louden et al (2000) noted that for numeracy, early identification was much less common. Additionally, numeracy assessments were less directly related to current syllabus documents. “We found in most States there are only isolated instances at the school level of any systematic focus and provision of specific help for children with learning difficulties in numeracy” (Milton, 2000, p. 109). However, since this report was published, there is substantial evidence of the widespread identification of numeracy difficulties in the early years in Australasia (Groves, this volume). The Louden report adopted the terminology of first, second and third wave intervention for considering strategies for learning support. “First wave” referred to the initial teaching to whole class groups and “second wave” to early intervention programs. Provision for upper primary school children
still experiencing difficulties was referred to as “third wave” intervention. The report noted that third wave provision often lacked sufficient resources and suffered from poorer quality of programs.

Louden et al. (2000, p. 12) recommended, “(that) schools need to adopt a clear focus on early numeracy teaching in regular classroom settings and ensure that the focus is maintained in second and third wave numeracy programs.” The earlier findings of the report – that many children are not identified with learning difficulties until they are well into their primary school years and then many teachers accept that some children just won’t be able to do mathematics – suggests the lack of provision for third wave intervention may not be challenged. The development of programs for third wave numeracy intervention and then appropriate professional development to support their implementation should be a priority.

The second report, “Literacy, Numeracy and Students with Disabilities” (van Kraayenoord, Elkins, Palmer, & Rickards, 2000), investigated the provision of literacy and numeracy teaching for children with special needs in regular classrooms. This report highlighted successful strategies but also identified areas of concern. A key concern was the lack of training for the teachers, some of whom had undertaken no studies in special education. As a mathematics teaching profession, the issue raised in the report of adequate preparation of teachers for diverse learners is of relevance.

An additional concern centred on the role of teacher aides – often being responsible for all literacy and numeracy teaching for some students. This concern related to the preparation and training of assistants and the roles they were allowed to undertake (van Kraayenoord et al., 2000). Stephens (2000) noted a related problem:

Typically these approaches (remediation and withdrawal) involved withdrawal from the regular classroom of children in the middle and upper years of primary school, thus removing the classroom teacher from the cycle of intervention and support. This focus on withdrawal undervalues the importance of classroom instruction in detecting and overcoming difficulties, and the support of classroom peers. (p. 27)

The provision of key areas of education by teacher aides, rather than professional teachers could be seen to be another aspect of excluding students from adequate opportunities to learn mathematics.

The third report, “Assessment and Reporting of Student Achievement for Students with Specific Educational Needs against Literacy and Numeracy Benchmarks”, provided background information to support policy decisions relating to the use of the National Literacy and Numeracy Benchmarks. The project, conducted by the Australian Council for Education Research [ACER], identified critical issues in the use of the benchmarks for students with specific educational needs (Zammit, Meiers, & Frigo, 1999). In agreement with the Louden Report (2000), the ACER report highlighted the need for consistent definitions and means for identifying students with specific educational needs. In addition, they noted inconsistencies in practices across the states concerning which students were exempted from state-based assessment. They also suggested the need for further research for supporting students who do not achieve the National Numeracy Benchmarks.

In New Zealand, the Special Education 2000 policy (Ministry of Education, 1996) aims to improve the learning outcomes of all students with special needs. Provisions include specialist support from Resource Teachers: Learning and Behaviour [RTLB] for students with a ‘moderate’ level of intellectual disability. One of the goals of the RTLB initiative is “to provide support to regular teachers through advising and assisting them to provide the best learning conditions for the student (e.g., adapting instruction, adapting the
classroom organisation)” (Ryba & Annan, 2000, p. 269). RTLBs have reported a close interrelationship between learning difficulties and behavioural difficulties. Generally, RTLBs consider that students with learning difficulties require more one-to-one assistance and more highly skilled teachers and teacher aides. The effectiveness of provision “depends on the teachers’ ability/willingness to adapt the curriculum to meet students’ learning needs” (Bourke, Kearney, Poskitt, & McAlpine, 2001, p. 52).

These Australasian documents and reports have been complemented by large-scale numeracy programs designed to monitor and support numeracy attainment.

**Australasian Numeracy programs**

The considerable interest in the ways to support all students to become numerate has spawned a number of state-based or national programs, which have reported success with students or teachers. *Mathematics Recovery* (Phillips, Leonard, Horton, Wright, & Stafford, 2003; Wright, 2000) claims considerable success in assisting students who are experiencing difficulties learning mathematics. This program formed the basis for the widespread and well established New South Wales *Count Me in Too* program (Mulligan, Bobis, & Francis, 1999). A New Zealand Numeracy Development Project developed from the New South Wales program is in the early stages of implementation and evaluation with respect to achievement levels and students’ progress. This program “continues to be informed by developing understandings about students learning of number and what constitutes effective professional development and effective facilitation” (Thomas & Ward, 2002, p. ii). The diagnostic interview tool used in the New Zealand project has the potential to help teachers identify exceptional students and specific areas of difficulty in number knowledge and strategies. Evaluation reports on this project have addressed student achievement levels and progress, teacher attitudes, classroom practice, identification and provision (Higgins, 2001, 2002; Irwin & Niederer, 2002; Thomas & Ward, 2002).

The Victorian *Early Numeracy Research Project* also utilises diagnostic interviews (Clarke, 2001). Students should benefit from improvements to teaching as teachers incorporate the strategies they have been trained to use in the project interviews into their classroom practice:

> The features (of effective teachers) were increasingly evident over the three years of the project in other classrooms, as teachers took what they had learned from the interviews about children’s mathematical thinking, and, working with colleagues, endeavoured to provide the kinds of activities and tasks that enhanced learning for all students. (Clarke & Clarke, 2003, p. 315)

Australasian numeracy programs are discussed in more detail elsewhere (Groves, this volume).

**Issues in the education of students with learning difficulties in mathematics**

There are six contemporary issues in the mathematics education of students with learning difficulties. The first issue is the timing of the identification of students with learning difficulties and interventions. A number of the numeracy programs in Australia are characterised by early identification and intervention in the early years of schooling (e.g., Clarke, 2001; Clarke & Clarke, 2003; Mulligan, Bobis & Francis, 1999; Wright, 2000). Gervasoni (2002) considered the issue of timing of interventions in the early years. She reported on the *Extending Mathematical Understanding* [EMU] Program and whether grade level is a critical variable for its impact on mathematics learning of children who
have difficulty learning mathematics. Gerversoni concluded that specialised assistance at critical points assists such children at both Grade 1 and 2 with no clear indication as to whether intervention at one grade was more effective than at the other. However, parents and teachers believed that it was more appropriate to begin in Grade 1.

The second issue relates to effective strategies for inclusive classrooms. In line with the considerable research evidence supporting the value of children with special educational needs being taught in regular classrooms (e.g., Buckley & Bird, 2000; Forman, 2001), much of the research in the last few years has investigated effective strategies for inclusive classrooms. Murik and Murik (2001) investigated the use of Slavin’s (1990) strategy of Team-Assisted Individualisation [TAI] in inclusive mathematics classrooms. TAI refers to a cooperative learning model where the teacher presents the material, students work in groups to understand concepts and then sit for tests / quizzes individually. Students are given individual improvement scores with their aim being to improve over past performances. Students may earn points for their team based on how much their test scores exceed their base scores. The results of the study – through test results and student and teacher comments – would suggest the strategy to be successful for all students.

A whole-class survey was conducted with the students at the end of the term when three topics from the syllabus had been taught. … It was clear from the survey that TAI is well regarded by the students, and that they perceive it to be of benefit to them. The survey seemed to confirm what the review of the literature has shown to be the advantages of using the cooperative learning approach, in terms of positive social interactions and better academic outcomes. (Murik & Murik, 2001, p18-19)

Criticisms of this strategy could be levelled at the emphasis on knowledge transmission and assessment by written tests. However, there would appear to be scope for adapting the strategy to suit contemporary mathematics teaching and assessment practices. Interestingly, this study indicates the value to all students in the class from the adoption of a practice originally implemented to support students with learning difficulties. Dole (2003) also advocated the benefits of an inclusive approach for middle years students with learning difficulties. She concluded from a comparison of inclusive and withdrawal models that inclusive models offered more scope for learning, however she also acknowledged that an inclusive approach was demanding of teachers.

Students in the withdrawal classes as seen through the snapshots presented here, appear to enjoy developing skills and completing individualised skill-building programs. Yet in the inclusive model we see a much richer mathematical environment where students are engaged in thinking and communicating at a higher mathematical level. The range of productive pedagogies utilised is far-reaching, compared to those in the withdrawal mode. The inclusion model is highly demanding of the teacher in terms of effort and energy, but the cognitive, (and) social (benefits) and interest in learning of the students was palpable in this class compared to that exerted by students in the withdrawal class. (Dole, 2003, p. 285)

A profile of exemplary inclusive practices in the middle years are presented in Luke et al., (2003). In particular, they emphasised the positive role of student talk in an exemplary classroom, and the absence of textbooks.

A third issue attempts to determine the nature of specific learning difficulties with mathematics. Hopkins (2000) investigated the addition strategies used by adolescents with learning difficulties. Her aim was to determine if the source of the difficulties could be attributed to developmental delay or some other processing deficit. Hopkins’ results suggest that developmental delay could be occurring as the students were using
fundamentally similar strategies to younger students without learning difficulties. She also noted that:

These students had moved on to develop a reliance on an efficient counting strategy. Their lack of decomposition strategies was more a result of not knowing useful facts, than not applying facts. The greatest difficulty faced by these students was developing a reliance on retrieval. (Hopkins, 2000, p.329)

A fourth issue concerns the applicability of research on special groups, such as students with learning difficulties, for all students. Willis (2002) cautions against viewing ways of learning or teaching mathematics for special groups as curiosities, only of relevance to students in those groups and the teachers trained to teach them. She referred to an aspect of the development of counting which was first noticed in Indigenous Australian children. It was only after noticing the same development in non-Indigenous children that it was considered that more could be learned about the process of counting. Previously Ainscow (1994) commented:

I have spent considerable time and energy attempting to find special ways of teaching that will help special children to learn successfully … My conclusion now is that no such specialized approaches are worthy of consideration. (p. 19)

This would suggest that attention should be focused on what can be learned about all students from considering the learning of mathematics by special groups.

The fifth issue focuses on the impacts of ability grouping or streaming on students. Zevenbergen (2001) investigated the experiences of a representative sample of 128 students from Years 9 and 10. She reported that these students experienced qualitatively different learning environments according to whether they were in high or low stream classes. However, Zevenbergen pointed out that although streaming was designed to support students who were struggling with mathematics, the practices in these schools effectively worked against students in the lower streams through relatively inexperienced teachers, off-task behaviour, lower content and expectations, and a restricted curriculum (Zevenbergen, 2001):

The students in the high streams often had the more experienced teachers … The students in the lower streams reported different experiences with their teachers, often not seeing them in a favourable way. They offered comments that described their teachers as unsupportive. (p. 566)

The upper streams were more on-task whereas there were considerable behaviour management issues in the lower streams (p. 567) … the ethos in the lower streams is a hindrance to quality learning – and is identified by students in both high and low streamed classes as being such. The students who were instigators of the “mucking around” often recognised their implication in the hindering of students … Students were quick to realise that with the lower content and expectations of the lower streams, this effectively excluded them from participating or moving out of their current grades. Furthermore, students realised that the behaviour of their peers reduced what could be achieved in these classes. (p. 568)

Students in the lower streams were exposed to a restricted curriculum so that when they came to sit the Year level exams, they encountered new content and could not work out the problems, thereby restricting the grades they could achieve (p. 569)

Zevenbergen (2001) attributed the poor quality learning experiences in the lower streamed classes to the practice of streaming itself:

The practice of streaming is seen to be a positive one for the high streamed classes with the converse being the case for the lower streamed students. (pp. 569-570)
However, her argument for mixed ability groups rather than homogeneous ability groups merely shifts the grouping disadvantage from low ability students to high ability students without addressing the fundamental issues of teacher quality and student responsibility that appear to underpin the disadvantages in the lower stream classes. For example, a shift in disadvantage occurs when the instructional pacing of the class is too slow for gifted students (Diezmann & Watters, 2002b). In contrast, some students with learning difficulties find a slower pace helpful (Zevenbergen, 2001). Whether or not classes are streamed, if students experience unsupportive teachers, low expectations, disrupted lessons, and a restricted curriculum, their learning environment is unlikely to yield high quality student outcomes. Thus, there seems to be substantial issues of quality instruction, teacher professionalism, and school behaviour management that should inform the ability grouping debate.

The final issue is the relationship between the literature on special education and mathematics education. Although some work on students with learning difficulties in mathematics has been published in the special education literature, these authors are not usually part of the mathematics education research community. The problem with this demarcation between special education and mathematics education is that there are few opportunities for transfer of ideas between fields, which denies students with learning difficulties the benefits of cross-disciplinary expertise.

Suggestions for further research on students with learning difficulties

Australasian research in the field of learning difficulties in mathematics is scarce, especially at the secondary school level. In contrast, there have been several recent significant government sponsored reports. Typically, these reports have reviewed the literature, studied existing practice, distilled effective strategies and provided recommendations. Subsequent reports have summarised and referred to previous reports while providing a different focus on the same broad topic. In essence, they tell us what is happening at the moment. However, some fundamental questions need investigation to ensure that students who experience difficulty in the learning of mathematics become numerate citizens.

“Why do some learners have difficulties with mathematics? There is an urgent need for fundamental research into sources of students’ difficulties learning mathematics. There is scope for a range of small-scale projects, perhaps case studies, to fill the void in understanding why some students have difficulty with mathematics. Findings from these studies could be used to investigate mechanisms for making the curriculum more accessible to all students. Research is also needed to establish why some learning difficulties persist even with considerable intervention support. These persistent difficulties may underlie the view held by some students, parents and teachers that some individuals cannot learn mathematics. This research could build on recent studies that describe the characteristics of effective learning environments for students with learning difficulties (e.g., Dole, 2003).

“What is effective support for students with learning difficulties in mathematics?” This question has various facets. While some withdrawal programs and special teaching methods have been shown to be successful, it is difficult to predict whether funding and expertise will always be available when and where they are needed. Investigation of in-class methods of support may be a more useful on-going mechanism. The demonstrated enhancement to the general teaching practice of teachers trained in these programs also
requires investigation and comparison with alternatives. In considering in-class support, the role of teacher aides and other assistants such as parent helpers needs further investigation from both an educational and industrial perspective. Research is also needed to compare the cost effectiveness of withdrawal programs with practices that support students within the classroom.

“What is effective support for older students with learning difficulties in mathematics?” Effective support in the secondary school is arguably the area of greatest need. Many students are not identified as having difficulties with mathematics until late in their primary school years. But, by the time they reach secondary school, little intervention provisions may be available. Well-researched and carefully designed “third wave” programs for older students are needed to support these students. Successful programs in New South Wales provide guidance for the design of effective intervention programs for older students (Luke et al., 2003) (Groves, this volume).

An associated question to the issue of in-class support for students with learning difficulties is “How do classroom strategies that support students with learning difficulties affect other students?” Studies that support the inclusion of students with special needs in regular classrooms typically focus on the benefits to the students with special needs and rarely mention the effects on the other students.

The following section focuses on understanding the needs of mathematically gifted learners.

**Gifted Students**

The importance of educating adequate number of students with high-level mathematical capability has been highlighted in recent years (MacGillivray, 2000; Thomas, 2000). These gifted students, who comprise approximately 10 to 15% of the population, are variously described as “talented”, “highly able,” “promising” or having “a mathematical cast of mind.” However, despite the potential of mathematically gifted students for high performance, leadership, or creative achievement, current educational provisions for the gifted are unlikely to provide adequate challenge, opportunities, and support for many of these students.

The Australian Senate report into the education of gifted children (Collins, 2001) acknowledged the inadequacy of current educational provisions and highlighted the fallacy that gifted students do not need special educational provision:

> (Gifted) children have special needs in the educational system; for many their needs are not met; and many suffer underachievement, boredom, frustration, and psychological distress as a result ....

The common belief that the gifted do not need special help because they will succeed anyway is contradicted by many studies of underachievement and demotivation among gifted children. (p. xiii)

Similarly, in New Zealand, the needs of many gifted and talented students have been overlooked and they have received scant opportunities to develop their strengths and abilities. Recently, the New Zealand government developed a policy that acknowledged gifted and talented students as a group who require identification and provision (Ministry of Education, 2002).

The issues of identification and adequate provision for gifted students in Australasia are complex and need to be considered at the societal and individual levels. At the societal level, there are cultural and socio-economic issues.
Teachers need to be trained to identify gifted children… this training should pay particular attention to the need to identify gifted students who have disadvantages such as low socio-economic status, rural isolation, physical disability or Indigenous backgrounds. (Collins, 2001, p. xiv)

Giftedness and talent can mean different things to different communities and cultures in New Zealand, and there is a range of appropriate approaches towards meeting the needs of all such students. Schools need to develop multi-categorical approaches to giftedness that are flexible enough to include the many characteristics that are typical of gifted and talented learners. (Ministry of Education, 2002, p. 2)

At the individual level, Plunkett (2000) found that teachers’ attitudes towards identification, social attitudes, grouping and acceleration and provisions for gifted students was affected by whether or not they had specialist training in gifted education issues, and subsequently recommended that all teachers receive some training.

**Issues in the education of mathematically gifted students**

Published Australasian research on the mathematically gifted over the last four years is scant. However, the limited research has explored the identification of the mathematically gifted; educational provisions for gifted students; challenging tasks, and skill development.

Identifying the mathematically gifted

Traditionally, scores on intelligence, aptitude or achievement tests have been used to identify mathematically gifted students. However, there is potential insight to be gained by analysing student responses to problems. Additionally, some tests are inappropriate for the identification of mathematically gifted students.

Neiderer and Irwin (2001) compared the results of 66 ten year olds, some of whom had been previously identified as gifted, on a national standardised multiple-choice test (Progressive Achievement Test [PAT]), which is commonly used in New Zealand to identify gifted students, and on a set of problems. Students’ PAT results differed from the problem solving results, which gave greater insight into the mathematical ability of these students. Niederer, Irwin, Irwin and Reilly (2003) also reported that the accuracy of using PAT to identify gifted students was low and raised concerns about the mis-identification and lack of identification of mathematically gifted students:

> With an overall accuracy rate of 78% correct, use of PAT to identify mathematically gifted children will lead to many errors of commission (false alarms) and omission (gifted children). (p. 80)

Such a degree of accuracy will lead to many gifted students being overlooked, or many being mistakenly identified as gifted. (p. 71)

However, Niederer et al (2003) acknowledged that PAT was not designed to identify gifted students but rather was a test of basic skills and understandings:

> Use of the test to identify mathematically gifted students is therefore remote from its primary purpose. (p. 81)

Niederer et al (2003) also highlighted how teachers’ lack of training in identifying mathematically gifted students may lead them to rely on PAT for identification purposes.
The average primary teacher in New Zealand has very little tuition, and presumably very little understanding of what it means to be mathematically gifted. For teachers, who lack this tuition and understanding, it must be very tempting to regard PAT as an easy and scientifically respectable instrument for measuring mathematical giftedness. (p. 80)

They also raised the issue of the trade-off between the “hit rate” — the accurate identification of gifted students — and the “false-alarm rate” — the misidentification of students as gifted. Their study also discussed the accuracy rates of parent nominations of gifted students, which also had limited accuracy. The results of this study support the need for a multi-dimensional approach to identification of mathematically gifted students and highlight the importance of problem solving within the identification “toolbox”.

Mathematically gifted students can vary substantially in their analytical and spatial abilities (Kruteskii, 1976). Hence, identification measures need to accommodate spatial ability, which is fundamental to creative achievement in mathematics (e.g., MacFarlane Smith, 1964). Diezmann and Watters (2000c) explored the development of spatial intelligence over time in one mathematically gifted child. Their research identified distinctive stages in this child’s mathematical work and interests that were consistent with the stages proposed by Gardner (1993) in his developmental trajectory of intelligence. A comparison of the child’s work samples over time shows substantial changes in ability to communicate spatial information through drawing. Their conclusions were that spatial intelligence needs to be valued in mathematics, that spatially gifted children need to be identified, and that there is a need for support in the development of spatial ability. These findings have wide-reaching implications that extend to teachers, teacher education, and curriculum developers. The identification of changes in the manifestation in spatial ability over time highlights the value of longitudinal research on gifted students.

Educational provisions for gifted students

The most recent Australasian research into educational provision has focused on early entry to school, enrichment activities, secondary school programs, and university provision for gifted female students. The studies described in this review raise awareness of the need to consider how best to provide challenging instruction and support for students gifted and talented in mathematics.

**Early entry to school:** Early entry is one option for meeting the needs of judiciously selected gifted children. In Australia early admission to school is determined at state level with varying requirements. Diezmann, Watters, and Fox (2001) explored the rhetoric, research and reality of early entry in the Australian context. An overview of policies, practices and guidelines is presented as background to one parent’s perspective of gaining early entry for her child. This parent’s story raises issues of identification, evidence, accessibility, advocacy, and the bureaucratic process. It also highlights the importance of ensuring gifted students have appropriately challenging learning opportunities. The study concludes that early entry is an effective option for carefully selected students. Early entry to school is currently not a legally viable option in New Zealand.

**Enrichment:** Enrichment programs or activities can support the education of mathematically gifted students. Diezmann and English (2001) described a set of mathematically rich enrichment experiences that were provided for a group of 20 young gifted children to assist them to develop multi-digit number sense in conjunction with a science unit on Space. These activities focused on reading large numbers in symbolic form, developing referents for large numbers, appreciating the relative magnitude of numbers to one million, and understanding large numbers in relation to quantity, distance, and money.
Given physical referents for large numbers, the children developed an appreciation of numbers beyond those modelled and could make links to various problem-solving contexts and use large numbers in their reasoning. This study also highlighted the active role of the teacher in working with mathematically gifted students:

Although a student might be gifted he or she still needs appropriate teacher support in dealing with challenging tasks that extend mathematical understanding. (Diezmann & English, 2001. p. 13)

The enrichment experiences described support the notion of providing gifted students with the knowledge and skills that enable them to pursue topics of interest (e.g., space travel) with mathematical meaning. Here, the development of multi-digit number sense in young children supported gifted students’ learning and highlights a situation where the content needs of gifted students differ substantively from their non-gifted peers.

Secondary school programs: A study by Anthony, Rawlins, Riley and Winsley (2002) provided a comprehensive overview of support currently being offered to gifted secondary mathematics students in New Zealand. In the first instance, a nationwide survey of practices across all secondary schools in New Zealand was undertaken (Winsley, 2000). Additionally, information about the existence of policies was gathered through open-response questions. Focus group interviews (involving 64 students) were also conducted in order to gain insights into student perceptions of accelerated programs. Triangulation occurred through follow up interviews with the respective Head of Department at each school. The survey found that provision for mathematics programs for gifted students was varied and strongly influenced by school size, location and decile (socio-economic) ranking. The participating students’ perceptions of accelerated programs were very favourable. Students felt that such programs relieved boredom however very few students explicitly discussed benefits to mathematical learning. Additionally, they maintained that accelerated programs did not affect their friendship base. One implication of the study was that programs should be more flexible, incorporating aspects of both enrichment and acceleration in order to accommodate the students’ mathematical needs.

University provision for gifted students: The University of South Australia offers the ‘Hypatia Scholarship Program for Mathematically Gifted Women’, for undergraduate women within the School of Mathematics. The Hypatia model is an affirmative action strategy that encourages women with high abilities to pursue academic studies in mathematics. This scholarship offers a moderate financial reward, office space and a computer, and paid summer employment relevant to their degree programs. The program provides a collaborative learning environment and the opportunity for easier liaison with academics. Lucas and Underwood (2002) conducted semi-structured interviews with seven Hypatia students and three other Hypatia students provided written feedback to evaluate the benefits of the program. The women reported that the scholarship was a major incentive to study a mathematics degree. Students accepted the scholarship for financial, learning and career reasons. Some students expressed greater awareness of career opportunities and research directions. To date, the majority of Hypatia students have been successful in their course of study. However, this study also acknowledged difficulties of attracting students to the program and some students’ ambivalence about the program as an affirmative action strategy. Despite the apparent success of the Hypatia program, there are moves to terminate affirmative action programs (Damarin, 2000a).
Challenging tasks

The research on challenging tasks and the mathematically gifted encompasses strategy use, learning environments, task difficulty, and collaboration.

Lowrie (2000) reported on a single case study that explored a gifted student’s strategy use on an open-ended task. The eleven-year-old Grade 6 student’s task was to produce a map of the main street in her town. Although the student had an excellent understanding of scale and proportion (and possessed the content knowledge), she was unable to produce even a simple bird’s eye representation of shops in the street. This student’s inability to solve the problem in an effective manner and capitalise on visual processing was associated with her beliefs about mathematics and her reluctance to use imagery. The student did not complete the problem because she was concerned that her map would not be a true representation of the street and maintained that it was necessary to be accurate when solving problems. For example, she said:

I tried to estimate how long the main street was and wide an average shop front would be … I remembered that I had to consider the side streets off the main street. There are two off to the right and one to the left. That meant there were more shops on the left hand side of the road. It got too confusing … I don’t worry about where the shops are when I go down the street. I walk down the street and then go in if I need to. (Lowrie, 2000, p. 23)

Thus, even though this student possessed high level mathematical knowledge, her beliefs about the need for accurate representations impeded her problem solving. This study is discussed elsewhere (Owens, this volume).

Diezmann and Watters (2000b, 2001, 2002b) reported on a series of studies that investigated challenging tasks from various perspectives. In the first study, Diezmann and Watters (2000b) investigated the relationship between the level of challenge of a task and the learning opportunities it provided by examining one gifted ten-year-old’s response to a typical mathematics task (i.e., The Handshake problem) and a more difficult variation of this task incorporating larger numbers. They concluded that tasks needed to be sufficiently challenging to provide adequate learning opportunities for gifted students (Diezmann & Watters, 2000b):

Tasks of sufficient difficulty need to be carefully chosen or existing classroom tasks need to be adapted, that is, “problematized” … However once the task was appropriately challenging, the teacher needed to provide support for the student. (p. 18)

In the second study, Diezmann and Watters (2002b) investigated the learning opportunities that a series of typical problem solving tasks provided for twenty gifted 11 to 12 year olds from four mixed ability classes. Although most classroom teachers were responsive to the gifted students’ needs and provided them with opportunities to share their solutions, these students had considerable unproductive class time (Diezmann & Watters, 2002b):

They completed the set tasks quickly and occupied themselves aimlessly until it was time for the discussion … It is noteworthy that most of these students did not advise the teacher that they had completed the tasks nor did any students seek extra work. In all classes, gifted students were observed to engage in undesirable behaviors, including lack of group cooperation, boredom, disinterest in the task, and unwillingness to share answers with group peers. (p. 80)

These students were also studied worked in friendship pairs in a clinical setting on similarly challenging tasks to their regular classwork and on problematised tasks. Diezmann and Watters (2002b) reported that students were more likely to display higher-order thinking and work collaboratively with a partner when working on problematised
tasks rather than regular tasks and concluded that challenging tasks are fundamental to
gifted students’ learning:

It is only when the task becomes sufficiently problematic that students have the opportunity to
engage in productive mathematical activity through higher-level cognition, and to develop and
demonstrate intelligent behaviors (sic). (p. 83)

In the third study, Diezmann and Watters (2001) explored the relationship between the
difficulty of tasks and gifted students’ preference for working collaboratively. This study
involved six 11 to 12 year old gifted students working on tasks that involved Time
concepts and which varied in difficulty. The students were provided with opportunities to
work independently, collaboratively, or with a teacher present. It was evident that task
difficulty was an important factor in the students’ preference for solving tasks with the
support of others. These gifted students preferred minimal interaction with others when
they completed relatively easy tasks whereas they sought help from peers or the teacher as
task difficulty increased (Diezmann & Watters, 2001):

Collaboration was preferred only when the task was sufficiently challenging. However, in the
context where collaboration was encouraged and students took advantage of working with peers,
there was the development of mutual scaffolding, shared cognition, critical thinking, and the ability
to discern and monitor goal states for the problems. (p. 7)

The role of the teacher in the problem-solving process becomes increasingly important
as task complexity is increased. Lowrie (2002) and Diezmann and Watters (2000b) have
found that gifted students (as young as six years of age) are able to complete quite
sophisticated open-ended tasks provided they are supported with appropriate scaffolding.
In these studies the excitement of engaging in rich tasks that were both challenging and
motivationally interesting resulted in quite sophisticated solution paths. Since gifted
students are able to complete tasks quite quickly and are generally able to solve open-ended
problems with more sophistication than other members of the class (Lowrie, Francis, & Rogers, 2000), it is advisable that these students do fewer and more complex
problems. It is also important that teachers provide gifted students with quality feedback
on all aspects of the problem-solving process, highlighting successful strategy use and
encouraging peers to provide support and reflective feedback (Diezmann & Watters,
2000b).

Skill development

Two studies of gifted students focused on skill development necessary for particular
learning situations. The first study focuses on an enrichment program and the second on
learning in an online environment.

Diezmann and Watters (2000a) investigated ways to assist young gifted students (aged
5 to 8 years) to become autonomous learners during a ten-session enrichment program.
The teacher implemented strategies that encouraged students to be both critical and
creative in their reasoning. Scaffolded support included strategies for fostering ideas,
developing collaborative skills and nurturing creativity. Not surprisingly, many of the
parents surveyed at the completion of the program argued that their child had become more
self reliant, motivated and prepared to work on an independent basis. One of the most
interesting findings in the study was that over a number of weeks these young students
engaged with each other in quite volatile and productive discussions typical of “a
community of inquiry”.

Clarke and Bana (2001) reported on the development of gifted secondary students’
skills in social interaction while learning mathematics in an online environment. In this
study, 11 participants aged 14 years were monitored while they completed a mathematics subject in an Academic Talent Program. This subject was delivered through Telematics, which enables isolated students to participate as a virtual class in real time via computer and telephone links. The teacher attempted to promote an interactive learning environment by encouraging student explanation and discussion, reducing teacher input into the lessons, and encouraging students to respond to each other’s queries and discuss each other’s errors. The results indicated that the teacher was able to foster social interaction online among gifted students (Clarke & Bana, 2001):

This endeavour was generally found to be successful, with most students recognising the benefits of such an approach, even if some were reluctant to participate actively. (p. 163)

However, in parallel with the need for students to develop particular skills for productive interaction, the teachers also needed to develop their skills in the use of this learning medium (Clarke & Bana, 2001):

The teacher consistently tried to increase the amount and quality of interaction during the program … In order to attain success in learning via Telematics, it is important that teachers are given specialised training in teaching via this medium, which requires different skills from regular face-to-face teaching. (p. 169)

These studies highlight some of the additional skills sometimes required by gifted students in particular mathematical learning environments. The results of these studies have implications for other students who also need to develop the same skills for particular reasons. In particular, the ability to foster interaction online has particularly broad applicability.

Suggestions for further research on gifted students

The paucity of research and publication on the mathematically gifted in the past four years mirrors the status of research on this topic over the past decade. For example, a document analysis of Australasian research published between 1992 and 2001 in Mathematics Research Group of Australasia Conference [MERGA] Proceedings and the Australasian Journal of Gifted Education [AJGE] revealed that mathematically gifted students are the focus of less than one percent of these MERGA publications and six percent of articles in AJGE (Diezmann & Watters, 2002a). Given the limited knowledge base on understanding and providing for the needs of gifted students, there is an urgent requirement for research to support the development of adequate numbers of citizens with high-level mathematical capability. The following strategic research questions are indicated from this four yearly review of the literature.

“What are the most effective methods of identifying mathematically gifted students and monitoring their progress?” Teachers’ limited knowledge of mathematically gifted students indicates an urgent need to identify and disseminate relatively simple measures that have a high degree of accuracy in identifying mathematically gifted students and tracking their progress.

“What are the current provisions for gifted students and how effective are they?” The limited research on the mathematically gifted is problematic because the current provision for these students is difficult to determine as is the short-term and long-term effects of particular forms of provisions.

“What are the short-term and long-term effects of an accelerated mathematics program?” Acceleration is a commonly used strategy to cater for mathematically gifted
students through streaming, vertical curricula, grade skipping, and early entry. However, there is scant information on the comparative effectiveness of these approaches for the mathematically gifted. Longitudinal studies that document and analyse the effectiveness of these methods for highly gifted students in mathematics would be particularly informative.

“What are the short-term and long-term effects of the practice of early entry at various junctures in education?” The impact of early entry to primary school and mathematics learning needs investigation. Similarly, there is a need to investigate the impact of early entry to secondary school and university for mathematically gifted students to determine the effectiveness of this provision.

“What mathematical content is appropriate for mathematically gifted students?” Given the need to develop citizens with high-level mathematical capability, there is a need to understand how mathematics should be taught to support and maintain this interest in gifted students. A key issue is whether mathematics content should be learned through discipline-oriented (e.g., advanced algebra) or interdisciplinary approaches (e.g., cybernetics).

“How can teachers support gifted students to undertake challenging tasks within an inclusive classroom?” Many gifted students receive their mathematics education in a mixed ability classroom, hence, there is a need to identify and develop teaching strategies and resources that are adequately challenging but can be used within a regular classroom in which the teacher needs to cater for a diversity of students.

“What are the long-term effects of common curriculum practices that accommodate or overlook the needs of mathematically gifted students?” Long-term studies of different types of mathematically gifted students and particular provisions have been particularly informative elsewhere (e.g., Benbow, 2001).

“Why does the community hold negative attitudes towards the gifted including the mathematically gifted?” Collins (2001) argues specifically for research into the negative community attitudes towards the gifted, which should extend to developing positive attitudes towards these individuals. The sources of negative attitudes towards the mathematically gifted may be associated with the perception of them as a “marked” group or “deviant” population, because the general population finds mathematics difficult and holds negative attitudes towards it (Damarin 2000b).

Conclusion

The Information Age has increased numeracy demands for all citizens and requires adequate numbers of citizens with high-level mathematical capability. Although mathematics education acknowledges the existence of exceptional students, the Australasian literature base to inform their mathematics education is very limited. Thus, there are two central areas for research. Firstly, specific studies on critical issues for distinct groups of exceptional students are needed, as indicated by the proposed research questions. This research can be complemented by studies that focus on the needs and performance of exceptional students within broad student cohorts across a range of mathematics education issues. Secondly, inclusive practices in the classroom place a high demand on teachers to use strategies that support students with learning difficulties and gifted students. Thus, one of the most challenging research areas in mathematics education is how to simultaneously provide a high quality learning environment for the diversity of students within the classroom. For example, although streaming seems beneficial for gifted students, it seems contra-indicated for students with learning difficulties.
References


Diezmann, C. M., & Watters, J. J. (2002a). Summing up the education of mathematically gifted students. Proceedings of the Twenty-Fifth Annual Conference of the Mathematics Education Research Group of Australasia (pp. 219-226), Auckland: MERGA.


