A WEIGHTED GEOMETRIC INEQUALITY AND ITS APPLICATIONS

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ABSTRACT. A new weighted geometric inequality is established by Klamkin's polar moment of inertia inequality and the inversion transformation, some interesting applications of this result are given, and some conjectures which verified by computer are also mentioned

1. Introduction

In 1975, M.S.Klamkin [1] established the following inequality: Let ABC be an arbitrary triangle of sides a, b, c, and let P be an arbitrary point in space, the distances of P from the vertices A, B, C are R_1, R_2, R_3. x, y, z are real numbers, then

$$(x+y+z)(xR_1^2+yR_2^2+zR_3^2) \geqslant yza^2+zxb^2+xyc^2, \tag{1.1}$$

with equality if and only if P lies the plane of $\triangle ABC$ and $x:y:z=\overrightarrow{S}_{\triangle PBC}:\overrightarrow{S}_{\triangle PCA}:\overrightarrow{S}_{\triangle PAB}$ ($\overrightarrow{S}_{\triangle PCA}$ denote the algebra area, etc.)

Inequality (1.1) is called the polar moment of inertia inequality, it is one of the most important inequality for the triangle, there exist many consequences and applications for this one, see [1]-[5]. In this paper, we will apply Klamkin' inequality (1.1) and the inversion transformation to deduce a new weighted geometric inequality, then we discuss applications of our results. In addition, we also pose some conjectures.

2. Main Result

In order to prove our new results, first of all, we give the following lemma.

Lemma Let ABC be an arbitrary triangle, and let P be an arbitrary point in the plane of the triangle ABC, if the following inequality:

$$f(a,b,c,R_1,R_2,R_3) \geqslant 0. \tag{2.1}$$

holds, then we have the dual inequality:

$$f(aR_1,bR_2,cR_3,R_2R_3,R_3R_1,R_1R_2) \geqslant 0. \tag{2.2}$$

Indeed, the above conclusion can be deduced by using inversion transformation, see[6] or [3], [7].

Now, We state and prove main result.

Theorem Let x, y, z be positive real numbers, then for any triangle ABC and arbitrary point P in the plane of $\triangle ABC$ holds the following inequality:

$$\frac{R_1^2}{x}+\frac{R_2^2}{y}+\frac{R_3^2}{z} \geqslant \frac{aR_1+bR_2+cR_3}{\sqrt{yz+zx+xy}}, \tag{2.3}$$

with equality if and only if $\triangle ABC$ is acute-angled, P coincide with its orthocenter and $x:y:z=\cot A:\cot B:\cot C$.

Proof. If P coincide with one of the vertices of $\triangle ABC$, for example $P \equiv A$, then $PA=0, PB=c, PC=b$, (2.3) becomes trivial inequality and we easily to know it is holds true. In this case, equality in (2.3) obviously cannot occur.

Next assume P does not coincide with the vertices.

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If x, y, z are positive real numbers, then by the polar moment of inertia inequality (1.1) we have

$$(xR_1^2 + yR_2^2 + zR_3^2)\left(\frac{1}{yz} + \frac{1}{zx} + \frac{1}{xy}\right) \geqslant \frac{a^2}{x} + \frac{b^2}{y} + \frac{c^2}{z}.$$

On the other hand, from Cauchy-Schwarz inequality we get

$$\frac{a^2}{x} + \frac{b^2}{y} + \frac{c^2}{z} \geqslant \frac{(a+b+c)^2}{x+y+z},$$

with equality if and only if $x : y : z = a : b : c$.
Combining these two above inequalities, for any positive real numbers x, y, z, the following inequality holds:

$$(xR_1^2 + yR_2^2 + zR_3^2)\left(\frac{1}{yz} + \frac{1}{zx} + \frac{1}{xy}\right) \geqslant \frac{(a+b+c)^2}{x+y+z}. \tag{2.4}$$

and we easily to know that the equality if and only if $x : y : z = a : b : c$ and P is the incenter of $\triangle ABC$.

Now, applying the inversion transformation in the lemma to inequality (2.4), we obtain

$$\left[x(R_2R_3)^2 + y(R_3R_1)^2 + z(R_1R_2)^2\right]\left(\frac{1}{yz} + \frac{1}{zx} + \frac{1}{xy}\right) \geqslant \frac{(aR_1 + bR_2 + cR_3)^2}{x+y+z},$$

or equivalently

$$\frac{(R_2R_3)^2}{yz} + \frac{(R_3R_1)^2}{zx} + \frac{(R_1R_2)^2}{xy} \geqslant \left(\frac{aR_1 + bR_2 + cR_3}{x+y+z}\right)^2. \tag{2.5}$$

where x, y, z are positive numbers.

For $x \to xR_1^2, y \to yR_2^2, z \to zR_3^2$, then holds

$$\frac{1}{yz} + \frac{1}{zx} + \frac{1}{xy} \geqslant \left(\frac{aR_1 + bR_2 + cR_3}{xR_1^2 + yR_2^2 + zR_3^2}\right)^2. \tag{2.6}$$

Take again $x \to \frac{1}{x}, y \to \frac{1}{y}, z \to \frac{1}{z}$, then we get the inequality (2.3) of the theorem.

Notice that the conclusion in [7]: If equality in (2.1) occurs only P is the incenter of $\triangle ABC$, then equality in (2.2) only $\triangle ABC$ is acute-angled and P is its orthocenter. According to this and the condition which occurs equality in (2.4), we easy know equality in (2.3) if and only if $\triangle ABC$ is acute-angled, P is its orthocenter and holds

$$\frac{R_1}{xa} = \frac{R_2}{yb} = \frac{R_3}{cz}. \tag{2.7}$$

Since when P is the orthocenter of the acute triangle ABC, we have $R_1 : R_2 : R_3 = \cos A : \cos B : \cos C$. Hence from (2.7) we have $x : y : z = \cot A : \cot B : \cot C$ in this case. Thus, there is equality in (2.3) if and only if $\triangle ABC$ is acute-angled, P coincide with its orthocenter and $x/\cot A = y/\cot B = z/\cot C$. This complete the proof of the Theorem. □

Remark 2.1. If P does not coincide with the vertices, then inequality (2.4) is equivalent to the following result in [8]:

$$x\frac{R_2R_3}{R_1} + y\frac{R_3R_1}{R_2} + z\frac{R_1R_2}{R_3} \geqslant 2\sqrt{\frac{xyz}{x+y+z}}s, \tag{2.8}$$

where s is the semi-perimeter of $\triangle ABC$, x, y, z are positive real numbers. In [8], the proof of (2.8) without using the polar moment of inertia inequality, So does not start from Klamkin's inequality (1.1) we can deduce the inequality of the theorem.

3. Applications of the theorem

Besides the above notations, as usual, let R and r denote the radii of the circumcircle and incircle of triangle ABC, respectively, Δ denote the area, r_a, r_b, r_c denote the radii of excircles. In addition, when point P lies interior of triangle ABC, let r_1, r_2, r_3 denote the distances of P to the sides BC, CA, AB.

According to the theorem and the well-known inequality for any point P in the plane

$$aR_1 + bR_2 + cR_3 \geqslant 4\Delta, \tag{3.1}$$

We get

Corollary 1 For any point P in the plane and arbitrary positive numbers x, y, z, the following inequality holds:

$$\frac{R_1^2}{x} + \frac{R_2^2}{y} + \frac{R_3^2}{z} \geqslant \frac{4\Delta}{\sqrt{yz + zx + xy}}, \tag{3.2}$$

with equality if and only if $x : y : z = \cot A : \cot B : \cot C$ and P is the orthocenter of the acute angled triangle ABC.

Remark 3.1. Clearly, (3.2) is equivalent with

$$xR_1^2 + yR_2^2 + zR_3^2 \geqslant 4\sqrt{\frac{xyz}{x+y+z}}\Delta. \tag{3.3}$$

The above inequality is first given in [9] by Xue-Zhi Yang. The author [10] obtained the following generalization:

$$x\left(\frac{a'}{a}\right)^2 + y\left(\frac{b'}{b}\right)^2 + z\left(\frac{c'}{c}\right)^2 \geqslant 4\sqrt{\frac{xyz}{x+y+z}}\Delta', \tag{3.4}$$

where a', b', c' denote the sides of $\triangle A'B'C'$, Δ' denote its area.

If, in (2.3) we put $x = \frac{1}{a}, y = \frac{1}{b}, z = \frac{1}{c}$, note that $\frac{1}{bc} + \frac{1}{ca} + \frac{1}{ab} = \frac{1}{2Rr}$, we get the result:

Corollary 2 For arbitrary point P in the plane of $\triangle ABC$, the following inequality holds:

$$\frac{aR_1^2 + bR_2^2 + cR_3^2}{aR_1 + bR_2 + cR_3} \geqslant \sqrt{2Rr}, \tag{3.5}$$

equality holds if and only if the triangle ABC is equilateral and P is its center.

Remark 3.2. The conditions for equality that the following inequalities of corollary 4-8 are the same in the statement of Corollary 2.

In the theorem, for $x = \dfrac{R_1}{a}, y = \dfrac{R_2}{b}, z = \dfrac{R_3}{c}$, after reductions we obtain

Corollary 3 If P is arbitrary point which does not coincide with the vertices of $\triangle ABC$, then

$$\frac{R_2R_3}{bc} + \frac{R_3R_1}{ca} + \frac{R_1R_2}{ab} \geqslant 1, \tag{3.6}$$

equality holds if and only if $\triangle ABC$ is acute-angled and P is its orthocenter.

Inequality (3.6) first proved by T.Hayashi (see [11] or [3]), the author gave its two generalizations in [12].

Indeed, assume P does not coincide with the vertices, put $x \to \frac{R_1}{xa}, y \to \frac{R_2}{yb}, z \to \frac{R_3}{zc}$ in (2.2), then we can get the weighted generalization form of Hayashi inequality:

$$\frac{R_2R_3}{yzbc} + \frac{R_3R_1}{zxca} + \frac{R_1R_2}{xyab} \geqslant \left(\frac{aR_1 + bR_2 + cR_3}{xaR_1 + ybR_2 + zcR_3}\right)^2. \tag{3.7}$$

For $x = \frac{1}{a}, y = \frac{1}{b}, z = \frac{1}{c}$, we have

$$(R_2R_3 + R_3R_1 + R_1R_2)(R_1 + R_2 + R_3)^2 \geqslant (aR_1 + bR_2 + cR_3)^2. \tag{3.8}$$

Applying the inversion transformation of the lemma to the above inequality, then divide both sides by $R_1R_2R_3$ we get

Corollary 4 If P is arbitrary point which does not coincide with the vertices of $\triangle ABC$, then

$$(R_2R_3+R_3R_1+R_1R_2)^2\left(\frac{1}{R_2R_3}+\frac{1}{R_3R_1}+\frac{1}{R_1R_2}\right)\geqslant 4s^2. \tag{3.9}$$

It is not difficult to know that the above inequality is stronger than the following result which the author obtained many years ago:

$$\sqrt{\frac{R_2R_3}{R_1}}+\sqrt{\frac{R_3R_1}{R_2}}+\sqrt{\frac{R_1R_2}{R_3}}\geqslant\sqrt{2\sqrt{3}s} \tag{3.10}$$

Now, Let P be an interior point of the triangle ABC, then we have the well known inequalities(see[13]):

$$aR_1\geqslant br_3+cr_2, bR_2\geqslant cr_1+ar_3, cR_3\geqslant ar_2+br_1,$$

Summing them up, note that $a+b+c=2s$ and the identity $ar_1+br_2+cr_3=2rs$ we easily get

$$aR_1+bR_2+cR_3\geqslant 2s(r_1+r_2+r_3)-2rs. \tag{3.11}$$

Multiplying both sides by 2 then adding inequality (3.1) and using $\Delta=rs$, then

$$3(aR_1+bR_2+cR_3)\geqslant 4s(r_1+r_2+r_3),$$

that is

$$\frac{aR_1+bR_2+cR_3}{r_1+r_2+r_3}\geqslant\frac{4}{3}s. \tag{3.12}$$

According to this and the equivalent form (2.5) of inequality (2.3), we get immediately the result:

Corollary 5 Let P be an interior point of the triangle ABC, then

$$\frac{(R_2R_3)^2}{r_2r_3}+\frac{(R_3R_1)^2}{r_3r_1}+\frac{(R_1R_2)^2}{r_1r_2}\geqslant\frac{16}{9}s^2. \tag{3.13}$$

From inequality (3.8) and (3.12) we infer that

$$(R_2R_3+R_3R_1+R_1R_2)(R_1+R_2+R_3)^2\geqslant\frac{16}{9}s^2(r_1+r_2+r_3)^2,$$

Note that again $3(R_2R_3+R_3R_1+R_1R_2)\leqslant(R_1+R_2+R_3)^2$, we get the following inequality:

Corollary 6 Let P be an interior point of triangle ABC, then

$$\frac{(R_1+R_2+R_3)^2}{r_1+r_2+r_3}\geqslant\frac{4}{\sqrt{3}}s. \tag{3.14}$$

Letting $x=r_a, y=r_b, z=r_c$ in (2.3) and note that identity $r_br_c+r_cr_a+r_ar_b=s^2$, hence

$$\frac{R_1^2}{r_a}+\frac{R_2^2}{r_b}+\frac{R_3^2}{r_c}\geqslant\frac{1}{s}(aR_1+bR_2+cR_3). \tag{3.15}$$

This inequality and (3.12) lead us get the following inequality:

Corollary 7 Let P be an interior point of the triangle ABC, then

$$\frac{R_1^2}{r_a}+\frac{R_2^2}{r_b}+\frac{R_3^2}{r_c}\geqslant\frac{4}{3}(r_1+r_2+r_3). \tag{3.16}$$

Adding (3.1) and (3.11) then dividing both sides by 2, we have

$$aR_1+bR_2+cR_3\geqslant s(r_1+r_2+r_3+r). \tag{3.17}$$

From this and (3.15), we get again the following inequality which similar to (3.16):

Corollary 8 Let P be an interior point of the triangle ABC, then

$$\frac{R_1^2}{r_a}+\frac{R_2^2}{r_b}+\frac{R_3^2}{r_c}\geqslant r_1+r_2+r_3+r. \tag{3.18}$$

When P locates interior of the triangle ABC, let D, E, F be the feet of the perpendicular from P to the sides BC, CA, AB respectively. Take $x = ar_1, y = br_2, z = cr_3$ in equivalent form (2.6) of inequality (2.3), then

$$\frac{1}{bcr_2r_3} + \frac{1}{car_3r_1} + \frac{1}{abr_1r_2} \geqslant \left(\frac{aR_1 + bR_2 + cR_3}{ar_1R_1 + br_2R_2 + cr_3R_3}\right)^2,$$

Using identity $ar_1 + br_2 + cr_3 = 2\Delta$ and well known identity(see[7]):

$$ar_1R_1^2 + br_2R_2^2 + cr_3R_3^2 = 8R^2\Delta_p, \tag{3.19}$$

(where Δ_p is the area of pedal triangle DEF)We get

$$abcr_1r_2r_3(aR_1 + bR_2 + cR_3)^2 \leqslant 64\Delta R^4\Delta_p^2.$$

Let s_p, r_p denote the semi-perimeter of the triangle DEF and radius of incircle respectively. Note that $\Delta_p = r_ps_p, aR_1 + bR_2 + cR_3 = 4Rs_p$, from the above inequality we obtain the following inequality which established by the author in [14]:

Corollary 9 Let P be an interior point of the triangle ABC, then

$$\frac{r_1r_2r_3}{r_p^2} \leqslant 2R, \tag{3.20}$$

equality holds if and only if P is the orthocenter of the triangle ABC.

It is well known, the inequality related to a triangle and two points is very few. Several years ago, the author guessed the following inequality holds:

$$\frac{R_1^2}{d_1} + \frac{R_2^2}{d_2} + \frac{R_3^2}{d_3} \geqslant 4(r_1 + r_2 + r_3), \tag{3.21}$$

where d_1, d_2, d_3 denote the distances from an interior point Q to the sides of $\triangle ABC$.

Inequality (3.21) is very interesting, the author have been trying to prove it. In what follows, we will prove the stronger result. To do so, we need a corollary of the following conclusion(see[15]):

Let Q be an interior point of $\triangle ABC$, t_1, t_2, t_3 denote the bisector of $\angle BQC, \angle CQA, \angle AQB$ respectively, $\triangle A'B'C'$ is an arbitrary triangle, then

$$t_2t_3 \sin A' + t_3t_1 \sin B' + t_1t_2 \sin C' \leqslant \frac{1}{2}\Delta, \tag{3.22}$$

with equality if and only if $\triangle A'B'C' \sim \triangle ABC$, and Q is the circumcentre of $\triangle ABC$.

In (3.22), letting $\triangle ABC$ be equilateral, then we immediately get

$$t_2t_3 + t_3t_1 + t_1t_2 \leqslant \frac{1}{\sqrt{3}}\Delta. \tag{3.23}$$

From this and the simple inequality $s^2 \geqslant 3\sqrt{3}\Delta$, we have

$$t_2t_3 + t_3t_1 + t_1t_2 \leqslant \frac{1}{9}s^2. \tag{3.24}$$

According to the inequality (2.3) of the theorem and (3.24), we can see that

$$\frac{R_1^2}{t_1} + \frac{R_2^2}{t_2} + \frac{R_3^2}{t_3} \geqslant \frac{3}{s}(aR_1 + bR_2 + cR_3). \tag{3.25}$$

By using inequality (3.12), we obtain the following stronger of inequality (3.21)

Corollary 10 Let P and Q be two interior points of the $\triangle ABC$, then

$$\frac{R_1^2}{t_1} + \frac{R_2^2}{t_2} + \frac{R_3^2}{t_3} \geqslant 4(r_1 + r_2 + r_3). \tag{3.26}$$

with equality if and only if $\triangle ABC$ is equilateral, P and Q both are its center.

Analogously, from inequality (3.17) and inequality (3.25) we get

Corollary 11 Let P and Q be two interior points of $\triangle ABC$, then

$$\frac{R_1^2}{t_1} + \frac{R_2^2}{t_2} + \frac{R_3^2}{t_3} \geqslant 3(r_1 + r_2 + r_3 + r). \tag{3.27}$$

with equality if and only if $\triangle ABC$ is equilateral, and P and Q both are its center.

4. **Some conjectures**

In this section, we will state some conjectures in allusion to the inequality which appeared in this paper.

Inequality (3.8) is equivalent to

$$R_2R_3 + R_3R_1 + R_1R_2 \geqslant \left(\frac{aR_1 + bR_2 + cR_3}{R_1 + R_2 + R_3}\right)^2. \tag{4.1}$$

Enlightened by this one and the well known inequality:

$$R_2R_3 + R_3R_1 + R_1R_2 \geqslant 4(w_2w_3 + w_3w_1 + w_1w_2), \tag{4.2}$$

We pose the following

Conjecture 1 Let P be an arbitrary interior point of the triangle ABC, then

$$\left(\frac{aR_1 + bR_2 + cR_3}{R_1 + R_2 + R_3}\right)^2 \geqslant 4(w_2w_3 + w_3w_1 + w_1w_2). \tag{4.3}$$

We consider the stronger of corollary 5, the author posed these two conjectures:

Conjecture 2 Let P be an arbitrary interior point of the triangle ABC, then

$$\frac{(R_2R_3)^2}{w_2w_3} + \frac{(R_3R_1)^2}{w_3w_1} + \frac{(R_1R_2)^2}{w_1w_2} \geqslant \frac{4}{3}(a^2 + b^2 + c^2). \tag{4.4}$$

Conjecture 3 Let P be an arbitrary interior point of the triangle ABC, then

$$\frac{(R_2R_3)^2}{r_2r_3} + \frac{(R_3R_1)^2}{r_3r_1} + \frac{(R_1R_2)^2}{r_1r_2} \geqslant 4(R_1^2 + R_2^2 + R_3^2). \tag{4.5}$$

For the inequality of corollary 6, we guess the following stronger inequality holds:

Conjecture 4 Let P be an arbitrary interior point of the triangle ABC, then

$$\frac{R_2R_3 + R_3R_1 + R_1R_2}{r_1 + r_2 + r_3} \geqslant \frac{4}{3\sqrt{3}}s. \tag{4.6}$$

On the other hand, for the acute-angled triangle, we pose the following conjecture:

Conjecture 5 Let $\triangle ABC$ be acute-angled and P is an arbitrary point in its interior, then

$$\frac{(R_1 + R_2 + R_3)^2}{w_1 + w_2 + w_3} \geqslant 6R. \tag{4.7}$$

Two years ago, Xue-Zhi Yang proved the following inequality (in a private communication with the author):

$$\frac{(R_1 + R_2 + R_3)^2}{r_1 + r_2 + r_3} \geqslant 2\sqrt{a^2 + b^2 + c^2}. \tag{4.8}$$

which is stronger than (3.14). Here, we further put forward the following

Conjecture 6 Let P be an arbitrary interior point of the triangle ABC, then

$$\frac{(R_1 + R_2 + R_3)^2}{w_1 + w_2 + w_3} \geqslant 2\sqrt{a^2 + b^2 + c^2}. \tag{4.9}$$

In [14], the author point out the following phenomenon(so-called $r - w$ phenomenon): If holds the inequality for r_1, r_2, r_3(this inequality can be also includes R_1, R_2, R_3 and otherwise geometric elements), then after changing r_1, r_2, r_3 into w_1, w_2, w_3 respectively, the stronger inequality often holds or often holds for the acute triangle. Conjecture 6 was proposed based on this kind of phenomenon. Analogously, we pose these following four conjectures:

Conjecture 7 Let $\triangle ABC$ be acute-angled and P is an arbitrary point in its interior, then

$$\frac{aR_1 + bR_2 + cR_3}{w_1 + w_2 + w_3} \geqslant \frac{4}{3}s. \tag{4.10}$$

Conjecture 8 Let $\triangle ABC$ be acute-angled and P is an arbitrary point in its interior, then

$$\frac{aR_1 + bR_2 + cR_3}{w_1 + w_2 + w_3 + r} \geqslant 2s. \tag{4.11}$$

Conjecture 9 Let P and Q be two interior points of the $\triangle ABC$, then

$$\frac{R_1^2}{t_1} + \frac{R_2^2}{t_2} + \frac{R_3^2}{t_3} \geqslant 4(w_1 + w_2 + w_3). \tag{4.12}$$

Conjecture 10 Let P and Q be two interior points of the $\triangle ABC$, then

$$\frac{R_1^2}{t_1} + \frac{R_2^2}{t_2} + \frac{R_3^2}{t_3} \geqslant 3(w_1 + w_2 + w_3 + r). \tag{4.13}$$

Remark 4.1. If conjecture 7 and 8 are proofed, then we can proof conjecture 9 and 10 valid for the acute triangle ABC.

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