A Two Level Representation for Spatial Relations, Part I

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Abstract:

A model to represent spatial relations is presented. It is used for the definition of common sense knowledge of rational agents in a multi-agent-scenario. The main idea is, that it is structured in two levels: the representation of relations may be accomplished in terms of predicate logic at one level or in expressions of Cartesian coordinates at the other. Hence reasoning is possible with common rules of deduction as well as via exact calculations of the positions.

Here we give an overview on the whole structure and then investigate in the the definition of a set of spatial relations at the “Logical Level”. Finally special features like the handling of the context and the problem of multiple views are discussed.
1 Introduction

Though we are mostly not aware of it, we are permanently confronted in dealing with spatial properties. So spatial knowledge is very important in real life and thus it is essential in the science of cognition. The discussion of ‘internal representation’ and the means of reasoning about space aspects are therefore as well of general interest in AI ([McDermott87]).

Our objective is the modeling of multi-agent-systems. Consider a multi-agent world involving both human and artificial agents. The communication between these agents has to be more restricted than the communication in fully natural language between two humans. The artificial agent (robot, expert system,...) simply lacks in human intuition, which has been gathered in learning processes lasting the whole life long. Artificial agents have no "feeling" for domains where humans got familiar with since they began to reflect. We are especially interested in the phenomenon of space [McDermott87].

Humans do not often think deeper about their spatial inferences. Artificial agents, however, need formalized rules to derive "knowledge" about spatial occurrences. The rules may be considerable complex and adaptable but in near future they will never gain the capabilities of human perception and cognition. We want to present a well-founded approach towards the desirable formalization of spatial knowledge.

But by the way, what is SPACE ? Following [Lang,Carstensen89] there are essentially two theories about its constitution,

- the Container-Theory and
- the Configuration-Theory.

The first intends the space to constitute the objects. For example imagine a large box which shall contain all objects. Then the space in the box is the absolute space of the world, all objects must be adapted to it. The second theory takes the objects and their relative position to each other to constitute the space. Consider a system of axis, one for each dimension, to denote the position and dimension values of the objects. These axis can be regarded as spanning a three-dimensional room, the world's space. Both of the two theories make sense in certain situations, none of them should be preferred to the other. Spatial knowledge depends on knowledge of the objects as well as on the ability of orientation.

Spatial reasoning and the representation of spatial relations can be located at the point where two branches of AI research meet: Computer Vision and Natural Language Processing. This is most obvious in recently developed AI-Systems like CITYTOUR [Andre,Rist,Herzog87], SOCCER [Schirra89] or EPEX [Walter89] which all shall construct a natural language description of images (scenes) or even of sequences of images. All these systems use an explicit knowledge base. They may be called spatial-knowledge based systems.

From the viewpoint of computer vision a hierarchy of representations may be considered (from the level of pixels up to the extraction of episodes, cf. [Walter89]). At the upper levels of this hierarchy linguistic expressions, i.e. prepositions, are used to describe scenes. On the other hand linguists and AI researchers are heavily interested in finding a well-defined sense of these relations. This implies that also their representation should be most adequate [Herskovits86] [Retz-Schmidt88] [Habel,Pribbenow89]. In this field of tension concerned scientists fought out a so called imagery debate more than ten years: Do human beings use merely a propositional representation of space from which we deduce some ‘mental images’ from time to time when needed or do a propositional and a analogical (pictorial) representation coexist rather
independently having the same influence on reasoning [McDermott87] [Pribbenow90]. This later mentioned ‘as well as’ position is the most appreciated direction of current research. The final goal is to find a common base for these two kinds of representation.

For instance, the project LILOG-R which was initiated from IBM Germany and several German universities (mainly the University of Hamburg) combines propositional and analogical knowledge in a hybrid system [Habel,Pribbenow89]. The analogical part is called ‘depictorial’ which means a representation in the sense of coarse drafts of real images [Khenkhar89]. Likewise the work of Güsgen [Güsgen89] who deals with intervals.

However, up to now no complete and satisfying calculus for connecting them and getting new relations has been found. Therefore also language-researchers are heavily interested in a spatial logic.

But we do not want to conceal that many researchers believe that there will never be such a ‘calculus’. Especially linguists insist on the ambiguity and the idiosyncrasy of natural language. Herskovits proposes a variety of ‘ideal meanings’ and ‘use types’ for spatial relations which fit in ‘normal situations’. But she heavily doubts that all the special meanings in all special situations will be computable without a large amount of experience with spatial occurrences [Herskovits86].

Our approach will perhaps not satisfy the linguists, for we present a rather restricted interpretation of spatial relations, which could crudely be treated as equivalent to Herskovits’ ideal meanings. But we hope to provide a step towards a useful application of spatial knowledge as a part of ‘common sense knowledge’ in an AI-system.

We will introduce a logic based two level space representation. The main issue is using a Cartesian system of coordinates to define spatial primitives. This is our ‘Coordinate Level’ in the sense to provide a unique and well-defined semantic meaning for the primitives. The other level evolves a higher niveau of abstraction and uses these exactly defined primitives to specify other spatial relations in terms of logical formulae (‘Logical Level’). Seen through the eyes of a logician this means working in a logical calculus with an underlying ‘theory of space’.

After the general introduction of our model we investigate in the specification of almost-exact spatial relations at the Logical Level. Almost-exact spatial relations are, for instance, spatial relations with an almost clear semantic when used in natural language, e.g. if humans use the relation RIGHT_OF it is rather obvious, in opposition to the use of the “fuzzy relation” NEAR_BY, what is meant. We discuss three possible specifications, namely

1. Conceptualization with primitives,
2. Dependent on regions and
3. Definition with interval relations,

for each of the spatial relations IN, UPON, OVER, ABOVE, RIGHT_OF, IN_FRONT_OF, NORTH_OF, AT, BETWEEN and AROUND. Further we investigate in various properties of the relations like reflexivity, symmetry, transitivity, inverse relations, orthogonality and other correlations.

If there are many autonomous agents cooperating in the scenario, the logic must be enriched by special features such as deictic interpretations.
2 The Context of Our Investigations

We consider multi-agent-systems as a society of autonomous agents which act cooperatively towards the objective of solving a complex task. These agents are ‘intelligent’, their knowledge is stored in their individual knowledge bases. But intelligence not only requires knowledge about the special aspects of the problem domain, it becomes more and more apparent that a huge part of ‘real intelligence’ consists of common sense knowledge, knowledge which is used by human beings often without much consideration. Spatial knowledge surely has its place among this common sense knowledge.

The agents encounter the problems together, hence they must communicate about the sequence of steps to solve it. No cooperation without communication! But also we, the human beings, may be involved in the process of problem solving, for instance working hand in hand with robots in a task of re-ordering chests in a room. Thus we have to understand the communication of artificial agents, it must take place in a human oriented way. But we want to avoid the mass of difficulties emerging from natural language processing. So we demand a restriction to fragments of sentences. Consider the following dialog:

Human: "Stack red block into blue box."
Robot: "Red block is not unique. Need more information."
Human: "Red block is left of sphere."
...etc...

Here we clearly see that spatial knowledge (what means into, what left of...) must be a part of the common sense knowledge base whereas the knowledge about blocks, boxes and ‘stacking’ objects belongs to the special aspects of the problem domain. The importance of space becomes obvious: The robots have to coordinate their own behaviour in space and have to refer to objects arranged in certain spatial relations.

Again, a two level representation of spatial occurrences is straightforward: The agents shall communicate in terms of spatial relations in ‘pseudo-natural’ language (at the Logical Level) but their very own inference processes should be as fast and exact as possible. Thus calculations at the Coordinate Level are most convenient.

The short dialog example above reveals a main problem in multi-agent-scenarios: The interpretation of different points of view. Let us take a second look at the dialog. What is ‘left of’ the sphere ? Is it an absolute statement or is it meant relatively from anyones point of view ? We will discuss these aspects later in Chap. 5.

A short comment to the term ‘agent’. Beware of always imaging robots when hearing it. These multi-agent-scenarios are only one special kind among many others. Also a group of expert systems which coordinate, say, transports in a certain area via an electronic communication network (thus representing transport agencies) can be regarded as a multi-agent-system.
3 A Two Level Space Representation

We want to construct a spatial knowledge base in terms of a logic based knowledge representation language. The spatial relations used in natural language provide the names of the relations (predicates) in order to simplify man-machine communication. Several inherently different problems arise:

a.) It is a hard task to specify primitive relations like *on* sufficiently precise. The syntactical statement ON (A,B) could be semantically interpreted as: *A is married to B*. We could add more characteristics to yield our intended meaning, for instance irreflexibility (not (ON (A,A))), asymmetry (not (ON (A,B) iff ON (B,A))) etc. We, however, can only restrict the possible interpretations; there will always be a lot of two-place relations in the real world satisfying all constraints but having nothing in common with our intended meaning of *on*.

b.) Artificial agents calculate and do syntactic manipulations much faster than any human, but they lack in doing many ‘every day’ spatial inferences and do not reach the capabilities even of a child. Idiosyncrasy and ambiguity of natural language space descriptions [Herskovits86] contribute to this poorness.

c.) Some given spatial relations (for instance between two objects) often cannot be mapped directly onto a given definition in the predicate calculus. Several artificial supplying predicates having no adequate pendant in reality must be used therefore.

In order to solve the problems mentioned above we suggest a two level representation of space.

The higher level specifies the spatial relations abstractly in terms of the predicate calculus. The lower level consists of the ‘concrete’ specifications of spatial primitives. These primitive relations are defined via arithmetic expressions in terms of coordinates and demand knowledge about the absolute position of an object as well as about its extension in the Cartesian space.

The Transformation Interface translates spatial relations from one representation level to the other. Thus constraints to coordinates at the lower level may be converted into a formula consisting of appropriate predicates and, vice versa, arbitrary complex formulae may be simplified by mapping them onto relations of coordinates.
3.1 The Logical Level

Now we take a closer look at the two levels. At the higher one - the ‘logical’ level - every spatial relation consists of a definition (DEF) and a list of restricting properties (PROP). The definition is founded either on already defined relations on the same level or on spatial primitives.

Let ON be already defined, a specification of OVER could be:

\[
\text{DEF: all X all Y } \text{OVER}(X,Y) \text{ iff (} \text{ON}(X,Y) \text{ or (exist Z } \text{ON}(Z,Y) \text{ and OVER}(X,Z)))))
\]

\[
\text{PROP: not } \text{OVER}(X,X) \text{ - not reflexive}
\]

\[
\text{not } \text{OVER}(X,Y) \text{ iff OVER}(Y,X) \text{ - not symmetric}
\]

\[
[\text{OVER}(X,Y) \text{ and OVER}(Y,Z)] \impl \text{OVER}(X,Z) \text{ - transitive}
\]

3.2 The Coordinate Level

At the lower level we rely on the three-dimensional Cartesian system of coordinates, as mentioned above. If we know position and extension of the objects it is easy to calculate the absolute extreme points of the objects in space. Every physical object therefore is idealized as a trihedral block with 90° angles only.

Given the list of the extreme points as ((x1,x2), (y1,y2), (z1,z2)) where x is the breadth, y the height and z the dimension of depth (see picture below), furthermore .1 the min-value and .2 the max-value. An appropriate specification of \text{on} will be:

\[
\text{ON}(X,Y) \text{ iff } y_1 (X) = y_2 (Y) \text{ and } x_1 (X) \leq x_2 (Y) \text{ and } x_2 (X) > x_1 (Y) \text{ and } z_1 (X) \leq z_2 (Y) \text{ and } z_2 (X) > z_1 (Y)
\]

Hence, one of our problems listed above is solved, that is a.). If we watch our two level model not longer from the logical point of view but turn towards more operational aspects, we soon become aware of of the need of an intelligent link between the two levels. It must be possible to transfer logical specifications from the Logical Level to the Coordinate Level in order to attack problem b.). A solution of c.) demands the opposite direction: If we also allow defined relations on the Coordinate Level which are too complex to be primitives, the propagation of these definitions to the higher level for logical deductions must be possible.
3.3 The Transformation Interface

The Transformation Interface (TI) translates relations from one representation level to the other. Thus constraints to coordinates at the lower level may be converted into a formula consisting of appropriate predicates and, vice versa, arbitrary complex formulae may be simplified by mapping them onto relations of coordinates.

Normally a specification of a relation only at one level is sufficient. If, however, there are relations defined at both levels the TI has the duty to check the consistency of the two specifications. The over-predicate defined above thus can be specified in the following way:

\[
\text{OVER}(X,Y) \iff y_1(X) \geq y_2(Y) \text{ and } x_1(X) \leq x_2(Y) \text{ and } x_2(X) > x_1(Y) \text{ and } z_1(X) \leq z_2(Y) \text{ and } z_2(X) > z_1(Y)
\]

Though we like to have a consistent representation (in the logical sense) of the relations in the combination of both levels, it seems to be impossible to get really equivalent definitions. The TI has to notice that the later specification of \(\text{over}\) is more universal than the one on the Logical Level given more above. \(\text{OVER}(A,B)\) is true at the Coordinate Level even if \(A\) hangs in the air \(\text{over} B\). But this is forbidden on the higher level where the demand of strict support is hidden in the definition.

The Transformation Interface seems to be the toughest point of the proposed construction. It has, as mentioned above, the task to translate definitions of spatial relations between the two levels. It has to ensure the correspondence between the ordering relations used on the Coordinate Level and the properties of the relations as given in the property lists on the Logical Level. On the other hand there must be a kind of abstraction mechanism to code a set of coordinate terms into one term in the Logical Level. This is because on the Logical Level we do not want to get lost in too many details. But the abstractions on the other hand may not omit too much information.

Further on a more technical level we need improved term and formula handling mechanisms to avoid an explosion of the definitions.

3.4 Discussion

The advantages of the introduced two level representation of space are straightforward. On the one hand there is the logical foundation and the possibility to make abstract deductions. If, for instance, \(\text{ON}(A,B)\) is already known, \(\text{OVER}(A,B)\) can directly be inferred without doing expensive computations on the Coordinate Level. Beyond these inference aspects logical specifications are useful for communication in ‘pseudo-natural’ language. But on the other hand the model is exact. Each logical definition can be transfered into ‘hard’ coordinates which at least becomes important when an agent acts physically in the world. This may easily be achieved by an access to the Coordinate Level via the Transformation Interface as mentioned above.

In the next chapter we will go into the details of representing a special kind of spatial relations -the almost exact spatial relations- on the Logical Level.
4 Almost Exact Spatial Relations on the Propositional Level

Common English prepositions, as well as some composed phrases, may induce *spatial relations*, which mostly have as arguments a location entity (LE) and a reference object (RO) (cf. [Habel,Pribbenow88], [Habel,Pribbenow89], [Habel89], [Pribbenow90]). Hence we often use the abstract term "spatial relations" instead of "spatial prepositions". These relations may be grouped in different ways. Herskovits distinguishes spatial relations in the classes of "basic topological relations" and "projective relations" [Herskovits86].

Despite of the consensus in linguistics and cognitive science that all spatial relations used by humans are more or less fuzzy, we propose a categorization in "fuzzy relations" and "non-fuzzy relations". These non-fuzzy relations shall be called "almost-exact relations", for better convenience.

In the multi agents setting we prefer *almost-exact spatial relations* for multi-agent communication. These are easier to formalize and handle than fuzzy relations. As a matter of fact, also humans get less confused in their use. For example, it seems to be mostly simple to refute that an object X is *right of* another object Y, rather than to refute that X is *near by* Y. The reason for this is the existence of a frame of reference for *right of* which standardizes its interpretation in a certain way, whereas *near by* does not use such a frame. Near by is more influenced by contextual factors and individual perceptions.

4.1 The Class of Almost-Exact Relations

Let $\mathcal{R}$ be the class of the almost-exact relations, containing:

- IN, IN_THE_MIDDLE_OF, OUTSIDE_OF; AT; UPON, DIRECT_UNDER; OVER, UNDER;
- ABOVE, BELOW, RIGHT_OF, LEFT_OF, BEHIND, IN_FRONT_OF; BETWEEN;
- NORTH_OF, SOUTH_OF, WEST_OF, EAST_OF; AROUND.

Each of these relations have got two parameters$^1$: REL(X,Y), where X is the location object (LO) and Y is the reference object (RO). Our objective is to define the meaning of relations in such a way that the human's intuitive conception is widely appreciated to gain a high amount of acceptance, linguistically and cognitively. But it is not possible, as mentioned above, to model the whole variety of human spatial interpretations. If there are doubtful situations or obscure scenes, the agents have to negotiate. Like in "real life" they must find a consensus so that all involved agents talk about the same things in the same manner.

Before going into details for some typical relations we propose a definition:

> A spatial relation is called **primitive** iff it cannot be defined by means of other definitions of relations which are themselves founded on primitive relations.

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$^1$The reference object of BETWEEN is a complex object which consists at least of two simple objects.
4.2 Possible Specifications

We present three possibilities to define and specify spatial relations at the Logical Level.

1. The definition is based on already defined or “primitive” relations. This could be a very elegant way, however, in most times it is not possible because of the manifold of primitive spatial relations. Primitive relations cannot be conceptualized in expressions of other simpler relations, they are just as they are. In some cases additional predicates like “INTERSECTS” provide means for a rather coarse specification, but these have to be proven procedurally. If the inspection is limited to certain domains, higher and richer hierarchies of relations may be defined (RIGHT_OF_A $\preceq$ RIGHT_OF_B $\preceq$ RIGHT_OF_D $\subseteq \ldots$ where A,B,D are objects in the domain). But this way has a strong artificial touch and seems to be appropriate only for the internal representation of spatial relations.

2. Functions are introduced which designate certain parts of the objects or generate special regions around the involved objects ([Habel,Pribbenow88], [Pribbenow90]). These functions shall be called region-describing functions, slightly according to [Habel,Pribbenow88]. They refer to spatial parts of the whole region of the scene and may be conceptualized as follows: REL_REGION (Y) provides the region where the LO X is in spatial relation REL to the RO Y. Linguists traditionally use expressions like LOC (X,REL_REGION (Y)) where LOC is the location predicate. The whole expression is strongly accompanied by side effects, for instance REL_REGION (Y) must have informations about size and form of the corresponding region which strongly depend on contextual factors.

3. Relations of intervals in the tradition of Allen [Allen85] are extended to the three dimensions of space (cf. also [Güsgen89] who treats this aspects fundamentally). But this corresponds closely to relations between the ending points of objects at the Coordinate Level. The main drawback is that the dimensions and extensions of objects, as well as their positions, have to be known precisely. A second issue is the fast-growing amount of fuzzyness when relations of intervals are composed several times. This point is even harder in the three-dimensional space than in the one-dimensional time.

A few words to the notation employed. As a basis for the interval representation we demand a three-dimensional Cartesian system of coordinates with dimensions x (breadth), y (height) and z (depth). The respective intervals have got the name of the dimension indexed by an object identifier, for instance $x_A$ is the interval of breadth of object A.

The 13 relations between intervals according to Allen ([Allen84]) are denoted as follows:

- o (overlaps), oi (overlapped_by), s (starts), si (started_by),
- m (meets), mi (met_by), f (finishes), fi (finished_by),
- < (before), > (after), = (equal),
- d (during), di (contains),
4.3 The Concrete Specifications

4.3.1 UPON

Meaning:
LO is located on top of RO, directly (contiguous) on RO. RO thus is a necessary support for LO.

Definition:
① UPON is a primitive relation. By means of auxiliary predicates CONTIGUOUS and SUPPORTS it could be defined as follows:
UPON (A,B) ::= CONTIGUOUS (A,B) and SUPPORTS (B,A). For the procedural computation needed here we have to take into account centers of gravity and mass of both the objects.
② UPON (A,B) ::= LOC (A, UPON_REGION (B)). Objects located in this region can, but need not necessary be located upon B. Additionally to the localization function which creates a region somewhere over B, a test for support is mandatory.
③ UPON (A,B) ::= mi( z_A ,z_B) and
   (=x_A ,x_B) or d(x_A ,x_B) or dl(x_A ,x_B) or s(x_A ,x_B) or
   si(x_A ,x_B) or f(x_A ,x_B) or fl(x_A ,x_B) or o(x_A ,x_B) or oi(x_A ,x_B) ) and
   (=y_A ,y_B) or d(y_A ,y_B) or dl(y_A ,y_B) or s(y_A ,y_B) or
   si(y_A ,y_B) or f(y_A ,y_B) or fl(y_A ,y_B) or o(y_A ,y_B) or oi(y_A ,y_B) )

Properties:
�行 not reflexive, not symmetric, not transitive.
�行 inverse relation: UPON (A,B) ↔ DIRECT_UNDER (B,A)
�行 orthogonal to the following relations:
   OVER: OVER (A,B), UPON (C,A) → OVER (C,B)
   ABOVE: ABOVE (A,B), UPON (C,A) → ABOVE (C,B)
�行 further properties:
   UPON (A,B) → ¬AT (A,B). This is rather doubtful, but we preliminary state it.
   UPON (A,B)→ ¬NEAR_BY(A,B)。No one would normally categorize a relation as near by when objects are in contact to each other.

The most important aspect of the relation UPON is the concept of support. It has to be ensured and must normally be verified in a procedural manner (one issue is to compute the center of gravity of the objects).

1The interval relations in italic style have to be additionally checked with regard to the center of mass of the respective objects.
2We consider AT as a contiguity from the side.
3NEAR_BY belongs to the class of Fuzzy Relations.
4.3.2 OVER

Meaning:

We introduce two possible conceptions of OVER. The first is a weaker one, more adequate to normal use. The second is strictly adapted to a certain kind of scenario where stacks of objects are important and frequent (e.g. Blocks World scenarios). The latter makes a distinction between OVER and ABOVE, the first conception does not: OVER and ABOVE describe equivalent relations of objects.

(I) LO is somewhere, somehow over RO, meaning in a region beginning on top of B and then stretching up. The LO might hang in the air or be supported by anyone, that does not matter. (II) LO is in a stack over RO. This means there are zero or more objects in the same stack between A and B. Thus A has contiguity to B (i.e. lies upon B) or has to contiguity to a third object over B. Suspension is explicitly excluded.

Definition: (for (II), definition of meaning (I) see at ABOVE)

1

OVER is not primitive, it is based on ON and recursively defined:
OVER (A,B) ::= ON (A,B) or Exists(Z): ( ON (A,Z) and OVER (Z,B) )

OVER (A,B) ::= LOC (A, OVER_REGION (B)). This creates a region similar to ABOVE_REGION (see below) resp. to the region mentioned above in meaning (I). Therefore the substantial difference (support in a stack of objects) must procedurally be maintained.

OVER (A,B) ::= ( mi( y_A , y_B ) or >( y_A , y_B ) ) and

( =(x_A ,x_B ) or d(x_A ,x_B ) or di(x_A ,x_B ) or s(x_A ,x_B ) or
si(x_A ,x_B ) or f(x_A ,x_B ) or fi(x_A ,x_B ) or o(x_A ,x_B ) or oi (x_A ,x_B ) ) and

( =(z_A ,z_B ) or d(z_A ,z_B ) or di(z_A ,z_B ) or s(z_A ,z_B ) or
si(z_A ,z_B ) or f(z_A ,z_B ) or fi(z_A ,z_B ) or o(z_A ,z_B ) or oi (z_A ,z_B ) )

Properties:

≧ not reflexive, not symmetric, transitive.
≧ inverse relation: UNDER: OVER (A,B) ↔ UNDER (B,A)
≧ orthogonal to ABOVE : ABOVE (A,B), OVER (C,A) → ABOVE (C,B)
≧ Subrelation: ON: ON (A,B) → OVER (A,B)
≧ further property: not [OVER (A,B) → ON (A,B)]

The statements made about the inversity/symmetry of ON - DIRECT_UNDER hold also for OVER - UNDER in an analogical manner.

The stronger definition (II) can be justified only in special scenarios as mentioned above. Humans prefer definition (I) and decide depending on certain situations when to employ over and when above.

The pure interval definition is the same as the for ABOVE. This implies a fundamental difference to the propositional definition (II). Thus interval-definition and region-describing function both need a procedural attachment to check the location in the identical stack.

---

1Normally the region has the form of a reverse cone or pyramid (Cf. ABOVE).
4.3.3 ABOVE

Meaning:
The LO is located in a bounded region above the RO. The region is stretched between the upper edge of the RO and is bounded by the upper edge of the LO. It has the form of a cone or a pyramid. The angle of the "ending in a point" side walls may be fixed, say 45° to the vertical, or, depending on certain contextual factors (e.g. distance of the respective objects).

Definition:

1. ABOVE is a primitive relation. It nearly cannot be defined by means of other relations, nor by auxiliary predicates. What is important after all is the form of the region. Hence 2 and 3 play an important role rather than a propositional definition.
2. ABOVE (A,B) ::= LOC (A, ABOVE_REGION (B)). ABOVE_REGION must generate a region of just that size and shape mentioned above.
3. same as OVER (see comments there).

Properties:
- not reflexive, not symmetric, not transitive.
- Transitivity is doubtful, but in some cases the rule may be convenient. Nevertheless, a general rule does not hold because of the displacement of the ABOVE-regions.
- inverse relation: ABOVE (A,B) ↔ BELOW(B,A).
- Subrelations:
  - UPON: UPON (A,B) → ABOVE (A,B)
  - OVER: OVER (A,B) → ABOVE (A,B)
- further properties:
  - OVER (A,B) ↔ ABOVE (A,B) (according to definition (I) of OVER).
  - not [ABOVE (A,B) → OVER (A,B)] (according to definition (II) of OVER).

The statements made about the inversity / symmetry of ON - DIRECT_UNDER resp. OVER - UNDER hold also for ABOVE - BELOW in an analogical manner.

The primacy of the geometric object which constitutes the ABOVE-region is important. A propositional representation is not sufficient anymore. The demand for a pictorial medium grows and, as a consequence, an interface between the two representation formats becomes necessary.

Physical laws are obsolete in this interpretation of ABOVE. Objects may even be suspended from support, a conception which seems to be very natural.

4.3.4 RIGHT_OF, IN_FRONT_OF

Meaning:
We consider these relations as with ABOVE. Again there are symmetric pairs of inverse relations where the properties of the other can be inferred from the properties of the first by swapping the arguments. The region plays an important role again. Likewise ABOVE, it is created by stretching a cone or a pyramid from one "side" of the object.

Definition:
Analog to ABOVE by using the appropriate dimensions. Example: For RIGHT_OF the cone stretches from the right side of the RO instead of the upper edge used with ABOVE.
Properties:

- not reflexive, not symmetric, not transitive (cf. ABOVE).
- inverse relations:
  - RIGHT_OF (A,B) ↔ LEFT_OF (B,A).
  - IN_FRONT_OF (A,B) ↔ BEHIND (B,A)
- Subrelations:
  - none, except some artificial constructions like DIRECT_RIGHT_OF or NEAR_RIGHT_OF which can be regarded as restrictions of the angle of the region cone resp. the depth of the cone.
- Further properties:
  - In the domain of geographic maps or, more general, if the intrinsic or deictic frame of reference is aligned to the north \(^1\), RIGHT_OF is equivalent in some sense to EAST_OF. The same holds for LEFT_OF and WEST_OF.

RIGHT_OF and LEFT_OF are inverse to one another without any restriction. This fact can be explained by emphasis on the inherent left-right symmetry of the human body. For IN_FRONT_OF - BEHIND, however, the same remarks like for ABOVE - BELOW must be added.

4.3.5 IN

Meaning:
Reference object "contains" location object. The demand of containment in all of the dimensions would be a very strict definition. This must often be weakened with respect to the third dimension, the height.

Definition:

1. IN is a primitive relation. Based on other relations it is only possible to describe when IN does not hold. For each relation \( R \) from \( \mathfrak{R} \): If \( R \) is not IN and not \( \text{IN\_THE\_MIDDLE\_OF} \) then:
   \[
   R (A,B) \rightarrow \neg \text{IN} (A,B).
   \]
2. \( \text{IN} (A,B) ::= \text{LOC} (A, \text{IN\_REGION} (B)) \)
   \( \text{IN\_REGION} \) generates a geometric object, which represents the space in \( B \). Detailed specifications depend on the domain.
3. \( \text{IN} (A,B) ::= (\text{d}(x_A,x_B) \text{ or } s(x_A,x_B) \text{ or } f(x_A,x_B) \text{ or } =x_A,x_B)) \) and
   \[
   (\text{d}(y_A,y_B) \text{ or } s(y_A,y_B) \text{ or } f(y_A,y_B) \text{ or } =y_A,y_B)) \)
   and
   \[
   (\text{d}(z_A,z_B) \text{ or } s(z_A,z_B) \text{ or } f(z_A,z_B) \text{ or } =z_A,z_B)) \)
   and
   \[
   (\neg (=x_A,x_B) \text{ and } =y_A,y_B) \text{ and } =z_A,z_B))
   \]

Properties:

- not reflexive, not symmetric, transitive.
  - Transitivity has to be regarded as a default rule. It is only sound if the "IN-Categories" of both the IN relations in the premise are compatible [Habel,Pribbenow89]. A "classic example" where this is not the case: The rent in the paper, the paper in the drawer, the rent in the drawer? Here the categories "hollow space" and "solid substance" are not compatible.

---

\(^1\)This simply means the observer looks from south to north (deictic frame) or from the front side of the RO one would look southward (intrinsic frame).
inverse relation: \( \text{IN} (A,B) \leftrightarrow \text{AROUND} (B,A) \) holds merely in exceptional cases.

It would be more common to claim \( \text{IN} (A,B) \leftrightarrow \text{CONTAINS} (B,A) \), but \text{CONTAINS} is not a genuine spatial relation.

orthogonal to the following relations:

\begin{itemize}
  \item \text{OVER}: \text{OVER} (A,B), \text{IN} (C,A) \rightarrow \text{OVER} (C,B).
  \item \text{ABOVE}: \text{ABOVE} (A,B), \text{IN} (C,A) \rightarrow \text{ABOVE} (C,B).
  \item \text{UNDER}, \text{BELOW} in analogy to \text{ABOVE}.
  \item \text{RIGHT_OF}: \text{RIGHT_OF} (A,B), \text{IN} (C,A) \rightarrow \text{RIGHT_OF} (C,A).
  \item \text{LEFT_OF}, \text{BEHIND}, \text{IN_FRONT_OF}, \text{NORTH_OF}, \text{\ldots} analog.
  \item \text{BETWEEN}: \text{BETWEEN} (A,B), \text{IN} (C,A) \rightarrow \text{BETWEEN} (C,B); \text{B} is a complex object.
  \item \text{AROUND}: \text{AROUND} (A,B), \text{IN} (C,B) \rightarrow \text{AROUND} (A,C).
\end{itemize}

further properties:

\begin{itemize}
  \item \text{IN} (A,B) \rightarrow \text{IN_THE_MIDDLE_OF}(A,B); \text{this does not hold reversely}.
  \item \text{IN} (A,B) \rightarrow \neg \text{OUTSIDE_OF} (A,B)
\end{itemize}

The implications mentioned above are sound in a technical manner, however, they are normally not used in normal human speech. Reasons for this are essentially influences from the context, e.g. salience and tolerance aspects [Herskovits86]. Mostly objects \text{in} other objects will be not regarded with respect to relations to "distant" objects.

No physical laws are obeyed, it is possible that the objects are hanging \text{in} other objects. Actually the definitions of \text{IN} are too restrictive. A containment is demanded in all three dimensions. But it is a matter of fact that \text{IN} does not usually concern each dimension equally. The condition for height is often weakened. Example "The flowers are \text{in} the vase". An appropriate alternative could be an extension of the interval definition: To allow for the interval of height the interval relation \text{si}, too.

If \text{IN} holds at most \text{IN_THE_MIDDLE_OF} may hold at the same time.

\subsection{4.3.6 NORTH_OF, SOUTH_OF, WEST_OF, EAST_OF}

\textbf{Meaning:}

The frame of reference is fixed, the earth with its absolute points north and south, the north pole and south pole. A grid system of horizontal and vertical straight lines (degrees of latitude, degrees of longitude) provides a north - south and east - west orientation.

The objects \text{LO} and \text{RO} are conceptualized as points (i.e. zero-dimensional) due to the huge distances covered by the relations.

\textbf{Definition (exemplified by NORTH_OF):}

\begin{enumerate}
  \item NORTH_OF (and all the others) is a primitive relation. We assume some auxiliary predicates \text{NLA}, \text{SLA} (northern/southern latitude) and \text{ELO}, \text{WLO} (eastern/western longitude) and functions which return the longitude (lo) or the latitude (la), respectively.

  \[
  \text{NORTH_OF} (A,B) := ( \text{NLA} (A) \text{ and } \text{SLA} (B)) \text{ or } \\
  ( \text{NLA} (A) \text{ and } \text{NLA} (B) \text{ and } \text{la} (A) > \text{la} (B)) \text{ or } \\
  (\text{SLA} (A) \text{ and } \text{SLA} (B) \text{ and } \text{la} (A) < \text{la} (B)).
  \]

  This is a very crude proposal because this relation rather expresses \text{A is more northern than} \text{B}. A further refinement would be to allow only restricted deviations with respect to the degrees of longitude. This restriction can be triggered by the grain of the scene.

  The correspondence between \text{A} and \text{B}, however, can be better expressed in \text{2}, which directs to the pictorial representation.
\end{enumerate}
NORTH_OF (A,B) ::= LOC (A, NORTH_OF_REGION (B)).

The form of the region to be generated is two-dimensional, a 90 degree sector with the corner in the zero-dimensional RO. Each corresponds to one quarter of the heaven, in the fullest sense of the word. It is doubtful if the region should be restricted. We claim that there is no restriction due to distances of objects, nor to different categories of objects.

A definition by means of intervals is not possible because of the zero-dimensional conception of both the objects (at least the RO is zero-dimensional).

Properties:

- not reflexive, not symmetric, transitive.
  Transitivity only if certain presumptions hold. The deviation with respect to the longitude may not step over permissible limits. If one object is at the north pole, no other can be north of it. RO’s at the north pole have all conceivable LO’s south of them. There is an analogy to the east-west direction. Always the shortest connection between A and B is chosen. If A and B are just opposite on the earth globe, it is a personal opinion if we say: A is as well west of and east of B, or neither the first nor the second.

- inverse relation:
  NORTH_OF (A,B) ↔ SOUTH_OF (B,A).
  WEST_OF (A,B) ↔ EAST_OF (B,A).
  For convenience, A and B are both conceived as zero-dimensional.

- Because of the different frame of reference no points of contact to other relations are given a priori.

- Subrelations:
  The only subrelations emerge from a finer partition of the 360-degree circle. Thus, e.g. NORTH_EAST_OF ⊆ NORTH_OF ∪ EAST_OF, we get appropriate subrelations.

Further properties:

NORTH-EAST_OF (A,B) → NORTH_OF (A,B) or EAST_OF (A,B),
not [REL₁ (A,B) and REL₂ (A,B)] REL₁ and REL₂ relations from {NORTH_OF, EAST_OF, NORTH_EAST_OF, SOUTH_EAST_OF, SOUTH_OF,…}, REL₁ ≠ REL₂.

The propositional definition rather provides a representation in terms of coordinates. A pictorial format would be more convenient.

In [Cao90] another approach to the representation of the quarters of heaven is proposed. The author projects the three-dimensional earth globe to a two-dimensional plane. He maintains an isomorphism between the grids on the globe and the grids in his plain projection. Thus he gets an homogeneous representation for the northern and southern hemisphere.

4.3.7 AT

Meaning:

RO and LO meet one another. We demand a "side-contact" because the upper contact is covered with ON and the lower contact with DIRECT_UNDER.

Definition:

AT (A,B) ::= (LEFT_OF (A,B) or RIGHT_OF (A,B) or IN_FRONT_OF (A,B) or BEHIND (A,B)) and CONTIGUOUS (A,B)
AT (A,B) ::= LOC (A, AT_REGION (B)). The AT-region may be the fusion of the LEFT_OF, RIGHT_OF-…regions. The condition of contiguity must be checked procedurally because only a location in the generated region does not point this out.

\[ AT (A,B) ::= \left\{ \begin{array}{l}
\text{not \((< (z_A , z_B) \text{ or } > (z_A , z_B) \text{ or } m(z_A , z_B) \text{ or } mi(z_A , z_B))\)} \text{ or } \\
\text{\((m(z_A , z_B) \text{ or } mi(z_A , z_B))\)} \text{ and } \\
\text{not \((< (x_A , x_B) \text{ or } > (x_A , x_B) \text{ or } m(x_A , x_B) \text{ or } mi(x_A , x_B))\)} \text{ and } \\
\text{not \((< (x_A , x_B) \text{ or } > (x_A , x_B) \text{ or } m(x_A , x_B) \text{ or } mi(x_A , x_B))\)} \end{array} \right. \]

This simply means there must be a contact \textit{at} \ the x- or y-dimension and in the z-dimension an "overlapping" is necessary.

Properties:
- not reflexive, symmetric, not transitive.
- inverse relation: symmetric relation, thus self-inverse.
- orthogonal to the following relations:
  - Normally orthogonal to NORTH_OF, WEST_OF,…. assumed considerable distances: NORTH_OF (A,B), AT (C,A) NORTH_OF (C,B) and so forth. At smaller distances to no other relation orthogonal.
- further property: AT (A,B) \textit{not} [UPON (A,B)].

In English there is still another interpretation possible. We have chosen the "stronger" one which demands contact, according to the German "an". But \textit{at} can also be regarded similar to NEAR_BY, which would be going into the direction from the German "bei".

4.3.8 BETWEEN

Meaning:
The RO is a complex object, i.e. it consists at least of two simple objects. The LO then is located \textit{in the middle of} the area which is stretched by the simple objects RO\textsubscript{1},…,RO\textsubscript{n}.

Definition:
1. BETWEEN is a primitive relation. Its meaning just urges a pictorial interpretation. Thus a propositional definition is hard to do. We could try to use the auxiliary predicate OVERLAPS, but this demands a region between the RO\textsubscript{i}'s. Regions, anyhow, are created by region-describing functions, defined in \(2\).
2. BETWEEN (A,B) ::= LOC (A, BETWEEN_REGION (B)). The size and form of the region depends almost fully on the complex RO B. If B consists merely consists of two objects, a kind of rhomboid must be generated between the objects. Three objects stretch a triangle, four a quadrangle and so forth. It is assumed that the objects lie in the same plane, the created region hence is two-dimensional.
3. A definition in terms of intervals is very difficult because of the potentially oblique-angled form of the BETWEEN-regions. Because we have restricted our investigations to the Cartesian system of coordinates with angles of 90°, the interval definition of BETWEEN shall be omitted.
Properties:
- not reflexive, not symmetric, transitive.

Between (A,B), BETWEEN (B,C) \(\rightarrow\) BETWEEN (A,C) holds in a restricted sense. As mentioned above, B is a complex object. If it can be regarded as a solid object, possibly at a higher level of abstraction, and C is a complex object with respect to both A and B, the rule holds.

inverse relation:
There cannot be an inverse relation because the arguments are normally not of identical types (the RO is a complex object).

It may make sense to work with degrees of BETWEEN-relations. If an LO A is completely in the BETWEEN-region created by the complex RO B, we can say "A is well between B". Otherwise "A is somewhat between B", or "A is hardly between B". This must be computed through the degree of overlapping of A and the BETWEEN-region.

BETWEEN may be conceptualized in quite another way. Similar to AROUND (see below) we could also make use of the path-concept (cf. [Habel89]).

4.3.9 AROUND

Meaning:
The RO is conceptualized as zero-dimensional. The LO around it describes a closed path. In the case it is a dynamic object, we consider its trajectory as a static object. Anyhow, the RO is enclosed in the "inner space" stretched by the LO.

Definition:
1. AROUND is a primitive relation, based on other concepts than the relations treated before, the path-concept (cf. [Habel89]). Hence we could present path-describing functions:
   AROUND (A,B) ::= IN_THE_MIDDLE_OF (B, path-area (A)). "path-area" creates a virtual object (a plain area) which then is tested to lie in the middle of B.
2. AROUND (A,B) ::= LOC (A, AROUND_REGION (B)).
   Again the region describing functions are the most convenient way to tackle the problem of defining AROUND adequately.
3. A definition in the interval setting is not possible directly, because the inner space of A will be rarely aligned to the three dimensions of the Cartesian system of coordinates (only angles of 90˚ allowed). A possibility is to investigate the inner space of the LO A if the RO lies in the middle of it. The the problem of testing AROUND is reduced to the computation of IN_THE_MIDDLE_OF.

Properties:
- not reflexive, not symmetric, transitive.

AROUND (A,B), AROUND (B,C) \(\rightarrow\) AROUND (A,C) holds if B can be conceptualized zero-dimensional as well as at least two-dimensional (a kind of a path object).

inverse relation:
we do not discuss it further, but it may be called SURROUNDED_BY:
AROUND (A,B) \(\leftrightarrow\) SURROUNDED_BY (B,A).
Further properties:

Normally there must be a kind of *near*-relation between the two involved objects, otherwise it would not be appropriate to mention that the one is *around* the other. But it is too strict to claim AROUND (A,B) \(\rightarrow\) NEAR_BY (A,B).

IN (A,B) \(\rightarrow\) AROUND (B,A) (??). At least doubtful and not used in common languages.

AROUND (A,B) \(\rightarrow\) OUTSIDE_OF (A,B).

We have to distinguish between two-dimensional and the three-dimensional case. The three-dimensional AROUND (i.e. *around* with respect to all three dimensions) is equivalent to CONTAINS and this implies the inverse relation IN. In the two-dimensional case however, this is not always the case.

4.3.10 Other Relations

A few words to the relations not treated explicitly. We have omitted symmetric relations to the spatial relations described in chapter 4.3.1 - 4.3.9. These are DIRECT_UNDER (symmetric to UPON), UNDER (OVER), BELOW (ABOVE), LEFT_OF (RIGHT_OF), and BEHIND (IN_FRONT_OF). We claim the same attributes as for the respective relation in brackets, swapped parameters assumed.

IN_THE_MIDDLE_OF resembles IN, but is often restricted to two-dimensional RO's. The spatial relation OUTSIDE_OF, however, holds in a multitude of situations and is rather inexact. For the multi-agent setting its definition therefore seems to be not very important.
5 Further Aspects of Spatial Reasoning

We have only inspected a very restricted world up to now. The relations were very simple, they could be computed by means of coordinates or intervals. But this will not be the case in most ‘real world relations’. Also we developed only intrinsic definitions of spatial relations. No point of a spectator has been regarded so far. If we want to do this, we have to add a deictic interpretation.

5.1 Handling the Context

Linguistic research is interested in investigations about the ‘deeper’ meaning of spatial relations. The use and the meaning of a relation depends on a multitude of contextual factors, for instance intrinsic properties of the objects referred in the utterance, other objects in the scene or properties of relations between objects [Lang, Carstensen90] [Pribbenow88]. A first approach to the problem of context handling is the definition of a region in which a given spatial relation shall be regarded as true. The extension and form of the region depends on the size of the involved objects, the location object (LO) and the reference object (RO). A specification of RIGHT_OF at the Coordinate Level might be:

\[
\text{RIGHT\_OF}(X, Y) \text{ iff } \begin{cases} 
  x_1(X) > x_2(Y) & \text{and} \\
  y_1(X) > y_1(Y) - \varepsilon (x_1(X) - x_2(Y), \gamma(x_1, x_2, y_1, y_2, z_1, z_2)(X, Y)) & \text{and} \\
  y_2(X) < y_2(Y) + \varepsilon (x_1(X) - x_2(Y), \gamma(x_1, x_2, y_1, y_2, z_1, z_2)(X, Y)) & \text{and} \\
  z_1(X) > z_1(Y) - \varepsilon (x_1(X) - x_2(Y), \gamma(x_1, x_2, y_1, y_2, z_1, z_2)(X, Y)) & \text{and} \\
  z_2(X) < z_2(Y) + \varepsilon (x_1(X) - x_2(Y), \gamma(x_1, x_2, y_1, y_2, z_1, z_2)(X, Y)) \end{cases}
\]

The first condition ensures that \( X \) might possibly be right of \( Y \). A projection to the dimension of breadth (x-dimension) would really conclude this relation if distances are unlimited. The other conditions limit the displacement of \( X \) in the y- and z-direction. Also the distance in x-direction is limited indirectly. The function \( \varepsilon \) calculates the absolute deviation tolerated in the depth and height-direction. Its arguments are the absolute distance between the objects \( (x_1(X) - x_2(Y)) \) and a factor which is a measure for the ratio of the size of LO and RO. This factor is delivered by the function \( \gamma \) which takes the six extreme points of both objects as its parameters \( (\gamma(x_1, x_2, y_1, y_2, z_1, z_2)(X, Y)) \).
A glance at the coordinate system makes evident that the ‘right_of-region’ is merely a truncated pyramid rotated by a 90-degree angle. The bigger the distance between the objects is, the broader the pyramid will be up to a certain threshold distance. Then the pyramid ‘closes’, no object in further distances may be called ‘right of Y’ any longer. The height of the pyramid is also biased by the ratio of sizes.

This is an first attempt, the $\varepsilon$-function may be refined with more parameters in order to refine the context more precisely (cf. [Habel,Pribbenow88]).

5.2 The Problem of Multiple Views

If there are ‘intelligent’ agents in the scene, the intrinsic approach of defining the spatial relations cannot be upheld. Up to now no point of any spectator has been regarded. But each agent really is a spectator, each agent has his own view [Hernández90] [Güsgen,Fidelak90]. Hence a deictic interpretation also would be necessary ([Pribbenow88], [Retz-Schmidt88], [Lang, Carstensen90]).

Each agent has his own viewpoint, so each agent might see other things, say, “right of Block_2”. We need a transformation between the sending agent X (the one who tells a spatial occurrence) and the receiving agent (the one who is told) in the following way. Y checks the position of X, decodes X's statement and computes a spatial range for himself where he then tries to act in the sense of the statement.

Let us discuss this problem again on the picture above. There are two agents in the scene. One agent refers to the glass at the “right of the bottle” and the other one, who stands at the opposite side, has to recognize which glass is meant. To solve the problem we might argue in several directions.

From a linguistic point of view one might say: Well, this is no question concerning space reasoning. The problem lies inherently in the unprecision of language. The sentence “The glass right of the bottle” is underdetermined. So the solution would be to use a more detailed description like “The glass right of the bottle from my point of view.” Then the procedure of detecting which
glass is meant is the one discussed above. But if we take this solution, we would have all problems from linguistics to decide when a sentence contains enough information to understand it.

Following [Lang, Carstensen90], a bottle has an intrinsic front and thus it is clear by definition what is meant by right of, namely in looking at the front of the bottle there is only one space region defined to be at the right of the bottle. This would be a good solution from the bottles point of view, but it does not seem to be natural at all.

Then there is the cognitive approach by accepting the problem as it is and by using a negotiation process if sentences are underdetermined. That is, the agents should act like humans in such cases. The listener has just to question “Do you mean right of from your point of view or from mine?”. Now this is a fine concept for the multi agent scenario, but it does not help to find a solution for the spatial representation problem of RIGHT_OF.

People from vision might argue, that the whole discussion is senseless. Since we deal with computers the best way is to say “The object at coordinates (5.2, 0.7, 2.3)" and that’s it. But this is not natural either and moreover it would be very hard for humans to following the discussions of those agents. This example also gives rise to our discussion on the two level approach from above.

The solution we have in mind provides a compromise with the ‘cognitive approach’ mentioned above using our introduced two-level space representation. We demand that the agents always refer to objects in the scene from their very own point of view as a default assumption which is stored in each agent's knowledge base. Therefore the agents know (assume) that all the others are egocentric in describing spatial relations. If a significant contradiction to this assumption occurs or an utterance is ambiguous because of other reasons the agents will negotiate to clarify the situation.

Furthermore we propose rules at the Logical Level to support the deduction of deictic interpretations. The meaning of of the prepositions hence can be relativized in order to the positions of the agents. The rule concerning the scene in the example above is:

\[
\text{BEHIND}(X,Y) \text{ relative_to } \text{SELF} \quad \text{and} \quad \text{RIGHT_OF}(Z,Y) \text{ relative_to } X \\
\rightarrow \quad \text{LEFT_OF}(Z,Y) \text{ relative_to } \text{SELF}
\]

The receiving agent thus can deduce that the speaker refers to the glass left of the bottle from his (the hearers) point of view.

The Transformation Interface translates this rule and induces an overlapping of the ‘left_of_bottle-region’ of the hearer. With all this knowledge the hearer is able to detect the exact position of the glass.

5.3 Fuzzy Relations

How can we define relations of a kind like near_by? This seems to become a very subjective interpretation, one may say NEAR_BY has a range of a certain dimension, one other may claim that this range must be expanded or restricted. A third opinion is that it is not sensible at all to define any range a priori. The fuzzyness namely emerges from the matter of fact that the range
may alter from one context to the other. This is basically the position of AI scientists today ([Habel89], [Pribbenow88], [Schirra89]).

The context is the important foundation to determine the scope of fuzzy relations. For example, consider a big castle [Habel89]. We would surely imagine a much larger area when we hear 'near by the castle...' than we would do if we hear 'near by the telephone cell at the castle ...'. Simply we may say: The bigger the RO's, the larger the 'NEAR_BY-AREA'. Other fuzzy relations have to be treated in an analogical way.

There might grow a feeling that it is rather difficult to evaluate such relations in a propositional way. How can we describe a more or less large region? How can we define transitions from a state 'of course is A near by B' to 'we really cannot say that A is near by B, but also we cannot deny it'? Therefore a pictorial representation would be more suitable. We could generate 'floating' transitions between 'NEAR_BY(A,B) holds' and 'NEAR_BY(A,B) does not hold'. This argumentation mainly comes from the AI researchers of the university of Hamburg, FRG. The project LILOG-R tries to couple pictorial and propositional representation as mentioned above several times ([Habel89], [Khenkhar89]).

[Schirra89] suggests another proceeding. He introduces 'T-values' which determine the applicability of relations (0 ≤ T ≤ 1). A T-value shall characterize their typicality. At each point of the image a relation has got certain T-values which can efficiently be computed. This field of T-values is regarded as a potential field. Schirra calls it 'Typicality Potential Field' (TyPoF). In any potential field (local) maxima can be found easily.

Despite of the discussion, we introduce the fuzzy relations NEAR_BY and DISTANT_TO, which both gain their fuzzyness by the subjective perception and conception of distances between objects, in the way we specified the almost exact relations. As mentioned above, almost each spatial relation partly contains some uncertainty. But these two are "fuzzier" than the others.

The context plays a more important role than it did for the almost-exact functions. Hence an absolute, context- and domain-independant definition like in the chapter before is nearly impossible. We will state the essence and the key ideas of such a definition, the rest must be done in combination with a concrete application.

5.3.1 NEAR_BY

Meaning:
The LO is not at the RO, but "almost at". There is a distance between them, small in relation to the size of the respective objects.

Definition:

1. NEAR_BY cannot be defined based on other relations, nor simply be defined propositional at all. We could present several auxiliary functions and predicates, but lately all efforts lead to region-describing functions.

2. NEAR_BY (A,B) := LOC (A, NEAR_BY_REGION (B)).
A refinement is possible with respect to the context of the scene. The created region may be a "narrow", a "normal", or a "wide" region around the RO (the terminology according to
The correct invocation of the appropriate function has to be triggered by contextual factors, such as distance of the objects, or size and salience.  

We introduce tolerance intervals, i.e. intervals which describe the range where we still accept a relation NEAR_BY. These intervals are, likewise the region-describing functions, constituted by the context. Thus we call the functions $T$ "interval-describing functions". The function $T_x$ returns a tolerance interval for the breadth and the functions $T_y$ and $T_z$ tolerance intervals for depth and height resp.

$$\text{NEAR\_BY} (A,B) := ( <(x_A,x_B) \text{ or } >(x_A,x_B) \text{ or } <(z_A,z_B) \text{ or } >(z_A,z_B) ) \text{ and }$$
$$\text{not } [ <(x_A,T_x(A,B)) \text{ or } >(x_A,T_x(A,B)) ] \text{ and }$$
$$\text{not } [ <(y_A,T_y(A,B)) \text{ or } >(y_A,T_y(A,B)) ] \text{ and }$$
$$\text{not } [ <(z_A,T_z(A,B)) \text{ or } >(z_A,T_z(A,B)) ].$$

The respective dimension of the RO B is always in the middle of the corresponding tolerance interval.

**Properties:**

- not reflexive, symmetric, not transitive.
- We claim NEAR\_BY a priori symmetric thought it is strictly spoken not. For example we say "the telephone cell is near by the station", but normally not "the station is near by the telephone cell". Anyhow, also the second proposition is "true", therefore is shall be allowed, too.
- Limited to small distances, NEAR\_BY can be regarded transitive. But generally the distances between the involved objects become too large to accept NEAR\_BY.
- inverse relation: NEAR\_BY is symmetric, hence self-inverse.
- To no relation orthogonal.
- Subrelations: none.
- further Properties:
  
  $$\text{NEAR\_BY} (A,B) \Rightarrow$$
  $$\text{not } [\text{IN (A,B) or AT (A,B) or ON (A,B) or DIRECT\_UNDER (A,B)}]$$

We omitted explicit parameters for the context in the definitions. This was done for the sake of clarity. To implement NEAR\_BY however, the context must be formalized. Various influences are conceivable. The most important are $^2$: distance, size, salience, and relevance (cf. [Herskovits86]). The parameters of the context are conveniently handled in context-vectors.

### 5.3.2 DISTANT\_TO

**Meaning:**
LO and RO are "far away" from each other. This implies that no other spatial relation holds, except one of the quarters of heaven like NORTH\_OF.

**Definition:**

DISTANT\_TO is a primitive spatial relation. Its fuzzyness emerges from the individual conceptions of distance from distinct humans and in different domains. Again, the propositional definition is almost impossible because of the missing anchor in appropriate auxiliary functions and predicates.

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$^1$We will not go into detail further. The problem of the context is one of the hottest issues in spatial reasoning. Herskovits, for instance, has collected a manifold of contextual factors and different situations and checked the meaning of the prepositions from this perspective [He86].

$^2$The context factors in the following refer to the objects of the investigated relation, i.e. to LO and RO.
Also the definition in terms of region-describing functions appears full of problems. A DISTANT_TO-region cannot be closed, just on the contrary, one principal feature of it is the infinity of distant to. DISTANT_TO_REGION (B) would create an infinite three-dimension object, which by no means can be explicitly be computed. A solution would be to create a region NOT_DISTANT_TO and check the non-location of the LO.  
DISTANT_TO (A,B) ::= not [LOC (A, NOT_DISTANT_TO_REGION (B))].

The problem of ② does not occur in a definition with means of intervals. Similar to NEAR_BY (see ③ there) we use tolerance intervals $T_x$ $T_y$ $T_z$.

DISTANT_TO (A,B) ::= $(<(x_A ,x_B) \text{ or } >(x_A ,x_B) \text{ or }<(z_A ,z_B) \text{ or } >(z_A ,z_B))$ and 
$(<x_A ,T_x (A,B)) \text{ or } >(x_A ,T_x (A,B)))$ and 
$(<y_A ,T_y (A,B)) \text{ or } >(y_A ,T_y (A,B)))$ and 
$(<z_A ,T_z (A,B)) \text{ or } >(z_A ,T_z (A,B)))$.

The intervals are surely much wider than the NEAR_BY-intervals. Their exact length depends on the context.

Properties:
- not reflexive, symmetric, not transitive.
- To no relation orthogonal.
- Subrelations: none.
- further Properties: none.

The spatial relation DISTANT_TO will normally not be needed in rather closed scenarios like multi-agent-worlds we have in mind. For exact, fine grained, spatial reasoning it it not necessary.

6 Conclusion and Future Work

We presented a two level representation for spatial relations and investigated the meaning of almost exact spatial relations with respect to a two level representation for intelligent agents. We tried to find most general definitions to formalize spatial knowledge independent from any domain. Procedural knowledge to check important conditions, however, is needed almost always. These investigations shall put forward the implementation of the agents‘ individual knowledge bases. The work concerning spatial reasoning is embedded in a larger project, RATMAN, which shall provide a testbed to configure a wide variety of multi-agent-systems [Bürckert,Müller90].

There are two points we have not discussed yet. The first one is the use of the relations in a linguistic context as done in [Habel89] and [Herskovits86]. The question is, if we represent the prepositons as described above, is it enough to be accepted from a linguistic point of view. The second point deals with our multi agent scenario. Usually different agents have different definitions of spatial relations, so how do they handle the prepositons when “talking” about them. And further, an important question is how to combine different spatial knowledge bases?

There is still much to do to realize the spatial knowledge base of the agents. At this time we are implementing the Coordinate level and want to gain some experience in its properties. The next step is to implement the Logical Level, i.e. to formalize and program the thoughts of this paper. The Transformation Interface between the two levels, however, seems to be the toughest point in our project. Its objective is to be considerable smart to support a sophisticated exchange of data between the Logical and the Coordinate Level. If this works our agents in the multi-agent system make a wide step towards intelligent spatial reasoning.
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