Sound Propagation in Gas-Vapor-Droplet Suspensions with Evaporation and Nonlinear Particle Relaxation

Max Kandula

ESC - Team QNA, NASA Kennedy Space Center
FL 32899, USA

Abstract. The Sound attenuation and dispersion in saturated gas-vapor-droplet mixture in the presence of evaporation has been investigated theoretically. The theory is based on an extension of the work of Davidson (1975) to accommodate the effects of nonlinear particle relaxation processes of mass, momentum and energy transfer on sound attenuation and dispersion. The results indicate the existence of a spectral broadening effect in the attenuation coefficient (scaled with respect to the peak value) with a decrease in droplet mass concentration. It is further shown that for large values of the droplet concentration the scaled attenuation coefficient is characterized by a universal spectrum independent of droplet mass concentration.

Keywords: Sound Attenuation, Gas-droplet-vapor suspensions
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INTRODUCTION

The propagation of sound in a gas containing suspended droplets is of considerable interest in many important technical applications such as combustion chamber instability, and jet noise mitigation by water injection. Theories of sound propagation in saturated gas-vapor-droplet mixtures have been presented in [1-4] on the basis of linear laws for particle momentum, heat and mass transfer (relaxation). The linear theories do not account or explain the attenuation behavior characterized by a spectral peak experimentally observed in the spatial absorption coefficient [5-7]. This work addresses the role of nonlinear relaxation on the attenuation and dispersion of sound in evaporating gas-droplet mixtures initially under thermodynamic equilibrium.

PROPOSED THEORY WITH NONINEAR RELAXATION

The present analysis extends the work of Davidson[4] to accommodate the effect of nonlinear relaxation on the mass, momentum and heat transfer between the droplet and the surrounding gas. Without any loss of generality the propagation of sound accounting for particle nonlinearity may be expressed by:

$$K = k_1 + ik_2 = f\left(\tau_{d1}, \tau_{t1}, \tau_{c1}, C_m, C_v, H_p, H_v, H_g, \mathrm{Pr}, Le, \gamma, \gamma_v, L, R_v / R_g\right) \tag{1}$$

where $\tau_{d1}, \tau_{t1}, \tau_{c1}$ are related to the linear relaxation times τ_d, τ_t, τ_c as follows:

$$\tau_{d1} = \psi_d \tau_d, \quad \tau_{t1} = \psi_t \tau_t, \quad \tau_{c1} = \psi_c \tau_c \tag{2a}$$

where $$\psi_d = C_{D1} / C_D, \quad \psi_t = Nu_1 / Nu, \quad \psi_c = Sh_1 / Sh \tag{2b}$$

and $$\tau_d = d_p^2 \rho_p / (18\mu_0), \; \tau_t = (3/2)\Pr \tau_d, \; \tau_c = (3/2) Sc \tau_d = Le \tau_t \tag{3}$$

with C_{D1}, Nu_1, Sh_1 stand respectively for the drag coefficient, Nusselt number and Sherwood number correspond to nonlinear particle relaxation. Also C_m, C_v refer to droplet and vapor mass loading respectively. See [4,8] for a detailed nomenclature.

The factors ψ_d, ψ_t, ψ_c corresponding to nonlinear relaxation can be expressed by

$$\psi_d(\mathrm{Re}_p) = 1 + \frac{\mathrm{Re}_p}{24}\left(\frac{6}{1+\sqrt{\mathrm{Re}_p}} + 0.4\right) \tag{4a}$$

$$\psi_t(\mathrm{Re}_p, \Pr) = 1 + 0.3\,\mathrm{Re}_p^{0.5} \Pr^{0.33}, \quad \psi_c(\mathrm{Re}_p, Sc) = 1 + 0.3\,\mathrm{Re}_p^{0.5} Sc^{0.33} \tag{4b,c}$$

Equations (8a)-8(c) are respectively based on [9],[10] and [11]. On the basis of a comparison between the theory (in the absence of mass transfer) and data on supersonic jets with water droplets, the particle Reynolds number is correlated as [5]

$$\mathrm{Re}_p = 10(\omega \tau_d)^3 \tag{5}$$

The dimensionless energy attenuation coefficient α and the propagation speed c are related to the complex wave number K by the relation

$$\alpha = 2k_2, \quad c / c_0 = 1 / k_1 \tag{6}$$

noting that plane wave solutions of the form $f = f_0 \exp\{i(K\omega x / c_0 - \omega t)\}$.

RESULTS AND DISCUSSION

Comparison with Data of Cole and Dobbins

Figure 1a shows a comparison of linear and nonlinear theories with the data of Cole and Dobbins[12] corresponding to an average gas temperature of $T_0 = 276\,\mathrm{K}$ (Davidson[4]): $C_m = 0.00763$, $C_v = 0.00807$, $\Pr = 0.715$, $Le = 0.839$, $\gamma_v = 1.32$, γ=1.4, R_v / R_g=1.61, $H_g = 1$, H_v=1.9, H_p=4.2, L=19.7.

These parameters describe mean fog conditions in the experiment. The results suggest that the nonlinear theory shifts the curve of the linear theory to the left, but the differences at this concentration level are not appreciable. The attenuation peak for the nonlinear theory slightly exceeds that for the linear theory.

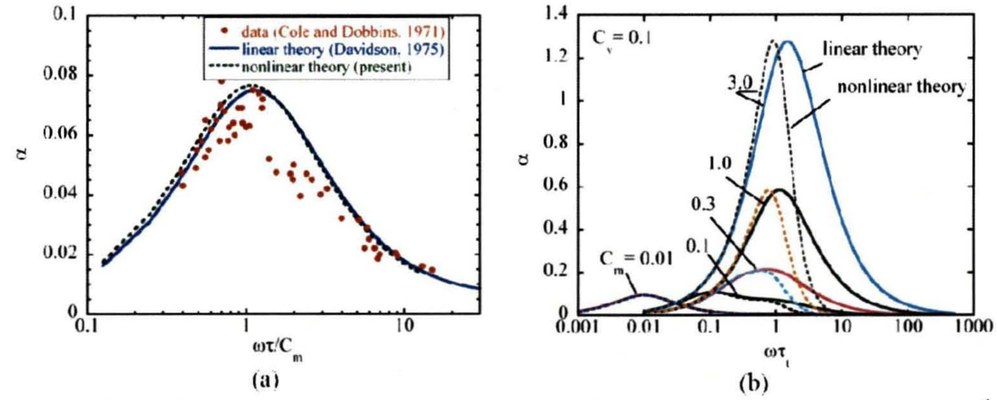

FIGURE 1. Comparison of dimensionless attenuation coefficient from Davidson's linear theory[4] and the present nonlinear theory with the experimental data of Cole and Dobbins[12]; (b) Effect of nonlinear particle relaxation on the dimensionless attenuation coefficient at various droplet mass concentrations in evaporating gas-vapor-droplet mixtures.

Predictions at High Droplet Concentration

Calculations are performed for sound propagation in air-water droplet mixtures at high droplet mass concentration The parameters are: $C_v = 0.1, \Pr = 0.7$, $Le = 0.85, \gamma = 1.4$, $\gamma_v = 1.32$, $R_v / R_g = 1.61$, $H_g = 1$, $H_v = 2.0$, $H_p = 4.15$, $L = 15.7$.

Attenuation

Figure 1b presents the effect of nonlinear particle relaxation on the variation of the dimensionless absorption coefficient with the particle relaxation time for a wide range of droplet concentration. For C_m above 0.3, only one distinct attenuation peak is observed, suggesting the existence of interaction between mass, momentum and heat transfer.

Figure 3 presents the spectral distribution of the scaled attenuation coefficient $\alpha / \alpha_{\max}$ as a function of $\omega / \omega_{\max}$ for various values of c_m ranging from 0.1 to 3.0. Here the quantities $\alpha_{\max}, \omega_{\max}$ correspond to the peak attenuation. For frequencies below the peak value, the shape of the spectrum of the scaled attenuation broadens with a decrease in value of C_m. For large values of C_m = 0.3 and above, the shape of the scaled attenuation curve is characterized by a universal spectrum independent of c_m.

Dispersion Coefficient

Figure 4 displays the variation of the dispersion coefficient $\beta = (c_0 / c)^2 - 1$ with particle relaxation time $\omega\tau_t$. It is evident that the nonlinear effects become important for $\omega\tau_t$ exceeding about 0.2 to 0.4, where viscous and heat conduction effects become important. In this range of $\omega\tau_t$, the dispersion coefficient according to the nonlinear theory is smaller than that provided by the linear relaxation.

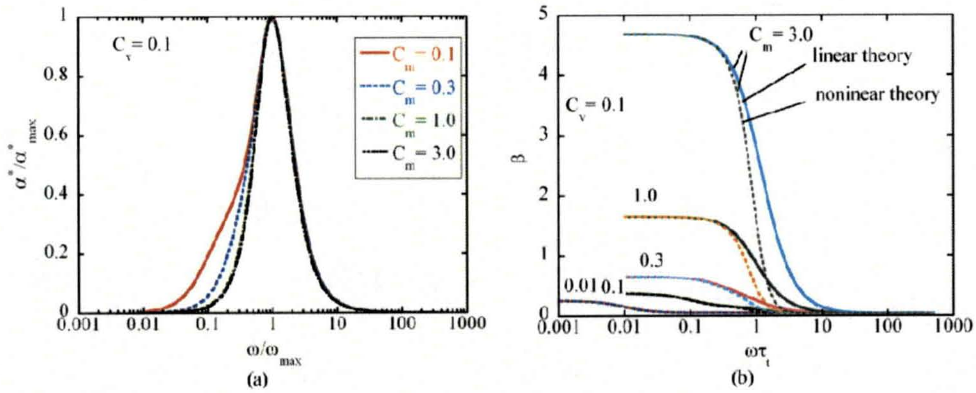

FIGURE 2. (a) Spectral distribution of the scaled attenuation illustrating spectral broadening effect in evaporating gas-vapor-droplet mixtures. (b) Effect of nonlinear particle relaxation on the dispersion coefficient at various droplet mass concentrations in evaporating gas-vapor-droplet mixtures.

CONCLUSIONS

The results suggest that at sufficiently high droplet concentration, significant interaction between the transport processes result in only one distinct attenuation peak. It is shown that with nonlinear relaxation the peak frequency is reduced relative to that indicated by the linear theory. At large values of the droplet mass concentration, the scaled attenuation spectrum exhibits a universal shape independent of the droplet mass concentration. The results also point to the existence of spectral broadening for low droplet mass concentration.

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