Abadir, K.M., and Talmain, G. (2009) Macro and financial markets: The memory of an elephant? In: 15th Spring Meeting of Young Economists 2009, 23-25 Apr 2009, Istanbul, Turkey.

MACRO AND FINANCIAL MARKETS:
The memory of an elephant?

Karim Abadir and Gabriel Talmain

Background, REStud 2002:

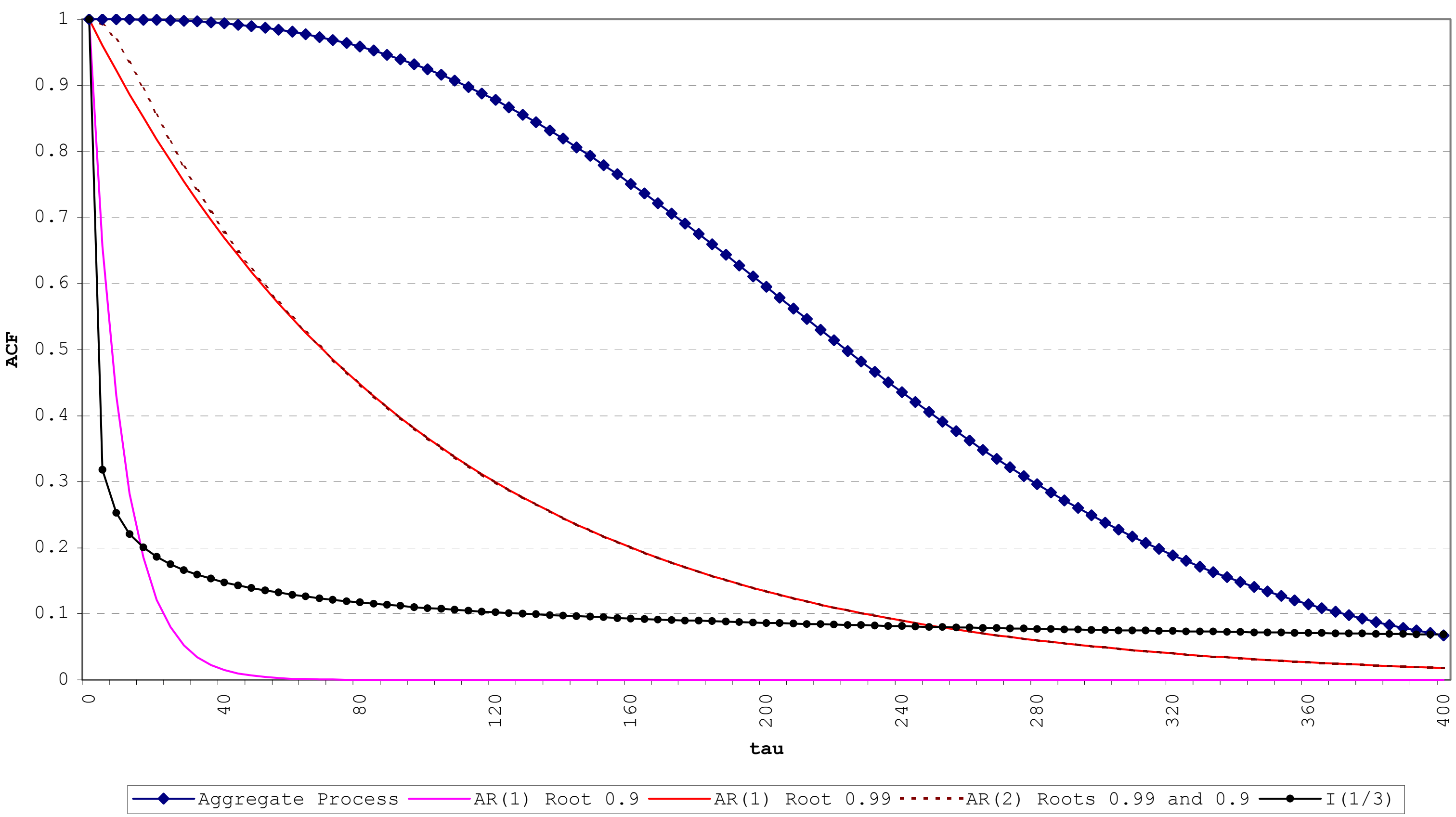
- Model and solution:
  - Micro-founded macro model.
  - Standard RBC + heterogeneity: drop "representative firm" assumption.
  - General Equilibrium yields explicit dynamic equation for GDP etc.

Background, REStud 2002:

- Model and solution:
  - Micro-founded macro model.
  - Standard RBC + heterogeneity: drop "representative firm" assumption.
  - General Equilibrium yields explicit dynamic equation for GDP etc.
- When $\exists$ heterogeneity, aggregation $\Longrightarrow$ long-memory; e.g. Robinson (1978), Granger (1980), and TS lit.
- But in economics, $\exists$ an inherent nonlinearity. Decompose GDP as

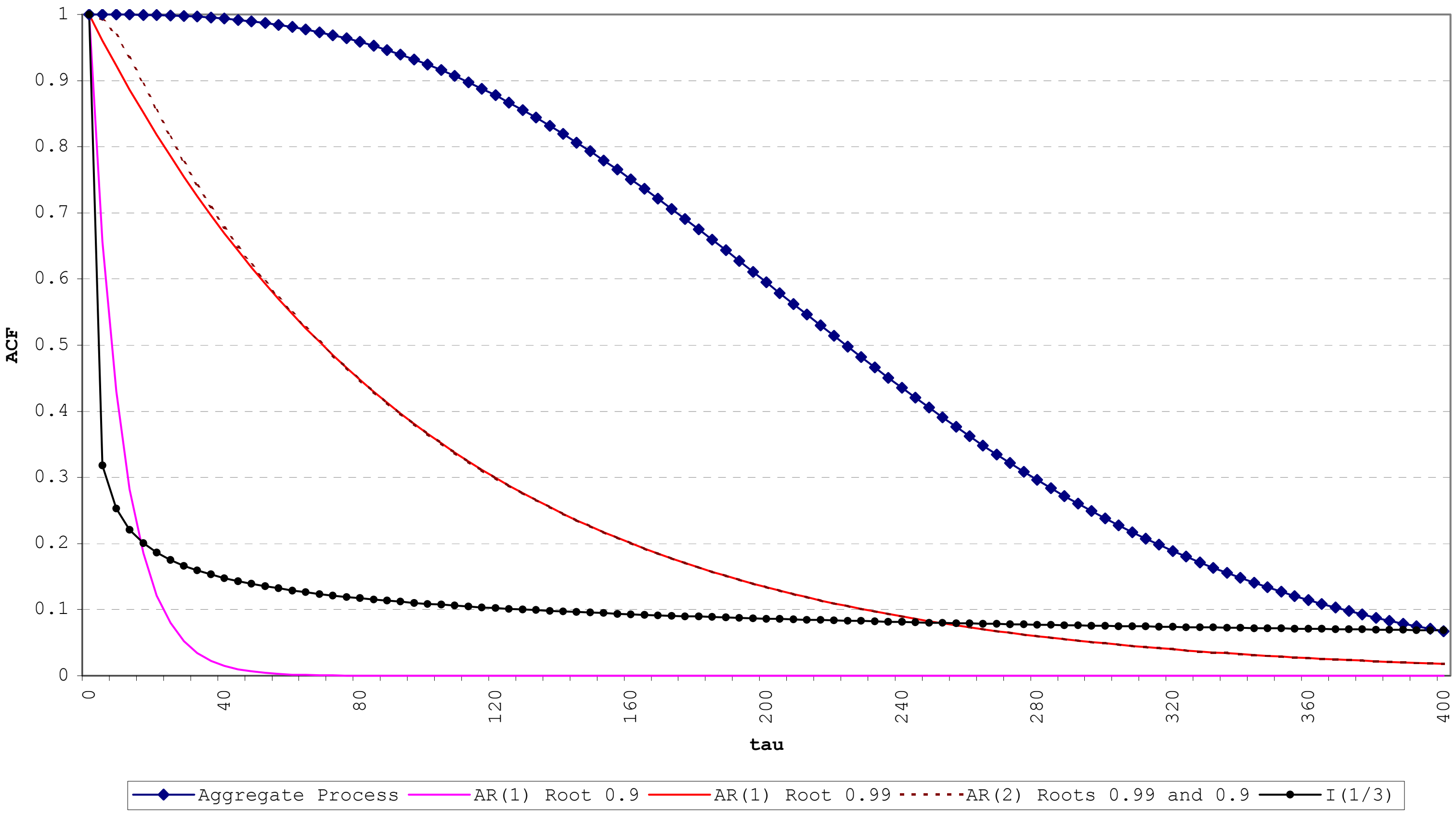
$$Y \equiv Y_1 + Y_2 + \cdots = \mathrm{e}^{\log Y_1} + \mathrm{e}^{\log Y_2} + \cdots \neq \mathrm{e}^{\log Y_1 + \log Y_2 + \ldots}$$

• The result is a new auto-correlation function (ACF) ρ_τ:



with different macropolicy implications.

- The result is a new auto-correlation function (ACF) $\rho_\mathcal{T}$:



with different macropolicy implications.

- But if $\nexists$ integration, what about the modification of co-integration?

1 UIP and forward premium puzzle

- Fisher (1930): "speculation" equates expected returns after conversion to the same currency (UIP); e.g.
  - £1 invested in domestic (UK) bond yields £$(1 + I_t)$ at maturity; vs.
  - £1 invested in foreign (US) bond converted into \$$1/S_t$, which yield \$$\left(1 + I_t^*\right)/S_t =$ £$\left(1 + I_t^*\right) S_{t+1}/S_t$ at maturity.
- Define $i := \log(1 + I)$, $s := \log S$, and

$$r_{t+1} := \Delta s_{t+1} + i_t^* - i_t,$$

as the excess return from investing in the foreign asset. Then, $\mathrm{E}_t[r_{t+1}] = 0$ and r_{t+1} should not be predictable.

- Typical empirical implementation: regress r_{t+1} on the forward premium $(f_t - s_t)$, where f and s are the forward and spot rates in logs:

$$r_{t+1} = \alpha + \beta\,(f_t - s_t) + u_{t+1},$$

where α is the average risk premium and β is the informational content of the forward premium.

  – UIP hypothesis implies $\mathrm{H}_0 : \alpha = 0$ and $\beta = 0$.

  – If one believes $f_t = \mathrm{E}_t\,[s_{t+1}]$, then $(f_t - s_t) < 0$ indicates that the US$ should depreciate.

  – Routine finding: $\beta < 0$. As more US$ depreciation is *expected*, higher returns are *actually* made on the US$!

    Investors are ready to pay more for an asset which, according to their expectations, should have become less attractive!

  – See Cumby and Obstfeld (1981, J Fin) or Engel (1996, J Emp Fin).

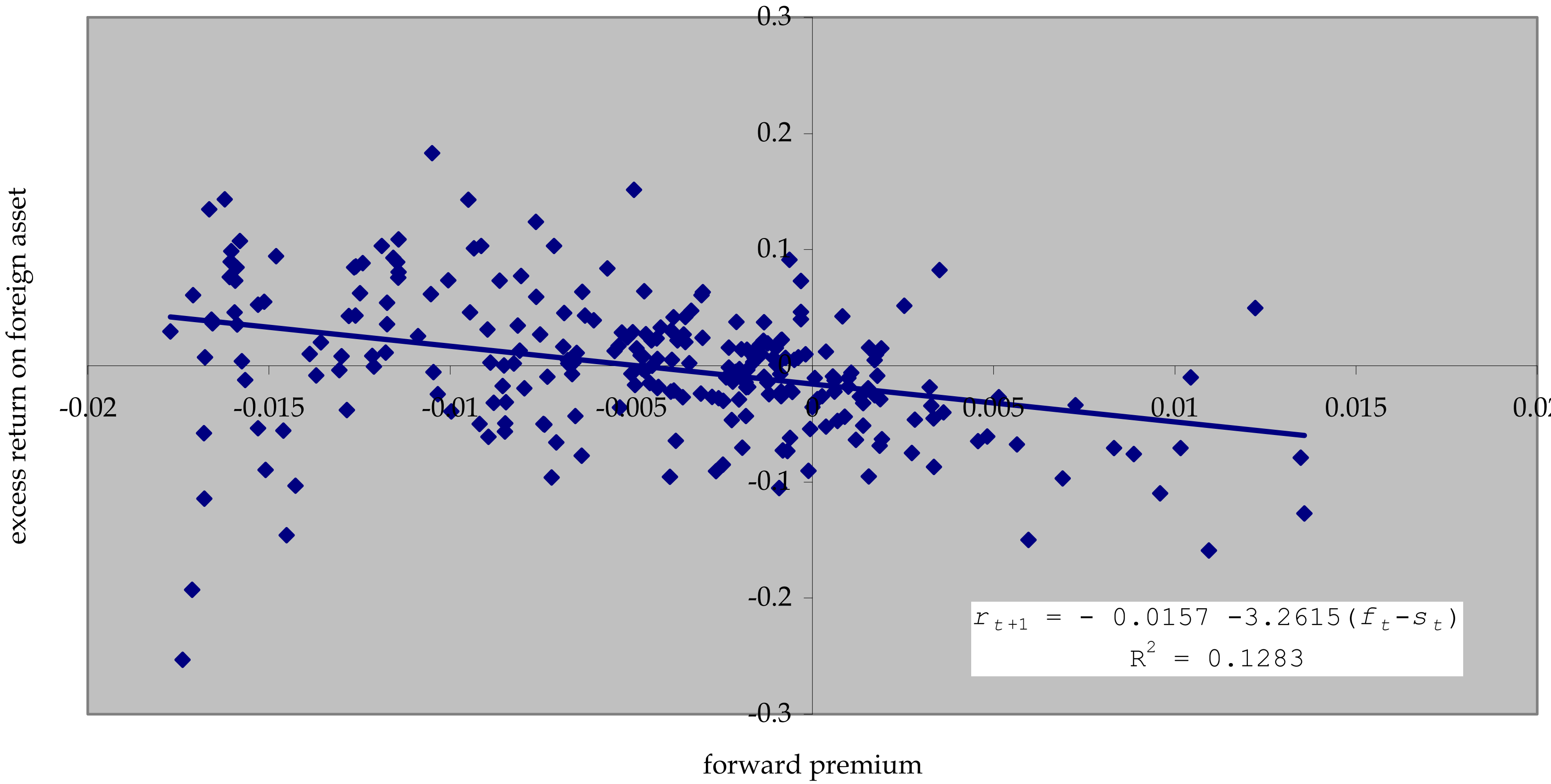
- Running this regression with our data (3-month rates on US$ and UK£ deposits in London and 3-month forward rate)

$$
\begin{array}{lccc}
\hat{r}_{t+1} & = -0.0157 & -3.26 & (f_t - s_t) \\
\text{t-ratio} & (-3.67) & (-6.14) & \\
\text{HAC t} & [-2.85] & [-2.90] &
\end{array}
$$

where HAC = heteroskedastic and autocorrelation-consistent,

$$
\begin{aligned}
\text{Durbin-Watson statistic} &: 0.65 \\
\text{ARCH(7) test, } F(7, 242) &: 24.10\ \{0.0\%\} \\
\text{RESET (omitted nonlinearities), } F(1, 255) &: 9.66\ \{0.2\%\}
\end{aligned}
$$

• Scatter plot of data



• For years, it has defied economic logic to find such a result: how could investors' expectations be so systematically wrong? Or are they?!

2 Solution to the puzzle: coping with non-linear long-memory

- Co-movements vs. own dynamics.
- Incomplete modelling of dynamics can make the estimation of co-movements biased and inconsistent.
- Example (just an illustration): consider autoregressive process

$$\begin{aligned} y_t &= \alpha y_{t-1} + \beta x_t + u_t, \\ \text{with } u_t &= \rho u_{t-1} + \varepsilon_t \quad \text{and } \varepsilon_t \sim \text{IID}\left(0, \sigma^2\right). \end{aligned}$$

Running OLS on the first equation only $\Longrightarrow$ biased and inconsistent estimators; e.g. Maddala and Rao (1973, Ecta).

- Usual approaches:
  - GLS on the augmented first equation; or

– error correction mechanism (ECM), autoregressive distributed lag (ADL)

$$y_t = (\alpha + \rho)\, y_{t-1} - \alpha\rho y_{t-2} + \beta x_t - \beta\rho x_{t-1} + \varepsilon_t,$$

estimating

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + a_3 x_t + a_4 x_{t-1} + e_t$$

then testing for the restriction

$$a_1 = \frac{a_2 a_3}{a_4} - \frac{a_4}{a_3}.$$

– error correction mechanism (ECM), autoregressive distributed lag (ADL)

$$y_t = (\alpha + \rho)\, y_{t-1} - \alpha\rho y_{t-2} + \beta x_t - \beta\rho x_{t-1} + \varepsilon_t,$$

estimating

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + a_3 x_t + a_4 x_{t-1} + e_t$$

then testing for the restriction

$$a_1 = \frac{a_2 a_3}{a_4} - \frac{a_4}{a_3}.$$

- Long-memory times series models: very persistent time series ("Joseph effect").
- Problem of ECM/ADL for data with long memory: too many lags.
- How about the GLS route?

- We would like to estimate the relation

$$z_t = \tilde{z}_t + u_t, \quad t = 1, 2, \dots T,$$

where the $T \times 1$ vector $\widetilde{\boldsymbol{z}} = \boldsymbol{X}\boldsymbol{\beta}$ is the "fundamental" value of $\boldsymbol{z}$, but $\boldsymbol{z}$ and $\widetilde{\boldsymbol{z}}$ (hence possibly $\boldsymbol{u}$) have long memory which needs to be accounted for. A possible 2-step procedure (à la GLS):

  – decompose the autocorrelation matrix of $\boldsymbol{z}$ as $\boldsymbol{R} = \boldsymbol{L}\boldsymbol{L}'$, where $\boldsymbol{L}$ is lower-triangular and invertible;

  – the transformed data $\boldsymbol{L}^{-1}\boldsymbol{z}$ and $\boldsymbol{L}^{-1}\widetilde{\boldsymbol{z}}$ do not contain long memory and can be regressed by traditional methods.

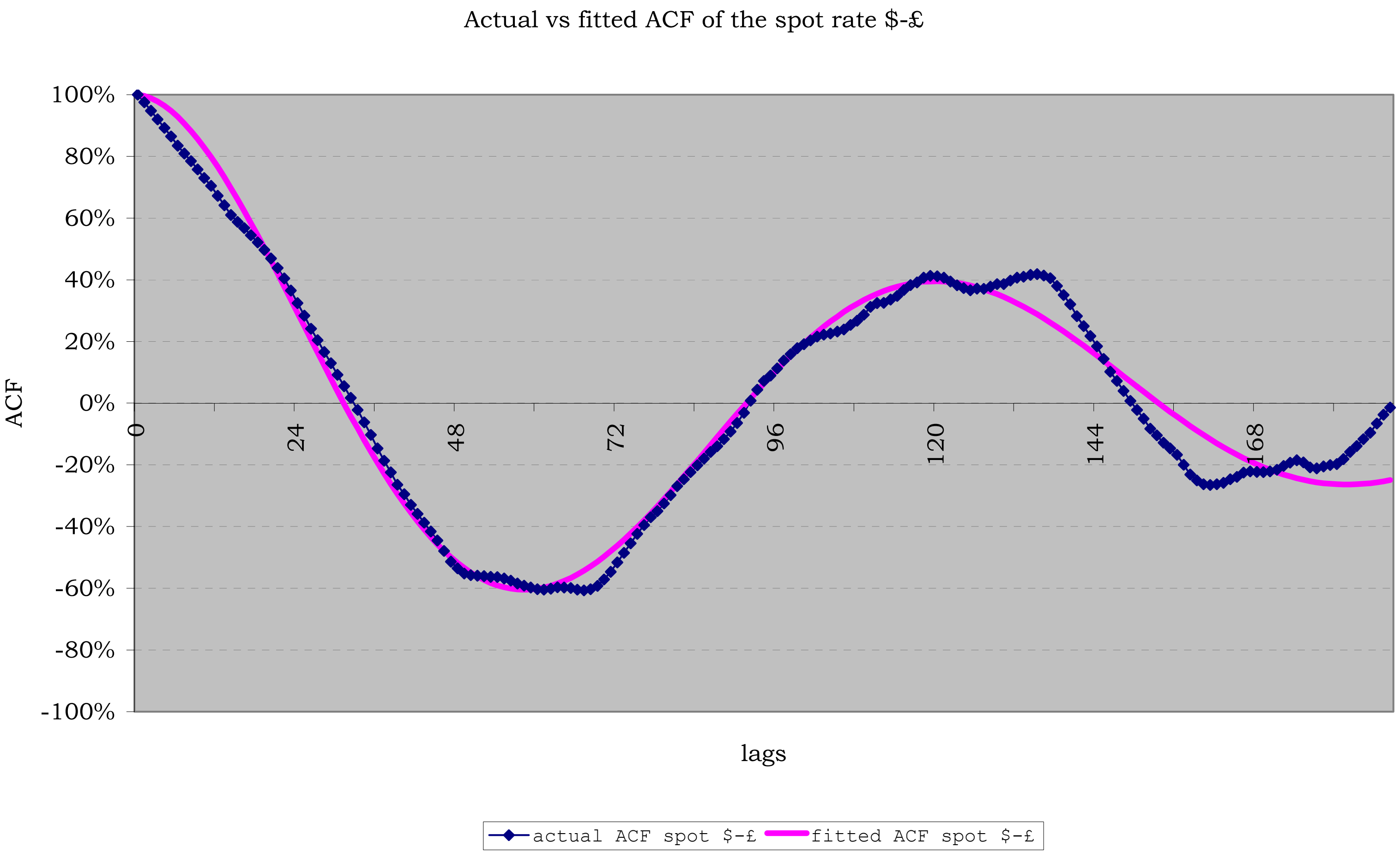
- Unfortunately, estimating $\boldsymbol{R}$ requires estimating $T-1$ parameters: same as infeasible GLS!

  Solution: estimate the ACF of $\boldsymbol{z}$, using a variant of the functional form in Abadir and Talmain (2002, R E Stud)

$$\rho_\tau \approx \frac{1-a\left[1-\cos\left(\omega\tau\right)\right]}{1+b\tau^c},$$

  with only 4 parameters to fit. (Note: denominator controls decay of memory.)

• UIP example: fit is excellent for $\boldsymbol{s}$



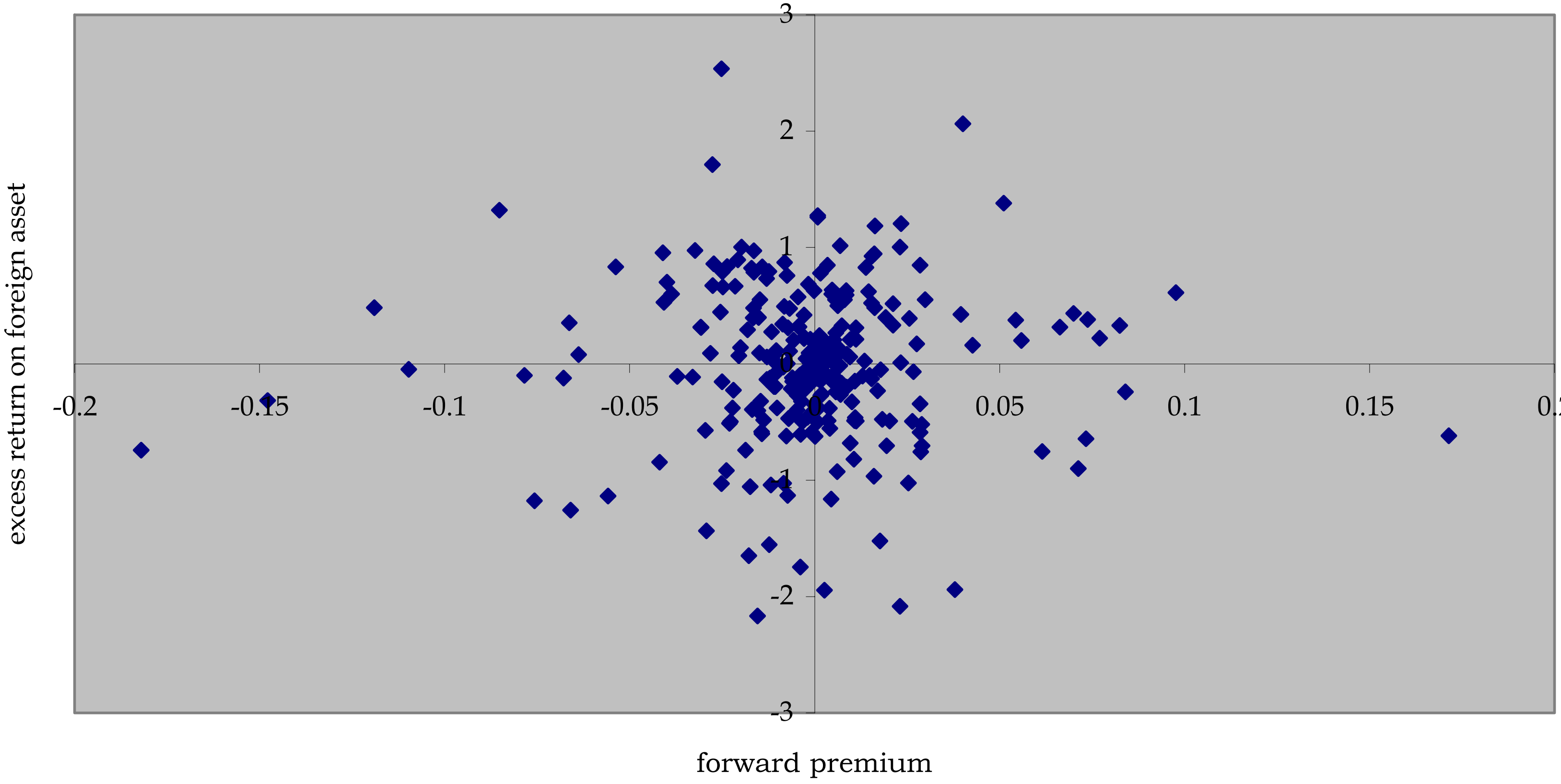
– The estimated $T \times T$ correlation matrix is then

$$\widehat{\boldsymbol{R}} \equiv \begin{pmatrix} 1 & \hat{\rho}_1 & \hat{\rho}_2 & \cdots & \hat{\rho}_{T-2} & \hat{\rho}_{T-1} \\ \hat{\rho}_1 & 1 & \hat{\rho}_1 & \cdots & \ddots & \hat{\rho}_{T-2} \\ \hat{\rho}_2 & \hat{\rho}_1 & \ddots & \ddots & \ddots & \ddots \\ \ddots & \ddots & \ddots & \ddots & \hat{\rho}_1 & \hat{\rho}_2 \\ \hat{\rho}_{T-2} & \ddots & \ddots & \hat{\rho}_1 & 1 & \hat{\rho}_1 \\ \hat{\rho}_{T-1} & \hat{\rho}_{T-2} & \cdots & \hat{\rho}_2 & \hat{\rho}_1 & 1 \end{pmatrix}.$$

– Find the Cholesky decomposition (matlab) $\widehat{\boldsymbol{R}} = \widehat{\boldsymbol{L}}\widehat{\boldsymbol{L}}'$; and

– calculate $\boldsymbol{s}^{\mathrm{acf}} = \widehat{\boldsymbol{L}}^{-1}\boldsymbol{s}$ and $\boldsymbol{f}^{\,\mathrm{acf}} = \widehat{\boldsymbol{L}}^{-1}\boldsymbol{f}$.

– The scatter plot of the transformed data is a nice spherical cloud

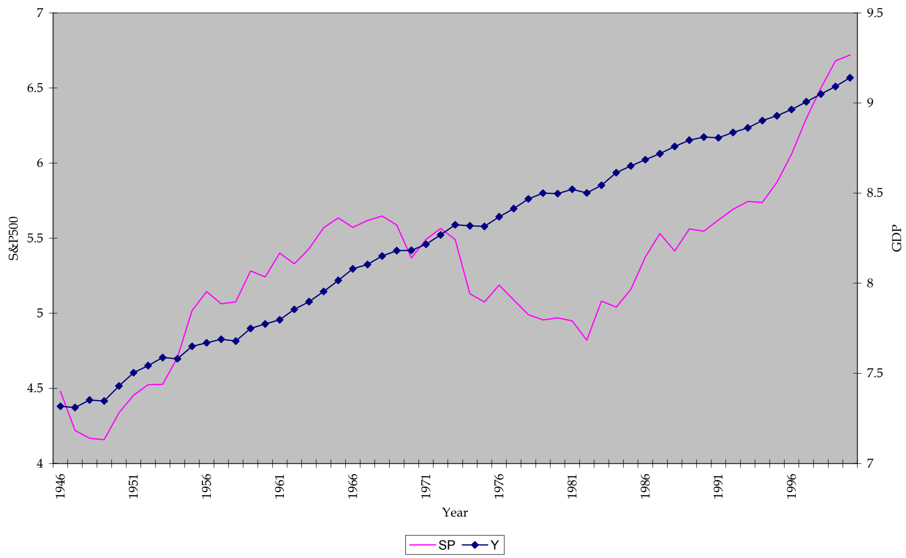


– and the regression with transformed data becomes

$$\begin{array}{lccc}\hat{r}^{\text{acf}}_{t+1} & = 0.00582 & +0.0604 & \left(f^{\text{acf}}_t - s^{\text{acf}}_t\right) \\ \text{t-ratio} & (0.14) & (0.05) & \\ \text{HAC t} & [0.19] & [0.04] & \end{array}$$

$$\begin{aligned} \text{Durbin-Watson statistic} &: 2.20 \\ \text{ARCH(7) test, } F(7, 242) &: 6.54\ \{0.0\%\} \\ \text{RESET (omitted nonlinearities), } F(1, 255) &: 0.82\ \{36.6\%\}\,. \end{aligned}$$

3 Stock market application



- Unit roots?! (NB: unit root = permanent memory and no mean-reversion to any regular pattern or trend!)
- Lack of unit roots shown for US and UK GDPs in Abadir and Talmain (2002, R E Stud).
- How about Stock indices?

– Jegadeesh and Titman (1993 and 2001, J Fin) find momentum;

– De Bondt and Thaler (1985 and 1987, J Fin) find long cycles;

– Cavaliere's (2001, Ect J) non-parametric test detects long-memory.

  - Jegadeesh and Titman (1993 and 2001, J Fin) find momentum;
  - De Bondt and Thaler (1985 and 1987, J Fin) find long cycles;
  - Cavaliere's (2001, Ect J) non-parametric test detects long-memory.

- GE theory: long-run proportionality relationship between the aggregate real value of firms and GDP. On a balanced growth path:
  - real interest rate is a function of capital/output, which is constant $\Rightarrow$ rate at which future aggregate profits are discounted is fixed;
  - but share of aggregate profits in GDP is constant;
  - hence the discounted stream of future profits (i.e. capitalized value of the stock market) is proportional to GDP.

- Jegadeesh and Titman (1993 and 2001, J Fin) find momentum;
- De Bondt and Thaler (1985 and 1987, J Fin) find long cycles;
- Cavaliere's (2001, Ect J) non-parametric test detects long-memory.

- GE theory: long-run proportionality relationship between the aggregate real value of firms and GDP. On a balanced growth path:
  - real interest rate is a function of capital/output, which is constant $\Rightarrow$ rate at which future aggregate profits are discounted is fixed;
  - but share of aggregate profits in GDP is constant;
  - hence the discounted stream of future profits (i.e. capitalized value of the stock market) is proportional to GDP.

- Empirically-testable version: log of stock market index (S&P500) corrected for inflation, s_t, should be in a long-term proportionality with the log of real GDP, y_t.

- Error-correction model of s_t on y_t,

$$\begin{aligned}\Delta s_t = \; & \alpha + (\beta_1 \Delta s_{t-1} + \cdots + \beta_m \Delta s_{t-m}) + (\gamma_0 \Delta y_t + \cdots + \gamma_n \Delta y_{t-n}) \\ & -\delta\,(s_{t-1} - y_{t-1}) + \delta_1 y_{t-1} + \varepsilon_t.\end{aligned}$$

  – The term $-\delta\,(s_{t-1} - y_{t-1}) + \delta_1 y_{t-1}$ is the ECM.

  – It represents the long-run 'equilibrium' relationship between s and y:

$$s_\mathrm{e} = \left(1 + \frac{\delta_1}{\delta}\right) y_\mathrm{e}, \qquad \delta \neq 0.$$

  – H_0: $\delta_1 = 0$ for long-run proportionality between S_e and Y_e.

  – Define $d_{t-1} := s_{t-1} - s_\mathrm{e}$ as the deviation of s_{t-1} from its long-term value s_e.

  – ECM: this deviation will pull s_t back towards its long-term equilibrium value by δd_{t-1}, where $\delta > 0$.

- For S&P500 over 1958-2000, we obtained the regression

$$\begin{aligned}\widehat{\Delta s_t} = & \underset{(-2.33)}{-0.728} \underset{(4.35)}{+0.539} \Delta s_{t-4} \underset{(2.83)}{+0.380} \Delta s_{t-6} \\ & \underset{(4.30)}{+3.11} \Delta y_t \underset{(-2.41)}{-1.72} \Delta y_{t-1} \underset{(2.58)}{+1.84} \Delta y_{t-2} \underset{(-1.77)}{-1.15} \Delta y_{t-6} \\ & \underset{(-2.11)}{-0.112} (s_{t-1} - y_{t-1}) \underset{(0.92)}{+0.0396} y_{t-1}\end{aligned}$$

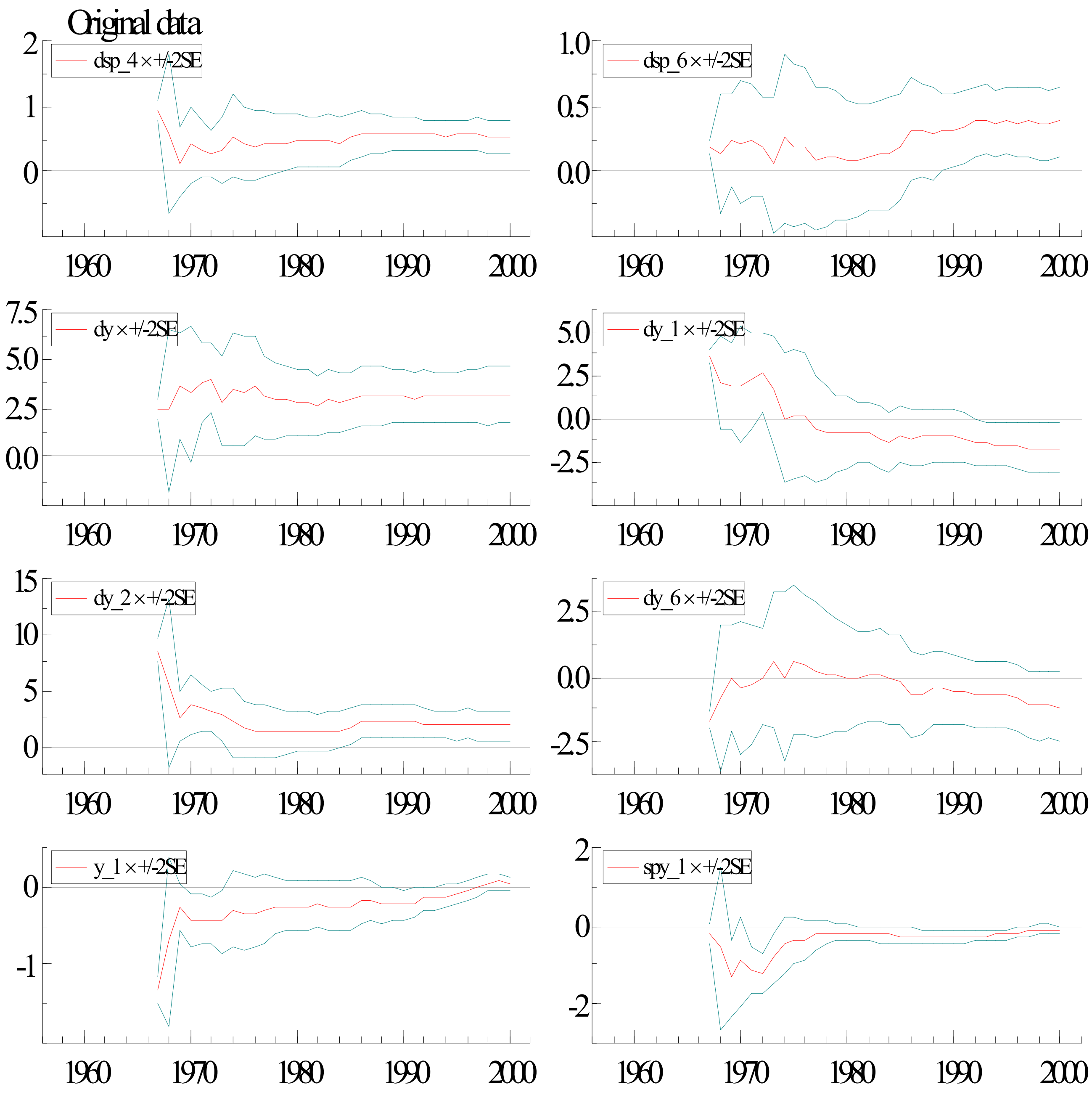
where the t-ratios are in parentheses, and we have $R^2 = 57.5\%$,

$$\begin{aligned}\text{AR(2) test, } F(2,32) &: 0.43 \ \{65.4\%\} \\ \text{ARCH(1) test, } F(1,32) &: 0.17 \ \{68.1\%\} \\ \text{RESET, } F(1,33) &: 1.36 \ \{25.3\%\}.\end{aligned}$$

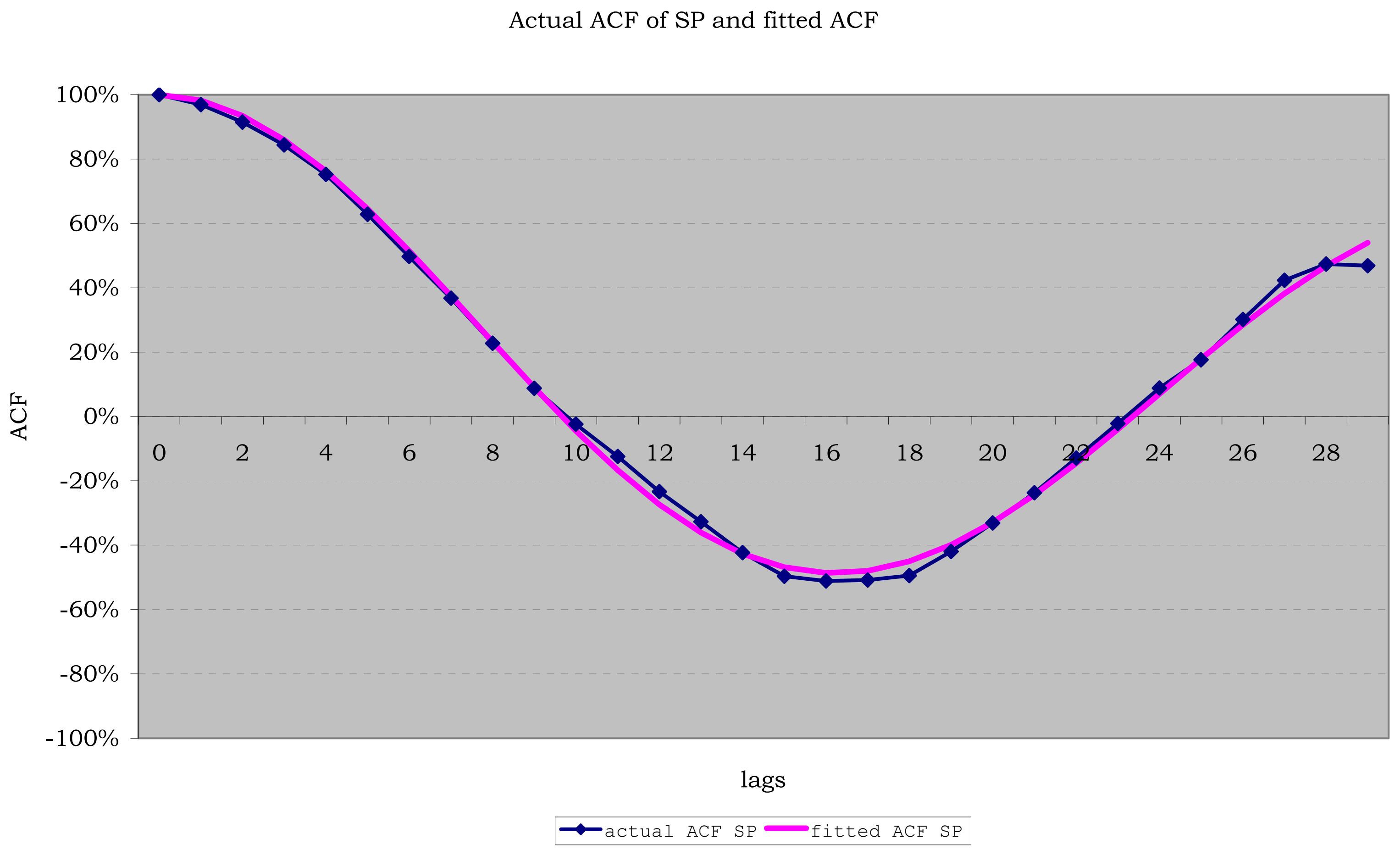
- H_0: $\delta_1 = 0$ is supported, but:

- by the end of the period, 1995-2000, the fit is poor;
- the coefficient of y_{t-1} is unstable and H_0 would be rejected on a sample ending in 1994;
- including more lags of Δs in the regression worsens rather than improves stability, while not improving the fit.

The recursive parameter estimates are...


Original data
dsp_4 × +/-2SE
dsp_6 × +/-2SE
dy × +/-2SE
dy_1 × +/-2SE
dy_2 × +/-2SE
dy_6 × +/-2SE
y_1 × +/-2SE
spy_1 × +/-2SE
1960
1970
1980
1990
2000

- Remember memory?!
  - Fit the ACF of $\boldsymbol{s}$;



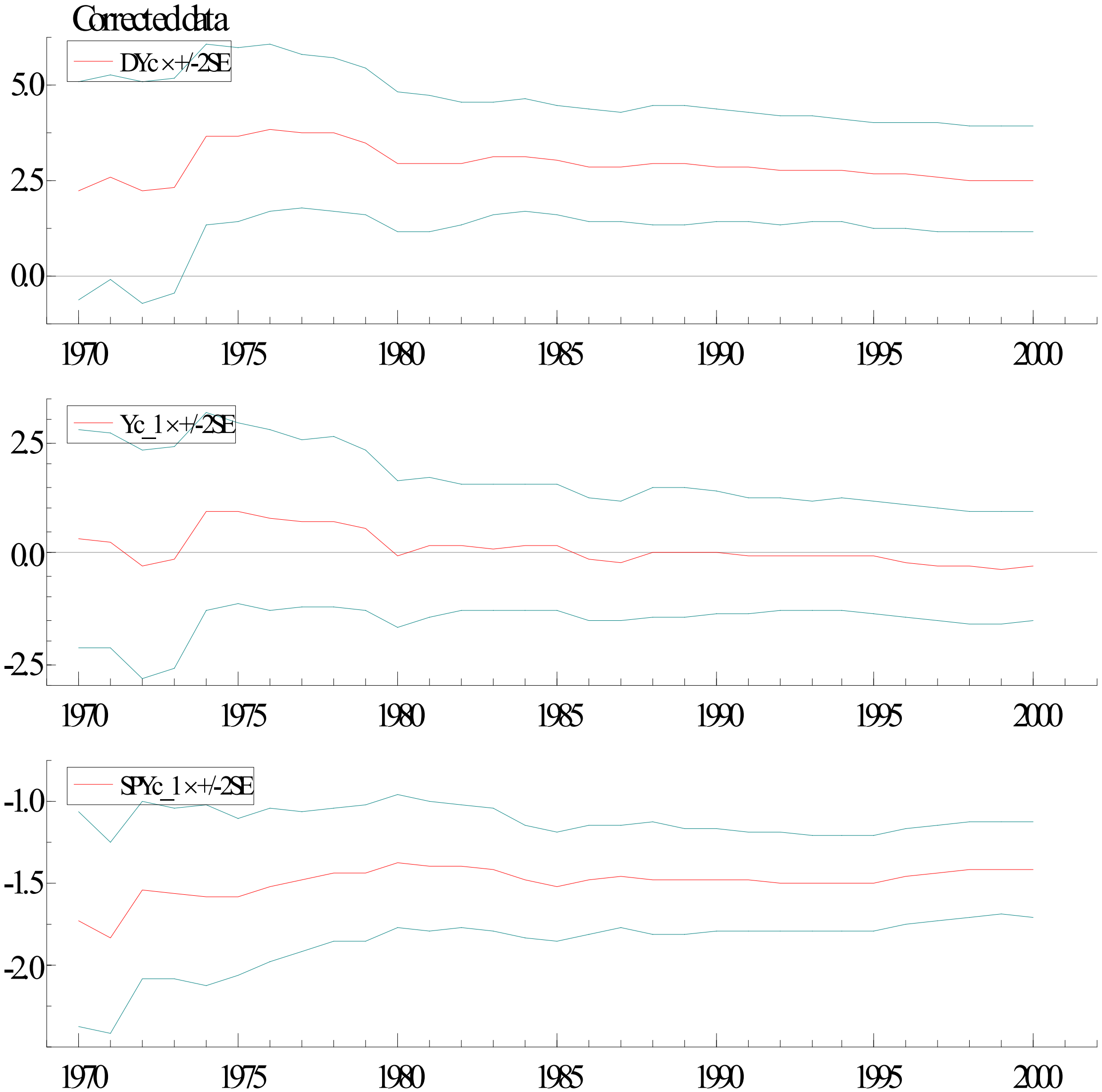
– run the regression with transformed variables over 1960-2000

$$\widehat{\Delta s_t^{\text{acf}}} = \underset{(-0.27)}{-0.363} \; \underset{(3.67)}{+2.52}\, \Delta y_t^{\text{acf}} \; \underset{(-10.0)}{-1.41} \left(s_{t-1}^{\text{acf}} - y_{t-1}^{\text{acf}} \right) \underset{(-0.49)}{-0.301}\, y_{t-1}^{\text{acf}}$$

where $R^2 = 76.3\%$,

$$\begin{aligned} \text{AR(2) test, } F\left(2,35\right) &: 2.47 \; \{9.9\%\} \\ \text{ARCH(1) test, } F(1,35) &: 1.72 \; \{19.9\%\} \\ \text{RESET, } F(1,33) &: 0.72 \; \{40.2\%\}\,. \end{aligned}$$

$-$ H_0: $\delta_1 = 0$ is supported throughout the sample



- deviations from cycles around the long-run fundamental values are restored well within a year;
- good fit.

4 Extensions

- The two-step procedure is not the most efficient estimation method:
  - we provide formulae for full GLS, QMLE, etc.;
  - but qualitative results remain unchanged.

  Example: QMLE case. For any given $\boldsymbol{R}$, define

$$\widehat{\boldsymbol{\beta}}_{\boldsymbol{R}} \equiv \left(\mathbf{X}'\boldsymbol{R}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\boldsymbol{R}^{-1}\boldsymbol{z}$$

  as a function of $\boldsymbol{R}$. The QMLE of $\boldsymbol{R}$ is obtained by maximizing

$$-\log\left|\left(\boldsymbol{z}-\mathbf{X}\widehat{\boldsymbol{\beta}}_{\boldsymbol{R}}\right)'\boldsymbol{R}^{-1}\left(\boldsymbol{z}-\mathbf{X}\widehat{\boldsymbol{\beta}}_{\boldsymbol{R}}\right)\boldsymbol{R}\right|$$

  with respect to the parameters of the ACF: the optimization of the joint likelihood (for $\boldsymbol{R}$ and $\boldsymbol{\beta}$) now depends on only 4 parameters that determine the whole autocorrelation matrix $\boldsymbol{R}$. Once the optimal value $\widehat{\boldsymbol{R}}$ of $\boldsymbol{R}$ is obtained, the MLE of $\boldsymbol{\beta}$ is $\widehat{\boldsymbol{\beta}} \equiv \widehat{\boldsymbol{\beta}}_{\widehat{\boldsymbol{R}}}$.