A Bayesian Framework for Reliability Analysis of Spacecraft Deployments

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Abstract – Deployable subsystems are essential to mission success of most spacecraft. These subsystems enable critical functions including power, communications and thermal control. The loss of any of these functions will generally result in loss of the mission. These subsystems and their components often consist of unique designs and applications, for which various standardized data sources are not applicable for estimating reliability and for assessing risks. In this study, a Bayesian framework for reliability estimation of spacecraft deployment was developed for this purpose. This approach was then applied to the James Webb Space Telescope (JWST) Sunshield subsystem, a unique design intended for thermal control of the observatory's telescope and science instruments. In order to collect the prior information on deployable systems, detailed studies of "heritage information", were conducted, extending over 45 years of spacecraft launches. The NASA GSFC Spacecraft Operational Anomaly and Reporting System (SOARS) data were then used to estimate the parameters of the conjugative beta prior distribution for anomaly and failure occurrence, as the most consistent set of available data and that could be matched to launch histories. This allows for an empirical Bayesian prediction for the risk of an anomaly occurrence of the complex Sunshield deployment, with credibility limits, using prior deployment data and test information.

Index Terms – NASA GWST, Deployment subsystems, Bayesian Reliability, reliability test planning

Acronym[1]

PDF probability density function

ML maximum likelihood

[1] The singular and plural of an acronym are always spelled the same.

Notation

JWST	James Webb Space Telescope
p_f	Probability of deployment failure
$p_{f\,prior}$	Prior probability of deployment failure
$I_x(a, b)$	Incomplete beta function
$\Gamma(m)$	Gamma function

1. Introduction. Spacecraft Deployable Subsystems and Their Reliability Estimation

Deployable subsystems are essential to mission success of most spacecraft. These subsystems enable critical functions including power, communications and thermal control. The loss of any of these functions will generally result in loss or significant degradation of the mission [Freeman 1993, Saleh and Castet 2011, de Selding 2012]. These subsystems and their components often consist of unique designs and applications, for which various standardized data sources are not applicable for estimating reliability and for assessing risks.

From the reliability standpoint, deployable subsystems are best modeled as one-shot systems, for which probability of a failure/success event is governed by the binomial distribution. The mathematically correct classical (frequentist) maximum likelihood (ML) estimate of the *probability of deployment failure* p_f is the simple common sense estimate which is given by

$$\hat{p}_f = \frac{n}{N} \tag{1}$$

where N is the total number of trials (deployments), n is the number of unsuccessful trials, and $\widehat{p}_f$ is the estimate of the p_f.

As a rule, one is interested in the upper $(1 - \alpha)$ confidence limit on the probability of deployment failure, which is given as a solution with respect to p of the following equation

$$I_{1-p}(N-n, n+1) \leq \alpha \qquad (2)$$

where the incomplete beta function is given by [Lawless, 2003]

$$I_t(a,b) = \begin{cases} \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^t x^{a-1}(1-x)^{b-1}dx, & 0 \leq t \leq 1, a>0, b>0 \\ 0, \quad if\ t<0 \\ 1, \quad if\ t>1. \end{cases} \qquad (3)$$

and $\Gamma(x)$ is the gamma function given by:

$$\Gamma(a) = \int_0^\infty x^{a-1} e^{-x} dx \qquad (4)$$

2. Bayesian Approach to Reliability Estimation Deployable Subsystems

In the given Bayesian approach, the standard beta distribution is applied as the prior distribution of the probability of deployment failure. Its probability density function (PDF) is defined over the interval [0, 1], and it is given by

$$f(t;\alpha,\beta) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} t^{\alpha-1}(1-t)^{\beta-1}, & 0 \leq t \leq 1, \alpha>0, \beta>0 \\ 0, \quad otherwise \end{cases} \qquad (5)$$

Note that depending on its parameters, the beta distribution has very different shapes as illustrated by the Figure 1, thereby allowing flexibility in characterizing uncertainty.

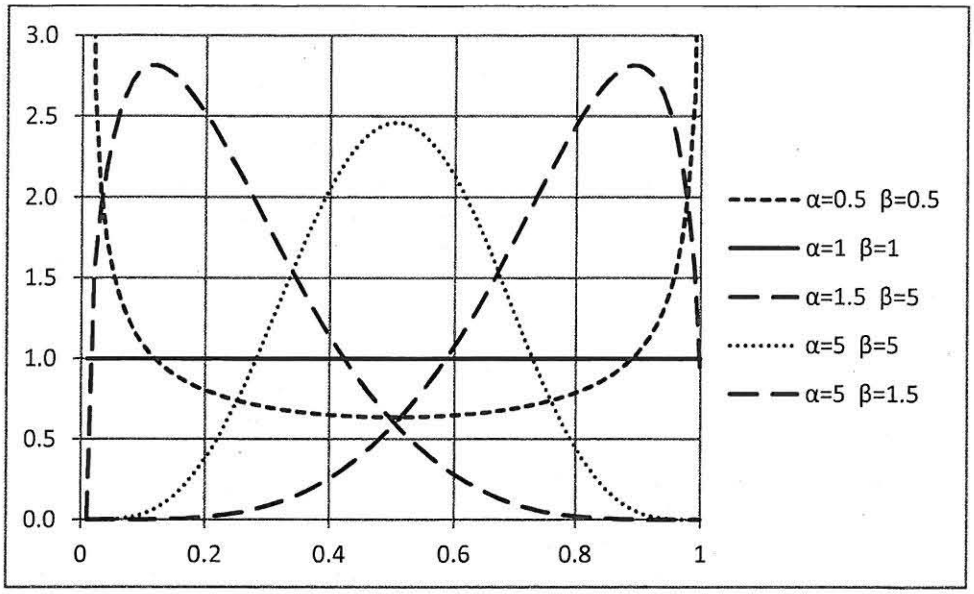

Figure 1. Probability density functions of beta distribution

It is interesting that the standard uniform (flat) distribution is a particular case of the beta distribution with $\alpha = 1$ and $\beta = 1$.

It should be noted that the beta distribution as a prior distribution in binomial probability estimation is the *conjugative* distribution, which means that the posterior estimate of interest is also the beta distribution. This allows for by- passing complex numerical integrations.

In this study, the prior distribution is estimated based on some appropriate data. This approach is known as the *empirical* Bayesian as opposed to Bayesian estimation based on elicitation of expert opinion.

In the framework of the empirical Bayesian approach, the prior information might be a set of one-shot system failure/success data based on historical performance. Let's assume we have n_0 trials out which x_0 are failures. In this case, the conjugate prior distribution is the beta distribution with parameters $\alpha = x_0$ and $\beta = n_0 - x_0$. At this point, it is important to note that the mean of the prior beta distribution $p_{f\,prior}$ is given by

$$p_{f\,prior} = \frac{x_0}{n_0}, \tag{6}$$

which coincides with classical estimate (1) of the probability of deployment failure. Thus, if there are available data on success/failure deployment related to some similar (from engineering standpoint) subsystems, these data can be used to estimate the parameters of the beta prior distribution.

Next, let's assume that we have the test deployment results (data) related to the subsystem of interest, which are x failures out of n deployments (trials). Based on the Bayes' theorem, the posterior PDF of the probability of deployment failure can be written as

$$f(p|x) = \frac{\Gamma(n+n_0)}{\Gamma(x+x_0)\Gamma(n+n_0-x-x_0)} p^{(x+x_0)-1} (1-p)^{(n+n_0-x-x_0)-1}, \tag{7}$$

which is obviously the PDF of the beta distribution.

The corresponding posterior mean (which is the Bayesian point estimate of the failure probability) is given by

$$p_B = \frac{x+x_0}{n+n_0} \tag{8}$$

It should be noted that when $n >> n_0$ and $x >> x_0$, the Bayesian estimate (8) is getting closer to the classical estimate (1) based on the test data. In other words, the classical statistical inference tends to dominate over the Bayesian one. Analogously, if $n_0 >> n$ and $x_0 >> x$, the Bayesian inference tends to dominate.

Based on the posterior PDF (7), the $(1 - \alpha)$ upper limit $p_{B\,up}$ of Bayes' probability interval (the Bayesian analog of the classical upper confidence limit) is a solution of the following equation with respect to p

$$I_p(x + x_0, n + n_0 - x - x_0) = \alpha \tag{9}$$

Consider the following numerical example. Let the collected prior information be summarized as 100 deployments with, say, 2 failures, i.e., $n_0 = 100$ and $x_0 = 2$. The test data for a given deployable subsystem is limited to 10 failure-free deployments i.e., $n = 10$ and $x = 0$.

In this case, based on the test data classical point estimate (1) of probability of deployment failure is 0, which is not very informative. The classical upper 90% confidence limit on the failure probability calculated using Equation (2) is 0.206. Based on the prior and test data, the respective Bayesian upper 90% limit is 0.035, which looks consistent with the data it is based on.

3. Prior Data Sources for Deployable Subsystems Reliability Estimation

In analyzing deployments, several sources of information may be used for the construction of a prior distribution. In this study, sources of data analyzed, included the Spacecraft Mechanism Handbook and the Goddard Space Flight Center Spacecraft Operational Anomaly Reporting System (SOARS). SOARS is a demonstrated consistent source of historical data for NASA Goddard Space Flight Center projects [Robertson and Stoneking 2003]. This provided a look at 45 years of deployment history. The total number of failures reviewed included 52 known failures. Figures 2 and 3 show a classification of all 52 failures by subsystems and assignable causes. Failures on the same spacecraft, appearing in both data sets, are treated as only 1 failure.

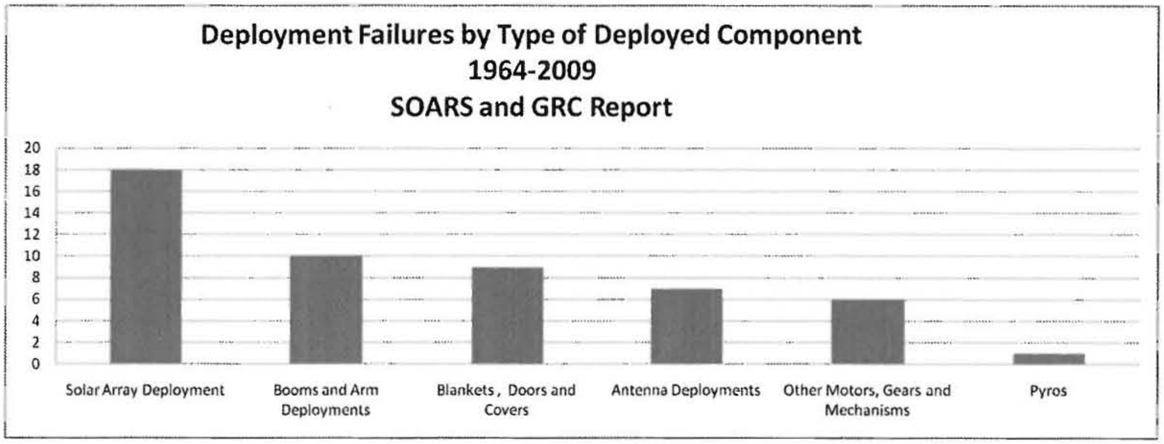

Figure 2. Classification of failures by deployed Component Type.

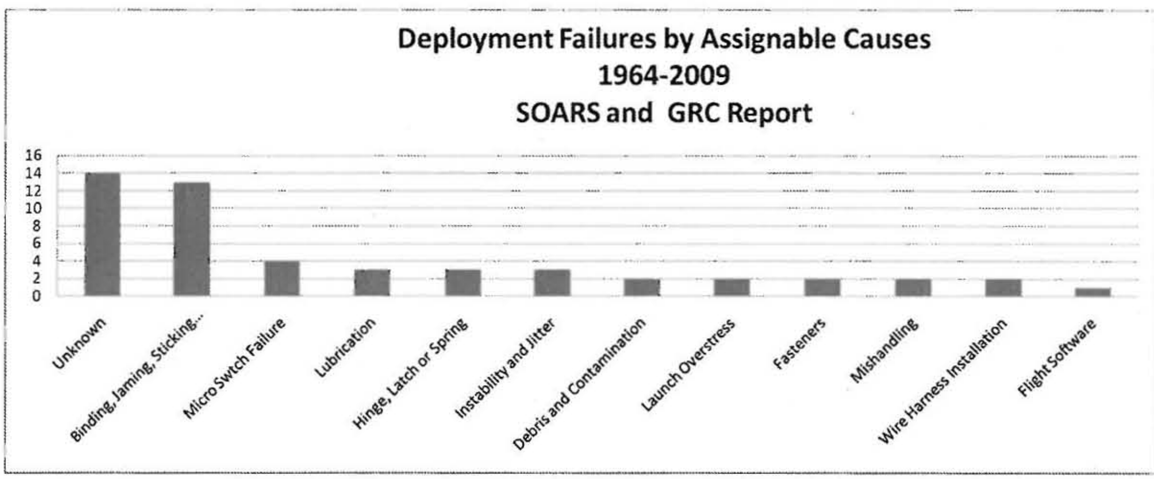

Figure 3. Classification of failures by Assignable Causes.

Studies to support documentation of lessons learned for the Spacecraft Mechanism Handbook reflect failures occurring on military and civil spacecraft launched between 1964 and 1997. These data showed 34 failures. The exact population of spacecraft is not known for this data. However, there were approximately 1262 civil and military missions launched by the United States in this period. With a few exceptions, the data reflect largely mission ending failures, which were not overcome by operational workarounds and may not represent a complete anomaly record. The failure records can be examined in [Fusaro, 1998].

The SOARS records reflect NASA GSFC civil spacecraft developed and launched from 1978 to the present. The data reflected 19 failures including both mission ending and failures which were overcome by operational workarounds. During this period, there were 123 spacecraft successfully launched into orbit by NASA GSFC. This provides the most consistent data set for the construction of a prior distribution. Note that data were not segregated by severity for this example. This is of course an option in applying this methodology to test design.

4. Case Study — JWST Sunshield Deployment

The James Webb Space Telescope is the next generation space telescope, which will view deep space in the infrared, beginning with its launch in 2018. JWST will be one of the most complex deployable structures ever launched and will enable NASA to peer to the epoch of the formation of the very first luminous objects after the primordial Big Bang. The JWST is shown in Figure 4, as it will be deployed in the Sun-Earth L2 orbit, in which it will serve its mission.

4.1 The JWST Sunshield and its Deployment

Central to the success of the mission is the sunshield structure, a tennis court size, multi-layer, gossamer film structure, which enables the telescope and science instruments to cool to cryogenic temperatures, while blocking light from the sun.

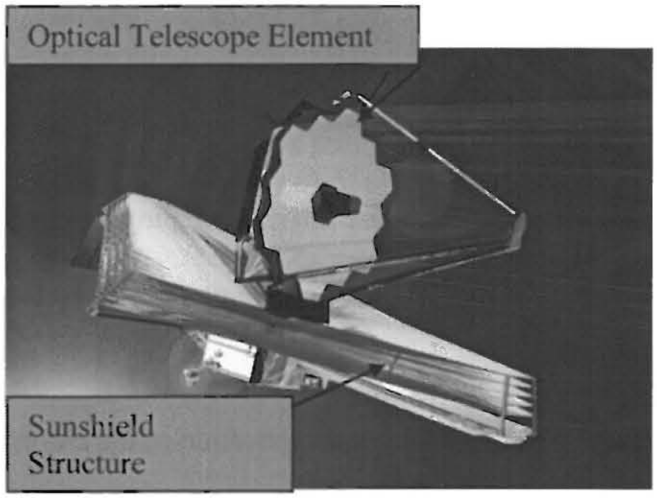

Figure 4. The James Webb Space Telescope in its deployed configuration showing the optical telescope element and sunshield.

The sunshield deployment from the stowed launch configuration consists of several key steps. Figure 5 shows how the deployment progresses from the launch to operational configurations. The deployment steps can be classified into 3 major deployment sequences. This includes deployment of the structural supports, membrane release and tensioning of the 5 membrane layers.

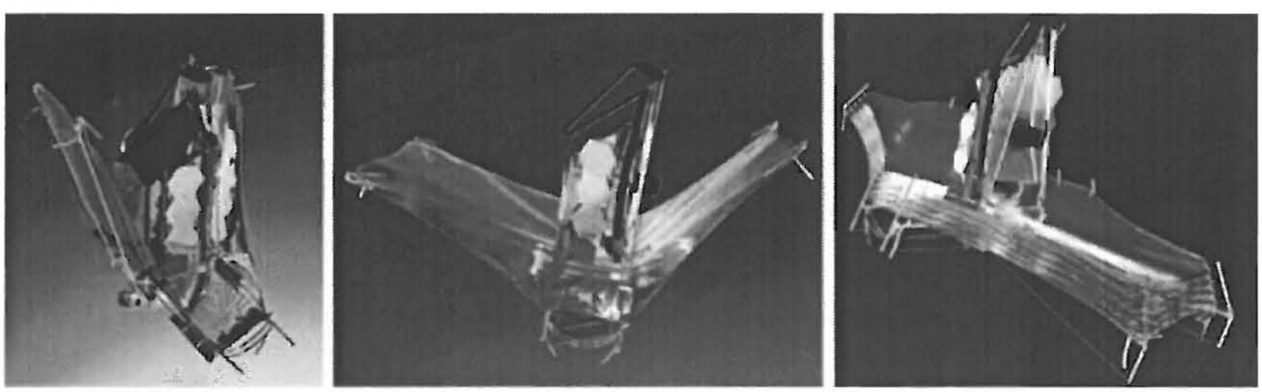

Figure 5. Deployment of the sunshield from launch to operational configuration.

The SOARS records from 1978 through 2009 were analyzed to generate a prior distribution for this analysis. Out of these records, 123 missions were selected as having the deployable subsystems, which can be used as the prior data for the JWST sunshield Bayesian reliability analysis. In 19 of these missions, deployable subsystem anomalies occurred, ending the mission, degrading the mission or creating an operational contingency.

In this case study, we are considering application of the Bayesian approach to test design. Let's assume that a test sequence of 10 deployments has been run and the test results are failure free. Based on the prior data, the prior PDF is depicted in Figure 6.

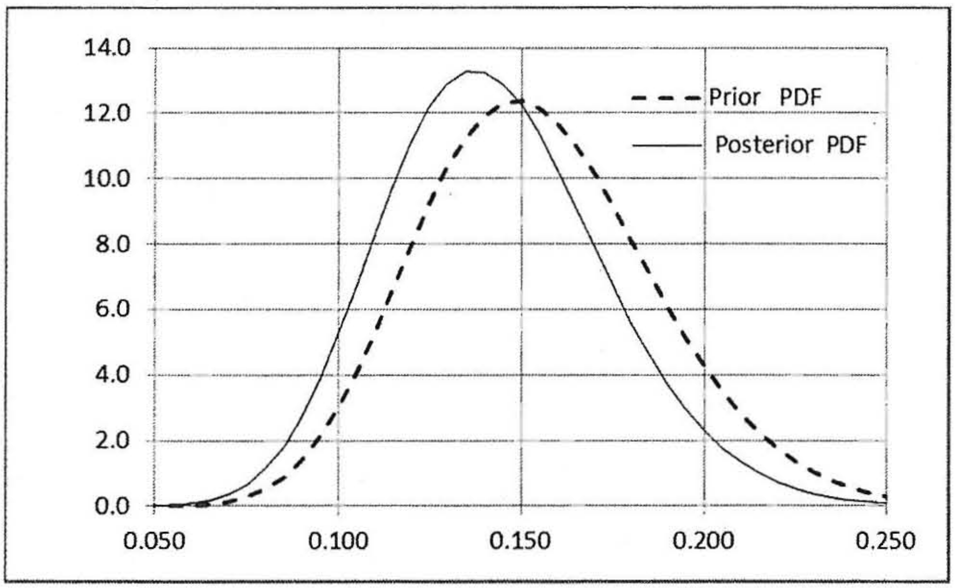

Figure 6. Probability density functions of prior and posterior distributions of probability of deployment failure.

The prior mean coinciding with the classical maximum likelihood (ML) estimate (1) is $\frac{19}{123} = 0.154$. Using Equation (8), the Bayesian point estimate is evaluated as

$p_B = \frac{19}{123+10} = 0.143$. Based on the prior and test data, the respective Bayesian upper 90% limit is 0.182. Clearly, the minimum test sequences to run for the system can be targeted based upon the desired risk reduction using this approach.

Now, we assume that the test result is one failure out of 10 deployment sequences. In this case, the Bayesian point estimate is 0.154 and the Bayesian upper 90% limit is 0.191. If our analysis was limited to the classical approach, we could only compare the 90% upper confidence limit on failure probability for 0 out of 10 test result with the test having one failure out of 10, which are 0.205 and 0.337 respectively. We can see that using the prior data in the framework of Bayesian of reliability estimation is rather robust with respect to the test results. It can be explained by the dominance of the prior information over the test data, which is, to an extent, typical for the deployable systems of interest.

It should be noted that the Bayesian estimate of probability of deployment failure can be updated not only as a result of an additional test runs, but also through updating the prior information, as soon as new appropriate data come to SOARS.

5. Conclusions

In this paper we have presented an empirical Bayesian approach to analysis of deployment risk and reliability. The deployable system is treated as a one-shot system governed by the binomial distribution. This allowed for the use of conjugate beta distributions to explicitly treat the uncertainties in the probability of success. The application is demonstrated by treating an example test case using 10 deployment sequences for a complex deployable system. This methodology can also be used to establish test cycles needed to achieve a particular risk or reliability target. The methodology uses real data explicitly. However, the historical or prior data can be expected to dominate the results of the posterior estimates.

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