A Simple Asymmetric Herding Model to Distinguish Between Stock and Foreign Exchange Markets

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Abstract

Drawing on previous work of one of the authors, the paper takes an asymmetric variant of Kirman's ant model and combines it with an elementary asset pricing mechanism. The closed-form solution of the equilibrium probability distribution allows the specification of a tractable likelihood function for daily returns, which is then employed to estimate the model's behavioural parameters for a large pool of Japanese stocks. By way of Monte Carlo simulations it is found that most of these markets belong to the same class, which is characterized by a dominance of the stylized noise traders. In contrast, the model assigns a number of major foreign exchange markets to a different class, where on average the majority of agents follows the fundamentalist trading rule. Implications for the tail index are also worked out.

JEL classification: C 15, C 52, C 63, D 53.

Keywords: Speculative markets, herding dynamics, probability density estimation, tail index.

Contents

1 Introduction

Financial markets are known to be well characterized by a number of stylized facts concerning the conditional and unconditional properties of their time series. Most prominently, these are the excess volatility of prices when compared to the underlying fundamentals, volatility clustering of returns where periods of quiescence and turbulence tend to group together, and the non-Gaussian leptokurtic shape of the distribution of returns (the "fat tails"). It is now furthermore widely agreed that these robust findings are hard to reconcile with the Efficient Market Hypothesis (EMH) and the one-to-one relationship between price changes and new information that it implies, at least if the "relevant" information is conceived as an assortment of non-correlated economical, political and perhaps even meteorological news.

On the other hand, a growing body of literature has developed in recent times that is built on heterogeneous interacting agents with limited rationality, which has proved to provide a fruitful alternative paradigm to account for the stylized facts. However, while this approach allows for a behaviour of its agents that in itself and its implications for the price dynamics is far more satisfactory than the EMH, it goes at the price of a major drawback. Typically, the greater complexity arising in the agent-based models precludes an analytical closed-form solution which, specifically, could be used to subject them to direct econometric tests. Instead, numerical values for the structural parameters are inferred from Monte Carlo simulations and their comparison with several summary statistics of the empirical data. These methods are (still) mainly informal and rather mixed, so that the empirical merits of different models are difficult to assess.

As far as we know, there are presently only few models that have been tested empirically. One class puts up a behavioural interpretation of econometric time series models within a simple chartist–fundamentalist framework. Vigfusson (1997) introduces a Markov switching mechanism as an approximation to the Frankel–Froot (1986) model, and Westerhoff and Reitz (2003, 2005) use a smooth transition autoregressive (STAR) model in order to describe the switching between a fundamentalist and chartist regime. This work is so far somewhat sketchy since a possible connection of these results to the stylized facts has not yet been explored. In another contribution, Boswijk et al. (2006) compose an elementary variant along the lines of Brock and Hommes of adaptive beliefs on the stock market. Re-

formulating the model in such a way that it can be estimated with single-equation nonlinear least-squares (on annual data, though), they find a significant role for heterogeneity in the strategies and a substantial variation of the average sentiment over time.

The stylized facts are addressed more directly in a different class of models. Gilli and Winker (2003) are concerned with Kirman's (1991, 1993) seminal herding model and estimate its two key parameters by an indirect simulated method of moments approach, where they seek to match the empirical kurtosis and the first-order autocorrelation of the squared daily returns of DEM/USD data. The parameter estimates imply that the unconditional frequency distribution of the two groups of traders is bimodal, which is an interesting structural result. Since the paper largely focusses on the intricacies of tuning the optimization algorithm to the specific problem at hand, it might, however, also be read as evidence against the special objective function here employed, which does not appear to be well-behaved, and as a call for a better tractable estimation approach.

Alfarano et al. (2005a,b, 2007) have designed an extended variant of a Kirman-type model that allows for asymmetric transition probabilities between the two categories of traders (referred to as the ALW model in the following). In addition to the enhanced flexibility of the modelling framework, the great virtue of this approach is that it permits the parameters to be estimated by maximum likelihood, so that the results can be readily reproduced with standard software tools. Application of this method becomes here possible by a skillful but nevertheless extremely simple specification of the flow of orders that drive the asset price. As this demand is formulated by the two groups of fundamentalists and noise traders, the prices are closely linked to the herding mechanism, which in turn governs the endogenous evolution of the population shares. As a result, not only is it possible to derive closed-form expressions for many properties of the herding dynamics, but they also carry over to the statistical distributions of prices and returns. In particular, the model can be explicitly solved for the unconditional distribution of returns, from which subsequently a likelihood function can be set up for estimation. Furthermore, the estimated coefficients provide us with a rough indication of whether a market tends to be dominated by fundamentalists or noise traders.

As it turns out, the basic structure of the ALW model could be cast in terms of a stochastic volatility framework. These models can reproduce the key stylized facts of financial time series, but their formulation is mainly driven by mathematical convenience

and less by economic intuition. The ALW model, by contrast, develops a meaningful *behavioural framework* for the decisions of financial investors (though still a very frugal one). Given its econometric tractability, estimations of this model might thus eventually be used as a simple diagnostic tool to distinguish different classes of speculative markets, by identifying different investment attitudes prevailing there.

A first attempt in this direction has already been made in previous work (Alfarano et al., 2005a,b), where the model has been successfully tested with daily returns from a number of financial markets. It has there been noticed that foreign exchange and stock markets might be characterized by the preponderance of different types of traders, such that fundamentalists have a greater weight on foreign exchange markets and noise traders on stock markets. So far, however, this interesting hypothesis was supported by a few casual observations only. In the present paper we therefore want to continue this empirical work and put it on a broader basis.

To this end, we estimate the two central parameters of the model for 982 stocks from the Tokyo Stock Exchange (see the appendix for a description of the data). The results thereby obtained have to be discussed with respect to two complementary criteria: stability and sensitivity. Stability in this context means that similar markets should yield similar estimates. Accordingly, since the composition of traders and their speculative strategies can be expected to be quite comparable across many of these Japanese markets, the parameter estimates should not be scattered over the entire parameter plane. On the other hand and in conformity with our hypothesis, the parameter estimates should be sensitive enough to distinguish these stock markets from the main foreign exchange markets.

It may also be emphasized that although foreign exchange and stock markets are held to be different in many informal discussions, a rigorous and straightforward characterization still seems to be lacking. Despite its oversimplification, the results from applying the present model to the daily data can be claimed to provide a useful step for such a distinction.

The remainder of the paper is organized as follows. Section 2 introduces the model, that is, the probabilistic herding mechanism, the price determination of the asset, and the concept of the equilibrium probability density function of returns. The section concludes with the presentation of the maximum likelihood approach derived from the equilibrium distribution. Estimations themselves are carried out in Section 3, where we begin with the estimations of the 982 Japanese stocks just mentioned. On the basis of extensive bootstrap

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re-estimations we then argue that the great majority of the stocks can be considered to belong to the same class of markets. By estimating a number of major foreign exchange markets and similar bootstrap experiments, it is subsequently shown that these can be assigned to a different type of markets. In Section 4, we resolve a possible contradiction between the model's tail index of the equilibrium distribution and the markedly distinct empirical Hill estimators that we find. Section 5 concludes, and an appendix informs about the data.

2 The model and its solution

2.1 The herding mechanism

In the following a probabilistic herding process is formulated along the lines of Kirman's (1993) famous ant model. Consider a market that is populated by a fixed number of agents, N. Each agent is either a noise trader (which we also refer to a as type 1) or a fundamentalist (type 2). Over time agents may switch from one strategy to the other with certain probabilities, which take account of two principles: autonomous switches and switches arising from social interaction among the agents (but no feedback from market prices). The analysis is concerned with the evolution of the population composition over time. To begin with the total number n of noise traders at a point in time, and $N-n$ the number of fundamentalists, eq. (1) specifies the probabilities ρ that within a microscopic time interval $\Delta\tau$ the switching of one of the agents increases or decreases the number of noise traders by one:

$$
\begin{aligned}
\rho(n+1, t+\Delta\tau|n, t) &= (N-n)\left[a_1 + n\,b\right]\Delta\tau \\
\rho(n-1, t+\Delta\tau|n, t) &= n\left[a_2 + (N-n)\,b\right]\Delta\tau
\end{aligned}
\tag{1}
$$

where a_1, a_2, b are positive and constant coefficients. The interval $\Delta\tau$ is assumed so small that no more than one switching agent needs to be considered, and $\rho(n, t+\Delta\tau|n, t) = 1 - \rho(n+1, \ldots) - \rho(n-1, \ldots)$ is positive. Clearly, the two parameters a_1 and a_2 represent the autonomous component, the agents' idiosyncratic propensities to change their attitude, which, in contrast to the original Kirman model, need not be equally strong in both directions. The parameter b captures a herding effect. It may here be noted that the probabilities are supposed to increase with the absolute number of agents to switch to (and not with

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the fractions n/N and $(N-n)/N$. In the course of the mathematical analysis this will turn out to be a momentous specification detail, namely, the stochasticity of the herding dynamics is maintained even if the population size N tends to infinity (cf. Alfarano et al., 2007, Section 5).

The aggregate dynamics of the Markov chain constituted by the transition probabilities (1) can be analyzed by means of the so-called Master equation. It describes the time evolution of the probability $P(n,t)$ to find n agents of the noise trader type at time t, given the starting distribution $P(n,0)$. In the limit of a very large number of agents $N \gg 1$, one can work with the Fokker-Planck equation as a second-order approximation of the Master equation, which governs the evolution of the probability density $p(z,t)$ of the fraction z of noise traders over time, $z = n/N$. The details of this mathematical treatment in continuous time, where $\Delta\tau \to 0$, have been presented in Alfarano et al. (2005a, 2007).

The analysis makes it possible to derive closed-form solutions for a wide range of conditional and unconditional properties of the herding dynamics; in particular, the equilibrium distribution and the autocorrelation functions of the variable z. For our present purpose it is worth making the unconditional equilibrium density $p_z^e(\cdot)$ explicit, which depends only on the two ratios $\varepsilon_1 = a_1/b$ and $\varepsilon_2 = a_2/b$:

$$p_z^e(z) \;=\; \frac{1}{B(\varepsilon_1, \varepsilon_2)} \; z^{\varepsilon_1 - 1} \, (1-z)^{\varepsilon_2 - 1} \tag{2}$$

($B(\varepsilon_1, \varepsilon_2)$ being the beta function). The beta distribution in (2) is one of the most versatile distributions in probability theory. Therefore, despite the dependence on just two structural parameters, the unconditional distribution (2) is extremely flexible in describing different scenarios: unimodal or bimodal distributions, or monotonically increasing or decreasing distributions, depending on the relative magnitude of the two parameters. We may, however, anticipate that all our estimates of ε_1 and ε_2 will imply a unimodal distribution of the share of noise traders (in contrast to the estimates by Gilli and Winker, 2003, mentioned in the Introduction).

A sample path of the population share z_t in discrete time can be computed by means of the so-called Langevin equation (which is intimately related to the Fokker-Planck equation). Letting Δt be a small but fixed (macroscopic) time interval, it describes the changes in z_t

by the following stochastic difference equation,[1]

$$
\begin{aligned}
z_{t+\Delta t} &= z_t - \Delta t\,(a_1 + a_2)\,(z_t - \bar z) + \sqrt{\Delta t\, 2b\,(1 - z_t)\, z_t}\;\zeta_t \\
\bar z &= a_1\,/\,(a_1 + a_2) \qquad\qquad \zeta_t \sim N(0,1)
\end{aligned}
\tag{3}
$$

Our simulations of the full model below will indeed use this macroscopic adjustment equation for the herding dynamics. As $\bar z$ will turn out to be the average value of the fraction of noise traders, the stochastic process (3) is seen to exhibit a linear mean reversion toward this reference value. The tendency is spoiled by the random influences from the last term of the equation, but notice that these forces will become progressively weaker as z_t approaches one of the end-points of the unit interval.[2]

2.2   The price dynamics

Turning to the financial market, it is now time to specify the demand of fundamentalists and noise traders that is driving the asset price. Fundamentalists have an unanimous notion of the fundamental value of the asset. They expect that prices will return to it in a reasonable span of time and so buy (sell) when the asset is undervalued (overvalued), where the excess demand of a single fundamentalist agent is proportional with factor α_f to the percentage deviations. If p_t and $p_{f,t}$ denote the logs of the actual price and fundamental value, respectively, total excess demand on the part of fundamentalists at time t amounts to,

$$
\mathrm{ED}_{f,t} = -(N - n_t)\,\alpha_f\,(p_t - p_{f,t})
\tag{4}
$$

The excess demand of a representative single noise trader is proportional, with factor α_n, to a random variable λ_t, which represents the average "mood" of all noise traders. Their total excess demand is thus

$$
\mathrm{ED}_{n,t} = n_t\,\alpha_n\,\lambda_t
\tag{5}
$$

Individual noise traders might be quite inhomogeneous and follow different technical trading rules or irrational fads, hypes or other misperceptions. We abstain from any specific details

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[1]Thus, Δt is at a different conceptual level from $\Delta\tau$ in eq. (1), which must converge to zero as the population becomes arbitrarily large.

[2]Since a non-vanishing probability for the variable z_t to leave the unit interval nevertheless remains, we impose a condition of reflecting boundaries for these (rare) cases.

in this respect and simply assume that their average demand follows a random walk,

$$\eta_t := (\lambda_t - \lambda_{t-\Delta t}) \quad \sim \quad \text{UID}(-1, +1) \tag{6}$$

where UID means that the η_t are uniformly, independently and identically distributed on the interval $[-1, +1]$. The main reason why we have not not employed a Gaussian or Student's t distribution, say, is that the uniform noise in (6) will make the derivation of a closed-form solution for the distribution of returns a feasible task.

Regarding the determination of prices by these demands, we employ a Walrasian scenario and assume continuous market clearing,

$$\text{ED}_{f,t} \; + \; \text{ED}_{n,t} \;\; = \;\; 0 \tag{7}$$

for all t. Taking $z_t = n_t/N$ into account and defining $\rho := \alpha_n/\alpha_f$, the (log) price resulting from (4), (5), (7) is

$$p_t \;\; = \;\; p_{f,t} \; + \; \rho \, \frac{z_t}{1 - z_t} \, \lambda_t \tag{8}$$

For the corresponding rate of return over the interval Δt, $r_t = (p_t - p_{t-\Delta t})/\Delta t$, we obtain

$$r_t \;\; = \;\; r_{f,t} \; + \; \rho \left[\frac{z_t}{1 - z_t} \lambda_t \; - \; \frac{z_{t-\Delta t}}{1 - z_{t-\Delta t}} \lambda_{t-\Delta t} \right] \Big/ \Delta t \tag{9}$$

where $r_{f,t}$ represents the fundamental returns. We are from now on concerned with a trading period of a day and with daily returns; so put $\Delta t = 1$. Empirically, however, returns are systematically correlated only over intervals shorter than a trading day. This property can be readily built in by assuming that the mood λ_t of the noise traders changes much faster than the total composition z_t of agents. Such a separation of time scales is in physics called an adiabatic approximation. Here it allows us to neglect the lag in the population share in (9). Once this simplification is accepted, it is furthermore even more justifiable to omit the fundamental returns $r_{f,t}$, which on a daily basis are typically so small that they will be dominated by the other term.[3] Using (6), the daily returns are thus determined by

$$r_t \;\; = \;\; \rho \, \frac{z_t}{1 - z_t} \, \eta_t \tag{10}$$

---

[3]This includes the coefficient ρ, which will be specified as a normalization factor below. We have carried out a number of Monte Carlo experiments with a small constant r_f to verify the indeed negligible effect on the estimation results.

In Alfarano et al. (2005a) and Alfarano (2006) it has been shown theoretically as well numerically that this equation can generate the key stylized facts of financial returns, namely, fat tails, power-law decay of large returns, absence of memory in the returns as measured by their autocorrelations, and the presence of positive correlations for squared and absolute returns.

It should also be pointed out that (10) exhibits a so-called stochastic volatility structure, wich is given by the product of an iid noise and a stochastic variable σ_t that describes the time-dependence observed in the empirical data. In these models, σ_t follows a largely atheoretical stochastic process typically chosen for analytical tractability, and it is this process which is ultimately responsible for the good match of the main stylized facts such as, in particular, the power-law decay for large returns. By contrast, in eq. (10) the volatility factor $\sigma_t = \rho \frac{z_t}{1-z_t}$ derives from a theoretical herding model. The fact that our returns display similarly attractive properties to those from the stochastic volatility models can, therefore, be primarily ascribed to the stochastic herding dynamics. Moreover, in the original specification of the herding component one would hardly identify a strong potential to generate the ubiquitous empirical findings. In other words, the present model does not suffer from the usual "you have to start with GARCH to obtain GARCH" effect of many other models in the literature (see Pagan, 1996, p. 92).

2.3 The estimation approach

It has already been sketched how the stochastic process governing the noise trader share z_t can be analyzed. Since only the impact of z_t on prices and returns has been considered but no feedback in the opposite direction, the properties of the herding dynamics essentially carry over to the return equation (10), although the ratio $z_t/(1-z_t)$ and the multiplicative noise term η_t still require some analytical effort. Most important for us, one can also derive a closed-form representation for the equilibrium probability distribution of the returns (Alfarano et al., 2005a). Just as the distribution of z in (2), it is only dependent on the two ratios

$$\varepsilon_1 := a_1 \, / \, b \qquad \text{and} \qquad \varepsilon_2 := a_2 \, / \, b \tag{11}$$

and not on the size of a_1, a_2, b. Evaluated at a value r, the equilibrium probability density function of returns is given by the expression

$$p^e(r) = p^e(r; \varepsilon_1, \varepsilon_2) = \frac{\varepsilon_2}{2\rho(\varepsilon_1 - 1)} \left[1 - \beta\left(\frac{|r|}{|r| + \rho}; \varepsilon_1 - 1, \varepsilon_2 + 1\right) \right] \tag{12}$$

where β is the incomplete beta function. To have finite first and second moments, the parameter ε_2 must be larger than 2, while $\varepsilon_1 > 1$ implies a unimodal distribution (Alfarano, 2006, p. 117). These conditions will be understood in the following, and none of our estimations will even come close to these values. Nevertheless, there still remains a great degree of flexibility in describing the properties of financial returns. For example, a suitable choice of ε_2 can virtually achieve any level of excess kurtosis and fatness of the tail.

The coefficient ρ has not been included in the arguments of $p^e(\cdot)$ since before carrying out the estimation, the returns will be rescaled such that the mean of the absolute returns is unity. In the equilibrium distribution $p^e(\cdot)$ the corresponding normalization $E(|r|) = 1$ is brought about by putting

$$\rho = 2(\varepsilon_2 - 1)/\varepsilon_1 \tag{13}$$

(Alfarano et al., 2005a, p. 32). The analytically known equilibrium density of returns can, in particular, be used for an estimation approach *via* maximum likelihood. To this end it is straightforward to set up the following likelihood function (in logs, of course) for an empirical series $\{r_t^{emp}\}_{t=1}^T$ of returns,[4]

$$\ell(\varepsilon_1, \varepsilon_2; r_t^{emp}) := \sum_{t=1}^T \ln\left[p^e(r_t^{emp}; \varepsilon_1, \varepsilon_2)\right] \tag{14}$$

It should not go unnoticed, however, that the likelihood in (14) is an approximation of the 'true' likelihood. It pretends, in fact, that the realizations of the returns in (10), which are conditional on the noise trader fractions z_t, are independent and identically distributed, according to the unconditional distribution (12). The advantage of the approximation is the simplicity of its implementation and the reduced computational burden. The method also gives asymptotically consistent estimates if the sample size T is large enough and the sampling frequency is sufficiently small (cf. Genon-Catalot, 1999).

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[4]The notation r_t^{emp} on the left-hand side of (14) avoids the curly brackets and is to mean the entire series $\{r_t^{emp}\}$.

3 A characterization of speculative markets

The agent-based nature of the model described above gives us the possibility to distinguish between speculative markets being dominated by fundamentalists or by noise traders, respectively. In fact, as it was remarked on eq. (3), the expression $\bar{z} = a_1/(a_1+a_2)$, which by (11) equals $\varepsilon_1/(\varepsilon_1+\varepsilon_2)$, is the average share of noise traders in the population, so that estimating the two parameters $\varepsilon_1, \varepsilon_2$ also easily allows us to characterize a market in this respect. From a few estimations in previous work some evidence has been obtained that stock markets are dominated by noise traders (owing to $\varepsilon_1 > \varepsilon_2$) and foreign exchange markets by fundamentalists (since $\varepsilon_1 < \varepsilon_2$; cf. Alfarano et al., 2005b, p. 253). In the following we want to investigate the robustness of this result on a broader empirical and methodological basis, which, in particular, will require us to establish appropriate confidence regions to the point estimates of the parameter pairs ε_1 and ε_2.

3.1 The case of the Tokyo Stock Exchange

Our investigations are primarily based on a great number of markets for corporate shares, for which we can utilize the daily prices of 982 stocks traded at the Tokyo Stock Exchange over the years $1975 - 2001$. The availability of this large data set enables us to subject the model to an extensive test. Thus, we calculate the daily returns, normalize them and maximize the likelihood function (14) for each of these series. The resulting estimates of ε_1 and ε_2 are plotted as dots in the parameter plane of Figure 1.

The first impression from the scatter plot is that the estimated coefficients cover a wide range, especially if it is added that the diagram does not show all of the estimates; almost 9.0% of them yield a value of ε_1 above the diagram's upper boundary of 50.[5] Assuming that the empirical time series are different realizations of the common generating mechanism of the herding model, the variability of the estimates in Figure 1 can be attributed to two different sources: finite-sample properties of the estimation procedure, and a variability of

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[5]However, the large estimates of ε_1 should not be overrated since their effect on the shape of the equilibrium density p^e is disproportionally low. As a matter of fact, if we plot the density function $p^e(\cdot\,; \varepsilon_1, \varepsilon_2)$ of a pair $\varepsilon_1, \varepsilon_2$ with $\varepsilon_1 = 15$, say, then for all $\varepsilon_1 > 15$ a range of values ε_2 can be found such that the corresponding density functions are practically indiscernible from the original one. Alfarano et al. (2006) contains a formal analysis of the asymptotic version of the model as $\varepsilon_1 \gg 1$, which can make this visual impression more precise.
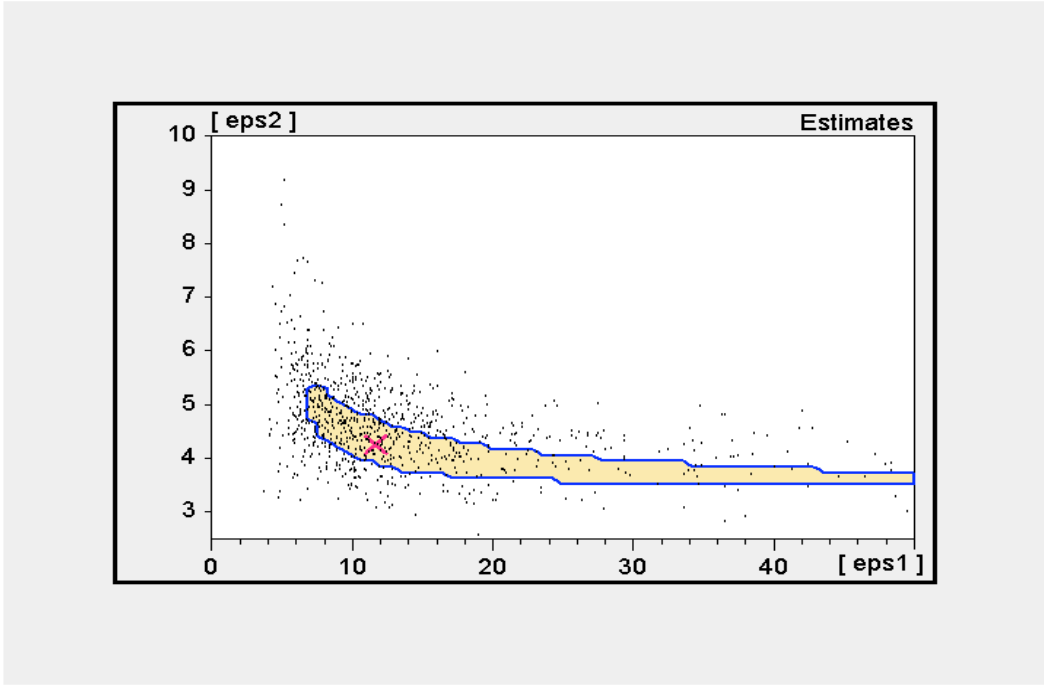
Figure 1: Estimates $\hat{\varepsilon}_1, \hat{\varepsilon}_2$ for 982 Japanese stocks at TSE.

Note: The diagonal (red) cross represents the median values $\varepsilon_1^m = 11.6$, $\varepsilon_2^m = 4.5$ of the estimates. The shaded area is the confidence region from the LR test statistic of eq. (15).

the 'true' parameters across the different shares. The following investigations can be seen as an attempt at telling the two sources apart.

For a first quantitative assessment of the variability beyond the naked eye, we can set up a confidence region based on the likelihood ratio test. To this end a reference series $\{r_t^{ref}\}$ is needed. We choose it from the 982 empirical series such that its estimated coefficients nearly coincide with the median values $\varepsilon_1^m = 11.6$ and $\varepsilon_2^m = 4.5$. Given the strong asymmetric distribution of the parameters that will be demonstrated below, the median is certainly a more appropriate candidate for a typical value than the mean. In addition, the sample size $T = 5,728$ of the reference series is close to the median sample size of $5,768$ data points.

In carrying out the likelihood ratio test LR (e.g., Davidson and MacKinnon, pp. 420f), we let $\{r_t^{ref}\}$ enter the log-likelihood function and first compute its value for the median values $\varepsilon_1^m, \varepsilon_2^m$. Another pair $\varepsilon_1, \varepsilon_2$ is then not statistically different from $\varepsilon_1^m, \varepsilon_2^m$ at a 5

percent significance level if we obtain

$$\text{LR}(\varepsilon_1, \varepsilon_2) := 2 \left[\ell(\varepsilon_1^m, \varepsilon_2^m; r_t^{ref}) - \ell(\varepsilon_1, \varepsilon_2; r_t^{ref}) \right] \leq \chi^2_{2;0.95} = 5.99 \qquad (15)$$

where the chi-square critical value is based on two degrees of freedom, too. All parameter pairs satisfying this inequality are contained in the shaded area of Figure 1. Interestingly, its extension to the right makes clear that even estimates with values of ε_1 as high as 50 and more may not be significantly different from estimates in a narrow vicinity of $\varepsilon_1^m, \varepsilon_2^m$.

So far, it can be said that the shaded area has identified a class of stocks at TSE, for which the median ε_1^m and ε_2^m cannot be rejected as the common underlying pair of parameters. While it captures a considerable proportion of stocks, this class is obviously not predominant. More precisely, only 321 of the estimated pairs happen to fall into the area, which is slightly less than one-third of the 982 stocks under consideration.

In evaluating this result it must be taken into account that the LR criterion (15) is based on asymptotic theory, which means it is valid if the underlying sample size T is sufficiently large. Although an order of magnitude of 5,000 observations is a comfortable amount of data for many aspects of time series estimations, our approach, which is based on an entire unconditional density function (including its tail properties), may require $T \gg 5,000$ to permit the application of asymptotic theory.

To assess whether the asymptotic LR test provides a good approximation for the present data, the small sample properties of the estimation approach have to be studied in isolation. This can be done by means of a bootstrap method, by which we generate the data ourselves. To this end we fix the model parameters at the values $\varepsilon_1^s = 11.0$ and $\varepsilon_2^s = 4.5$, which are essentially the median values of the empirical estimates, and get a series of returns by simulating eqs (3), (6), (10) over $T = 5,768$ days.[6] Subsequently, ε_1 and ε_2 are re-estimated by maximum likelihood from this sample. Using different random number sequences of η_t, we repeat this procedure 5,000 times and so obtain 5,000 bootstrap estimates $\hat{\varepsilon}_1^b$, $\hat{\varepsilon}_2^b$ ($b = 1, \ldots, 5000$). The bootstrap estimates are therefore, by construction, not only generated by the same mechanism but they also share the same underlying parameter values.

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[6]Regarding the parameter b in (3) we employ $b = 0.0025$, though the results are not sensitive to this choice. We have checked that it reproduces an empirically reasonable degree of long memory in the autocorrelation functions of absolute and squared returns; cf. Alfarano et al. (2005a, p. 37).
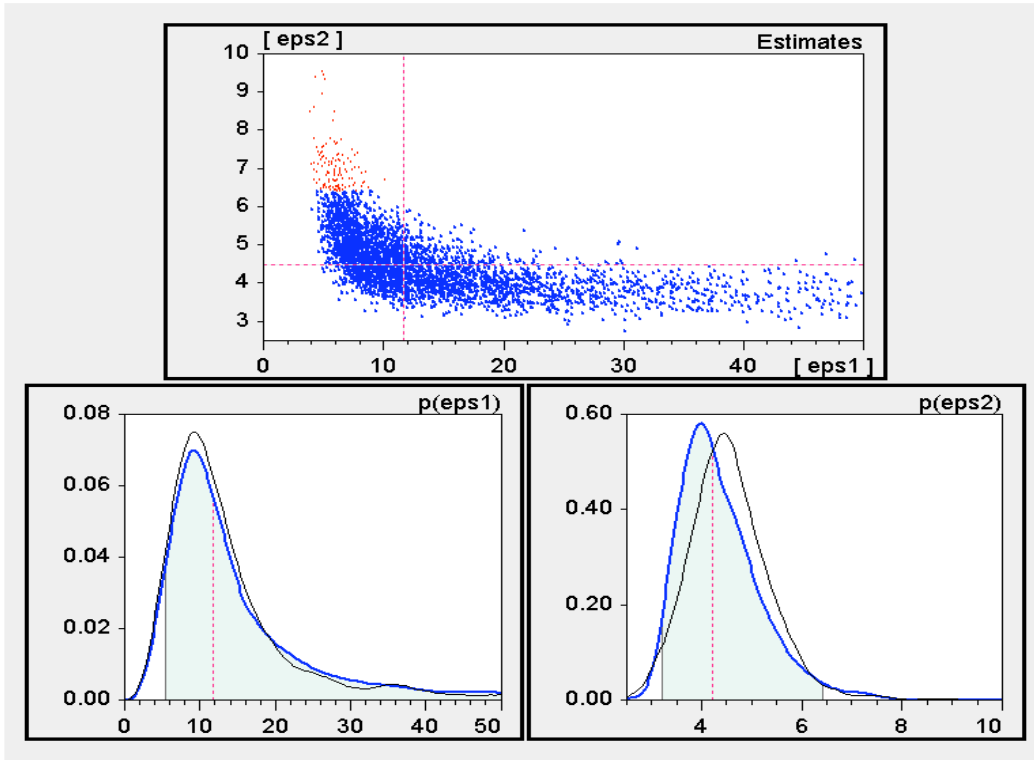
Figure 2: Scatter plot and marginal distributions from 5,000 bootstrap estimates.

Note: The thinner (red) dots in the top panel comprise 2.5% of the estimates. Bold lines (blue) in the lower panels depict the frequency distributions from the bootstrap estimations, solid lines (black) those from the empirical estimations. The dotted lines mark the median values.

The top panel in Figure 2 is analogous to Figure 1 and shows the estimated pairs as a scatter plot in the $(\varepsilon_1, \varepsilon_2)$-plane. The dotted horizontal and vertical lines indicate the median values of these estimates, which with 12.1 for ε_1 and 4.2 for ε_2 differ slightly from the true values. The dots of the pairs $\hat{\varepsilon}_1^b$, $\hat{\varepsilon}_2^b$ cover a similar range to the empirical estimates. In particular, the lower limits of the estimates seem to be quite alike.

Nevertheless, before turning to a more direct comparison of the joint estimates of ε_1 and ε_2, let us consider their marginal distributions. This is done in the lower two panels of Figure 2.[7] As was to be expected, both density functions, which are plotted as the bold (blue) lines, are not symmetrical but skewed to the right. The shaded area defines the 95% confidence interval of the parameters, that is, the white areas under the density function

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[7]The graphs of the functions are obtained from kernel density estimations of the frequency distributions. Specifically, the Epanechnikov kernel was used (cf. Davidson and MacKinnon, 2004, pp. 678ff).

to the left and to the right represent 2.5% each of the total area under the graph of that function. The fact that in the lower-left panel there is no such white area to the right demonstrates that also in the bootstrap experiments more than 2.5% of the $\hat{\varepsilon}_1^b$-estimates are larger than 50 (in fact, 13.1% of the $\hat{\varepsilon}_1^b$ exceed 50).

The most remarkable information in the lower two panels derives from a comparison of the frequency distributions based on the bootstrap experiments with those based on the 982 empirical estimations. The latter being drawn as the thin (black) solid line, it is seen that the two marginal distributions for ε_1 are almost identical, and also those for ε_2 are fairly close. Since the 5,000 bootstrap estimates belong to the same class by construction, the high similarity of the distributions suggests that most of the empirical estimates could be assigned to one class as well.

The marginal distribution can also be computed for the implied mean value $\bar{z}$ of noise traders, which recalling eqs (3) and (11) is given by $\bar{z} = \varepsilon_1 / (\varepsilon_1 + \varepsilon_2)$. Doing this we find a 95% confidence band for $\bar{z}$ (in percentage points) of

$$48.2 \;=\; 74.2 - 26.0 \;\leq\; \bar{z} \;\leq\; 74.2 + 23.5 \;=\; 97.7 \tag{16}$$

where 74.2% is the median $\bar{z}$ of the distribution. As the distribution of $\bar{z}$ implied by the empirical estimates is not much different from the bootstrap distribution, either, eq. (16) asserts that the empirical markets for the single shares can be characterized as being strongly influenced by the noise traders. Though certainly not in every episode of the price dynamics, but on average, fundamentalists tend to be in a minority position.

The high similarity of the empirical and bootstrapped marginal distributions as well as the qualitative agreement between the two scatter plots in Figures 1 and 2 indicate that the variability of the empirical estimates are to a large extent due to the finite sample properties of the estimation procedure, rather than to a variability of the underlying 'true' parameters. Thus, the asymptotic confidence region from eq. (15), i.e. the shaded area in Figure 1, appears to underestimate the sampling variability of the empirical estimates. This means that the 'class' of essentially equivalent pairs $\varepsilon_1, \varepsilon_2$ that this area defines is too narrow. It should better be specified on the basis of the scatter plot from the bootstrap experiments in Figure 2.

3.2 Constructing a two-dimensional confidence region

Our task is therefore to construct a whole confidence region from the cloud of points $\hat{\varepsilon}_1^b, \hat{\varepsilon}_2^b$ in Figure 2. Based on the concept of eq. (15), we can for that purpose design a 'modified' LR test. However, two aspects have now to be taken into account. First, in order to establish an appropriate critical value of chi-square, we gradually increase the right-hand side of the LR inequality until it is satisfied by exactly 95% of the bootstrap estimates. The resulting critical value is denoted as $\tilde{\chi}^2_{2;0.95}$.

The second issue concerns the fact that this LR test is dependent on a single artificial series $\{r_t^b\}$ against which the likelihood function is evaluated. Clearly, such a series must not be arbitrary. The problem is solved by choosing from the 5,000 random sequences underlying the simulations a sequence $b^\star$ that gives rise to a "consistent" return series $\{r_t^{b^\star}\}$, by which we mean that its estimation yields precisely the parameters from which it was generated: $(\hat{\varepsilon}_1^{b^\star}, \hat{\varepsilon}_2^{b^\star}) = (\varepsilon_1^s, \varepsilon_2^s) = (11.0, 4.5)$. Several series with this property exist (actually six out of the 5,000, with a precision of at least two digits after the decimal point). Employing one of them, a critical chi-square value of 34.4 is found and the LR criterion (15) becomes

$$
\begin{aligned}
\text{LR}(\varepsilon_1, \varepsilon_2) &:= 2\left[\ell(\hat{\varepsilon}_1^{b^\star}, \hat{\varepsilon}_2^{b^\star}; r_t^{b^\star}) - \ell(\varepsilon_1, \varepsilon_2; r_t^{b^\star})\right] \\
&= 2\left[\ell(\varepsilon_1^s, \varepsilon_2^s; r_t^{b^\star}) - \ell(\varepsilon_1, \varepsilon_2; r_t^{b^\star})\right] \leq \tilde{\chi}^2_{2;0.95} = 34.4
\end{aligned}
\tag{17}
$$

The resulting set is the shaded area in Figure 3, which for graphical reasons is there truncated at $\varepsilon_1 = 50$.[8] This area can be safely relied on as the most instructive part of an appropriate 95% confidence region for our model with parameters $(\varepsilon_1, \varepsilon_2) = (\varepsilon_1^s, e_2^s) = (11.0, 4.5)$, and finite sample size $T = 5,768$.

The original motive for constructing this set was to have a criterion that can decide which of the single empirical estimates belong to a common class. Counting the number of estimates within that class can give us an indication of how homogeneous the stocks traded at TSE might be. To this end the dots in Figure 3 reproduce the empirical estimates $\hat{\varepsilon}_1, \hat{\varepsilon}_2$ from Figure 1. Clearly, we can observe a lower number of these estimates outside the present confidence region; the precise percentage is 11.6% (in contrast to 67.3% in

---

[8]To check the robustness of this region, we carried out the same procedure for the other five consistent series $\{r_t^{b^\star}\}$. Their critical chi-square values are fairly similar to the value in (17) (ranging between 34.0 and 34.4), and the six areas themselves are only marginally different.
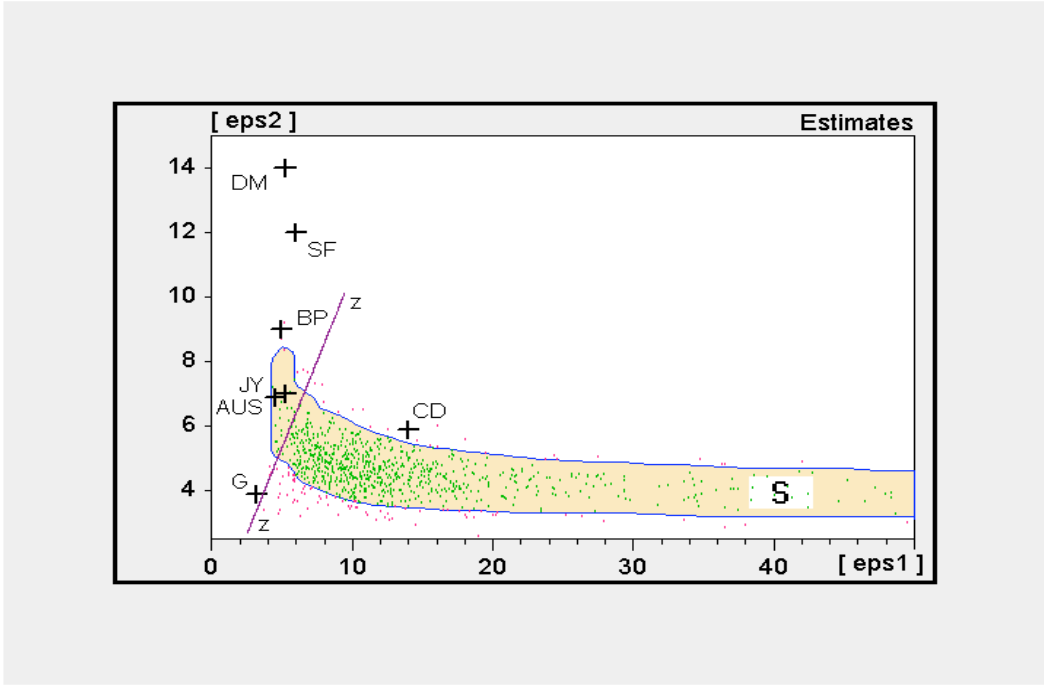
Figure 3: Class S of parameter pairs $\varepsilon_1, \varepsilon_2$ (the shaded area).

Note: The dots are the empirical estimates $\hat{\varepsilon}_1, \hat{\varepsilon}_2$ from Figure 1. 'G', 'DM', 'SF', etc., are the model's estimates of the gold and the foreign exchange markets; cf. Table 1. Points above the zz-line imply $\bar{z} < 48.2$; cf. eq. (16).

Figure 1). So we can summarize that the area in Figure 3 represents almost 90 percent of the Japanese stocks. That is, almost 90 percent of the daily return series at the Tokyo Stock Exchange are compatible with our model's data generation process under constant parameters $(\varepsilon_1, \varepsilon_2) = (\varepsilon_1^s, e_2^s) = (11.0, 4.5)$. We refer to this region in Figure 3 as area, region or class S, where S is meant to be mnemonic of 'stocks'.

The higher percentage of 11.6% of the empirical estimates outside class S, as compared to the benchmark of 5%, is a signal that there might be an intrinsic variability of the parameters across the time series, which is not directly linked to the finite sample effect. Nevertheless, the bootstrap experiment points out that the large range of estimates may not be exclusively, but *mainly* or even predominantly, due to the finite sample size of the empirical time series. We may therefore claim that the parameter pair (ε_1^s, e_2^s) is a good estimate characterizing most of the Japanese stock market.

To sum up, the modified, bootstrap-based LR criterion of eq. (17) and its representation as the shaded confidence region in Figure 3 constitute a reliable way to specify a

homogeneous type of markets to which, as it turns out, the great majority of share markets at TSE can be assigned. The return dynamics on these markets can thus be broadly characterized by a single common pair of the structural parameters ε_1 and ε_2, which is (practically) given by the median of the empirical estimates (neglecting the small difference between ε_1^s and ε_1^m). Given that ε_1^m exceeds ε_2^m by far, the conceptual background of our herding model allows us furthermore to reveal the noise traders as the, on average, dominating group *vis-à-vis* the fundamentalists.

3.3   Contrasting the stock markets with foreign exchange markets

The analysis that culminated in the shaded area in Figure 3 has not only demonstrated that similar share markets yield similar estimates, it has also established a criterion of "similar", or "essentially equivalent". We now have to look at this result from a different angle and ask whether the class defined by the confidence region in Figure 3 is not rather uninformative in that it is much too wide. The question does not only concern the estimation procedure by which we have arrived at the set, but also the structure of the model altogether. While it has been emphasized that the model's main merit just lies in its analytical tractability and the convenient estimation approach thus made possible, which would then be worth all the simplifying assumptions, the model would lose much of its attractiveness if nearly all estimated parameters from speculative markets turned out to be "essentially equivalent".

To check the model's ability to discriminate, we turn to foreign exchange markets. We estimate the daily returns of eight major currencies against the US Dollar: the Canadian Dollar (CD), Japanese Yen (JY), Deutsche Mark (DM), British Pound (BP), Swiss Franc (SF), French Franc (FF), Italian Lira (IL), and the Australian Dollar (AUS). In addition, the gold market (G) may be considered (see the appendix for further details). Table 1 reports the estimated values of $\varepsilon_1, \varepsilon_2$ and the corresponding mean value $\bar{z}$ (in percent) of the share of noise traders (the statistics $\hat{\alpha}_H$ and α in the last two rows will be discussed later).

Seven of the nine parameter pairs are also drawn as the (black) crosses in Figure 3. Only two of them, the Japanese Yen and the Australian Dollar, happen to fall into the shaded area, while the estimates of the other foreign exchange markets (including the omitted FF and IL) are consistently above it. The estimate of the Gold market lies outside

	CD	JY	DM	BP	SF	FF	IL	AUS	G
$\hat{\varepsilon}_1$:	14.0	5.2	5.2	4.9	6.0	4.2	4.2	4.5	3.2
$\hat{\varepsilon}_2$:	5.9	7.0	14.0	9.0	12.0	14.1	10.5	6.9	3.9
$\bar{z}$:	70.4	42.6	27.1	35.3	33.3	23.0	28.6	39.4	45.1
$\hat{\alpha}_H$:	4.0	3.5	4.9	5.0	5.1	3.9	4.1	4.1	2.9
α :	3.7	3.6	4.6	3.9	4.5	4.3	3.9	3.5	2.5

Table 1: Estimations of eight foreign exchange markets (currencies against USD)
and the gold market (G).

Note: $\hat{\alpha}_H$ and α are the Hill estimator and the Hill index characterizing the tail of
the distribution of returns, which are discussed in Section 4.

region S, too; remarkably to the south-west of it and also to the left of the 'outliers' of
the empirical estimates.[9] The main message to be derived from these features is that the
model does not tend to assign all markets to the same class; there are indeed important
speculative markets that are recognized as being significantly different.

Comparing the third row of Table 1 with eq. (16), it is seen that the differences have
also a structural interpretation. Except for the Canadian Dollar, the foreign exchange
markets as well as the Gold market distinguish themselves from the stock markets by
average shares $\bar{z}$ of noise traders that are well below 48%, which is the lower-bound of
the marginal distribution of $\bar{z}$ from the stock market bootstrap estimates reported in (16).
Figure 3 emphasizes this feature by the zz-line. It is (a segment of) the geometrical locus of
the pairs $\varepsilon_1, \varepsilon_2$ that entail $\bar{z} = \varepsilon_1/(\varepsilon_1+\varepsilon_2) = 48.2\%$, and pairs above the line are associated
with a lower average share of noise traders. If the average noise trader share were employed
as an alternative criterion to (17), the Japanese Yen and the Australian Dollar could no
longer be considered as possibly being generated by $(\varepsilon_1^s, \varepsilon_2^s)$; in exchange, so to speak, for
the Canadian Dollar and its high share $\bar{z} = 70.4\%$.

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[9]The Gold market is well-known to be quite different from the other markets studied here, which begins
with its intransparency and the special role of the central banks as major actors on it. The fact that
the Gold market is estimated so much differently from the other markets may therefore be pointed out as
another credit the model can claim.

Maintaining the LR criterion (17) for establishing a confidence region, we can put the results on the foreign exchange markets on a firmer basis by carrying out the same kind of bootstrap experiments as for the Japanese estimations. We pick out a representative pair of the parameters, simulate the model with respect to a sequence of random shocks b over $T = 5,768$ periods again, and then re-estimate $\varepsilon_1^b, \varepsilon_2^b$ from the resulting return series. As before, 5,000 replications are evaluated.
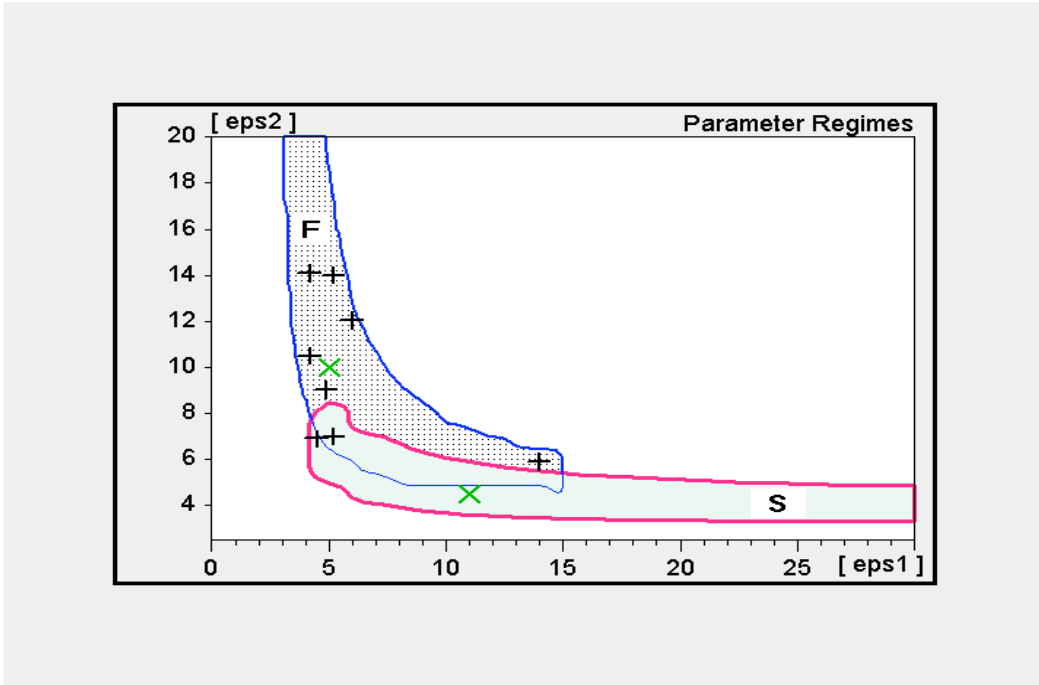

Figure 4: Two classes of parameter pairs $\varepsilon_1, \varepsilon_2$.

Note: The sets are the truncated 95% confidence regions S and F for, as indicated by the diagonal crosses, $(\varepsilon_1^s, \varepsilon_2^s)$ and $(\varepsilon_1^f, \varepsilon_2^f)$. The other crosses represent the currency estimates from Table 1.

Concretely, we choose $\varepsilon_1^f = 5$ and $\varepsilon_2^f = 10$ for these experiments, which are essentially the median values of the currency parameters from Table 1. From the simulation runs we again select a special random sequence $b^\star$ that gives rise to a return series $\{r_t^{b^\star}\}$, such that $(\varepsilon_1^{b\star}, \varepsilon_2^{b\star}) = (e_1^f, \varepsilon_2^f)$. Applying then eq. (17) to the 5,000 estimations (where now superscript 's' is to be replaced with 'f' and the critical value on the right-hand side is 28.2), the set F in Figure 4 results as the 95% confidence region to which the representative parameter pair $(e_1^f, \varepsilon_2^f) = (5.0, 10.0)$ gives rise.[10]

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[10]Regarding the second parameter, the confidence set extends to about $\varepsilon_2 = 50$.

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Labelling the newly generated area with a symbol of its own F (for foreign exchange markets) is justified since it contains the estimates of all currencies from Table 1 in its interior, including the French Franc and the Italian Lira. The only exception is the Australian Dollar, which is practically on the borderline in the south-west. Notice also that the Canadian Dollar, which in Figure 3 could have perhaps been regarded as an outlier, happens to be within set F (in the south-east corner).

Moreover, the two sets F and S are largely disjunct. Visually, the overlapping region would appear even smaller if the two sets had not been truncated. The degree of overlapping can be made precise as follows: (1) 20.3% of the estimates $\hat{\varepsilon}_1^b, \hat{\varepsilon}_2^b$ originating with $\varepsilon_1^f, \varepsilon_2^f$ fall into class S; (2) conversely, 16.3% of the estimates $\hat{\varepsilon}_1^b, \hat{\varepsilon}_2^b$ originating with $\varepsilon_1^s, \varepsilon_2^s$ fall into class F; (3) as concerns the empirical Japanese stock market estimates $\hat{\varepsilon}_1, \hat{\varepsilon}_2$, 20.4% of them fall into class F.

It can thus be said that the model is able to identify two different classes of speculative markets. It predicts that the $(\varepsilon_1, \varepsilon_2)$-estimates for stock markets tend to fall into the set S, and those for the foreign exchange markets into the set F. According to the structural interpretation of the model, markets in set S are dominated by noise traders and markets in the set F by fundamentalist traders. About $16-20$ percent of the estimates are, however, contained in an inconclusive region for which, if the origin of the return series were not known, we could not decide whether it comes from a stock market or a foreign exchange market. This summary is a concise hypothesis that we have derived from a large sample of stocks at TSE and a smaller sample of foreign exchange markets, and that can be tested in further empirical work.

The special result for the gold market is noteworthy in addition, and that so far we have no empirical example of an estimate lying above set S and to the right of set F.

Figure 5 illustrates the differences between stock and foreign exchange markets from a time series perspective. The diagram displays three normalized series of daily absolute returns, for the USD/DEM exchange rate, the Japanes Nikkei index, and a 'representative' stock from the TSE sample. Since the series are of equal length and the $|r|$-axes are identically scaled, their volatility can be directly compared. A simple visual inspection shows that the exchange rate in the top panel has the lowest, and the Japanese stock in the bottom panel the highest variability. The middle panel demonstrates that the greater variability of stocks persists after aggregation, though the Nikkei index exhibits less extreme
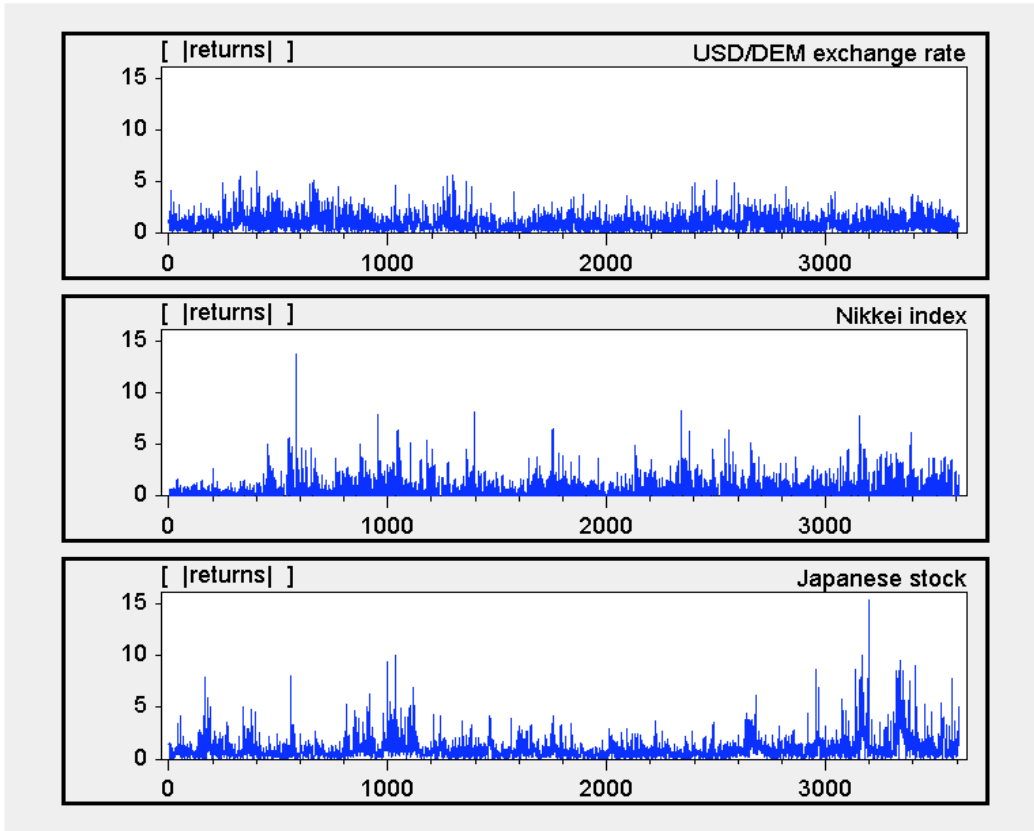
Figure 5: Absolute daily returns from three markets (normalized).

events than the single stock. Similar differences are observed for other empirical data from stock and foreign exchange markets, and the human eye has no great difficulty in recognizing them.

A connection of the empirical time series to our structural model is provided by the parameter ε_2. As already mentioned in a remark on the equilibrium distribution (12), ε_2 can be tailored to achieve an almost arbitrary degree of fatness in the tail. For example, parameter values $\varepsilon_2 \geq 10$, which are typical for the foreign exchange markets, can be shown to generate such a rapid decay in the tail that the event $|r| > 10\%$ has a practically negligible probability (in the empirical series in the top panel, it has actually never occurred within 3,607 days). The limited number of extreme events on these markets nicely fits in with our characterization that (because of the higher estimates of ε_2) they are dominated by fundamentalist traders, and the general economic intuition that fundamentalists tend to be stabilizing.

4 Parameter estimates and tail indices

An outstanding characteristic of the return distributions of speculative markets are the fat tails and their power-law decay. A possible theoretical measure of fatness, or the hyperbolic decay in the tail, is the tail index, which is defined as the highest finite absolute moment of the underlying distribution. The empirical estimates of the tail index across different markets, time periods and frequencies are now well-known to be contained in a limited interval centered around a value slightly higher than three, which has even been advocated as a trace of an underlying inverse "cubic law" for financial returns (Gopikrishnan et al., 1998). It seems to be a pleasant property of the present model that it indeed exhibits a power-law decay of the extreme returns, where the tail index can be proved to be equal to the behavioural parameter ε_2 (see Alfarano, 2006, pp. 120, 127). However, our estimated values of this parameter for both stock and foreign exchange data do not appear to be compatible with the interval just mentioned. In the following we want to inquire into this problem and reconcile the theoretical feature of the model with the empirically identified regularities.

4.1 The concept of the Hill tail index of a probability distribution

For empirical time series, the tail index can be conveniently estimated by the Hill estimator, denoted as $\hat{\alpha}_H$. For a few selected stocks it has already been noticed in previous work that $\hat{\alpha}_H$ tends to be lower than the estimated ε_2 (Alfarano et al., 2005a, p. 36). This also holds true for the abovementioned reference series $\{r_t^{ref}\}$ from TSE, for which we obtain $\hat{\alpha}_H = 3.16$ although the estimated ε_2 is 4.5. Even worse are the differences between ε_2 and $\hat{\alpha}_H$ for the foreign exchange markets in Table 1. The discrepancies are indeed so large and systematic that ascribing them to the finite-sample variability appears no longer convincing. To reestablish confidence in the model estimations the problem calls for a more profound resolution.

To this end we begin with a recapitulation of the definition of the Hill estimator (Hill, 1975). Since the equilibrium density function is symmetrical around zero and there is also little evidence in the empirical return series that their positive and negative tails are different, we can work with absolute returns. Denote them as $v_t := |r_t|$ and assume the v_t are already rearranged in ascending order, $v_t \le v_{t+1}$ for all t. It is furthermore presupposed

that the tail of the series has been specified in advance by the last m elements. The Hill estimator is then defined as

$$\hat{\alpha}_H \;=\; 1\,/\,\hat{\gamma}_H\,, \qquad \text{where} \quad \hat{\gamma}_H \;=\; \frac{1}{m}\sum_{k=0}^{m-1}\big[\ln v_{T-k}\,-\,\ln v_{T-m}\big] \tag{18}$$

Obviously, lower values of $\hat{\alpha}_H$ indicate a fatter tail of the data.

The Hill estimator derives from the semi-parametric assumption of a Pareto decay of the ordered returns, conditionally on being higher than a given threshold. In contrast, the estimation of the parameter ε_2 is based on a parametric approach, which utilizes the entire functional form of the model's equilibrium return distribution. Moreover, the Hill estimator is a function of 'few' extreme events, while the estimated value of ε_2 is, in principle, influenced by the entire data points. A direct comparison between the estimated ε_2 of a time series and its Hill estimator should therefore be taken with some caution.

For a careful study of the problem we want to carry over Hill's concept of eq. (18) to a situation where the values v_{T-k} need not be actually sampled, since their distribution is known and available in a closed-form representation. In the present case, the density function $p_v = p_v(v;\varepsilon_1,\varepsilon_2)$ of the absolute returns is linked to the equilibrium density $p^e = p^e(r;\varepsilon_1,\varepsilon_2)$ of the model's symmetrical raw returns in (12) by the relationship

$$p_v(v;\varepsilon_1,\varepsilon_2) \;=\; 2\,p^e(v;\varepsilon_1,\varepsilon_2) \tag{19}$$

With respect to a given threshold value v_o and an arbitrarily large maximum value v_{max}, let the tail support be given by the interval (v_o, v_{max}). To mimic the sampling process, divide the tail into m equally spaced subintervals (m likewise arbitrarily large) and consider their mid-points

$$u_k \;:=\; v_o \;+\; (k-1/2)\,(v_{max}-v_o)/m \qquad (k=1,\,\dots\,,m) \tag{20}$$

Instead of actually sampling from the tail and multiplying the log differences from $\ln v_o$ by $1/m$ as in (18), we can concentrate on the intervals, represent each one by its mid-point u_k, and multiply $\ln u_k$ by its probability weight within the tail; that is, by the factor $p_v(u_k)/\sum_{j=1}^{m} p_v(u_j) = p_v(u_k;\varepsilon_1,\varepsilon_2)\,/\sum_{j=1}^{m} p_v(u_j;\varepsilon_1,\varepsilon_2)$. In this way, the analogues of $\hat{\gamma}_H$ and $\hat{\alpha}_H$ in (18) are

$$
\begin{aligned}
\gamma \;=\; \gamma(v_o;\varepsilon_1,\varepsilon_2) &\;=\; \sum_{k=1}^{m} \frac{p_v(u_k;\varepsilon_1,\varepsilon_2)}{\sum_{j=1}^{m} p_v(u_j;\varepsilon_1,\varepsilon_2)}\left[\ln u_k \,-\, \ln v_o\right]\\
\alpha(v_o;\varepsilon_1,\varepsilon_2) &\;=\; 1\,/\,\gamma(v_o;\varepsilon_1,\varepsilon_2)
\end{aligned}
\tag{21}
$$

Once v_{max} and the tail density m are large enough, a further increase of these parameters will leave γ in (21) essentially unaffected. It is the advantage of having an entire distribution available that in this way α misses no rare events—a risk unavoidable for every sampled return series; eq. (21) can include them with the appropriate probability weights, even if the theoretical probabilities are getting arbitrarily close to zero.

The quantity α in (21) is a definite magnitude associated with a given (and computable) probability distribution, and no longer an estimator. For this reason the caret over α was omitted. The number $\alpha(v_o; \varepsilon_1, \varepsilon_2)$ may be referred to (not as "the" tail index and not as the Hill "estimator", but) as the "Hill tail index" of the model's equilibrium density $p^e(\cdot; \varepsilon_1, \varepsilon_2)$. The notation clarifies that α is a function of the two behavioural parameters ε_1, ε_2 and the chosen threshold.

After a few explorations, the computations of (21) were found to be completely safe if they are based on $v_{max} = 50$ (a value never observed in empirical nor simulated data) and a density of the subintervals in the tail like $m = 5,000$ (also $m = 500$ would have been good enough). It is established below that $v_o = 3$ is a reasonable threshold value to choose. The thus determined Hill tail indices for the two parameter pairs constituting classes S and F are reported in Table 2.

	$\varepsilon_1^s = 11,\ \varepsilon_2^s = 4.5$	$\varepsilon_1^f = 5,\ \varepsilon_2^f = 10$
$\alpha(v_o; \varepsilon_1, \varepsilon_2)$:	3.10	4.07

Table 2: Hill tail indices of the equilibrium distribution ($v_o = 3$).

The table clearly shows that the analytical tail index (ε_2) of the model's equilibrium density and its Hill tail (α) index are generally different. The results are rather quite in line with the empirical findings mentioned above. With $\alpha = 3.10$, the Hill index for $\varepsilon_1^s, \varepsilon_2^s$ is perfectly compatible with the Hill estimator $\hat{\alpha}_H = 3.16$ that we had obtained for our TSE reference series $\{r_t^{ref}\}$. The Hill index for $\varepsilon_1^f, \varepsilon_2^f$, which was taken to be representative for the foreign exchange markets, is with $\alpha = 4.07$ of a similar order of magnitude as the empirical estimators in the last row of Table 1. In particular, this α is distinctly lower than $\varepsilon_2^f = 10$.

A couple of additional examples quickly demonstrate that there is no systematic connection of $\alpha(v_o; \varepsilon_1, \varepsilon_2)$ to the model parameter ε_2 alone. The Hill tail index is actually dependent on a certain kind of "trade-off" between ε_2 and ε_1, in the sense that a value $\alpha(v_o; \varepsilon_1, \varepsilon_2)$ is maintained upon an increase of ε_2 if simultaneously ε_1 is suitably reduced. Figure 6 illustrates this interrelation by drawing two isoclines of $\alpha(v_o; \cdot, \cdot)$. As it is also indicated by the diagonal cross, the lower line is the locus of parameter pairs giving rise to $\alpha(v_o; \varepsilon_1, \varepsilon_2) = \alpha(v_o; \varepsilon_1^s, \varepsilon_2^s) = 3.10$ (setting $v_o = 3$). The upper contour line induces $\alpha(v_o; \varepsilon_1, \varepsilon_2) = \alpha(v_o; \varepsilon_1^f, \varepsilon_2^f) = 4.07$. Obviously, the trade-off between the two parameters is of a nonlinear nature. In the north-west of the parameter plane, the Hill index even stays (nearly) put if ε_2 increases and ε_1 is held constant.
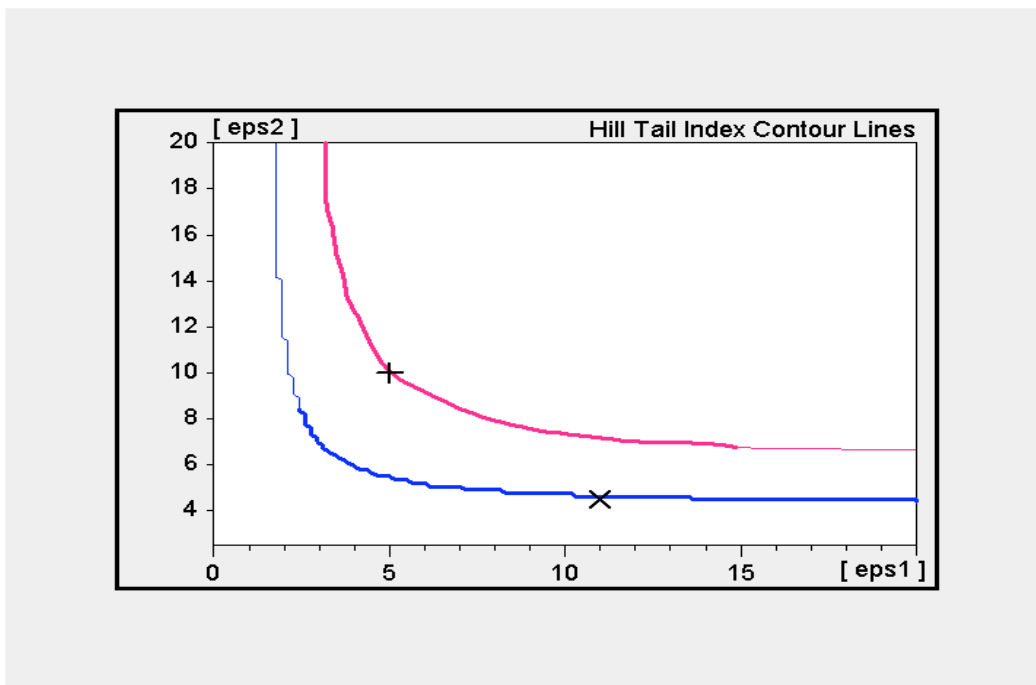

Figure 6: Isoclines of the Hill tail index $\alpha(v_o; \cdot, \cdot)$, with $v_o = 3$.

Note: Crosses indicate the two parameter pairs $(\varepsilon_1^s, \varepsilon_2^s)$ and $(\varepsilon_1^f, \varepsilon_2^f)$.

It is interesting to compare the isoclines of the Hill tail index in Figure 6 with the confidence regions constituting class S and F in Figure 4. To some qualitative degree, the two isoclines trace out the general shape of the regions. The fact that the isoclines leave these sets indicates, of course, that the latter incorporate more information than the Hill tail index.

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ε_1	ε_2	α	tail size
$v_o = 2$			
11.0	4.5 :	2.65	11.27
5.0	10.0 :	3.12	11.68
$v_o = 2.788$			
11.0	4.5 :	3.02	5.00
5.0	10.0 :	3.89	4.55
$v_o = 3$			
11.0	4.5 :	**3.10**	4.10
5.0	10.0 :	**4.07**	3.55
$v_o = 5$			
11.0	4.5 :	3.57	0.85
5.0	10.0 :	5.38	0.42
$v_o = 25$			
11.0	4.5 :	4.43	0.00
5.0	10.0 :	8.58	0.00

Table 3: Hill tail indices α under alternative threshold values v_o.

Note: Tail size in percent. Underlying is $m = 5,000$ and $v_{max} = 50$, except for $v_o = 25$ where v_{max} is increased to 80.

4.2   The role of the threshold value

After this first evidence the impact of different values of the threshold v_o should be checked. Let us therefore concentrate on the two representative pairs $(\varepsilon_1^s, \varepsilon_2^s)$ and $(\varepsilon_1^f, \varepsilon_2^f)$ and examine the dependency of the Hill tail index upon variations of v_o. This is done in Table 3. To begin with, for the thresholds $2 \leq v_o \leq 5$ the index α is within a range that is in line with what is known from empirical work on the Hill estimator (cf. also Table 1 for the exchange rates). Note that the corresponding tail sizes on a percentage basis are in conventional bounds, too (at least for $v_o \geq 2.5$, say). Over the given and even a much wider interval of v_o, however, α shows no tendency to settle down on a particular value: the index of a parameter pair rather rises if v_o rises. Obviously α can distinguish the tails of two data sets by saying that one is "fatter" than the other, but as long as we do not agree on a fixed value of the threshold, the index cannot provide an absolute measure of fatness.

While in the first four parts of Table 3 the Hill index is in a familiar range but considerably lower than the tail index ε_2, in the last part an extraordinarily high threshold $v_o = 25$ is chosen. In this way, the Hill index gets much closer to ε_2. We indeed checked that the correct *asymptotic* power-law behaviour of the tail is recovered, i.e. $\alpha \rightarrow \varepsilon_2$, if simultaneously v_o as well as v_{max} are suitably increased.[11]

The numerical exercise shows that the discrepancies between the Hill tail index α and the underlying value of ε_2 are related to the choice of the tail size. The application of the Hill tail index is in fact based on the semi-parametric approximation to a power-law decay of the distribution. Although this assumption is in principle satisfied for our equilibrium distribution (12), the 'pure' form of this law is an inadequate description of the tail if we consider a threshold v_o between 3 and 5, say, for the normalized returns. In that region, which corresponds to the usual choice in empirical investigations using the Hill estimator, the estimation of the 'tail' index is affected by entries that still form part of the shoulders of the distribution. Hence at least for the type of distribution with which we are working here, lower values of the threshold v_o introduce a downward bias of the Hill statistic.

As has also been demonstrated, by sufficiently increasing the threshold the remaining tail eventually produces a Hill index α that is similar to the theoretical decay parameter ε_2. The price for this reconciliation is that it relies on such extreme returns $|r_t| > 25\%$ that are hardly ever observed in reality. One might even suspect that this is not just a sampling problem but that these events would only be outliers that should not be part of a theory. In any case, the theoretical parameter ε_2 is of limited informational value to describe a power-law decay and we have to resort to the Hill index as a more reasonable summary statistic to characterize the tail behaviour of the available returns.[12]

Even if the power-law decay in the equilibrium probability distribution (12) is in its pure form too rigorous to be of practical use, there is still another remarkable property concerning the tail. If we take the estimated ε_1, ε_2 values, employ the Hill tail index to describe the 'fatness' of the tail of this distribution, and base its computation on an empirically relevant range of the threshold like $3 \leq v_o \leq 4$, then we obtain an order of

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[11]If v_{max} stays constant, α may happen to exceed ε_2.

[12]It is an open question after all whether the scaling properties suggested by the usual visual procedures are not spurious, and only due to the small-sample properties of these tests; cf. LeBaron (2001) and Lux (2001).

magnitude in the results that is quite compatible with the Hill estimators that one typically finds in the financial markets literature. Hence our estimations do not only support the model's characteristic probability distribution (12) in its entirety, but also in this more specific feature of its tail, as it is usually conceived in empirical work.[13]

5 Conclusion

In this paper we have dealt with an elementary agent-based model for a parsimonious description of an artificial financial market with fundamentalists and noise traders. As shown in earlier work (Alfarano et al., 2005a,b), its interplay between randomness and a herding component is capable of reproducing the key stylized facts of financial data, namely, volatility clustering, Paretian tails of the return distribution, unit root and positive dependence of the autocorrelations of absolute and squared returns.

In contrast to many contributions from the agent-based literature, the simple structure of the model makes it possible to express several conditional and unconditional properties of the return time series in a closed-form solution. Using this information, the (few) underlying parameters can be estimated by standard econometric techniques. This is a feature the model shares with the class of econometric time series models, i.e., GARCH and other stochastic volatility models. An important advantage of our approach, however, consists in the behavioural roots of its volatility features, which allow us to connect the abovementioned statistical properties of the returns to the stylized characteristics of the investors that we have postulated.

Our concern in this paper was to go beyond the previous casual estimations of the model. We examined as many as 982 corporate shares traded at the Tokyo Stock Exchange and contrasted them with results from the major foreign exchange markets. Considering the returns on a daily basis, the main finding from these empirical estimations and a battery of complementary Monte Carlo experiments is a tendency for the parameter estimates to come out differently for the two types of markets. The implied structural interpretation is

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[13]The relatively large difference between the two Hill tail indices 3.10 and 4.07 for the representative pairs $\varepsilon_1^s, \varepsilon_2^s$ and $\varepsilon_1^f, \varepsilon_2^f$ in Table 2, and the two distinct isoclines in Figure 6 suggest different tail properties for the two regimes, even if one takes the sampling variability into account. This issue is more systematically investigated in an extended version of this paper.

a dominance of noise traders on stock markets, and a dominance of fundamentalists on the foreign exchange markets. A degree of ambiguity that still remains can be loosely said to be about 20 percent.

Admittedly, a general assessment of the properties we obtained would be premature given that despite the large number of single stocks, our study was limited to the TSE data. So the predictions still need to be tested with data from the other great trading places. On the other hand, our approach opens up a new line of research on the design as well as estimation of agent-based models with a richer and theoretically more satisfactory structure, which may be complementary to the econometric time series modelling of financial data.

6   Appendix

The data set of the stocks time series consists of 982 stocks from the Japanese market at daily sampling intervals with a time horizon ranging from 4 January 1975 to 28 December 2001 (though not all of the series cover the entire period; the identification numbers can be provided upon request).

The currencies against the US Dollar of the Canadian Dollar (CD), Japanese Yen (JP), Deutsche Mark (DM), British Pound (BP), Swiss Franc (SF), French Franc (FF) and the Italian Lira consist of a total of 3,913 daily observations each, ranging from 15 December 1989 to 15 December 2004. The Australian exchange rate data (AUS) cover the period since the floatation of the Australian Dollar in December 1983 until the end of 2004 (amounting to 5,495 data points).

The time series of the gold price (G) extends from January 1974 until December 1998 (5,140 entries).

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