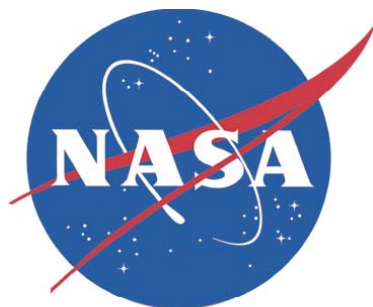

Inductive Pulsed Plasma Thruster Model with Time-Evolution of Energy and State Properties

19th Advanced Space Propulsion Workshop



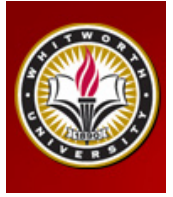
Kurt Polzin
NASA-Marshall Space Flight Center



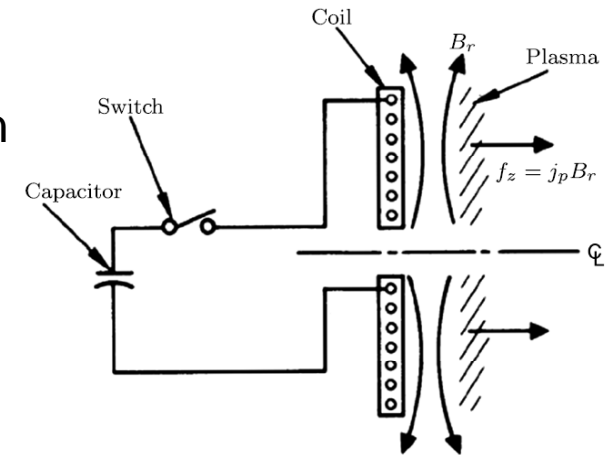
Kamesh Sankaran
Whitworth University



Inductive Pulsed Plasma Thrusters



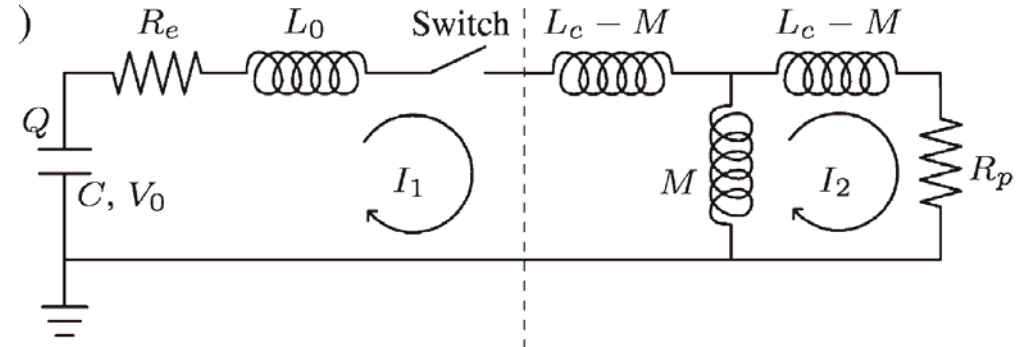
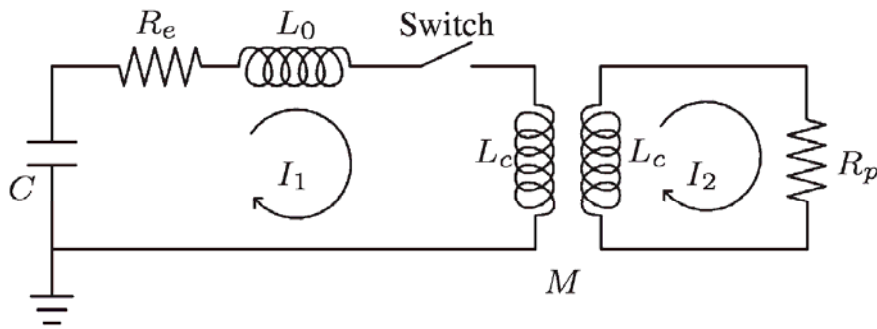
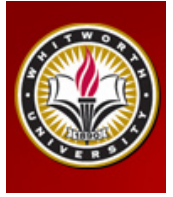
- Energy stored in capacitor banks
- High current switch(es) permit discharge through an inductive coil
- Fast-rising current ionizes/electromagnetically accelerates gas



- Pulsed Inductive Thruster (planar IPPT)
 - Since 1980 – 1-m diameter, Exclusively pulsed gas injection
 - Since 1990s – Marx generator spiral configuration, current IPPT SOA



Model / Motivation



$$\frac{dI_1}{dt} = \frac{VL_C + (MI_1 + I_2L_C)\frac{dM}{dt} - I_2MR_p - I_1R_eL_C}{L_C(L_0 + L_C) - M^2}$$

$$\frac{dI_2}{dt} = \frac{M\frac{dI_1}{dt} + I_1\frac{dM}{dt} - I_2R_p}{L_C}$$

$$\frac{dV}{dt} = -\frac{I_1}{C}$$

$$\frac{dM}{dt} = -\frac{L_C}{2z_0} \exp\left(-\frac{z}{2z_0}\right) \frac{dz}{dt}$$

$$m(t) = m_0 + \int_{t=0}^t \rho_A v_z dt$$

$$\frac{L_C I_1^2}{2z_0} \exp(-z/z_0) = \rho_A v_z^2 + m(t) \frac{dv_z}{dt}$$

$$\eta = 6 \times 10^{-4} T_e^{-3/2} \text{ ohm-m} \quad R_p \simeq \frac{\pi \eta (b + a)}{\delta_a (b - a)}$$

$$\delta_a(t) = \sqrt{\frac{\eta}{\mu_0} (t + t_0)}$$

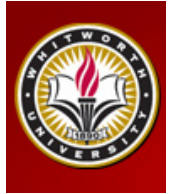
$$t_0 = \frac{\mu_0 \delta_s^2}{\eta}$$

- No self-consistent, time-dependent T_e
- No gas-dependent properties

Lovberg & Dailey, PIT Primer, 1994.



Modified Model



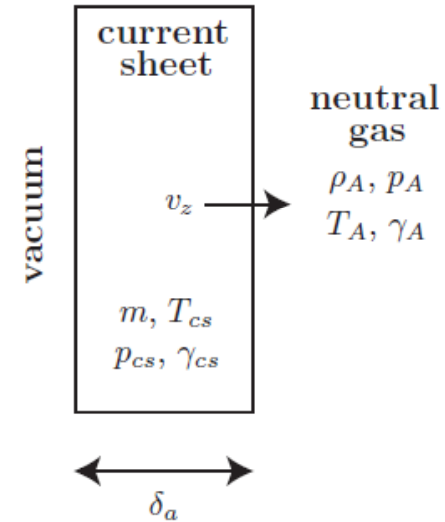
Circuit Equations (No Change)

$$\frac{dI_1}{dt} = \frac{V L_C + (MI_1 + I_2 L_C) (dM/dt) - I_2 M R_p - I_1 R_e L_C}{L_C (L_0 + L_C) - M^2}$$

$$\frac{dI_2}{dt} = \frac{M (dI_1/dt) + I_1 (dM/dt) - I_2 R_p}{L_C},$$

$$\frac{dV}{dt} = -\frac{I_1}{C}$$

$$\frac{dM}{dt} = -\frac{L_C}{2z_0} \exp\left(-\frac{z}{2z_0}\right) \frac{dz}{dt}$$



Continuity (No Change)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \longrightarrow \frac{dm}{dt} = \rho_A(z) v_z, \quad \text{where } \rho_A(z) = \begin{cases} \rho_0 (1 - z/\delta_m) \\ 0 \end{cases}$$

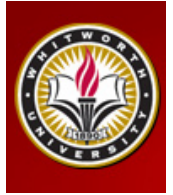
Momentum

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \mathbf{j} \times \mathbf{B}$$

$$\longrightarrow \frac{dv_z}{dt} = \left[\frac{L_c I_1^2}{2z_0} \exp\left(-\frac{z}{z_0}\right) - \rho_A(z) v_z^2 - p_A \pi (b^2 - a^2) \right] / m(t)$$

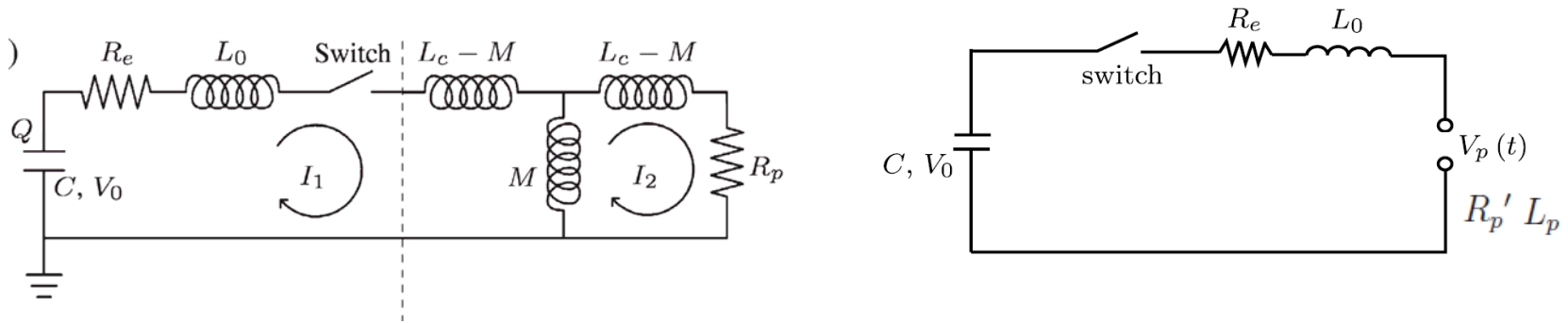


Adding Energy Equation



Energy

$$\frac{\partial \varepsilon}{\partial t} + \nabla \cdot [(\varepsilon + p) \mathbf{v} - \bar{\mathbf{B}}_M \cdot \mathbf{v}] = \nabla \cdot \left(\frac{-\mathbf{E}' \times \mathbf{B}}{\mu_0} \right) \quad \varepsilon = \frac{p}{\gamma - 1} + \frac{1}{2} \rho v^2 + \frac{B^2}{2\mu_0}$$



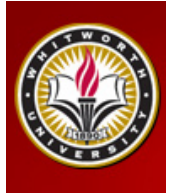
$$V_p = R_p I_1 + \dot{\phi}(t) = R_p' I_1 + L_p \frac{dI_1}{dt} + I_1 \frac{dL_p}{dt} \quad \text{Voltage across plasma}$$

$$P = \frac{d}{dt} \left(\frac{L_p I_1^2}{2} \right) + \frac{I_1^2}{2} \frac{dL_p}{dt} + I_2^2 R_p \quad \text{Net electrical power into plasma}$$

$$\frac{dL_p}{dt} = \frac{L_c}{z_0} \exp\left(-\frac{z}{z_0}\right) \frac{dz}{dt} \quad \text{Plasma inductance evolution}$$



Adding Energy Equation (cont)



Energy

$$\frac{\partial \varepsilon}{\partial t} + \nabla \cdot [(\varepsilon + p) \mathbf{v} - \bar{\mathbf{B}}_M \cdot \mathbf{v}] = \nabla \cdot \left(\frac{-\mathbf{E}' \times \mathbf{B}}{\mu_0} \right) \quad \varepsilon = \frac{p}{\gamma - 1} + \frac{1}{2} \rho v^2 + \frac{B^2}{2\mu_0}$$

$$\rightarrow \frac{dE}{dt} = \frac{L_c I_1^2}{2z_0} \exp\left(-\frac{z}{z_0}\right) v_z + I_2^2 R_p + \frac{d}{dt} \left(\frac{L_p I_1^2}{2} \right) - \left[\frac{1}{2} \rho_A v_z^2 + \frac{\gamma_A p_A}{\gamma_A - 1} \pi (b^2 - a^2) \right] v_z$$

① Electromagnetic work

② Ohmic Heating

③ Rate of change of electromagnetic field energy

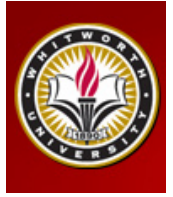
④ Power lost accelerating newly-entrained gas

⑤ Net internal power convecting into the current sheet +
work performed by ambient pressure against the current sheet face

$$E = \frac{p_{cs}}{\gamma_{cs} - 1} \delta_a \pi (b^2 - a^2) + \frac{1}{2} m v_z^2 + \frac{L_p I_1^2}{2}$$

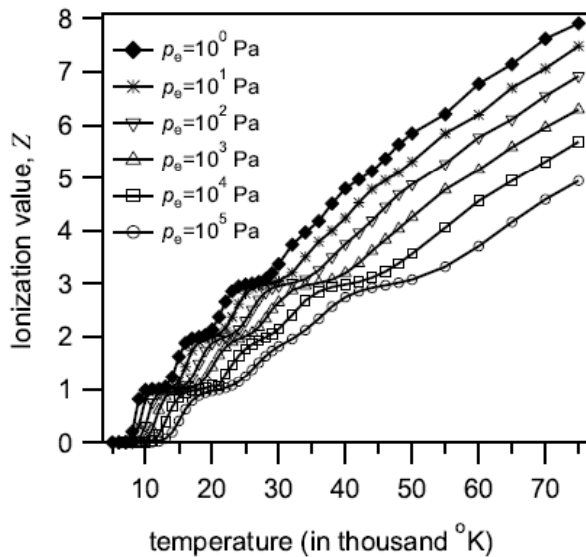


Plasma Model



Ionization (Equilibrium – Saha)

$$\frac{n_i n_e}{n_{i-1}} = \frac{2 (2\pi m_e k_B T)^{3/2}}{h^3} \frac{\sum_l g_l^i \exp(-\epsilon_l^i / k_B T)}{\sum_l g_l^{i-1} \exp(-\epsilon_l^{i-1} / k_B T)} = K_i \quad n_e^{N+1} + \sum_{l=1}^N \left[n_e^{N-l} (n_e - l n_0) \prod_{i=1}^l K_i \right] = 0$$

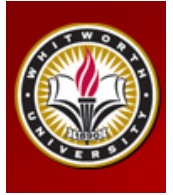


Transport (resistivity & collisionality)

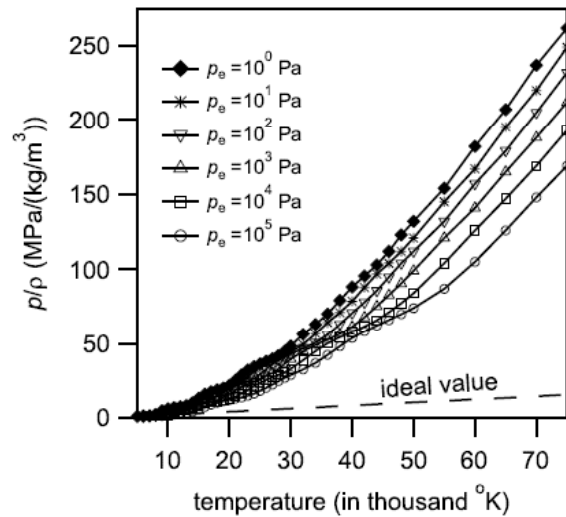
$$\eta = \frac{m_e \sum_s \nu_{es}}{n_e e^2} \quad \nu_{es} = n_s Q_{es} \sqrt{\frac{8k_B T_e}{\pi m_e}}$$



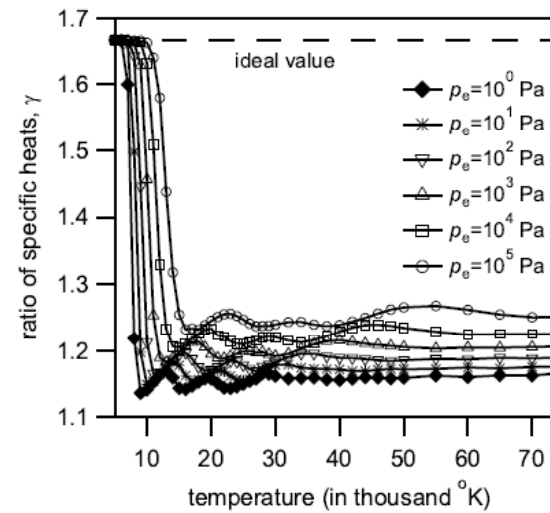
Plasma Model



Non-Ideal Equation of State



Specific Heat Ratio



$$\delta_a = \sqrt{\delta_s^2 + \frac{\eta}{\mu_o} t}$$

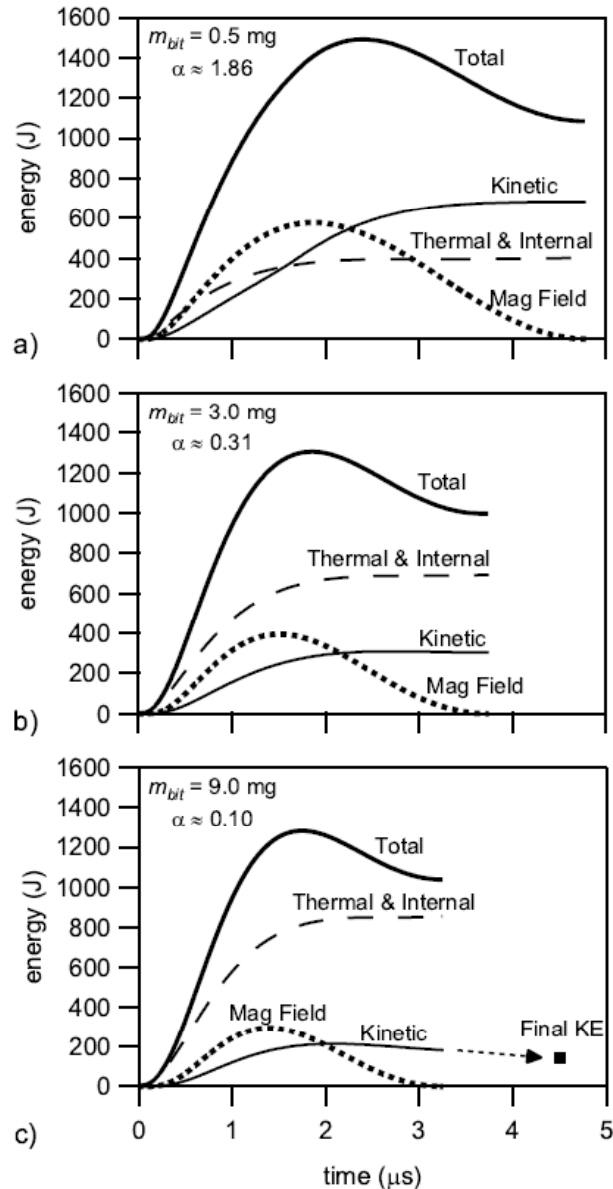
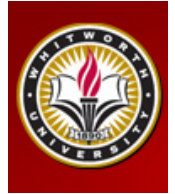
Sheet Thickness

$$R_p \approx \frac{\pi \eta (b + a)}{\delta_a (b - a)}$$

Plasma Resistance



Results I



$$\alpha = \frac{C^2 V_0^2 L_C}{2m_{bit} z_0^2}$$

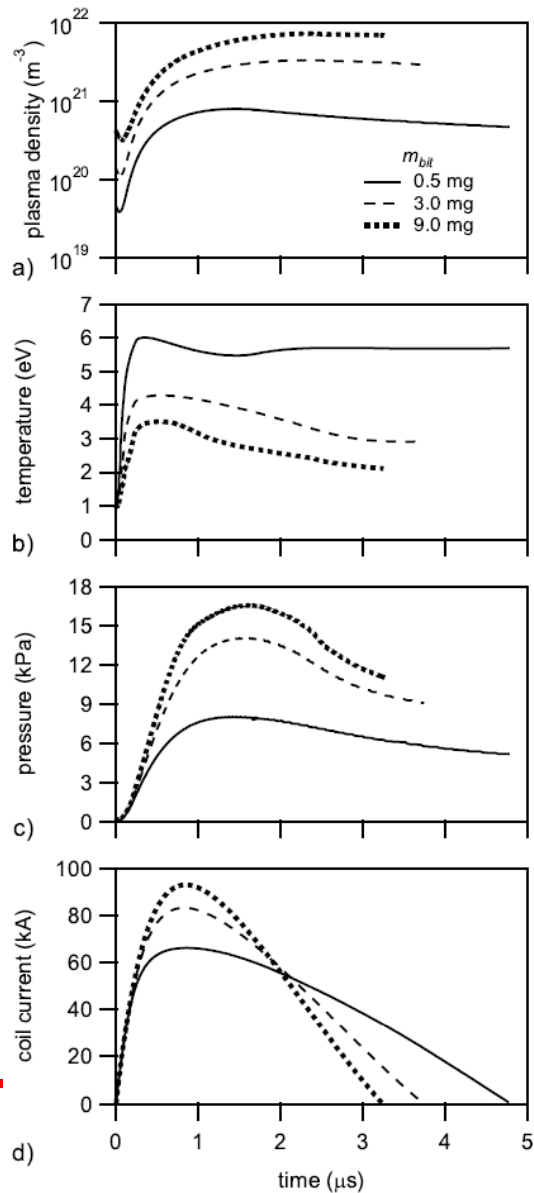
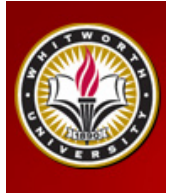
From top to bottom

Poorer dynamic impedance matching

- Kinetic and thermal energy increase early and level off (decoupling)
- Magnetic field energy oscillates, returning to zero at the end of the first half-cycle
- Shorter half-cycle in the poorer-matched case
- Energy shifts from kinetic to thermal/internal mode as mass bit increases



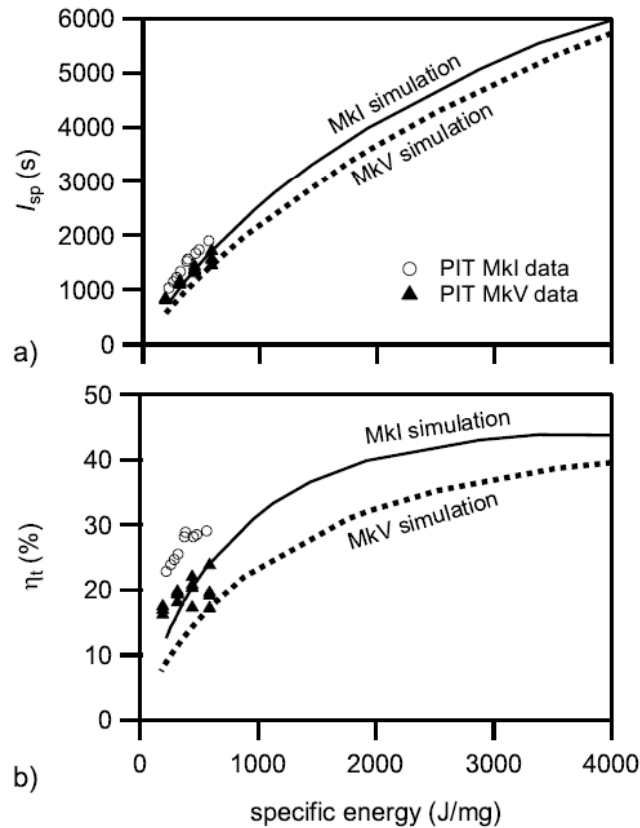
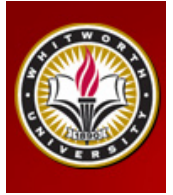
Results II



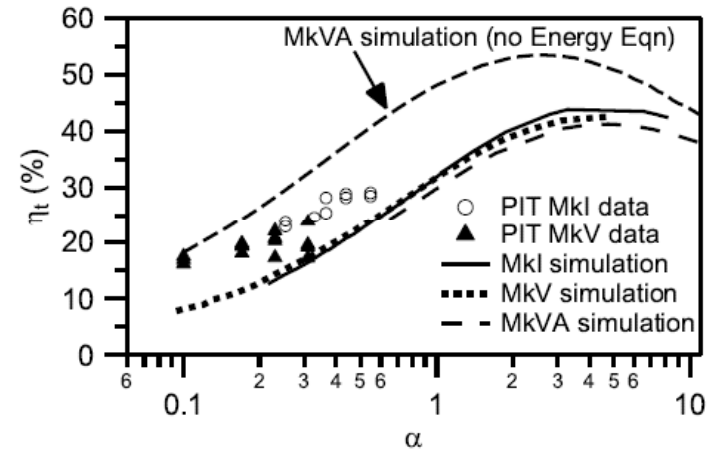
- Temperature, plasma density, and pressure
 - Increase early (Ohmic heating, multiple ionization, entrainment of gas)
- Temperature & plasma density
 - Stabilize in late-time (decoupled)
- Pressure
 - Decreases in late-time (sheet expansion)
- Current decreases in peak value, increases in 'half-cycle' duration
 - Asymmetric nature consistent with greater electromagnetic work



Results III



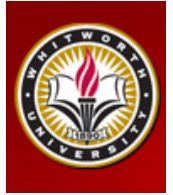
$$\alpha = \frac{C^2 V_0^2 L_C}{2m_{bit} z_0^2}$$



- I_{sp} – good quantitative agreement
- η_t – good agreement in form, magnitude difference suspected due to modeling assumptions
- New model
 - Shifts optimum α , lowers max achievable η_t relative to no E-eqn / plasma model simulation



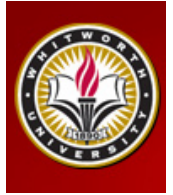
Conclusions



- Efficiency has a maximum value as a function of α , consistent with previous, more simplified IPPT modeling
- For same E_0 , higher efficiency achieved when the peak coil current was lower and more asymmetric, leading to significantly more energy being deposited in directed kinetic energy
- For argon, the plasma properties vary early in the discharge. As the sheet decouples, density and temperature reach approximately constant values while the pressure decreases through thermal sheet expansion
- Qualitatively and quantitatively the results from the model generally compare favorably with performance measurements. Disagreements can be attributed to the simplifying assumptions



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