Orbit determination of FedSat based on GPS receiver position solutions – first results

W. Enderle, Y. Feng, N. Zhou
Cooperative Research Centre for Satellite Systems, Queensland University of Technology, GPO Box 2434, 2 George Street, Brisbane, Qld 4001 Australia
Tel: +61 (0)7 3864 9132 Fax: +61 (0)7 3864 1516 Email: w.Enderle@qut.edu.au

ABSTRACT

On 14th December 2002, the Australian small satellite FedSat was launched with a Japanese HII-A rocket into an 800 km sun-synchronous orbit with an inclination of 98.6 degrees. One of the payloads is a dual frequency GPS BlackJack receiver from NASA Jet Propulsion Laboratory (JPL). This receiver provides a Position, Velocity and Time (PVT) solution and also raw measurements (code and carrier), which will be used in different ways for Precise Orbit Determination (POD), Orbit Determination (OD) based on GPS position solutions, 2-axis Attitude Determination (AD) and atmospheric experiments.

KEYWORDS: GPS Orbit Determination, Orbit Determination, ONS, GNSS, Space Flight Dynamics

1. INTRODUCTION

Using GPS measurements for satellite Orbit Determination (OD) has become more and more popular within the last decade. In this context, two main categories of GPS based orbit determination can be identified. One is the Precise Orbit Determination (POD), which needs, besides the GPS code and/or carrier phase measurements, additional information, e.g. atmospheric data etc. Methods of this category are mostly based on GPS double difference techniques and provide the highest accuracy with results in the range cm and dm level (e.g. Topex/Poseidon). These types of methods are very demanding in terms of the computation complexity, CPU load and also processing time. Besides the POD, another category/class exists which is much simpler than the POD, yet still produces results with an orbit determination solution accuracy sufficient for many spacecraft missions (e.g. Radcal, MOMS experiment on MIR station). These methods are based on GPS pseudo range measurements derived from code phase, carrier phase or directly from the on-board generated GPS receiver Position, Velocity and Time (PVT) solution. This paper describes a method based on the direct use of the position solution, generated by the BlackJack GPS receiver on-board FedSat.
The results of FedSat orbit determination are shown and discussed.

2. SATELLITE ORBIT DETERMINATION – BASIC PROBLEM

Orbit Determination is the process of calculating the state vector, or more precisely the position and velocity vector of a satellite at a specific epoch, based on a set of adequate measurements. The position solutions of the GPS Receiver, generated on-board the spacecraft can be used as such “measurements”. The basic principles of this process are illustrated in Figure 1. The relevant steps in the orbit determination process are: computation of a reference orbit, calculation of the observations, computation of partial derivatives, computation of the design matrix, formulation of the normal equations and finally calculation of the solution based on Kalman Filter (KF) or Least Squares (LSQ) techniques.

![Figure 1: Basic Principle of Satellite Orbit Determination](image)

3. FEDSAT ORBIT DETERMINATION CONCEPT BASED ON ON-BOARD GPS RECEIVER POSITION SOLUTIONS

One of the simplest methods of GPS based Satellite orbit determination is the use of the on-board GPS receiver position solution itself. The use of the GPS position data as a set of measurements requires only a simple measurement model as the GPS receiver coordinates are directly expressed in the Earth Centred Earth Fixed (ECEF) WGS84 coordinate system.

3.1 Orbit Dynamical Model

The orbits of artificial satellites around the Earth can be described in general by two body orbits up to a certain degree of accuracy. However, the use of a simple orbit dynamic model is very limited concerning the time of validity and as already mentioned its accuracy. For this reason more advanced satellite orbital models are normally used for the generation of the reference orbit. In the context of this study an Earth Gravity Model JGM-3, Tapley (1996)
with order and degree of 20x20 has been used. Other orbital perturbations like atmospheric
drag, Solar and Lunar gravity and also tidal effects have not been considered.

The dynamic equation of motion for a satellite reference orbit can be expressed as:

\[
\dot{\mathbf{r}}_I = -\frac{GM_\oplus}{r} \mathbf{r}_I + \mathbf{a}_I, \tag{1}
\]

where \(GM_\oplus\) is the Earth gravitational constant and \(\mathbf{a}_I\) the acceleration vector acting on
the satellite due to gravitational perturbations. The calculations of the spherical harmonic terms,
needed for the numerical integration of the dynamical equations of motion are conducted
according to Cunningham (1970). The numerical integration of equation (1) was done by
applying a 4\(^{th}\) order Runge-Kutta method.

3.2 Observation Model

The positions solutions of the GPS receiver on-board FedSat are expressed in the Earth
Centred Earth Fixed (ECEF) WGS84 coordinate system. These position solutions are used
directly as measurements in the orbit determination process. The numerical integration of the
dynamic equations of motion is conducted in the Earth Centred Inertial (ECI J2000)
coordinate system. The model for the GPS position fixes \(\mathbf{r}_E\) in the ECEF- WGS84 coordinate
system is given by:

\[
\mathbf{r}_{\text{calc}} = \mathbf{T}_{\text{EI}} \cdot \mathbf{r}_I, \tag{2}
\]

with

\[
\mathbf{T}_{\text{EI}} = \mathbf{PM} \cdot \mathbf{SidMat} \cdot \mathbf{N} \cdot \mathbf{P}. \tag{3}
\]

The individual terms of equation (2) are: \(\mathbf{r}_E\) is the position vector in the ECEF-WGS84
coordinate system, \(\mathbf{r}_I\) is the position vector in the ECI J2000 coordinate system and \(\mathbf{T}_{\text{EI}}\) is the
transformation matrix which transforms a vector expressed in the ECI J2000 coordinate
system into the ECEF-WGS84 coordinate system. The transformation matrix itself consists
of: PM - polar motion matrix, SidMat - Earth or Siderial rotation matrix, N - Nutation matrix
and P - the Precession matrix.

3.3 Orbit Determination Solution – Batch Least Squares

The orbit determination problem, namely solving the state vector \(\mathbf{x}_I = [\mathbf{r}_I, \mathbf{v}_I]^T\), consisting of
position and velocity expressed in the inertial coordinate system for a specific epoch based on
adequate measurements was performed by applying a Batch Least Squares method. The Batch
Least Square solution vector in the inertial system is given by:

\[
\Delta \mathbf{x} = \mathbf{H}^\mathsf{T} \mathbf{W} \mathbf{H} \cdot \mathbf{H}^\mathsf{T} \mathbf{W} \mathbf{l}. \tag{4}
\]

The terms \(\mathbf{H}^\mathsf{T} \mathbf{W} \mathbf{H}\) and \(\mathbf{H}^\mathsf{T} \mathbf{W} \mathbf{l}\) are given by the following equations:
\[ H^T W H = \sum_{t=0}^{t=n} \hat{H}^T \hat{W} \hat{H} \]  \hspace{1cm} (5)

\[ H^T W I = \sum_{t=0}^{t=n} \hat{H}^T \hat{W} \hat{I} \].  \hspace{1cm} (6)

The weight matrix \( W \) is a weighting for the measurements (GPS receiver positions). The values of the weighting matrix have to be known a priori. The elements of the weighting matrix can be calculated according to:

\[
W = \begin{bmatrix}
\frac{1}{\sigma_{rx}^2} & 0 & 0 \\
0 & \frac{1}{\sigma_{ry}^2} & 0 \\
0 & 0 & \frac{1}{\sigma_{rz}^2}
\end{bmatrix}.
\hspace{1cm} (7)
\]

\( \sigma_{rx}, \sigma_{ry} \) and \( \sigma_{rz} \) are the one sigma values for the position error. For all three components, in this work, the following value has been used: \( \sigma_n = 15 \) m.

The residual vector \( l \) is the difference between the calculated measurements (based on reference orbit dynamic model and observation model) and the real measurements. This vector can be calculated for each measurement epoch according to:

\[ l = \mathbf{r}_{Ecalc} - \mathbf{r}_{Eobs}. \hspace{1cm} (8) \]

The design matrix \( \mathbf{H} \) must be created from the observation matrix \( \frac{\partial \mathbf{r}_E}{\partial \mathbf{x}} \) and the state transition matrix \( \phi(t,t_0) \). The observation matrix relates the observation model to the state vector and the state transition matrix provides the information of the functional relationship between the state vector at the initial time \( t_0 \) and an arbitrary time \( t \).

The functional relationship for the design matrix \( \mathbf{H} \) is given by:

\[ \mathbf{H} = \frac{\partial \mathbf{r}_E}{\partial \mathbf{x}} \cdot \frac{\partial \mathbf{x}}{\partial \mathbf{x}_0}. \hspace{1cm} (9) \]

The observation matrix for each epoch can be expressed as:

\[ \frac{\partial \mathbf{r}_E}{\partial \mathbf{x}} = \begin{bmatrix}
\frac{\partial \mathbf{r}_E}{\partial \mathbf{r}_t}, \frac{\partial \mathbf{r}_E}{\partial \mathbf{v}_t}
\end{bmatrix}, \hspace{1cm} (10) \]

with
\[ \frac{\partial \mathbf{r}_{E}}{\partial t} = \mathbf{T}_{E}, \quad \frac{\partial \mathbf{r}_{E}}{\partial \mathbf{v}} = 0. \quad (11), (12) \]

The state transition matrix \( \phi(t, t_0) \) is hereby defined as:

\[ \phi(t, t_0) = \frac{\partial \mathbf{x}}{\partial \mathbf{x}_0}(t) = \begin{bmatrix} \frac{\partial \mathbf{r}}{\partial \mathbf{r}_0} & \frac{\partial \mathbf{r}}{\partial \mathbf{v}_0} \\ \frac{\partial \mathbf{r}_0}{\partial \mathbf{v}} & \frac{\partial \mathbf{r}_0}{\partial \mathbf{v}_0} \end{bmatrix}. \quad (13) \]

The state transition’s differential equation is defined by:

\[ \dot{\phi}(t, t_0) = \frac{d}{dt} \begin{bmatrix} \frac{\partial \mathbf{x}}{\partial \mathbf{x}_0} \end{bmatrix}. \quad (14) \]

Equation (14) relates the state vector to the accelerations represented by the orbit dynamic model used for the calculation of the state transition matrix. This dynamic model can be the same as the dynamic model used for the calculation of the reference orbit. However, with respect to simplification for a later on-board implementation of this algorithm, a simplified model has been used. In this work the state transition matrix was calculated based on an approach proposed by Markley (1986). In this approach, only the flattening of the Earth, \( J_2 \) was considered in the dynamic model for the state transition matrix. The development of Markley’s method is results in an approximation of the state transmission matrix. One can write:

\[ \phi(t, t_0) \approx \begin{bmatrix} \Phi_{rr} & \Phi_{rv} \\ \Phi_{vr} & \Phi_{vv} \end{bmatrix}. \quad (15) \]

This matrix is a 6x6 matrix with the following 3x3 matrices as elements:

\[ \Phi_{rr} = I + \frac{\Delta t^2}{6} (2\mathbf{G}_0 + \mathbf{G}) \quad (16) \]

\[ \Phi_{rv} = I\Delta t + \frac{\Delta t^3}{12} (\mathbf{G}_0 + \mathbf{G}) \quad (17) \]

\[ \Phi_{vr} = \frac{\Delta t}{2} (\mathbf{G}_0 + \mathbf{G}) \quad (18) \]

\[ \Phi_{vv} = I + \frac{\Delta t^2}{6} (\mathbf{G}_0 + 2\mathbf{G}) \quad (19) \]

where

\( \Delta t = t_i - t_0 \), \( I \) is the 3x3 identity matrix and the Jacobian matrix \( \mathbf{G}(t_i)_{6x6} \) is given by:
\[
G(t_i) = \begin{bmatrix}
\frac{\partial f_x}{\partial r_x} & \frac{\partial f_y}{\partial r_y} & \frac{\partial f_z}{\partial r_z}
\frac{\partial f_x}{\partial r_x} & \frac{\partial f_y}{\partial r_y} & \frac{\partial f_z}{\partial r_z}
\frac{\partial f_x}{\partial r_x} & \frac{\partial f_y}{\partial r_y} & \frac{\partial f_z}{\partial r_z}
\end{bmatrix}
\]

\(G_0 = G(t_0)\) and the acceleration based on the Earth flattening effect of \(J_2\) (no other effects have been taken into account) is given by:

\[
f_x = \ddot{r}_x = -\frac{GM_\oplus}{r^3} r_x \left[ 1 + \frac{3}{2} \frac{J_2 R_\oplus^2}{r^2} \left[ 1 - \frac{5r_x^2}{r^2} \right] \right]
\]

\[
f_y = \ddot{r}_y = \frac{r_y}{r_x} f_x
\]

\[
f_z = \ddot{r}_z = -\frac{GM_\oplus}{r^3} r_z \left[ 1 + \frac{3}{2} \frac{J_2 R_\oplus^2}{r^2} \left[ 3 - \frac{5r_z^2}{r^2} \right] \right]
\]

The vector length \(r\) can be calculated according to:

\[
r = \sqrt{r_x^2 + r_y^2 + r_z^2}
\]

The partial derivative as the elements of the Jacobian matrix can be found in the literature, e.g. Chiaradia (1999).

The final orbit determination solution could be calculated according to:

\[
x = x_0 + \Delta x.
\]

The vector \(x_0\) is an initial estimation of the state vector. This value can be obtained from the reference orbit, or alternatively directly from the GPS receiver PVT solution.

### 3.4 Applied Orbit Determination Process and Data

The orbit determination process implemented in this work is outlined in Figure 2.
Figure 2: FedSat orbit determination process based on GPS receiver position solutions

The total number of the BlackJack receiver position solutions obtained in the below outlined time interval was 1429, however, the number of position solutions used in the Batch Least Squares algorithm was 910. This number was the result of an implemented threshold for quality control of the used measurements (GPS receiver position solutions). Only such GPS receiver solutions were used, which residuals were smaller than 50m. The time intervals and the corresponding measurements are shown in Table 1.
Table 1: Measurements used in the Orbit Determination Process

<table>
<thead>
<tr>
<th>#</th>
<th>Measurements Start</th>
<th>Measurements End</th>
<th>Duration [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>2003/04/04 02:05:50.000 1212 439540</td>
<td>2003/04/04 02:26:30.000 1212 440790</td>
<td>1250</td>
</tr>
<tr>
<td>02</td>
<td>2003/04/04 03:52:10.000 1212 445930</td>
<td>2003/04/04 04:07:20.000 1212 446840</td>
<td>910</td>
</tr>
<tr>
<td>03</td>
<td>2003/04/04 05:25:00.000 1212 451500</td>
<td>2003/04/04 05:48:20.000 1212 452900</td>
<td>1400</td>
</tr>
<tr>
<td>04</td>
<td>2003/04/04 07:05:50.000 1212 457550</td>
<td>2003/04/04 07:29:10.000 1212 458950</td>
<td>1400</td>
</tr>
<tr>
<td>05</td>
<td>2003/04/04 10:26:00.000 1212 469560</td>
<td>2003/04/04 10:35:40.000 1212 470140</td>
<td>580</td>
</tr>
<tr>
<td>06</td>
<td>2003/04/04 12:27:10.000 1212 476830</td>
<td>2003/04/04 12:31:50.000 1212 477110</td>
<td>280</td>
</tr>
<tr>
<td>07</td>
<td>2003/04/04 13:52:20.000 1212 481940</td>
<td>2003/04/04 14:12:50.000 1212 483170</td>
<td>1230</td>
</tr>
<tr>
<td>08</td>
<td>2003/04/04 15:32:50.000 1212 487970</td>
<td>2003/04/04 15:53:40.000 1212 489220</td>
<td>1250</td>
</tr>
<tr>
<td>09</td>
<td>2003/04/04 17:09:10.000 1212 493750</td>
<td>2003/04/04 17:34:40.000 1212 495280</td>
<td>1530</td>
</tr>
<tr>
<td>10</td>
<td>2003/04/04 18:50:50.000 1212 499850</td>
<td>2003/04/04 19:15:30.000 1212 501330</td>
<td>1480</td>
</tr>
<tr>
<td>11</td>
<td>2003/04/04 20:34:20.000 1212 506060</td>
<td>2003/04/04 20:56:30.000 1212 507390</td>
<td>1330</td>
</tr>
<tr>
<td>13</td>
<td>2003/04/04 23:55:40.000 1212 518140</td>
<td>2003/05 00:07:20.000 1212 518840</td>
<td>700</td>
</tr>
</tbody>
</table>

Figure 3 shows the distribution/location of the GPS receiver solution data in the FedSat orbit. The measurements result from power on time (per orbit) of the GPS receiver in the order of 30 min or more, depending on the power conditions of FedSat.
4. RESULTS

The residuals between the calculated positions, based on a precise satellite orbit model and the position measured by the BlackJack GPS receiver on-board FedSat are shown in Figure 4. The residuals are expressed in the ECEF - WGS84 coordinate system for x, y and z directions. As can be seen in Fig.4, the maximum value for the residuals is 50 m. The $\sigma$ values for the residuals are provided in Table 2.

![Figure 4: Residuals between calculated FedSat positions based on a precise orbit model for the reference orbit and position solution generated by the GPS BlackJack receiver on-board FedSat](image)

**Table 2:** FedSat Orbit Determination results based on GPS BlackJack GPS receiver position solutions and a precise satellite orbit model for the FedSat reference orbit prediction
5. CONCLUSIONS AND OUTLOOK

This paper describes a satellite orbit determination concept, based on the GPS receiver position solution, used directly as a measurement. As can be seen in Table 2, the achieved position accuracy for FedSat, based on GPS receiver position solutions as measurements, lies in the order of 25 m for the x and y component and 17 m for the z component. The achieved 3-D position accuracy ($1\sigma$) for FedSat based on this method is 40 m.

The outlined GPS based satellite orbit determination concept has been applied successfully in the context of the Australian small satellite mission – FedSat. The processed BlackJack GPS receiver data showed that this concept is suitable for mission operations, however, it was also experienced that the applied orbit determination concept was sensitive for the number and quality of available position solutions. Further algorithm modifications will be conducted and implemented including a Kalman Filter. The aim of the algorithm modifications will be the improvement of accuracy, robustness and also to demonstrate the suitability for on-board orbit determination. The described concept will be tested in an on-board implementation within the Australian JaeSat mission.

ACKNOWLEDGEMENT: This work was carried out in the Cooperative Research Centre for Satellite Systems with financial support from the Commonwealth of Australia through the Cooperative Research Centre program and from the Queensland State Government.

REFERENCES


Gill E, Montenbruck O, The on-board navigation system for the BIRD small satellite, DLR Forschungsbericht 2006-06,


Jochim EF (1996) GPS based on-board and on ground orbit operations for small satellites, IAA Symposium on small satellites for Earth observation, November 4-7, Berlin, Germany


