Properties of the Hyperbolic Macrocell Channel Model: Path Power and Doppler Shift Statistics

Seedahmed S. Mahmoud*, Zahir M. Hussain*, and Peter O'Shea†

*School of Electrical and Computer Engineering
RMIT University, Melbourne, Victoria 3000, Australia
E-mails: s2113794@student.rmit.edu.au, zahir.hussain@rmit.edu.au

†School of Electrical and Electronic Systems Engineering
Queensland University of Technology, Brisbane, Queensland 4000, Australia
E-mail: pj.oshea@qut.edu.au

***Abstract*— In this paper we investigate the statistical properties of the path power and Doppler frequency shift for the recently-proposed hyperbolic macrocell channel model. It is well-known that the Doppler spectrum is dependent on the probability density function of the direction-of-arrival (DOA) of the multipath components at the mobile station and the direction of motion of the mobile. We derive and simulate the joint probability density functions (pdfs) of the power-DOA and the power-Doppler shift.**

I. Introduction

Geometrical-based channel models are defined as those models that specify the region in space where scatterers are distributed as well as the distribution of these scatterers [1]. These models are useful for both simulation and analysis purposes. Utilization of the geometric models for simulation involves randomly placing scatterers in the region according to a spatial probability density function. From the location of the scatterer we can determine the direction-of-arrival (DOA), time-of-arrival (TOA), and the signal amplitude. From the spatial probability density function of the scatterers, it is possible to derive the joint and marginal power and Doppler probability density functions [2]. Knowledge of these statistics can be used to predict the performance of an adaptive array. Furthermore, knowledge of the underlying structure of the resulting array response vector may be exploited by beam-forming and position location algorithms [3].

In [4] we devised a geometrical-based hyperbolic channel model for macrocell environment, which provides the directional information of the multipath components. This model assumes a circular distribution of scatterers around the mobile station (MS), and the distances between the MS and the scatterers are subject statistically to a hyperbolic distribution.

It is well known that the Doppler spectrum is dependent on the probability density function of the direction-of-arrival (DOA) of the multipath components at the mobile station and the direction of motion of the mobile. The Doppler spectrum is U-shaped as noted by Clark [5], [6], when the pdf of the DOA of the multipath components at the mobile is uniform.

In this paper we investigate the behavior of the model proposed in [4]. The joint probability density function (pdf) of the power/ DOA and the power/ Doppler frequency shift are derived and simulated. If an exponential path-loss model is assumed, the multipath components that travel a shorter distance will have greater power. Since the time-of-arrivals (TOA) vary with DOA, the signal power level (being a function of TOA) will be dependent upon DOA. Furthermore, since the Doppler frequency is a function of the DOA, the signal power level may also be viewed as a function of the Doppler frequency [6].

II. Joint Power/ DOA and Power/ Doppler Shift Probability Density Functions

In this section we derive the joint power/ DOA and the power/ Doppler shift pdfs for the hyperbolic macrocell channel model proposed in [4]. The geometry used to derive the pdf is shown in Fig. 1. This model assumes that the scatterers are arranged within a circle of radius R around the mobile. The distances r_k between the mobile and the scatterers are distributed according to the reciprocal cosh pdf [4]. The angle-of-departure ψ_k is uniformly distributed in the interval [0,2π]. D denotes the distance between the base station and the mobile unit. The angle θ_k is the direction-of-arrival at the base station, which is evaluated geometrically [4]. The base station is located at the origin (0,0).

The joint probability density function of scatterers inside a circle of radius R denoted by $f_{x,y}(x,y)$ is given by

$$f_{x,y}(x,y) = \begin{cases} \frac{a}{\tanh(aR)\cosh^2(ar_k)} & \text{for } 0 \leq r_k \leq R \\ 0 & \text{elsewhere} \end{cases} \quad (1)$$

where (x,y) denotes the location of the scatterer, R is the radius of the circle enclosing the scatterers, and the applicable values of a lie in the interval (0,1) [4], [7]. The value of the parameter a controls the spread of the scatterers around the mobile station. Increasing a reduces the spread of the pdf of r_k [7].

It will be useful to express the joint scatterer probability density function with respect to the polar coordinates, (l_k,θ_k) as an intermediate step before deriving the joint power/ Doppler frequency density function [1]. To determine the joint

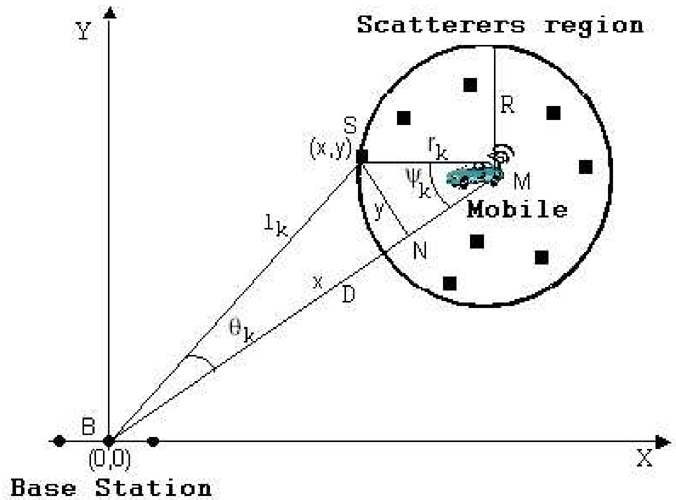

Fig. 1. Geometry of the hyperbolic macrocell channel model.

pdf $f(l_k, \theta_k)$, a transformation of the random variable (x, y) into the random variable (l_k, θ_k) is performed by

$$f_{l,\theta}(l, \theta) = |J(x, y)|\, f_{x,y}(x, y)\Big|_{x=Q(l_k,\theta_k),\; y=G(l_k,\theta_k)} \quad (2)$$

where $J(x, y)$ is the Jacobian of the transformation and we restrict it to be positive. From Fig. 1 we get

$$l_k = \sqrt{x^2 + y^2} \quad (3)$$

$$x = Q(l_k, \theta_k) = l_k \cos(\theta_k) \quad (4)$$

$$y = G(l_k, \theta_k) = l_k \sin(\theta_k) \quad (5)$$

The Jacobian $J(l_k, \theta_k)$ is given by

$$J(l, \theta) = \begin{vmatrix} \frac{\partial Q}{\partial l_k} & \frac{\partial Q}{\partial \theta_k} \\ \frac{\partial G}{\partial l_k} & \frac{\partial G}{\partial \theta_k} \end{vmatrix} = \begin{vmatrix} \cos(\theta_k) - l_k \sin(\theta_k) \\ \sin(\theta_k) \quad l_k \cos(\theta_k) \end{vmatrix} = l_k \quad (6)$$

By substituting (1), (3), (4), (5) and (6) into (2) we get

$$f(l_k, \theta_k) = \begin{cases} \frac{a\, l_k}{\tanh(aR)\cosh^2(a\, h(l_k, \theta))}, & l_1 \le l_k \le l_2 \\ 0 & \text{elsewhere} \end{cases} \quad (7)$$

where

$$h(l_k, \theta) = \sqrt{l_k^2 - 2\, l_k D \cos(\theta) + D^2} \quad (8)$$

$$l_1 = D \cos(\theta) - \sqrt{R^2 - D^2 \sin^2(\theta)} \quad (9)$$

$$l_2 = D \cos(\theta) + \sqrt{R^2 - D^2 \sin^2(\theta)}. \quad (10)$$

Limits on the parameter l_k were determined by fixing θ and then computing the points at which the resulting line intersected the scattering circle in Fig. 1.

We apply the law of cosines to the triangle BSM in Fig. 1 to derive the joint power/ Doppler pdf $f_{p,f_d}(p, f_d)$ where p is the power of the multipath components and f_d is the Doppler frequency. Applying this law gives

$$r_k^2 = D^2 + l_k^2 - 2\, l_k\, D \cos(\theta_k) \quad (11)$$

The total path propagation distance is given by

$$d = l_k + r_k = l_k + \sqrt{D^2 + l_k^2 - 2\, l_k\, D \cos(\theta_k)} \quad (12)$$

Squaring both sides of (12), and solving for l_k gives

$$l_k = \frac{D^2 - d^2}{2(D\cos(\theta_k) - d)} \quad (13)$$

The joint distance(d)/ DOA pdf is given by

$$f_{d,\theta}(d, \theta) = |J(l, \theta)|\, f_{l,\theta}(l, \theta)\Big|_{l=\frac{D^2-d^2}{2(D\cos(\theta)-d)}} \quad (14)$$

where $J(l, \theta)$ is the Jacobian transformation given by

$$J(l, \theta) = \left| \frac{\partial l}{\partial d} \right| = \frac{D^2 + d^2 - 2Dd\cos(\theta_k)}{2(D\cos(\theta_k) - d)^2} \quad (15)$$

Substituting (15) into (14) yields

$$f_{d,\theta}(d, \theta_k) = \frac{D^2 + d^2 - 2Dd\cos(\theta_k)}{2(D\cos(\theta_k) - d)^2} f_{l_k,\theta_k}\big(l(d, \theta_k), \theta_k\big) \quad (16)$$

The joint d/ DOA pdf $f_{d,\theta}(d, \theta_k)$ for the hyperbolic macrocell channel model is given by substituting (7), and (13) into (16)

$$f_{d,\theta_k}(d, \theta_k) = \frac{p(d,\theta_k)}{\cosh^2(g(d,\theta_k))} \quad (17)$$

where

$$p(d, \theta_k) = \frac{a(D^2 - d^2)(D^2 + d^2 - 2\, D\, d\, \cos(\theta_k))}{4\tanh(aR)(D\cos(\theta_k) - d)^3} \quad (18)$$

and

$$g(d, \theta) = a\sqrt{\frac{(n(d, \theta))^2}{4} - D\cos(\theta) n(d, \theta) + D^2} \quad (19)$$

where

$$n(d, \theta) = \frac{(D^2 - d^2)}{(D\cos(\theta) - d)}. \quad (20)$$

When an exponential path-loss model is assumed, the power will be related to the total path propagation distance d by [6], [2]

$$p_k = p_o \left(\frac{d}{D}\right)^{-n} \quad (21)$$

where p_k is the power level of the $\mathrm{k^{th}}$ path, p_o is power of the direct line-of-sight path, and n is the path loss exponent.

The above equation can be re-arranged to give

$$d = D\left(\frac{p_k}{p_o}\right)^{-\frac{1}{n}}. \quad (22)$$

The joint power (p)/ DOA pdf $f_{p_k,\theta_k}(p_k, \theta_k)$ for the channel model in [4] is given by

$$f_{p_k,\theta_k}(p_k, \theta_k) = |J(d, \theta_k)|\, f_{d,\theta_k}(d, \theta_k)\Big|_{d=D\left(\frac{p_k}{p_o}\right)^{-\frac{1}{n}}} \quad (23)$$

where the Jacobian transformation, $J(d,\theta_k)$, is given by

$$J(d,\theta_k) = \left| \frac{\partial d}{\partial p_k} \right| = \frac{D}{np_o \left(\frac{p_k}{p_o}\right)^{\frac{(n+1)}{n}}} \quad (24)$$

Substituting (24) into (23) yields

$$f_{p_k,\theta_k}(p_k,\theta_k) = \frac{D}{np_o \left(\frac{p_k}{p_o}\right)^{\frac{(n+1)}{n}}} f_{d,\theta_k}\big(d(p_k),\theta_k\big) \quad (25)$$

The joint power/ DOA pdf $f_{p_k,\theta_k}(p_k,\theta_k)$ is given by substituting (17), and (22) into (25)

$$f_{p_k,\theta_k}(p_k,\theta_k) = \frac{aD(D^2 - D^2\left(\frac{p_k}{p_o}\right)^{-\frac{2}{n}})h(p_k,\theta_k)}{4np_o \tanh(aR)\cosh^2(\Phi(p_k,\theta_k))} \quad (26)$$

where

$$h(p_k,\theta_k) = \frac{(D^2 + D^2\left(\frac{p_k}{p_o}\right)^{-\frac{2}{n}} - 2\,D^2\left(\frac{p_k}{p_o}\right)^{-\frac{1}{n}}\cos(\theta_k))}{\left(\frac{p_k}{p_o}\right)^{\frac{n+1}{n}}(D\cos(\theta_k) - D\left(\frac{p_k}{p_o}\right)^{-\frac{1}{n}})^3} \quad (27)$$

and

$$\Phi(p_k,\theta_k) = a\sqrt{\frac{\left(m(p_k,\theta_k)\right)^2}{4} - D\cos(\theta_k)m(p_k,\theta_k) + D^2} \quad (28)$$

where

$$m(p_k,\theta_k) = \frac{\left(D^2 - D^2\left(\frac{p_k}{p_o}\right)^{-\frac{2}{n}}\right)}{\left(D\cos(\theta_k) - D\left(\frac{p_k}{p_o}\right)^{-\frac{1}{n}}\right)}. \quad (29)$$

To evaluate the joint power (p)/ Doppler frequency shift (f_d) pdf, $f_{p,f_d}(p_k,f_d)$, we use the Doppler shift formula, which is given by [6]

$$f_d = f_m\cos(\theta_k - \theta_v) \quad (30)$$

where f_m is the maximum Doppler shift ($f_m = \frac{v}{\lambda}$), λ is the carrier wavelength, v is the speed of the mobile, θ_k is the DOA, and θ_v is the direction where the mobile is travelling [6]. Solving (30) for θ_k gives

$$\theta_k = \theta_v + \cos^{-1}\left(\frac{f_d}{f_m}\right) \quad (31)$$

The joint power/ Doppler frequency pdf, $f_{p,f_d}(p,f_d)$, is given by

$$f_{p,f_d}(p,f_d) = |J(p,\theta)|\, f_{p,\theta}(p,\theta)\Big|_{\theta=\theta_v+\cos^{-1}\left(\frac{f_d}{f_m}\right)} \quad (32)$$

where $J(p,\theta)$ is the Jacobian transformation given by

$$J(p_k,\theta_k) = \left| \frac{\partial\theta_k}{\partial f_d} \right| = \frac{1}{f_m\sin(\theta_k - \theta_v)} = \frac{1}{f_m\sqrt{1-\left(\frac{f_d}{f_m}\right)^2}} \quad (33)$$

Let $\xi = \left(\frac{p_k}{p_o}\right)^{-\frac{1}{n}}$, then the joint power/ Doppler frequency pdf, $f_{p_k,f_d}(p_k,f_d)$, is given by

$$f_{p_k,f_d}(p_k,f_d) = \sum_{i=1}^{2} \frac{aD^2(1-\xi^2)\Upsilon(\xi,f_d)}{4np_o\tanh(aR)\cosh^2(\beta(\xi))} \quad (34)$$

where

$$\Upsilon(\xi,f_d) = \frac{(1+\xi^2 - 2\xi\cos(\theta_i))}{\xi^{-(n+1)}(\cos(\theta_i)-\xi)^3 f_m\sqrt{1-\left(\frac{f_d}{f_m}\right)^2}} \quad (35)$$

and

$$\beta(\xi,\theta_i) = a\sqrt{\frac{\left(m(\xi,\theta_i)\right)^2}{4} - D\cos(\theta_i)m(\xi,\theta_i) + D^2} \quad (36)$$

where

$$m(\xi,\theta_i) = \frac{\left(D^2 - D^2\xi^2\right)}{\left(D\cos(\theta_i) - D\xi\right)} \quad (37)$$

and

$$\theta_i = \begin{cases} \theta_v + \cos^{-1}\left(\frac{f_d}{f_m}\right) & : \ i = 1 \\ \theta_v - \cos^{-1}\left(\frac{f_d}{f_m}\right) & : \ i = 2. \end{cases} \quad (38)$$

III. SIMULATION RESULTS

In this paper the joint power/ DOA pdf $f_{p_k,\theta_k}(p_k,\theta_k)$ (eq. (17)) and the joint power/ Doppler frequency pdf, $f_{p_k,f_d}(p_k,f_d)$ (eq. (34)) has been simulated using MATLAB. In this simulation we considered a path loss exponent of 2 (free space), a = 0.009, the distance between the mobile and the base station is set to D = 1.5 km, the scatters' circle radius is R = 0.5 km, and the power of the direct line-of-sight path, p_o, has been limited to 1 Watts.

Fig. 2 shows the joint power/ DOA pdf $f_{p_k,\theta_k}(p_k,\theta_k)$ for the hyperbolic macrocell channel model. From the figure, As this model is applicable for macrocell environment, it is clear that there is no power around the direct line-of-sight path. It is also evident that the powers of the other multipath components are less than the assumed direct line-of-sight power, p_o.

Fig. 3 shows the joint power/ Doppler frequency pdf $f_{p_k,f_d}(p_k,f_d)$ for the hyperbolic macrocell channel model. In this figure the directions where the mobile is travelling are set to $\theta_v = \frac{\pi}{2}$. Similarly, Fig 4 and 5 show the joint power/ Doppler frequency pdf when $\theta_v = 0$ and $\theta_v = \pi$ respectively. The Doppler frequency, f_d, is normalized by the maximum Doppler frequency, f_m. From the simulation results we observe that the joint power/ Doppler frequency pdf is highly dependent on θ_v.

IV. CONCLUSIONS

In this paper the joint probability density function (pdf) of the power/ DOA and the power/ Doppler frequency shift for the hyperbolic macrocell channel model are derived and simulated. Simulation results showed that the joint power/ Doppler frequency pdf is highly dependent on θ_v. It is also shown that the power pdf around the direct line-of-sight path is nearly zero (this is in accord with the assumptions of the hyperbolic macrocell channel model).

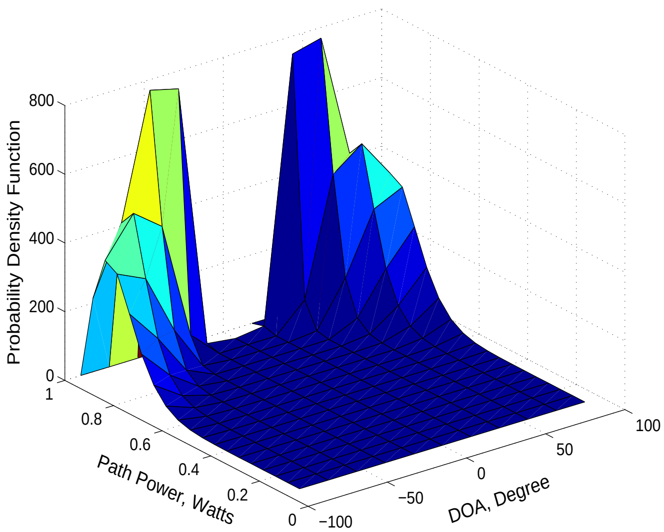

Fig. 2. The Joint power/ DOA probability density function for the hyperbolic macrocell channel model: (D = 1.5 km, R= 0.5 km, a = 0.009, n = 2, and p_o = 1 W).

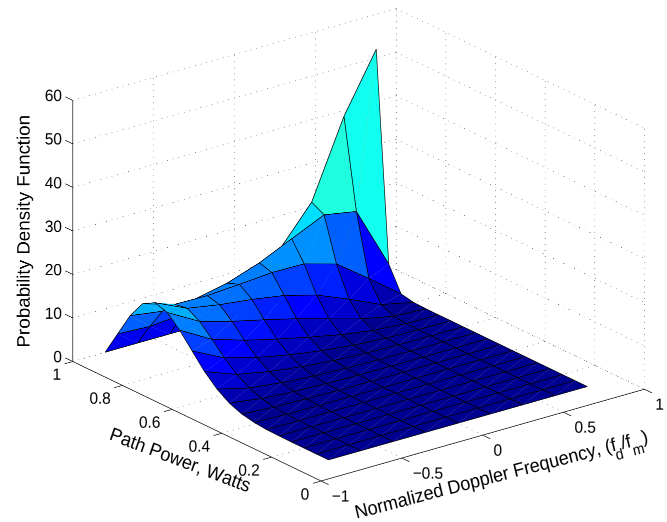

Fig. 4. The Joint power/ Doppler frequency probability density function for the hyperbolic macrocell channel model: (θ = 0, D = 1.5 km, R= 0.5 km, a = 0.009, n = 2, and p_o = 1 W).

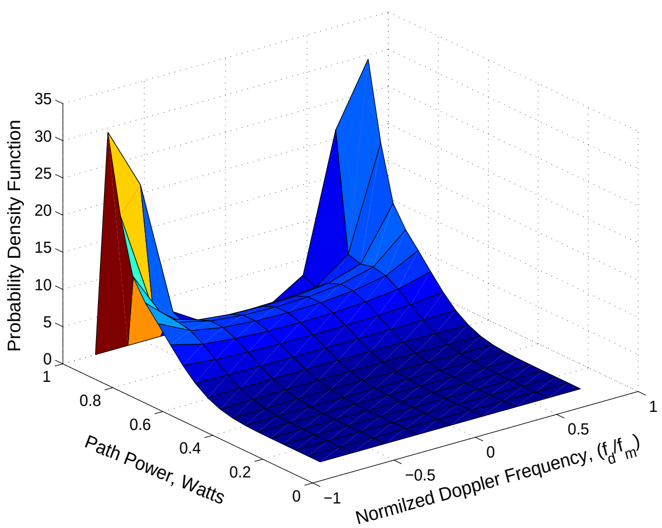

Fig. 3. The Joint power/ Doppler frequency probability density function for the hyperbolic macrocell channel model: (θ = $\frac{\pi}{2}$, D = 1.5 km, R= 0.5 km, a = 0.009, n = 2, and p_o = 1 W).

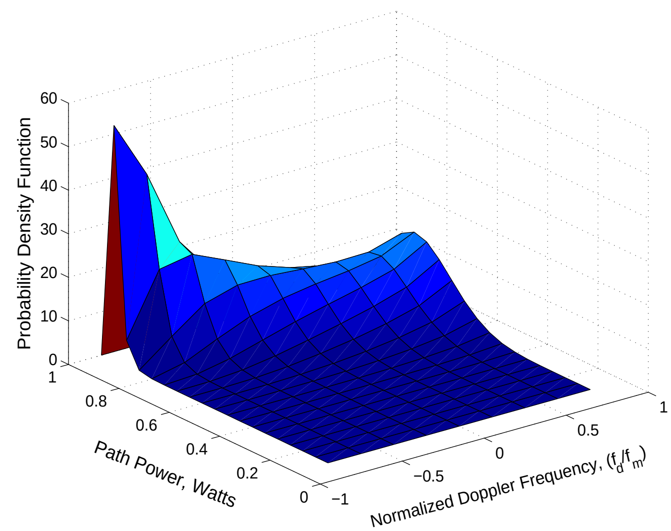

Fig. 5. The Joint power/ Doppler frequency probability density function for the hyperbolic macrocell channel model: (θ = π, D = 1.5 km, R= 0.5 km, a = 0.009, n = 2, and p_o = 1 W).

REFERENCES

[1] R. B. Ertel, and J. H. Reed, "Angle and time of arrival statistics for circular and elliptical scattering models," *IEEE Journal on Selected Areas in Communications*, vol. 17, No. 11, pp. 1829-1840, Nov. 1999.

[2] R. B. Ertel, and J. H. Reed, "Impact of path-loss on the Doppler spectrum for the geometrically based single bounce vector channel models," *IEEE Veh. Tech. Conf.*, pp. 586-590, 1998.

[3] R. B. Ertel, P. Cardieri, K. W. Sowerby, T. S. Rappaport, and J. H. Reed, "Overview of spatial channel models for antenna array communication systems," *IEEE Personal Communications*, vol. 5, pp. 1022, Feb. 1998.

[4] Seedahmed S. Mahmoud, Zahir M. Hussain, and Peter O'Shea, "Geometrical model for mobile radio channel with hyperbolically distributed scatterers," *8th IEEE International Conference on Communications Systems*, Singapore, Nov. 2002.

[5] W. C. Jakes, *Microwave Mobile Communication*, New York: Wiley, 1974.

[6] T. S. Rappaport, *Wireless Communication- Principles and Practice*, Prentice-Hall, 1996.

[7] Seedahmed S. Mahmoud, Zahir M. Hussain, and Peter O'Shea, "Space-time model for mobile radio channel with hyperbolically distributed scatterers," *IEEE Antennas and Wireless Propagation Letters*, vol. 1, no. 12, pp. 211-214, 2002.