Catering for mathematically gifted elementary students:

Learning from challenging tasks

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Diezmann, Carmel M and Watters, James J (2000) Catering for mathematically gifted elementary students: Learning from challenging tasks. *Gifted Child Today* 23(4):14-19.

Abstract

Mathematical learning in gifted children depends on the nature of the mathematical task and the implementation of effective teaching strategies. In this paper, we argue the importance of increasing the complexity of regular classroom mathematical tasks to provide gifted children with opportunities to experience greater challenge. The approach is illustrated by the experience of a young gifted girl who finds no challenge in solving a regular problem but with teacher support can be challenged by the same task made more complex. The advantages of this approach are argued in terms of enhanced interest, motivation, metacognition and the development of autonomy. The role of the teacher as a scaffolder and model is crucial to this process.

The person who really thinks, learns as much from his failures as his successes. John Dewey

INTRODUCTION

Boredom is a major concern of gifted students and stems from lack of challenge in academic tasks and a perception by these students of the limited value of the "learning" experience (Feldhusen & Kroll, 1991; Galbraith, 1985; House, 1987). Academic tasks constitute the "work" of the classroom and ideally, provide the necessary challenge that affords learning (Doyle, 1983, 1988). A key feature of challenging tasks is their authenticity within a domain. For example, an authentic mathematical task is characterized by its complexity, the obstacle to a ready-made solution, and the need for high-level thinking and reasoning. Thus, challenging mathematical tasks for gifted students should be authentic tasks that provide opportunities for them to emulate the practices of mathematicians, though at a less sophisticated level. The fundamental relationship between the level of challenge of a task and mathematical learning is recognised by gifted elementary students. For example, in response to the question "Do you like your schoolwork easy or hard?" a gifted twelve-year-old girl replied "I like it hard so I can learn new things". She went on to say "The perfect math lesson would be full of problems and hard questions".

Although mathematically gifted children are characterised by the quality of their reasoning (Johnson, 1983), these children require appropriate and challenging learning experiences to facilitate their cognitive development (Henningsen & Stein, 1997; Hoeflinger, 1998). It is unlikely that many classroom tasks that are selected for, or designed to suit, the majority of students in a heterogeneous class will be sufficiently challenging for gifted students. One response to this inherent lack of challenge in many classroom tasks is to supplement these tasks with enrichment activities. However for some gifted students, the level of challenge of enrichment tasks is still inadequate and more difficult work is required. Contrary to an intuitive reaction against providing students with very challenging work, there is evidence that gifted students crave such work (Stanley, 1991):

> Figuratively, they [gifted students] were starved for mathematics at the proper pace and level and rejoiced in the opportunity to take it straight rather than being "enriched" with math puzzles, social studies discussions, trips to museums, critical thinking training not closely tied to mathematics, and so forth. (p. 37)

Much has been written about instructional strategies and the pacing of content for the gifted, but less about the content itself (Shore & Delcourt, 1996; Tomlinson et al., 1997; Willard-Holt, 1994). The purpose of this paper is to develop a framework for enhancing the quality of mathematics learning in gifted children by considering the nature of the learning tasks and the teaching practices. Thus, we explore the suitability of the math content in a typical classroom elementary mathematics task by analysing the relationship between the level of challenge of the task and the learning opportunities that are provided for a gifted student. Additionally, the role of the teacher in monitoring and where necessary, modifying the level of challenge of a task, and in supporting a gifted student's learning is considered.

THEORETICAL BACKGROUND

The Nature of the Learning Task

All learners require challenging tasks to facilitate learning and develop autonomy. To realise their potential, gifted students should engage in challenging tasks for three reasons related to cognition, metacognition, and motivation.

First, challenging tasks facilitate the development of *cognition* because they provide opportunities for students to develop mathematical power through high-level thinking and reasoning (Henningsen & Stein, 1997). Such tasks cater for the preferences of mathematically gifted students for exploring patterns and relationships, producing holistic and lateral solutions, and for working abstractly (House, 1987). Even at the elementary level, gifted students' capability and preference for working abstractly should be accommodated because over-use of manipulatives can have a deleterious effect on mathematical ability (Marjoram, 1992).

Second, challenging tasks can encourage the use and development of *metacognitive skills*. Metacognition describes a person's knowledge and control of his or her cognitive functioning (Lester, Garofalo, & Kroll, 1989). Metacognitive performance is a crucial factor in high achievement (Schraw & Graham, 1997) necessary for success on challenging and novel tasks

(Betts & Neihart, 1986). Specifically, success is influenced by knowing how to exploit useful knowledge (Schoenfeld, 1985) and knowing when to discontinue with inappropriate or unproductive strategies (Taplin, 1995). Polya (1945/1973) argues that metacognition facilitates the development of the type of knowledge which is of particular value for "future mathematicians":

> For him [the future mathematician], the most important part of his work is to look back at the completed solution ... He may find an unending variety of things to observe. He may meditate upon the difficulty of the problem and about the decisive idea; he may try to see what hampered him and what helped him finally ... He may compare and develop various methods ... Digesting the problems he solved as completely as he can, he may acquire well ordered knowledge, ready to use. (p. 205)

Third, solving challenging tasks enhances motivation (Lupkowski-Shoplik & Assouline, 1994). Challenge develops appropriate dispositions to learning, achievement (Bandura, Barbaranelli, Caprara, & Pastorelli, 1996), and intrinsic motivation (Vallerand, Gagné, Senécal, & Pelletier, 1994). Furthermore, the importance of success on challenging tasks develops self-efficacy and self-esteem (Bandura, 1986). Thus, motivation is a crucial component in the realisation of giftedness (Gagné, 1985) and is a desirable goal for gifted education (Feldhusen & Hoover, 1986).

Challenging tasks facilitate the development of autonomy by capitalizing on students' cognitive and metacognitive abilities and motivation (Betts & Neihart, 1986). Autonomy is necessary for creativity in mathematics. To achieve autonomy, learning needs to be designed around rich and challenging problem situations that afford multiple opportunities for student construction of knowledge through inquiry, discussion and argument (Palincsar, Magnusson, Marano, Ford, & Brown, 1997).

Clearly challenging tasks are important for gifted students' learning, but equally important is the role of the teacher in providing appropriate tasks and to support the students as they explore these tasks.

The Role of the Teacher

The teacher has two key roles in supporting gifted students' learning. First, the teacher needs to select tasks that are appropriately challenging (Henningsen & Stein, 1997). If necessary, the teacher needs to moderate the difficulty of the tasks for particular students because the same task may not be of equivalent value for different students.

Second, to facilitate high-level cognition, the teacher needs to "proactively and consistently support students' cognitive activity without reducing the complexity and cognitive demand of the task" (Henningsen & Stein, 1997, p. 546). The teacher can provide extrinsic and intrinsic support to the students by engaging in the practice of cognitive apprenticeship, which is a suite of teaching strategies for developing expertise in domains, such as mathematics (Collins, Brown, & Newman, 1989).

Extrinsic support is provided to the learners through scaffolding, modelling, and coaching. Of these strategies, scaffolding and modelling are key factors in facilitating high-level thinking and reasoning (Henningsen & Stein, 1997). Scaffolding is based on the notion that social interaction and expert guidance facilitate learning (Vygotsky, 1978). It bridges the gap between what a student can do independently and then with support (Rosenshine & Meister, 1992), however, scaffolding is inherently temporary (Tobias, 1982). Scaffolds can be verbal cues, prompts or hints (Rosenshine & Meister, 1992). Modelling is a form of scaffolding (Rosenshine & Meister, 1992) and provides students with a demonstration of the thought processes of an “expert” (Collins, et al., 1989). Through cognitive modelling the teacher “exposes learners to the teacher’s ways of processing information by reasoning aloud while performing the procedures involved in a task” (Gorrell & Capron, 1990, p. 15).

Intrinsic support is provided by the teacher facilitating the processes of exploration and reflection on ideas and by scaffolding the learner's construction of meaning (Collins et al., 1989; Henningsen & Stein, 1997). However, the teacher needs to monitor and respond to the capability of the learner in order to maintain the challenge of the task (Palincsar & Brown, 1984). Listening plays a key role in monitoring and understanding students’ thinking. However, the teacher needs to engage in interpretive rather than evaluative listening. Interpretive listening leads to dialogue or questioning that is information-seeking with the teacher facilitating knowledge construction by probing responses, paraphrasing, providing opportunities for vocalisation and monitoring understanding (Davis, 1997). Interpretative listening implies that the teacher needs to be flexible and responsive to the student in implementing tasks.

The following example explores the level of challenge and the learning opportunities that a series of mathematical tasks provided for a mathematically gifted elementary student. This example highlights the role played by the teacher. Student-teacher interaction was crucial in responding to the capability of the learner and scaffolding the student to extend her achievement.

EXAMPLE

Comparing the Value of Mathematical Tasks

Ten-year-old Michelle completed three tasks. The first task was a novel problem that similarly aged students find challenging (Diezmann, 1999).

> *At a party five people met for the first time. They all shook hands with each other once. How many handshakes were there altogether?*

Michelle spontaneously solved this task rapidly and with ease. Her solution involved drawing a simple diagram and summing the numbers from one to four (See Figure 1). Her rapid and successful response suggests that this task was easy for her. Although this task would have been sufficiently challenging for some ten-year-olds, for Michelle, it was unchallenging, and hence, of limited value for mathematical learning.

Insert Figure 1 about here

After her ready success on the first task, Michelle was asked to calculate the number of handshakes for six people (Task 2). The number of people was increased in an attempt to make the task more challenging. Michelle also completed this task rapidly and was again successful (See Figure 2). Thus, a slight increase in the size of the numbers was not sufficient to increase the cognitive challenge of this task for Michelle.

Insert Figure 2 about here

Michelle was then presented with a further variation on the initial task. She was asked to work out how many handshakes there would be if 100 people each shook hands with one another (Task 3). Though she had used the same method for the first two tasks (See Figure 1 and Figure 2), she immediately dismissed her previous method as inappropriate for the third task because it would involve a lengthy computation. Hence, this latter task had evoked a metacognitive response.

Despite her previous successes, Michelle was initially unable to proceed with the third task. In response to Michelle's difficulty, the teacher provided scaffolding in the form of hints and cues to enable Michelle to proceed. For example, when Michelle was puzzled about how to add the numbers from one to 99 efficiently, she was reminded about the visual representation referred to as "rainbow tens". This representation is commonly used for learning the addition number facts to ten (Department of Education, 1991). The "rainbow tens" presents an analogous additive situation to Michelle's addition task albeit with smaller numbers. Using analogies is a helpful problem-solving strategy (Polya, 1945/1973). After the hint to think about the "rainbow tens", Michelle subsequently drew this representation (See Figure 3). Her diagram shows how the numbers from zero to ten can be arranged in pairs so that each pair totals 10.

Insert Figure 3 about here

The "rainbow tens" representation provided Michelle with a means of conceptualizing how to add the numbers from one to 99 by making repeated combinations of 100 (e.g., 99+1; 98+2). The teacher then provided further hints to Michelle about how to represent and calculate all the sums of 100 that would be formed. Subsequently, Michelle calculated the answer to the handshake problem from the number of multiples of 100 (See Figure 4) and by subtracting "50", so it was not used twice in the calculation. (See Figure 5).

Insert Figure 4 about here

Insert Figure 5 about here

The change in the number of handshakes from the second to third tasks (from 6 to 100) resulted in a substantial increase in the cognitive challenge of the task for Michelle. While she immediately recognized that her previous strategy would be inefficient for this task, due to the larger number of handshakes, she could not easily identify an alternative strategy. Although Michelle was unable to proceed with the task independently, she was able to proceed with the task with scaffolding. Thus, during this task Michelle was working at the optimal level for learning. Support from the teacher enabled Michelle to engage in an exploration of patterns and relationships in determining how to sum the numbers to 99, and to explore a range of solution paths. Thus, a consequence of optimising the cognitive challenge of this task for Michelle was that she was engaged in working mathematically, which involved using mathematical concepts and procedures, representations, rules, and reasoning (Greeno, 1994).

Although Michelle found the third task more challenging than the other tasks, she expressed a preference for this task stating that it was more "interesting" than the others. Thus, the final task had motivational advantages. This task also provided Michelle with an opportunity to employ metacognitive skills.

Although the final task was sufficiently challenging for Michelle, when Karl Gauss was set a similar task to the handshake problem of adding the integers from 1 to 100, he produced "an ingenious and instantaneous solution [and stated that]… there are 50 sums of 101" (D. Johnson, 1994, p. 244). Gauss' response to this task highlights the importance of considering the relative cognitive value of mathematical tasks for particular individuals.

CONCLUSION

Mathematics tasks that facilitate learning should be commensurate with the capability of the learner. For gifted students, this requires flexibility in the nature of task and appropriate support from others. Tasks of sufficient difficulty need to be carefully chosen or existing classroom tasks need to be adapted, that is, "problematized". In the handshake tasks, discussed earlier, it was necessary for the task to be modified by the teacher before it became a sufficient challenge for this particular student. However once the task was appropriately challenging, the teacher needed to provide support for the student. The need for support should be viewed positively, rather than negatively, because the more complex task provides an opportunity for mathematical learning that is not provided by an easier task. Furthermore, the teacher provides feedback to the student, highlighting successful strategies and acknowledging the student's capability. Peers may also provide support and feedback.

Appropriate time allocation for tasks is also an important consideration. Gifted students achieve mastery faster (House, 1987), and generally have more lengthy concentration spans than non-gifted students (House, 1987). However engaging in challenging tasks is time-consuming, and time is also required for the incubation of ideas, which is associated with insight into challenging problems (Boden, 1990). Thus, an effective goal should be that gifted students do fewer and more complex tasks over a longer period of time.

Problematizing tasks for gifted students is important to implement beyond the regular classroom and needs to be incorporated into curriculum differentiation practices, such as acceleration (e.g., Stanley, 1991), and enrichment programs (e.g., Lupkowski-Shoplik & Assouline, 1994; Parker, 1989). Curriculum differentiation practices should be of benefit to mathematically gifted students. However, if the tasks are inappropriate the anticipated benefit is not realised. For example, enrichment can be problematic if it consists of tasks related to irrelevant topics or "busy work" that lack challenge (House, 1987; Stanley, 1991; Worcester, 1979). However, acceleration can be equally problematic if the selection of academic tasks does not provide for the development of a comprehensive understanding of mathematics and consequently, there are gaps in students' foundational mathematical knowledge (Lupkowski-Shoplik & Assouline, 1993). Thus, understanding the contribution of academic tasks to learning and achievement is critical in effective curriculum differentiation for gifted students.

Clearly challenging tasks are essential for effective mathematical learning. Gifted students stand to benefit when the academic tasks are appropriately challenging, and the conditions for learning are optimised. This position has also been advocated by the National Council of Teachers of Mathematics [NCTM] which argues that these students "have a right to experience education as a relevant, challenging, and engaging enterprise" (House, 1987, p. 18). The NCTM has reiterated their advocacy for gifted students through their goal of "equity and excellence" in mathematics education (NCTM, 1998).

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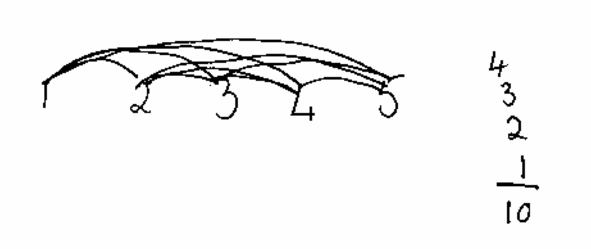

Figure 1. Five people shake hands.

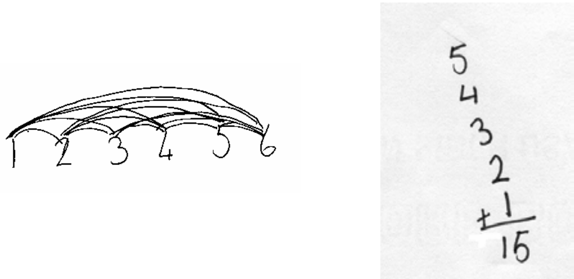

Figure 2. Six people shake hands.

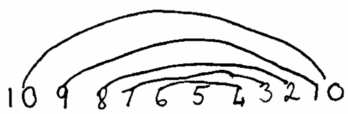

Figure 3. Rainbow tens.

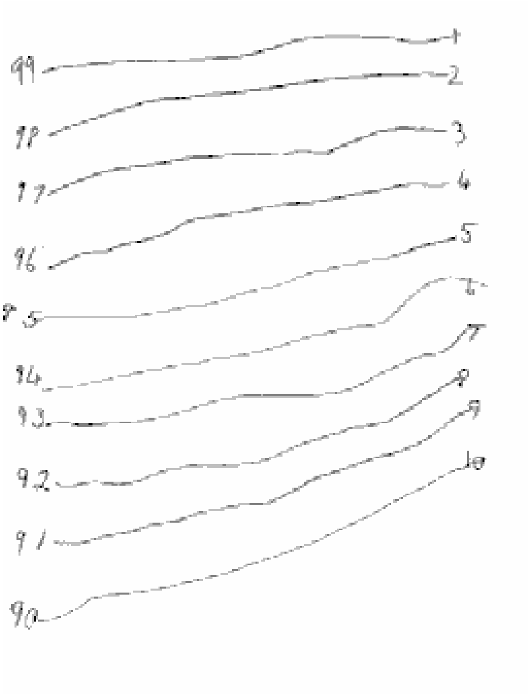

Figure 4. Sums to 100.

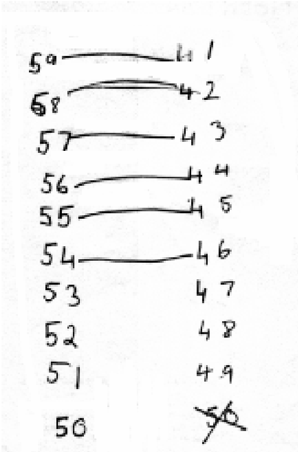

Figure 5. Excluding the second fifty.

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