

## Parameter Estimation of Spacecraft Fuel Slosh Model

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### **Abstract**

Fuel slosh in the upper stages of a spinning spacecraft during launch has been a long standing concern for the success of a space mission. Energy loss through the movement of the liquid fuel in the fuel tank affects the gyroscopic stability of the spacecraft and leads to nutation (wobble) which can cause devastating control issues. The rate at which nutation develops (defined by Nutation Time Constant (NTC)) can be tedious to calculate and largely inaccurate if done during the early stages of spacecraft design. Pure analytical means of predicting the influence of onboard liquids have generally failed. A strong need exists to identify and model the conditions of resonance between nutation motion and liquid modes and to understand the general characteristics of the liquid motion that causes the problem in spinning spacecraft. A 3-D computerized model of the fuel slosh that accounts for any resonant modes found in the experimental testing will allow for increased accuracy in the overall modeling process. Development of a more accurate model of the fuel slosh currently lies in a more generalized 3-D computerized model incorporating masses, springs and dampers. Parameters describing the model include the inertia tensor of the fuel, spring constants, and damper coefficients. Refinement and understanding the effects of these parameters allow for a more accurate simulation of fuel slosh. The current research will focus on developing models of different complexity and estimating the model parameters that will ultimately provide a more realistic prediction of Nutation Time Constant obtained through simulation.

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## Introduction

Fuel slosh in spacecraft has been a long standing concern within the aerospace community and has been a source of dynamics and control problems [1]. Movement of liquid fuel in the fuel tank causes a loss of rotational energy of the spacecraft. Such energy loss will cause the craft to nutate, or wobble, about its primary spin axis. The amount of time it takes for the cone angle of the nutation to increase by a factor of  $e^1$  is defined as the Nutation Time Constant (NTC) and is a primary factor in determination of the long term stability of the spinning spacecraft during the upper stage burn.

During the initial design of spacecraft, use of simple analytical models will not consistently yield accurate results in determination of the stability of the configuration. Analytical determination of the dynamic behavior of a spacecraft can be significantly different from the actual flight values. Computational fluid dynamics software packages provide some insight, but it turns out that they have several shortcomings. Their complexity and inability to accurately model the coupling effects of sloshing mass on the six degree-of-freedom motion experienced by the spacecraft make their application problematic. Accurately quantifying the fuel slosh effects using equations is not feasible during the initial phases of design. Simplified models will greatly help to reduce computational time and design costs [2].

Liquid oscillations in spinning tanks have been studied in the past. Liquid oscillations in spinning fuel tanks produce very different response characteristics compared to those of non-spinning fuel tanks [3]. An energy sink model was originally developed by Thomson [4] to include the effects of small, passive sources of energy dissipation. This model does not work well for spacecraft fuel slosh energy dissipation due to the fact that fuel mass is a large fraction of the total mass of the spacecraft.

Use of mechanical analogs to simulate sloshing mass is a common alternative to fluid modeling. Extensive analysis has been done on the different tank shapes and locations, as well as the use of propellant management devices (PMD). A summary of this analysis, like that reported by Hubert [11] shows the vast differences in possible behaviors of different designs. For off-axis, cylindrical tanks with hemispherical endcaps that have been popular in a number of spacecraft programs, a number of relatively simple mechanical models have been developed. Hubert also notes that one of the most difficult aspects of employing such mechanical models is in the selection of appropriate parameters in the model.

A homogeneous vortex model of liquid motions in spinning tanks and an equivalent mechanical model was developed by Dodge et al., [5]. An approximate theory of oscillations that predicts the characteristics of the dominant inertial wave oscillation and the forces and moments on the tank are described. According to Dodge et al., the pendulum model simulates a motion that does not involve an oscillation of the center of mass. Therefore, it is not a valid model of inertial wave oscillations. Weihs and Dodge [6] illustrate that the free surface effects can be ignored when the liquid depth is small.

A 3-D pendulum model was proposed by Green et al., [7]. There was evidence of liquid resonance from the experimental data. The resonance was closely tied to the tangential torque and to a lesser degree to the radial torque, and there was little or no resonance in the force measurements. Green et al., proposed a rotary oscillator concept to simulate the torque resonance in tangential and radial directions. This rotary oscillator model was superimposed on the pendulum model to provide the overall response of liquid oscillation in the tank.

Development of a more accurate model currently lies in a more generalized model incorporating masses, springs and dampers. Translational and rotational motion about three axes in a rectangular coordinate system allows the sloshing mass to move with less restriction. Parameters describing the model include the inertia tensor of the fuel, spring constants, and damper coefficients. Refinement of these parameters allow for the most accurate simulation of fuel slosh. The current research will focus on developing models of different complexity and estimating the model parameters from simulated model values and actual experimental values).

**Problem Definition**

Various parameters are estimated by matching the rotor model response to the experimental response from Southwest Research Institute's Spinning Slosh Test Rig (SSTR). The experimental set-up of the SSTR is shown in Figure 1. The SSTR can subject a test tank to a realistic nutation motion, in which the spin rate and the nutation frequency can be varied independently, with the spin rate chosen to create a centrifugal acceleration large enough to ensure that the configuration of the bladder and liquid in the tank is nearly identical to the zero-g configuration. A complete description of the actual tests, data acquisition and analyses of data is provided by Green, et al., [7]. The fuel motion is simulated using models with various parameters (inertia, springs, dampers, etc.,) and the problem reduces to a parameter estimation problem to match the experimental results obtained from SSTR.

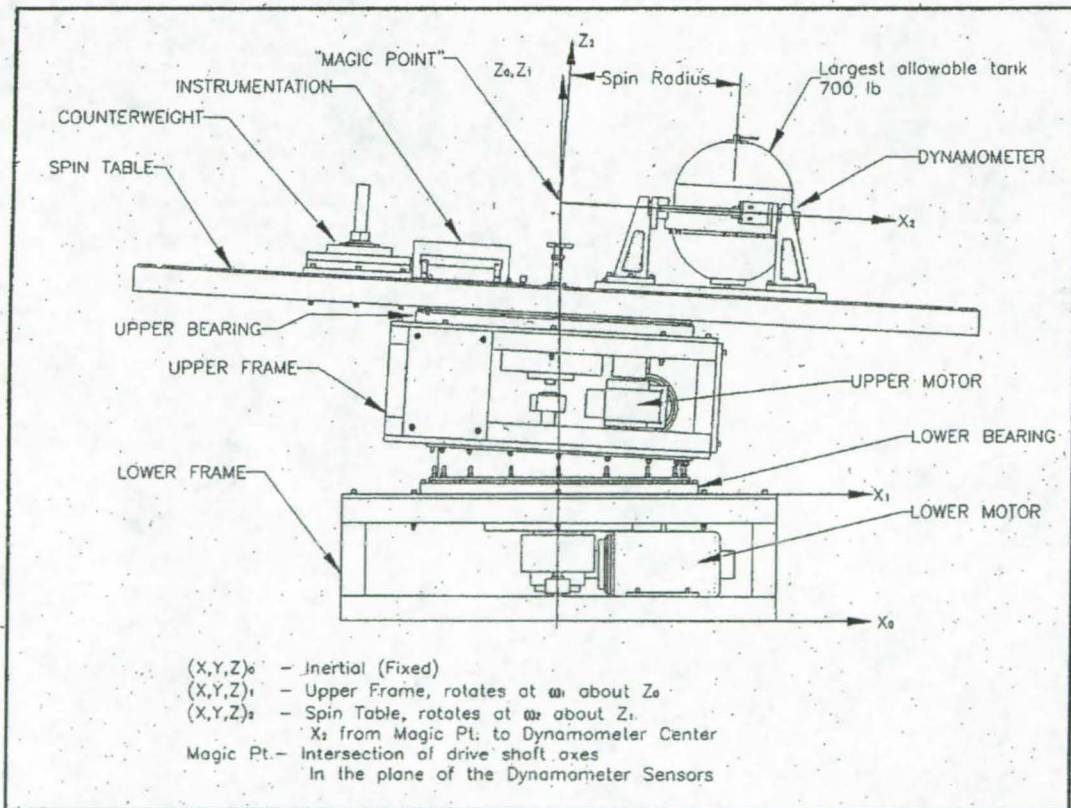


Figure 1. Schematic diagram of Spinning Slosh Test Rig (SSTR)



### Method of Approach

A computer simulation of the SSTR incorporating a 3-D Rotor Model is developed using SimMechanics software [8, 9] and is shown in Figure 2. This model has flexibility to include different parameters (inertia of rotor, translational and rotational stiffness and damping in the three mutually perpendicular directions, offsets, etc.,) in the parametric estimation process. The complexity of the 3-D Rotor Model depends on the parameters included in the estimation process. It is assumed that more parameters will provide better response characteristics closer to reality. The block diagram of the system identification procedure is illustrated in Figure 3. The identified parameters are input into the spacecraft model to obtain the Nutation Time Constant.

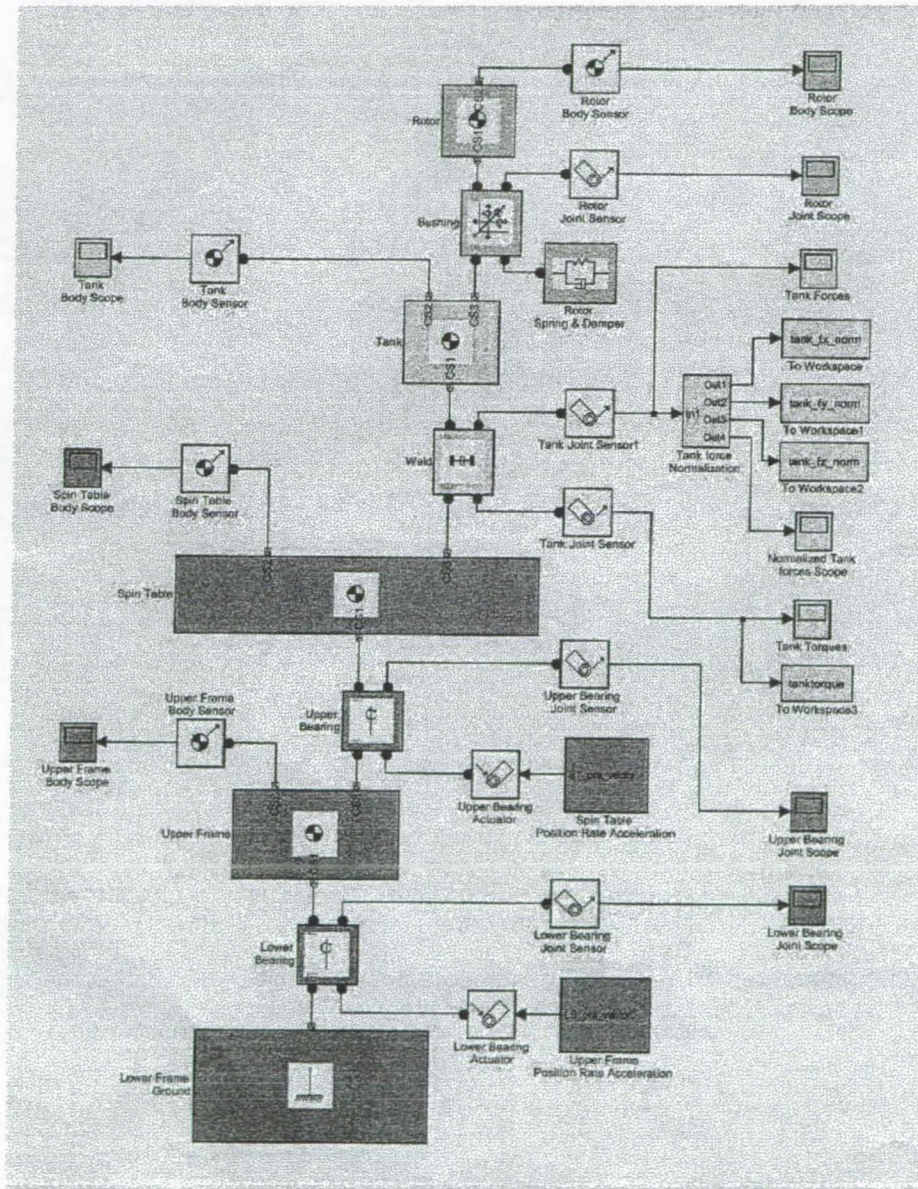


Figure 2. 3-D Rotor Model developed using SimMechanics software

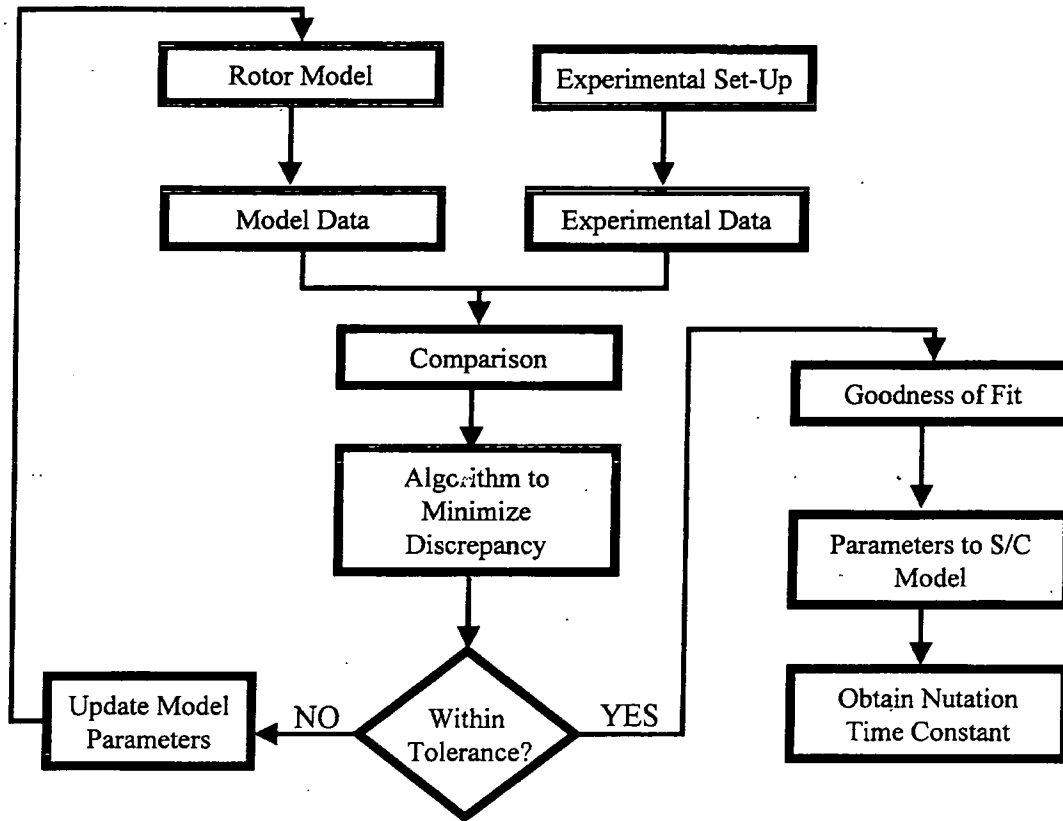


Figure 3. Block diagram of the parametric estimation process

### Optimization Concept

The parametric estimation problem reduces to an optimization problem of minimizing the residual which is the sum of the squares of the difference between the experimental and model response. This can be put in the mathematical form as:

*Minimize*

$$Residual R = \Sigma (Experimental Response - Model Response)^2$$

This minimization of residual is performed using the Newton's method for nonlinear least-squares or the Levenberg-Marquardt modification are applied to find the appropriate parameter values. This is accomplished through the LSQNONLIN algorithm of the optimization toolbox in MATLAB.

### Illustrative Example 1

A simple model of a mass-spring-damper was used to illustrate the concept (Figure 4). The parameters chosen are the stiffness of the spring ( $k$ ) and the damping coefficient ( $b$ ). In the "Experimental" model parameters are fixed while in the "SimMechanics" model, the initial values of the parameters are assumed and then estimated using LSQNONLIN algorithm. Table 1 shows the estimated parameters. The results indicate that the stiffness has a dominating effect on the response compared to damping.

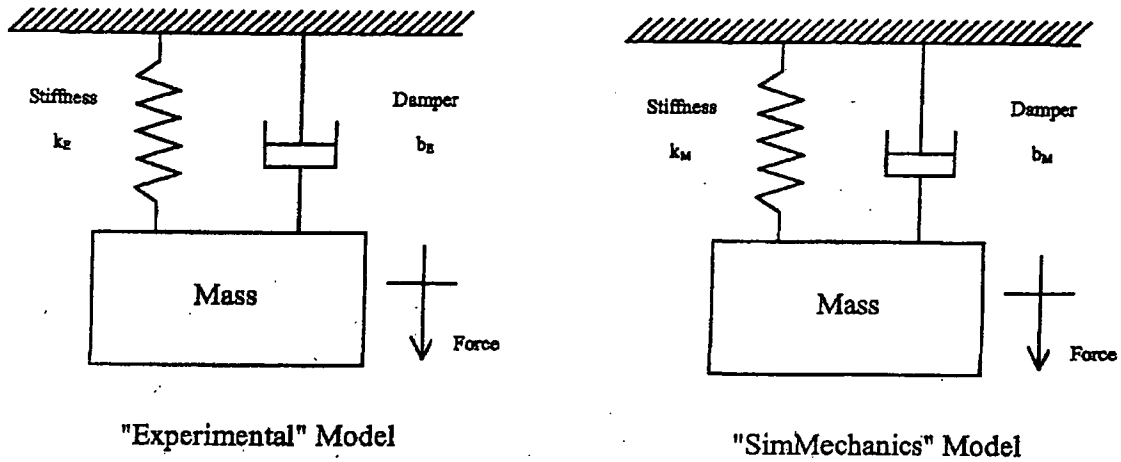


Figure 4: Simple model of a mass-spring-damper

Model	Stiffness Constant (N/m)	Damping Coefficient (N-s/m)
"Experimental" Model	150.0 (actual)	100.0 (actual)
"SimMechanics" Model	149.8 (estimated, dominant)	75.8 (estimated)

Table 1. Estimated parameters for the simple model of a mass-spring-damper

**Illustrative Example 2 (3-D Rotor Model)**

The construction of the 3-D Rotor Model using mass, springs and dampers is shown in Figure 5.

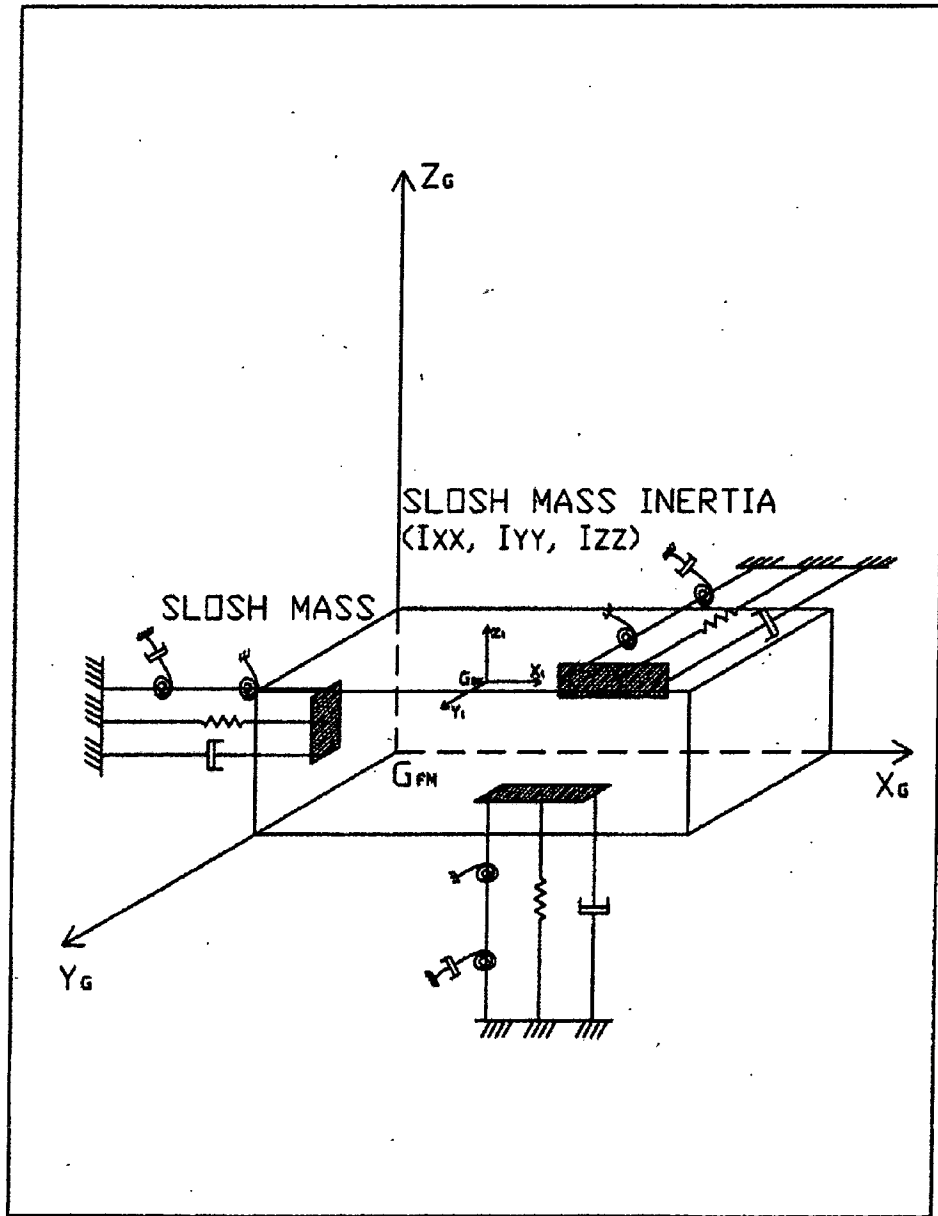


Figure 5. Schematic diagram of the 3-D Rotor Model with modeling parameters.

In the study, the experimental data is simulated using the SimMechanics model with different parameter values. There are various levels of complexity in the 3-D Rotor Model based on the number of parameters chosen. They are organized below in the order of increasing complexity.

- *Three inertia elements*
- *Three inertia and one mass element.*
- *Six stiffness elements*
- *Six damping elements*
- *Six stiffness and six damping elements*
- *Three inertia, one mass, six stiffness and six damping elements (sixteen total)*

Furthermore, offset distances and cross-coupling effects can be added to improve the model response. Overall the various combinations lead to a total of 31 possible parameters using the current 3-D Rotor Model. The validation of the models using the antioptimization concept developed by Gangadharan et al., [10] will be explored.

Finally, the entire estimation process is automated for efficient estimation of model parameters and incorporation into the NASA ELV Nutation Time Constant model. Three MATLAB m-files were created to run the estimation process. A detailed flow chart of the automation process is pictured below in Figure 6.

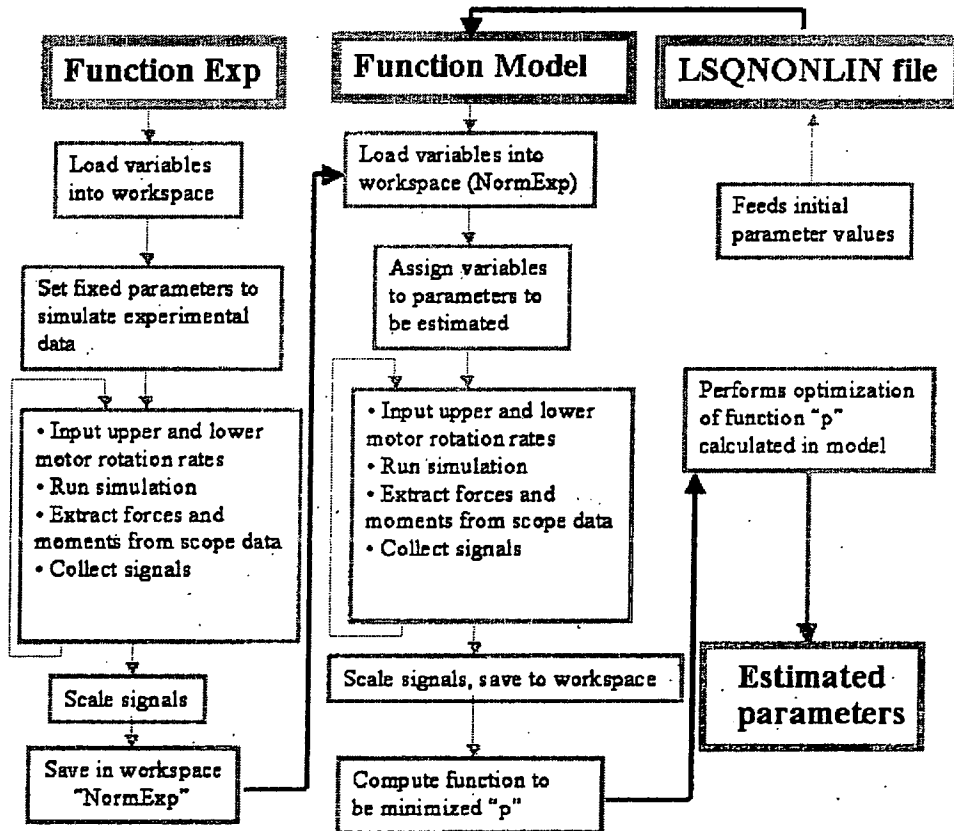


Figure 6. Detailed automated estimation process



## **Results and Discussion**

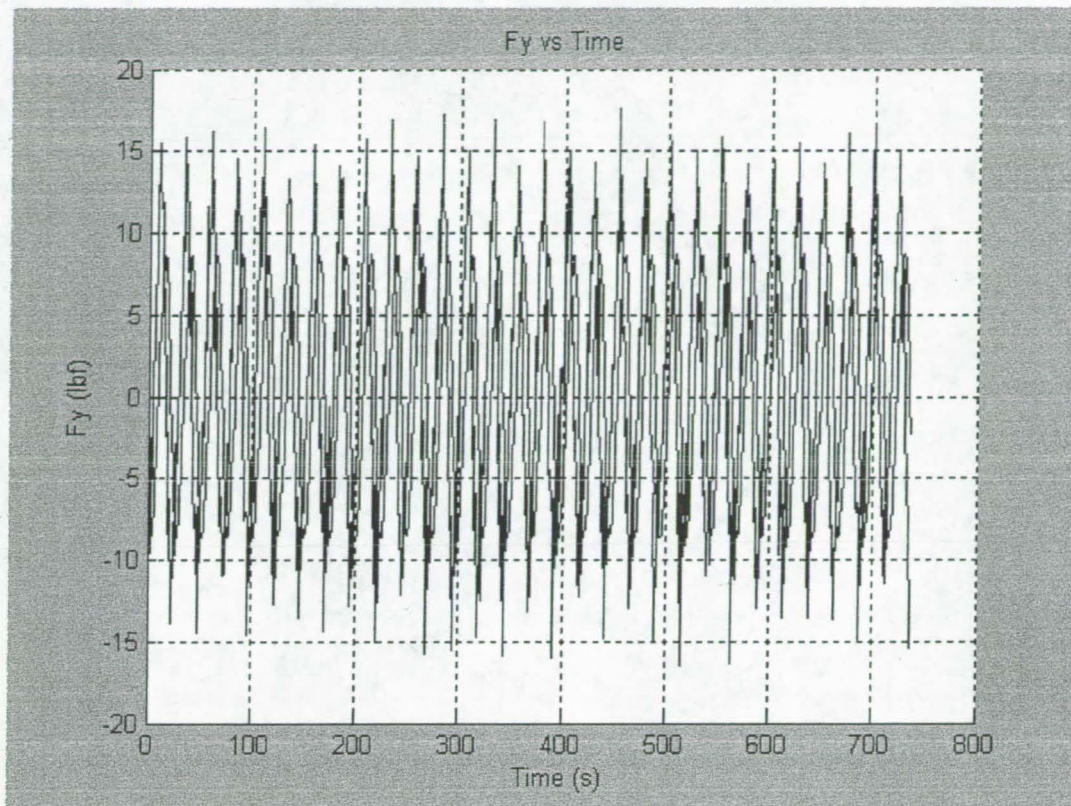
The 3-D rotor currently being explored contains a maximum of 16 model parameters. Starting with the most dominant three parameters, increasing levels of complexity have been analyzed by adding parameters. Several combinations of these parameters yield 31 total possibilities as noted by [12]. In the process of determining the dominance of each parameter, a simulated output data set is created with different parameter values. With different sets of initial guesses, the parameter optimization is then run in comparison to the simulated data. The amount of residual error between the optimized parameters and the actual values can be measured as a function of the amount each parameter changed.

Results obtained from the research to determine parameter dominance have shown that rotational spring constants do not change at all from the initial guesses. This shows they play no role on the outcome of the simulated results. Rotational dampening values showed a more chaotic behavior, and depending on initial guesses, diverge to large negative values or towards infinity. This trend indicates that they play little to no quantifiable effect on the simulation results. Some future alterations to the 3-D model may shed more light on the specific role of the rotational degrees of freedom of the mass.

Inertial tensor and mass parameters, when measured independently of other parameters, show consistent behavior. Inertial parameter values tended to move towards the actual values, but do not converge to the actual values. Even when parameter values had an error on the order of 10%, the overall accuracy of the simulation remained very high. This shows the low dominance of the inertial tensor. The mass parameter converges quickly and accurately as was expected. Accurate measurements of this value within 5% are commonly obtained.

## **Estimation Using Experimental Values**

Actual experimental values from SSTR and the model with 16 parameters were used in the estimation process. Figure 7 shows the response of force  $F_y$  obtained from the model using optimized parameters superimposed on the actual response from SSTR.



### **Conclusions**

A computerized 3-D model of the fuel slosh is developed and the system identification concept is illustrated using simulated experimental and actual experimental values. Various levels of complexity of the model in the system identification process are explored.

### **Future Work**

The system identification process will be applied using experimental data for various fill fractions and other spacecraft. Coupling effects between parameters will be studied. Model will be modified to include more than one mass to account for various resonant modes found during experimental testing. Finally, the overall system identification process will be automated using MATLAB software.

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