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**The effect of air density on atmospheric electric fields required for lightning initiation from
a long airborne object**

E.M. Bazelyan¹, N.L. Aleksandrov², Yu.P. Raizer³ and A.M. Konchakov⁴

¹Krzhizhanovskyy Power Engineering Institute, Moscow, Russia

e-mail: bazelyan@iristel.ru

²Corresponding author,

Department of Physical Mechanics, Moscow Institute of Physics & Technology,

Dolgoprudny, Moscow region, 141700 Russia

e-mail: nick-aleksandrov@yandex.ru

³Institute for Problems in Mechanics, Russian Academy of Science, Moscow, Russia

e-mail: raizer@ipmnet.ru

⁴Moscow Institute of Physics & Technology, Dolgoprudny, Moscow region, Russia

e-mail: alekon@rambler.ru

Abstract

The purpose of the work was to determine minimum atmospheric electric fields required for lightning initiation from an airborne vehicle at various altitudes up to 10 km. The problem was reduced to the determination of a condition for initiation of a viable positive leader from a conductive object in an ambient electric field.

It was shown that, depending on air density and shape and dimensions of the object, critical atmospheric fields are governed by the condition for leader viability or that for corona onset. To establish quantitative criteria for reduced air densities, available observations of spark discharges in long laboratory gaps were analyzed, the effect of air density on leader velocity was discussed and evolution in time of the properties of plasma in the leader channel was numerically simulated. The results obtained were used to evaluate the effect of pressure on the quantitative relationships between the potential difference near the leader tip, leader current and its velocity; based on these relationships, criteria for steady development of a leader were determined for various air pressures.

Atmospheric electric fields required for lightning initiation from rods and ellipsoidal objects of various dimensions were calculated at different air densities. It was shown that there is no simple way to extend critical ambient fields obtained for some given objects and pressures to other objects and pressures.

Keywords: Lightning initiation; Airborne object; Reduced air density; Streamer initiation; Viable leader

1. Introduction

Aircraft in flight are seldom struck by lightning developing from clouds. The probability of such events is comparable with the probability of lightning strike to grounded objects of similar dimensions. Most lightning strikes to aircraft in flight are initiated by the aircraft when penetrating into a region with sufficiently high electric fields (Fitzgerald, 1967; Cobb and Holitza, 1968; Bazelyan et al., 1980; Clifford and Kazemir, 1982; Mazur et al., 1984, 1986, 1989). As any conducting object not attached to the Earth, aircraft is polarized in an ambient electric field such that the local electric fields are enhanced near the extremities sufficiently to launch two leaders. A positive leader is initiated in the direction of the electric field from one aircraft extremity and a negative leader is initiated in the opposite direction from a different extremity. The negative leader requires larger electric fields and is launched with some delay, at the instant at which the length of the conducting system "aircraft plus positive leader" becomes sufficiently large. Bridging the cloud-to-ground gap is followed by lightning strike to the aircraft. The negative leader can direct also towards the positively charged region of a cloud and involve the aircraft in an intercloud lightning discharge.

Protection of present generation aircraft and other vehicles against potential catastrophic effects of lightning is an important problem. A number of airborne lightning-characterization studies with different instrumented aircraft have been carried out to gather information about lightning-aircraft interaction and to analyze its mechanism; a review of these studies is given in (Rakov and Uman, 2003). However, theoretical works have analyzed conditions for lightning initiation by aircraft only at 1 atm and generally neglected the effect of reduced air pressure at high altitudes (Lalande et al., 1999a, 1999b; Bazelyan and Raizer, 2000a). Pitts et al. (1987) studied this effect for aircraft; but they reduced the condition for lightning initiation to that for corona onset near aircraft extremities, whereas leader initiation and its viability were not considered. Lalande et al. (2002) assumed that the ambient electric field corresponding to a

steady leader development from a ground structure is proportional to air density; however, this assumption has not been substantiated.

The effect of pressure on lightning initiation from airborne objects seems to be important because the center of a negatively charged region in thunderclouds is generally located at altitudes of 3 – 5 km above ground, whereas the center of a positively charged region is located at altitudes of 6 – 10 km, the altitudes at which the relative air density is less than air density under standard conditions by a factor of 2 – 3. Such an essential reduction in air density leads to a hundreds percent decrease in breakdown voltage of air gaps. However, characteristics of the leader process for reduced air pressures are poorly known even on a laboratory scale. Information on initiation and development of a lightning leader at high altitudes is practically lacking. This forced us to develop a physical model of the process based on the most general concept of a spark discharge and to use meager experimental data of laboratory observations to check qualitatively at least some starting statements.

In this work, it was assumed that a leader initiated from a conductive object above ground is qualitatively similar to a leader developed (i) from a high-voltage electrode under laboratory conditions or (ii) from a grounded structure under thunderstorm conditions. At atmospheric pressure, all these discharges show similar structure elements and comparable currents, injected charges and rates of development. The fact that the leader process from an object above ground is bi-directional is not very important to understanding its nature as a whole. We proceeded from the electric field criteria, which have been used successfully to estimate conditions for corona onset, leader initiation and its steady development under standard conditions. The major problem was to determine parameters appearing in these criteria for reduced air densities.

There is reason to believe that the parameters are well known for atmospheric pressure. Here, the results of computer simulation based on these criteria agree well with measured breakdown voltage for long air gaps in laboratory (Bazelyan and Raizer, 2000a; Aleksandrov et

al., 2005a) and with critical atmospheric electric fields at which lightning is initiated from grounded objects (Aleksandrov et al., 2005b). In the first stage it was important to estimate main trends in changing critical electric fields corresponding to various phases of the leader process with decreasing air density.

The purpose of this work was to analyze theoretically conditions for lightning initiation from a long conductive object above ground and to study the effect of air density on these conditions. To simplify the problem under consideration, we studied only the objects of simple shape and neglected a number of effects, which could be important in real cases, among them, the effect of charge stored on the aircraft and the effect of high-temperature exhaust plume. All these effects should be taken into account to quantify atmospheric electric fields required for lightning initiation from real vehicles in flight. We also neglected the effect of space charge injected into air during the glow-corona phase, the effect important to leader initiation from fixed grounded objects under thunderstorm conditions (Bazelyan and Raizer, 2000b, Aleksandrov et al., 2001a, 2005b). This effect is of little importance to objects moving with a velocity of several tens of meters per second or larger and therefore leaving almost all injected space charge behind it (Bazelyan and Raizer, 2000b).

2. Conditions for initiation of a viable leader

2.1 Three necessary conditions

There is a sequence of discharge processes concluded by development of a viable leader from a conductive object in an atmospheric electric field and any one of these processes could play a decisive role in lightning initiation. They are initiation of streamers, leader initiation and its steady development. The onset of each process is characterized by corresponding critical ambient electric fields, whereas the atmospheric electric field required for lightning initiation is

the greatest magnitude among these critical fields. To initiate lightning from the object, the following three conditions must be fulfilled.

1. The local electric field near an extremity of the object must be enhanced sufficiently to initiate a streamer flash, which always precedes leader initiation. This field can be identified approximately with onset field for a stationary corona near the extremity (Bazelyan and Razhansky, 1988; Bazelyan and Raizer, 1998). The corona onset field is generally determined from the empirical Peek formula. For a hemispherically-tipped rod, a similar formula can be written as

$$E_{cor} = 27.8\delta \left(1 + \frac{0.54}{\sqrt{r_0\delta}} \right) \text{ [kV/cm]}, \quad r_0 - \text{[cm]} \quad (1)$$

where r_0 is the radius of curvature for the rod tip and δ is the relative air density.

Suppose a floating hemispherically-tipped rod of length d and radius $r_0 \ll d$ is subjected to an ambient uniform electric field E_0 . Then, the local electric field near the rod tip is

$$E_e = \beta \frac{\Delta U_e}{r_0}; \quad (2)$$

where

$$\Delta U_e = \frac{E_0 d}{2} \quad (3)$$

is the difference between the potential of the rod and potential produced by the external electric field at the location of the rod tip and $\beta = \beta(d/r_0)$ is the coefficient varying between 1 and 0.5 as the ratio d/r_0 varies in the practically important range 30-10000. To initiate streamers, the local electric field E_e must exceed E_{cor} ; from (2) and (3), this corresponds to the critical atmospheric electric field

$$E_{0cor} = 2r_0 E_{cor} / \beta d. \quad (4)$$

It follows from (4) that the critical electric field E_{0cor} is proportional to the ratio r_0/d and can be rather large if r_0/d is not very small.

2. The streamer flash must be followed by leader initiation. According to laboratory observations, to initiate a leader in air under standard conditions, it is necessary to apply a voltage of $U_{\min} = 400$ kV to a high-voltage electrode. At lower voltages, a leader is not initiated except in very short gaps in which initial streamers reach the opposite electrode prior to leader formation. The last case is not interesting in our consideration.

It was supposed by Bazelyan and Raizer (1998) that the threshold voltage U_{\min} is associated with a necessity to spend energy on the heating of a leader channel to gas temperatures required to maintain the plasma inside the channel in weak electric fields. However, it is difficult to calculate reliably the values of U_{\min} for various pressures. Fortunately, under practically important conditions, the criterion for leader initiation is less stringent than other criteria. Using (3), this criterion written as $\Delta U_e \geq U_{\min}$ is reduced to

$$E_0 \geq 2 U_{\min} / d \quad (5)$$

Inequality (5) is satisfied for short objects only in very large ambient electric fields. For instance, it follows from (5) that the required ambient field is $E_0 = 80$ kV/m for $d = 10$ m at 1 atm. No experimental data are available about density dependence of the threshold U_{\min} as opposed to the corona onset field E_{cor} for which the effect of density has been studied both experimentally and theoretically.

3. The leader must develop steadily; to do this, the potential of the leader tip, U_t , must exceed the potential of the external electric field at its location, $U_0(x_t)$, by U_{\min} or larger throughout the whole leader development. However, during this process, the voltage drop along the leader channel increases and hence the potential U_t tends to decrease. To satisfy the inequality $U_t - U_0(x_t) \geq U_{\min}$, the potential of the ambient field, $U_0 = -E_0 x + \text{const}$, must decrease in time faster than does U_t . (Here, we assumed that the field E_0 is uniform and is primarily produced by negative cloud charges.)

The condition for leader viability can be written as $E_L \leq E_0$ (Bazelyan and Raizer, 2000a; Lalande et al., 2002) regardless of air pressure, where E_L is the average electric field in the leader

channel. The problem is to determine (i) the field E_L for various pressures and fixed values of the leader current I and (ii) relationship between I and the difference $U_t - U_0(x_t)$. This problem has been approximately considered only under standard conditions (Aleksandrov et al., 2001b; Bazelyan and Raizer, 1998).

Thus, it may be concluded that the effect of reduced density is understood only for condition of streamer initiation. The effect of density on the criterion of leader initiation and that of its viability should be analyzed in detail. This could be done based on a particular physical model of the leader process.

2.2 Model of leader and condition for its viability

We consider the leader process using a simple semi-empirical model suggested by Bazelyan and Raizer (1998) and used to calculate breakdown voltage for long atmospheric pressure gaps. Based on this model, atmospheric electric fields required for initiation of triggered lightning during thunderstorms were also calculated (Aleksandrov et al., 2005b). A good agreement between calculations and observations justifies its application.

Suppose a positive leader develops in atmospheric-pressure air in a quasi-stationary manner such that the leader velocity and other parameters vary slightly for the time it takes for the leader tip to cover a distance comparable with the length of the streamer zone, $L_S = \Delta U_S / E_S$. Here, ΔU_S is the voltage drop along the zone and $E_S \approx 500$ kV/m is the average electric field in the channel of a long positive streamer under standard conditions.

The leader velocity v_L , current near the leader tip, I , and the linear charge of the leader, q , are related by the balance of charge as

$$I = qv_L. \quad (6)$$

The current I provides charge to a new section of the leader. This charge is accumulated primarily not in the leader channel but in its envelope. The charge accumulation causes an

increase in the difference $\Delta U_i = U_i - U_0(x_i)$, where U_i is the potential of the leader tip and $U_0(x_i)$ is the potential of the external electric field at the location of the leader tip, x_i . The relationship between q and ΔU_i is expressed as

$$q = C_1 \Delta U_i, \quad (7)$$

where C_1 is the effective capacitance per unit length of the leader near its tip.

Physically, the leader velocity depends on its current and the potential drop ΔU_s along the streamer zone in which the current is formed. However, the modern theory of the leader process has not been developed sufficiently to give simple and physically clear relations between v_L , I and ΔU_s . (Perhaps, there is no simple algebraic relation between these parameters.) Therefore, we use the semi-empirical relationship

$$v_L = a(\Delta U_i)^{1/2}, \quad (8)$$

where $a = 15 \text{ m s}^{-1} \text{ V}^{-1/2}$ under standard conditions. Equation (8) agrees with available laboratory measurements of the leader velocity and applied voltage (Les Renardieres Group, 1977). For short laboratory leaders, the difference between the applied voltage and ΔU_i is small because, in typical rod-to-plane gaps with a strongly non-uniform distribution of electric field, the potential $U_0(x_i)$ becomes small even at short distances from the stressed electrode from which the leader develops.

Equation (8) is supported by the following facts.

(a) It follows from (8) that $v_L \approx 2 \times 10^4 \text{ m/s}$ for an applied voltage of 2 MV, typical velocity and applied voltage for a leader in $\sim 10 \text{ m}$ air gaps (Les Renardieres Group, 1977).

(b) It follows from (6), (7) and (8) that

$$v_L = a^{2/3} C_1^{-1/3} I^{1/3} = c I^{1/3}, \quad (9)$$

in reasonable agreement with available observations (Les Renardieres Group, 1977). In particular, this agrees with the fact that a transition from typical laboratory leaders to typical lightning ones causes an increase (i) in the leader velocity by an order of magnitude, (ii) in the voltage by two orders of magnitude, and (iii) in the current by about three orders of magnitude.

(c) Breakdown voltages calculated using (6), (7) and (8) for laboratory gaps from 10 to 100 m in length agree well with available measurements (Bazelyan and Raizer, 1998; Aleksandrov et al., 2005a).

Although, physically, the leader velocity is governed by ΔU_S rather than by ΔU_t , this relation is represented by equation (8) because the ratio $\Delta U_S/\Delta U_t$ is approximately constant and depends, as well as the linear capacitance C_1 , on the electrostatic model used to describe the space distribution of charge in the leader. When simulating the streamer zone and the section of the charged leader envelope adjacent to its tip by a sphere of radius R and assuming a uniform electric field in the streamer zone, E_S , we have $\Delta U_S = \Delta U_t/2$ (Bazelyan and Raizer, 1998; 2000a). In this approximation, the total capacitance of the charged space under consideration, $C = 2\pi\epsilon_0 R$, is half the capacitance of a conductive sphere of the same radius, whereas the average effective linear capacitance of the leader section near its tip is

$$C_1 = C/(2R) = \pi\epsilon_0. \quad (10)$$

Using the spherical model of charge distribution in the vicinity of the leader tip and assuming a uniform distribution of space charge, we have $C_1 = 4\pi\epsilon_0/3$ and $\Delta U_S = \Delta U_t/3$. The last relation is close to that obtained in the model of Lalande et al. (2002) in which the charged leader envelope and streamer zone are represented by a uniformly charged cylinder rather than by a charged sphere. It should be noted that, at large distances from cloud charges, the ambient electric field E_0 is such that $E_0 \ll E_S$ and the potential U_0 of the ambient field remains almost constant along the length of the streamer zone, L_S . Therefore, the difference between $U_0(x_t)$ and $U_0(x_t+L_S)$ does not affect the ratio $\Delta U_S/\Delta U_t$.

To relate the average electric field in the leader channel, E_L , and current I in it, we used the following approximation (Bazelyan and Raizer, 1998)

$$E_L = \frac{b}{I}, \quad (11)$$

where $b = 3 \times 10^4$ VA/m under standard conditions. Here, it is assumed that the current varies little along the leader channel, which is somewhat similar to the channel of a long arc discharge. In the arc channel, electric field decreases with increasing current. Equation (11) agrees qualitatively with measured current-voltage characteristics of an arc discharge (Raizer, 1991) and takes into account that electric field in a 'new' section of the leader channel behind its tip has no time to decrease to typical arc fields.

It follows from (6), (7), (8) and (11) that the condition for leader viability is reduced to

$$E_0 \geq E_{ocr} = \left(\frac{2\sqrt{2}b}{aC_1} \right)^{2/5} \frac{1}{d^{3/5}} \approx \frac{5.3 \times 10^5}{d^{3/5}} \text{ [V/m]}, \quad d \text{ [m]}. \quad (12)$$

Here, the voltage difference near the tip of a rod object corresponding to the critical electric field E_{ocr} is expressed as

$$\Delta U_e = E_{ocr} d / 2 = \left(\frac{b}{2aC_1} \right)^{2/5} d^{2/5} \approx 2.65 \times 10^5 d^{2/5} \text{ [V]}, \quad d \text{ [m]}. \quad (13)$$

In (12) and (13), the coefficients a and b are taken under standard conditions and, from (10), $C_1 = \pi \epsilon_0 \approx 27.8$ pF/m.

For instance, from (12), we get $E_{ocr} = 51$ kV/m for $d = 50$ m, the length typical for modern aircraft. In this case, from (13), we have $\Delta U_e = 1.27$ MV, the voltage difference that is larger than 400 kV, the voltage required for leader initiation at 1 atm; that is, the condition for leader viability is more stringent than the condition for leader initiation. At atmospheric pressure the condition for leader initiation becomes more important only for objects with length $d < 2.8$ m. Such short objects are of no practical importance.

According to (1) and (2), the voltage difference $\Delta U_e = 1.27$ MV is sufficient to initiate streamers from a rod object with radius $r_0 < 20$ cm at atmospheric pressure. However, the criterion for streamer initiation is more stringent than the criterion for leader viability for objects with $r_0 > 20$ cm.

To make similar estimates for objects at high altitudes, it is necessary to determine the effect of reduced air density on the criterion for leader initiation and that for its viability. The second criterion seems to be more important also at small air densities. Concerning the first criterion, it could be sufficient to understand whether ΔU_{\min} decreases with decreasing density; if so, this criterion is not important also for objects at high altitudes.

3. Criterion for leader viability at reduced air densities

Let us consider the effect of air density on the criterion for viability of a leader initiated from an object of length d . The critical ambient electric field E_{0cr} corresponding to leader viability is determined by equation (12), which depends on parameters C_1 , a and b . These parameters can vary with air density. However, according to the spherical model for space charge near the leader tip, it follows from (10) that $C_1 \approx \pi\epsilon_0$ is independent of air density. A similar conclusion can be drawn approximating the distribution of space charge in the streamer zone and envelope of the leader by a uniformly charged cylinder and using the known formula for a long cylindrical conductor of length L and radius $R_C \ll L$:

$$C_1 \approx \frac{2\pi\epsilon_0}{\ln \frac{L}{R_C}}. \quad (14)$$

Then, assuming that R_C is the radius of the charged envelope of the leader channel, we have only a weak logarithmic dependence of C_1 on the reduced air density δ because, for a given value of the voltage drop ΔU_0 , the radius varies with δ as $R_C \sim \Delta U_0 / E_S \sim \delta^{-1.3}$, where the relation between the electric field in the streamer channel and δ , $E_S \sim \delta^{1.3}$, has been supported both experimentally and numerically (Phelps and Griffiths, 1976; Aleksandrov and Bazelyan, 1996, Bazelyan and Raizer, 1998).

Thus, attention should be focused on the effect of air density on the coefficients a and b , which determine the leader velocity and average electric field in the leader channel, respectively.

3.1 The effect of reduced air density on leader velocity

Modern theory of the leader process cannot predict the effect of air density on leader velocity and other parameters of the leader discharge. Experimental data on leader velocity at reduced pressures are incomplete. There have been some measurements in the Pamir outdoor high-voltage laboratory at an altitude of 3.35 km above sea level (Bazelyan et al., 1975). But, at such altitudes, air pressure is around 0.7 atm and the effect of reduced density is not profound. Therefore, the time for positive leader formation in air gaps up to 3 m in length was close to that under standard conditions.

More detailed observations were made in an optically transparent pressure chamber and the pressure range studied was 1 – 0.3 atm (Aleksandrov et al., 1984; Bazelyan and Razhansky, 1988). However, in this case, the length of the discharge gap did not exceed 50 cm and a leader developed from the outset in the final-jump phase in which the streamer zone was in contact with the opposite electrode. It is known that, in the final-jump phase, the discharge current in the leader channel is associated with charge transport by fast-propagating streamers towards the opposite electrode, rather than with charging a new segment of a relatively slow leader. Therefore, as a rule, the leader current increases sharply in the final-jump phase (Bazelyan, 1982; Bazelyan and Raizer, 1998). Concerning the processes near the leader tip, they do not vary qualitatively at the onset of the final jump, especially if the leader current is limited by placing an additional high-ohmic resistor in series with the discharge gap. This justifies our attempt to use observations in relatively short discharge gaps in order to determine a relationship between leader velocity and current.

Observations in the pressure chamber showed that, when placing a high-ohmic series resistor, the average current and average leader velocity of a positive leader were, respectively, 0.8 A and 2.4 cm/ μ s for 0.3 atm. These values of the current and velocity are close to those for

the leader process in atmospheric pressure air in the initial phase in which streamers of the leader have not reached the opposite electrode. For reduced pressures, doubling the leader current caused an increase in the leader velocity up to 2.8 – 3 cm/ μ s, in agreement with the relationship $v_L \sim I^{1/3}$ given by (9) for the leader process under standard conditions. In addition, observations in the same pressure chamber for 1 atm showed that, on average, the leader velocity was 4.2 cm/ μ s at an average leader current of 3.4 A, also in agreement with the relationship $v_L \sim I^{1/3}$, within the limits of experimental error. Thus, it follows from all these observations that the relationship between the leader velocity and current is adequately described by expression (9) in the range 0.3 – 1 atm, the value of c being pressure independent.

Physically, the weak pressure dependence of the leader velocity at a fixed leader current can be explained based on the mechanism of the leader elongation. We assume that a new segment of the leader channel forms within the leader tip that is similar in this regard to the stem of the impulse corona, within which the leader process is initiated. Most likely a new segment of the leader channel forms out of a package of numerous, young, yet-conducting streamers propagating from the forward end of the leader channel. The characteristic length of conducting section of streamers is $l_a \sim v_s \tau_a$, where v_s is the velocity of streamers in the streamer zone and τ_a is the lifetime of electrons in the streamer channel. The value of l_a depends on the relative air density as $l_a \sim \delta^2$. Indeed, $\tau_a \sim \delta^2$ because electron loss in the streamer channel is governed by three-body attachment to O₂ molecules, whereas v_s is independent of δ for weak streamers forming the streamer zone of a leader (Raizer et al., 1998; Bazelyan, Raizer, 2000a). Therefore, the radius of the leader tip, r_{tip} , that is close to l_a increases with decreasing δ . Under standard conditions, $r_{tip} \sim 1$ cm and the radius of the leader channel formed within the leader tip is an order of magnitude smaller. This means that the current carried by the leader tip contracts into much smaller volume. It was suggested by Bazelyan and Raizer (1998) that the possible mechanism of the current contraction is the development of an ionization-thermal instability. For

the characteristic time of instability development, Δt_{inst} , the leader forwards by the distance r_{tip} ; that is, the average leader velocity is $v_L \sim r_{\text{tip}}/\Delta t_{\text{inst}}$.

An important conclusion about a stepped manner of positive leader propagation can be made from this mechanism of leader development. In reality, a so-called continuous positive leader must forward by jerks, the length of each step being around the radius of the leader tip. This has been supported by streak photographs of the leader process made with high space resolution at 1 atm (Bazelyan and Raizer, 1998). The leader steps were more distinct for reduced pressures, for which their length is several times larger and the time of their formation is several times longer (see Figure 1).

A weak dependence of the leader velocity on reduced air density δ implies that the ratio $r_{\text{tip}}/\Delta t_{\text{inst}}$ is also pressure independent. This fact can be inferred from the properties of a long streamer (Bazelyan and Raizer, 1998; Aleksandrov et al, 2005b). Its radius r_s , being inversely proportional to δ , can be considered to be the initial radius of an overheated region during the instability development. The discharge current contracts into this region that is heated to some gas temperature, T_{th} . For fixed total current I , the Joule power deposited per unit length, P_J , is proportional to the electric field in the region. To compensate the loss of electrons in the overheated region, this field must increase to the threshold ionization electric field E_i at which the ionization rate is equal to the rate of electron loss. It is known that $E_i \sim \delta$; therefore, we have also $P_J \sim \delta$. Then, the power deposited per unit volume is $p_J \sim P_J/r_s^2 \sim \delta^3$ and the power deposited per one molecule within the overheated region is $p_m \sim p_J/\delta \sim \delta^2$. As a result, the characteristic time of instability development leading to the heating to the gas temperature T_{th} depends on density as $\Delta t_{\text{ins}} \sim \delta^2$. In this case, the leader velocity $v_L \sim r_{\text{tip}}/\Delta t_{\text{inst}}$ is independent of air density because $r_{\text{tip}} \sim \delta^2$. A more detailed analysis of the development of ionization-thermal instability within the leader tip for various air densities, taken into account ionization kinetics and hydrodynamic expansion, is given elsewhere (Bazelyan et al., 2007).

If the linear leader capacitance C_1 and relationship between I and v_L are independent of δ , it follows from (6) and (7) that the relationship between v_L and voltage difference ΔU_t (equation (8)) is also independent of δ , i.e., the coefficient $a = 15 \text{ m(sV}^{1/2})^{-1}$ obtained at 1 atm does not change in the range 0.3 – 1 atm. Thus, the criterion for leader viability (equation (12)) can vary with altitude only due to the density dependence of the coefficient b determining the electric field in the leader channel, E_L .

3.2 The effect of reduced air density on electric field in leader channel

The evolution of the characteristics of the plasma in the leader channel in atmospheric-pressure air was numerically calculated in (Aleksandrov et al., 2001b; 2003) taking into account thermal expansion of the channel, thermal conduction and non-equilibrium ionization in it. In this work, similar calculations were carried out for reduced pressures, to estimate the density effect on the evolution in time of the electric field in the channel. The evolution of the leader channel can be divided into two phases, the first one being the ionization and heating of the channel over a period of 30 – 50 μs after the current contraction and the second one being a slow expansion and cooling of the channel. Although the electron density on the channel axis slowly decreases in time in the second phase, the conductivity per unit length of the channel increases in time due to an increase in the cross section area of the channel. The evolution in time of the parameters in a given cross section of the channel can be approximately considered as the evolution of the parameters averaged along the length of the developing channel. The second phase is much more interesting for the present aim because a typical leader propagating with a velocity of 2 $\text{cm}/\mu\text{s}$ covers only a distance of 1 m in the first phase.

In the second phase in which the gas temperature T and gas number density N vary slowly the rate of ionization is compensated by the rate of electron-ion recombination; that is, the electron balance can be described by the quasi-steady-state equation. Under the conditions

considered, the main processes determining the electron density are the electron-impact ionization of neutral particles, the associative ionization in collisions between O and N atoms,



and the reverse process – the dissociative recombination of electrons with NO^+ ions. Here, electron diffusion and the processes involving negative ions can be neglected for $T > 1000$ K (Gallimberti, 1979; Aleksandrov et al., 1997). In this case, the balance equation for electrons is written as

$$k_i N n_e + k_{ai} N_O N_N = k_{ei} n_e^2 \quad (16)$$

where n_e is the electron density, N_O and N_N are respectively the densities of O and N atoms, k_i is the electron-impact ionization rate constant averaged over the main neutral components of the high-temperature air, k_{ai} is the rate constant of reaction (15), and k_{ei} is the rate constant of the reverse reaction.

The densities of neutral particles were assumed to be equal to their thermodynamically equilibrium values. The electron density tends to the thermodynamically equilibrium one and depends only on T and N as the gas temperature exceeds 6000 K.

The hydrodynamic model in question was constructed in the isobaric approximation, which gives a correct description of a leader because a number of experiments provide evidence for the subsonic radial expansion of the leader channel in air at discharge currents of 1-10 A (see, e.g., Gallimberti, 1979; Bazelyan and Raizer, 1998). The set of model equations includes the mass-balance equation

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho v) = 0 ; \quad (17)$$

the energy-balance equation

$$\rho c_p \left(\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial r} \right) = j E_L + \frac{1}{r} \frac{\partial}{\partial r} \left(r \lambda \frac{\partial T}{\partial r} \right); \quad (18)$$

and the ideal gas equation of state

$$p = N k T. \quad (19)$$

Here, ρ is the mass density, v is the radial velocity of the gas, c_p is the specific heat (heat capacity per unit mass) of the gas at constant pressure, λ is the thermal conductivity, p is the gas pressure, k is the Boltzmann constant, $j = en_e\mu_e E_L$ is the current density, e is the elementary charge, and μ_e is the electron mobility. The value of E_L was determined from equation

$$E_L = \frac{I}{2\pi e \int_0^{\infty} n_e(r) \mu_e(r) r dr} \quad (20)$$

for $I = \text{const.}$

It should be noted that equations (17) - (20) were used previously to simulate the decay of the leader channel after the discharge arrest in laboratory gaps (Gallimberti and Stangherlin, 1986) and the evolution of the properties of the leader channel during a lightning strike to an aircraft in flight (Larsson et al., 2000; Larsson, 2002). But, in contrast to these calculations, our kinetic model takes into account non-equilibrium ionization in the channel.

The initial conditions used corresponded approximately to the parameters of the plasma column after the current contraction at $t \sim 1-3 \mu\text{s}$; it was assumed that $T(t=0) = 1000 \text{ K}$ and $E/N(t=0) = 10^{-15} \text{ V cm}^2$ in a column of radius 0.03-0.1 cm. The values of $n_e(t=0)$ was determined from (20). The characteristics of the channel rapidly (for $\sim 50 \mu\text{s}$) 'forget' its initial state; that is, the evolution of the channel parameters is almost independent of its initial conditions. The boundary conditions were

$$\frac{\partial T(t, r=0)}{\partial r} = 0, \quad (21)$$

$$T(t, r=\infty) = T_\infty = 300 \text{ K}. \quad (22)$$

The values of k_i , k_{ai} , k_{ei} , N_O , N_N , c_p , λ , and μ_e were taken from available measurements and calculations (see (Aleksandrov et al., 2001b; 2003)).

Figures 2 and 3 show the evolution in time of the electric field E_L in a given section of the leader channel at various pressures for $I = 1$ and 4 A, respectively. At a given instant, the value of E_L decreases with decreasing pressure because of a more rapid expansion of the leader

channel, the effect causing an increase in the conductivity per unit length of the channel. A decrease of pressure from 1 to 0.3 atm leads only to a ~ 30 % decrease in the electric field. The pressure dependence of the average electric field in the channel can be approximated as $E_L(\delta) \approx E_L(\delta=1)\delta^{1/3}$. A similar relation can be written for the coefficient b in (11), $b(\delta) \approx b(\delta=1)\delta^{1/3}$.

Thus the coefficients a and b depend weakly on δ . Taking into account that these coefficients enter in (11) as $a^{2/5}$ and $b^{2/5}$, it may be concluded that the critical atmospheric electric field at which a leader initiated from a conductor can survive is almost insensitive to density variations, $E_{0cr} \sim \delta^{2/25}$, at least in the range 0.3 – 1 atm corresponding to altitudes to 10 km.

3.3 The effect of air density on threshold of leader initiation

The effect of reduced air density on the threshold of leader initiation can be estimated using the streak photographs of a positive leader in a rod-to-plane gap of length 5 m for 0.7 atm (Bazelyan et al., 1975). The time of rise of the applied voltage impulse was 220 μ s. The leader was initiated at 300 kV and propagated steadily towards the plane cathode. This threshold voltage is 100 kV less than the threshold voltage corresponding to leader initiation at 1 atm (Bazelyan and Raizer, 2000a). It follows from this that the effect of air density on leader initiation is not profound and that, most likely, the threshold U_{min} does not increase with decreasing pressure.

To produce the voltage difference $\Delta U = U_{min}$ near the ends of an object of length d , the ambient electric field must increase to $E_0 = 2U_{min}/d$. A leader initiated from the object in this case will survive if condition (12) is satisfied. From the condition $2U_{min}/d = E_{0cr}$, it is possible to determine the critical length d_{cr} such that for $d > d_{cr}$ the condition for leader viability is more stringent than that for leader initiation. Under standard conditions we have $U_{min} \approx 400$ kV and $d_{cr} \approx 2.76 \times 10^{-14} U_{min}^{2.5} \approx 2.8$ m. The critical length d_{cr} does not increase with decreasing air

density because in this case U_{\min} also does not increase and the coefficients C_1 , a and b in (12) are approximately independent of δ . This means that, for practically important objects that are much longer than 3 m, the condition for leader initiation can be neglected because it requires much smaller ambient electric fields than those necessary to satisfy the condition for leader viability. Therefore, we shall not consider here the effect of reduced air density on U_{\min} in more detail, although the problem deserves further study.

4. Calculation of critical electric fields and discussion

In the general case, atmospheric electric fields required for lightning initiation from a long conductive object above ground depend on geometric dimensions of the object, primarily on its length and radius of curvature of its tips. These parameters determine (i) the local electric field near the tips near which an initial streamer corona is initiated and (ii) voltage difference ΔU affecting leader initiation and viability. Critical atmospheric electric field $E_{0\text{cor}}$ for corona initiation is determined from equation (2) for $E_e = E_{\text{cor}}$. The intensification factor of electric field at the tip of an object, E_e/E_0 , can be found analytically only in exceptional cases; generally numerical calculations are required to determine this factor. However, it is easy to make approximate estimates because the ratio E_e/E_0 is not sensitive to the shape of an object, its length and radius of curvature of its tip being the same.

Indeed the intensification factor for an ellipsoid is expressed as (Landau and Lifshits, 1982)

$$\frac{E_e}{E_0} = \frac{s^{3/2}}{(1-s) \left(\frac{1}{2} \ln \frac{1+s^{1/2}}{1-s^{1/2}} - s^{1/2} \right)} \approx \frac{a_1^2}{b_1^2} \frac{1}{\left(\ln \frac{2a_1}{b_1} - 1 \right)}, \quad (23)$$

where $s = 1 - (b_1/a_1)^2$ and $a_1 = d/2$ and b_1 are its semi-major and semi-minor axes, respectively. In (23), the last approximation is valid with an accuracy of 1.6 % or better for $a_1/b_1 > 10$.

Substituting the ellipsoid length d and radius of curvature of its tip, $r_0 = b_1^2/a_1$, for a_1 and b_1 in (23), we get

$$\frac{E_e}{E_0} \approx \beta \frac{d}{2r_0}, \quad (24)$$

where

$$\beta = \frac{2}{\ln \frac{2d}{r_0} - 2}. \quad (25)$$

For a long, hemispherically tipped rod, the numerically calculated ratio E_e/E_0 is approximated within several percent by (24), the coefficient β being approximated in the range $10 < d/r_0 < 2000$ as

$$\beta \approx 6 \frac{r_0}{d} + 1.12 \left(\frac{r_0}{d} \right)^{0.08}. \quad (26)$$

The coefficient β turns out to be comparable with unity and not very sensitive to the ratio d/r_0 both for hemispherically tipped rods and for ellipsoidal objects. For instance, we have $\beta = 0.501$ for an ellipsoidal object with $a_1/b_1 = 10$ and consequently $d/r_0 = 200$. For a rod with the same value of d/r_0 , the value of β is equal to 0.763. The fact that the effect of object shape on the coefficients is not profound allows one to use (24) with (25) or (26) for rough estimates of local electric field near the tips of objects arbitrary in shape.

It should be noted that the coefficient β controls also the relation between the electric field E_e and the potential drop near the object tips, $\Delta U = \beta^{-1} E_e r_0$ (compare with (2)).

For sufficiently long (> 3 m) objects, corona onset near its extremities is inevitably followed by leader initiation and, consequently, lightning triggering electric field is governed only by the condition for corona onset and by the condition for leader viability. The critical atmospheric electric field corresponding to corona onset, $E_{0\text{cor}}$, is almost directly proportional to the radius of curvature of the object tip, r_0 , whereas the condition for leader viability is independent of r_0 (see (12)). Hence, for any given values of d and δ , there is the critical radius

$r_{0cr}(d, \delta)$ such that, for $r_0 > r_{0cr}$, the condition for corona onset becomes most stringent and determines the lightning triggering threshold that here increases with r_0 . For smaller radii, $r_0 < r_{0cr}$, the triggering threshold is governed by the condition for leader viability and does not depend on r_0 . This is depicted in Figure 4 that shows the calculated atmospheric electric field, E_{0light} , required for lightning initiation from a rod as a function of the radius of curvature of its tip for various rod lengths and δ . The threshold field is independent of r_0 for $r_0 < r_{0cr}$ and increases noticeably for larger r_0 .

For objects of fixed length, the dependence of lightning triggering field on the radius of curvature of the object tip is similar to the well-known dependence of the breakdown voltage for long laboratory gaps on the radius of a high-voltage electrode (Carrara and Thione, 1976). It has been shown that the breakdown voltage of an atmospheric-pressure rod-plane gap, subjected to impulses with a rise time of 0.1 – 1 ms, is almost independent of the electrode radius starting from very small radii up to the critical radius of the electrode. Investigations in gaps with up to 27 m spacing have shown that the critical radius increases weakly with the gap spacing and saturates around 35 cm.

Figure 5 shows the critical radius r_{0cr} calculated for lightning initiation from a hemispherically tipped rod versus its length for various air densities. The values of r_{0cr} are in the range 0.2 – 0.5 m for $\delta = 1$ and $d < 100$ m, in agreement with laboratory observations by Carrara and Thione (1976), and increase to 1.1 – 1.5 m for $\delta = 0.3$. This means that, for objects of practically important lengths and extremely large radii, lightning initiation at low altitudes is determined by the condition for corona onset near the object tips, rather than by the condition for leader viability.

In a similar way, at fixed r_0 , there is some critical length d_{cr} such that at $d > d_{cr}$ the condition for leader viability becomes more stringent than the condition for corona onset and $E_{0light} = E_{0cor}$, whereas $E_{0light} = E_{0cor}$ at $d < d_{cr}$. The critical fields E_{0cr} and E_{0cor} decrease with increasing d . However, the effect of object length on E_{0cor} is more profound than that on E_{0cr} .

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Figures 6 and 7 show the electric field $E_{0\text{light}}$ calculated for a hemispherically ripped rod as a function of its length for $\delta = 1$ and 0.3 , respectively. Under standard conditions, lightning initiation from objects with $r_0 = 0.1$ m is controlled only by the condition for leader viability. However, this is the case for objects with $r_0 = 0.3$ m only at $d > 40$ m. For objects with large (> 0.5 m) radii, lightning initiation is determined by the condition for corona onset even at $d = 100$ m. As air density decreases, the condition for leader viability remains almost the same, whereas the corona onset field decreases almost proportionally to δ . As a result, the effect of corona onset on the electric field $E_{0\text{light}}$ is important only at large (> 1 m) r_0 (see Figure 7).

It is interesting to compare the results of this work with those of airborne lightning studies involving instrumented aircraft. Ambient electric fields required for lightning initiation were measured with the CV-580 research aircraft of 24.7 m length and with the C-160 aircraft of 32.4 m length (Rakov and Uman, 2003). For the CV-580 the average ambient electric field just prior to the time of the lightning occurrence was 51 kV/m with a range from 25 to 87 kV/m and, for the C-160, 59 kV/m with a range from 44 to 75 kV/m (Lalande and Bondiou-Clergerie, 1997; Lalande et al., 1999a, 1999b). As a zero approximation, assume that aircraft can be simulated by a conductive rod of sufficiently small radius. Then, lightning initiation is determined by the condition for leader viability and it follows from (12) that the ambient electric field required for lightning initiation from a 30 m object is almost independent of altitude and is about 70 kV/m, in qualitative agreement with observations by means of instrumented aircraft. It is difficult to make a more reliable comparison between our theory and observations for the following reasons. A conductive rod is a too rough model for real aircraft being complicated in shape and having many sharp points. In addition, some effects have not been taken into account in our study. Aircraft can store electric charge on the surface leading to an enhancement of electric field near extremities. The occurrence of high-temperature exhaust plume can also favor lightning initiation, the effect that has not been considered till now even qualitatively. Additional research is required to take into account these effects.

Based on our analysis, it may be concluded that the condition for leader viability or that for corona onset can become most stringent, as shape and dimensions of an object vary. A transition from one condition to another one causes bends on the curves representing the lightning triggering field as a function of air density and dimensions of the object (see, for instance, Figure 8). Therefore, there is no general analytical relation to extend some experimental data obtained for a given object to other objects and other conditions. In particular, it is difficult to extend numerous observations of rocket-triggered lightning at $\delta \approx 1$ to real airborne objects and lower values of δ .

5. Conclusions

1. A lightning flash is triggered by an airborne conducting object in a thundercloud electric field if this field is large enough (i) for onset of a streamer corona, (ii) for initiation of a leader within the stem of the streamers, and (iii) for steady development of the leader.

2. The condition for leader initiation can be important only for very short (< 3 m) objects; in most practically important cases the ambient electric field required for lightning initiation is determined by the condition for leader viability and that for corona onset.

3. The critical atmospheric electric field corresponding to corona onset is approximately proportional to the relative air density δ , whereas the critical electric field corresponding to leader viability is almost independent of δ .

4. A critical radius of curvature of the object tip is determined; lightning initiation is governed by the condition for corona onset at larger radii of the object and by the criterion for leader viability at smaller radii. The critical radius increases with decreasing δ ; it lies in the range 0.2 – 0.4 m at $\delta = 1$ and in the range 1.1 – 1.5 m at $\delta = 0.3$, the length of the object being between 10 and 100 m.

5. Depending on pressure and shape and dimensions of objects, the ambient electric fields required for lightning initiation from them are controlled by different conditions. Therefore, there is no simple way to extent critical ambient fields measured or calculated for some given objects and pressures to other objects and pressures.

Acknowledgements

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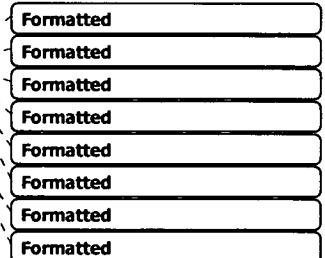
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Captions for illustrations

Figure 1. A streak photograph of the positive leader process in a 46.5 cm rod-to-plane air gap. Measurements were made for 0.305 atm and voltage amplitude of 110 kV.

Figure 2. The evolution of the electric field in the leader channel for $I = 1$ A and various gas pressures.

Figure 3. The evolution of the electric field in the leader channel for $I = 4$ A and various gas pressures.

Figure 4. The atmospheric electric field for lightning initiation from a hemispherically tipped rod as a function of the radius of curvature of its tip for various d and δ . Solid curves correspond to $\delta = 1$ and dash curves correspond to $\delta = 0.3$.

Figure 5. Calculated critical radius r_{0cr} for a hemispherically tipped rod as a function of its length for $\delta = 0.3$ and 1. The criterion for corona onset dominates at larger radii of curvature of rod tip and the criterion for leader viability dominates at smaller radii.

Figure 6. The atmospheric electric field required for lightning initiation from a hemispherically tipped rod as a function of its length for $\delta = 1$ and different radii of the rod tip.

Figure 7. The atmospheric electric field required for lightning initiation from a hemispherically tipped rod as a function of its length for $\delta = 0.3$ and different radii of the rod tip.

Figure 8. The lightning triggering field as a function of relative air density for a rod of length 20 m and radius 0.5 m.

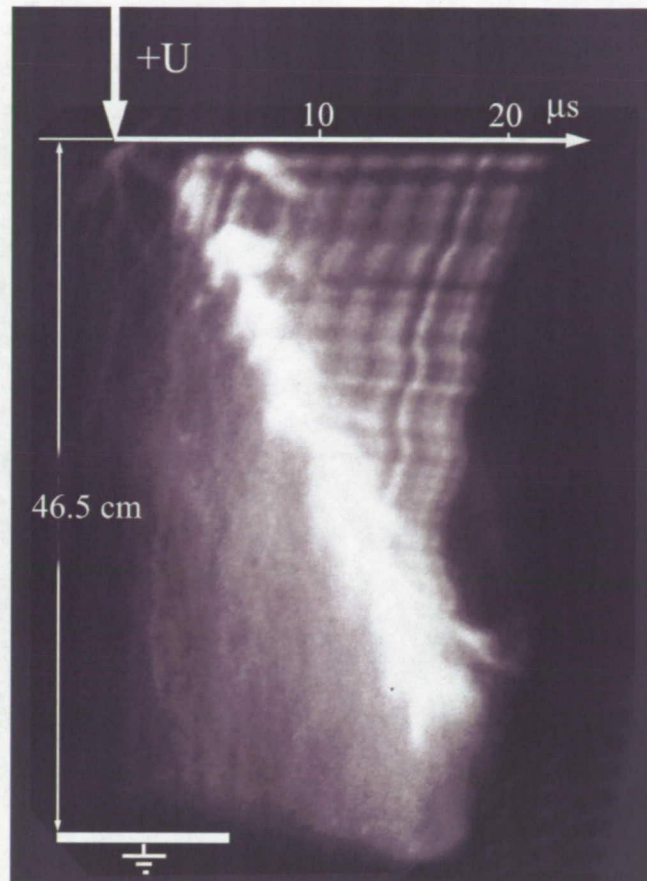


Fig. 1

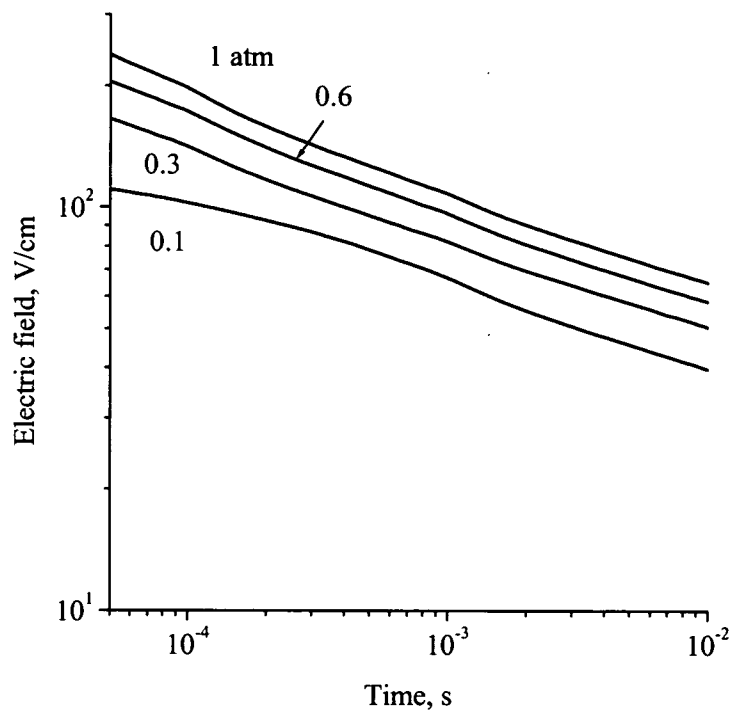


Fig.2

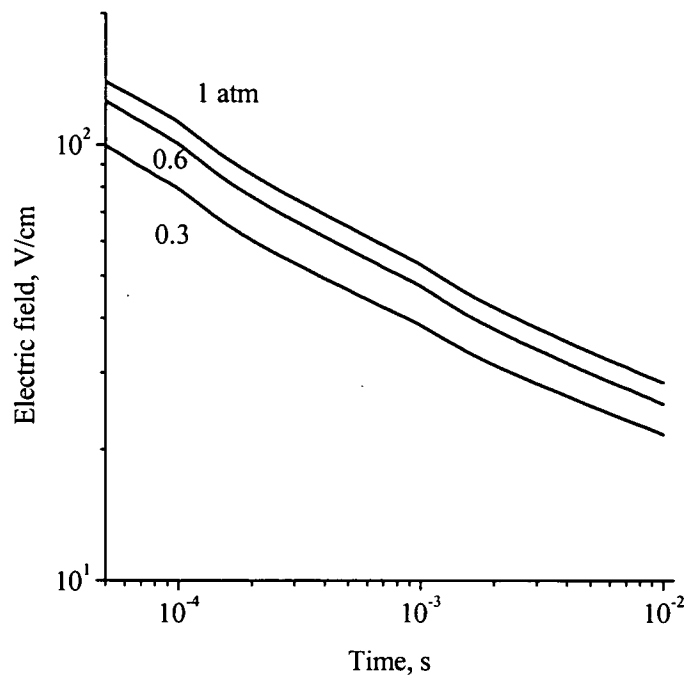


Fig. 3

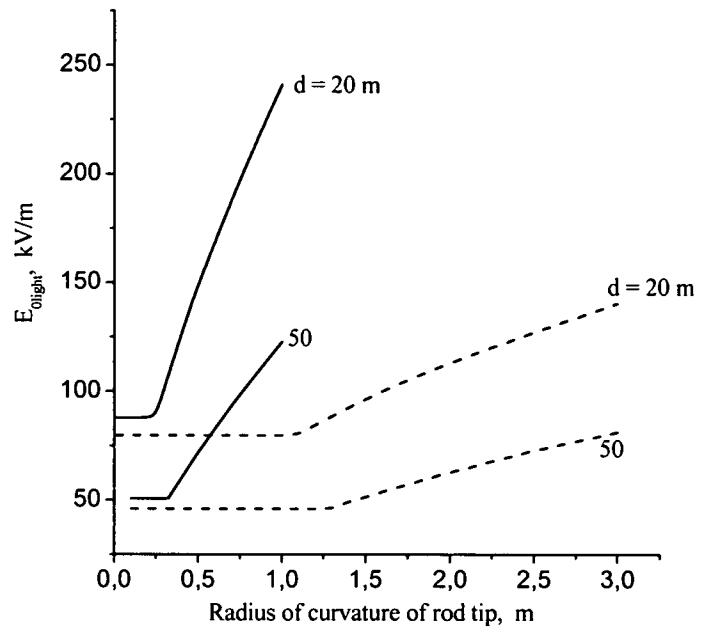


Fig.4

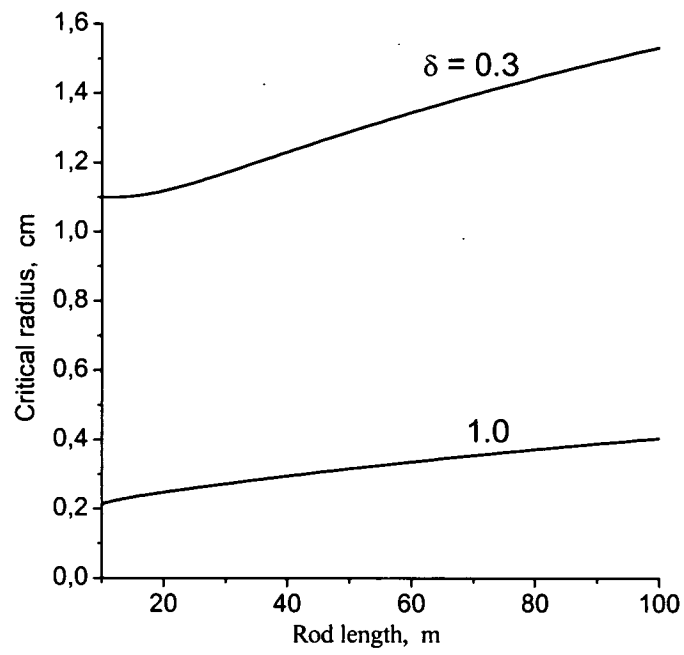


Fig. 5

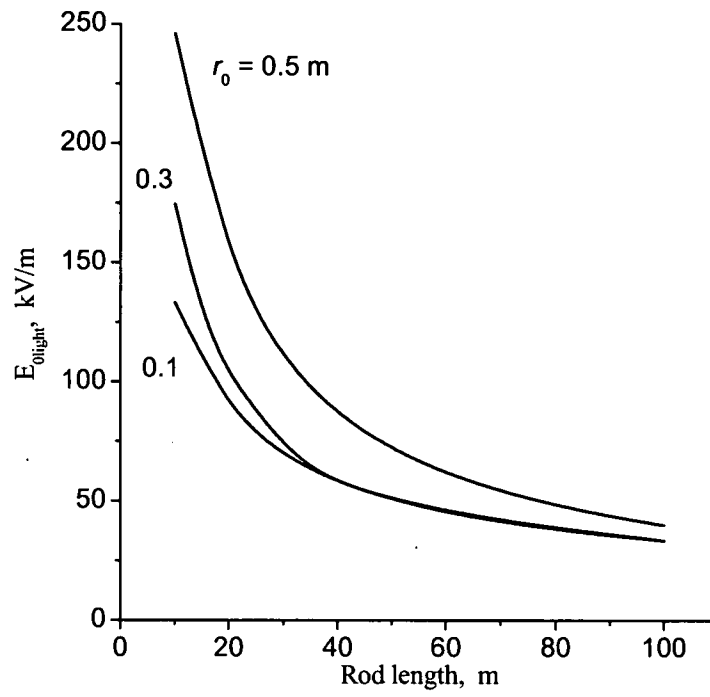


Fig. 6

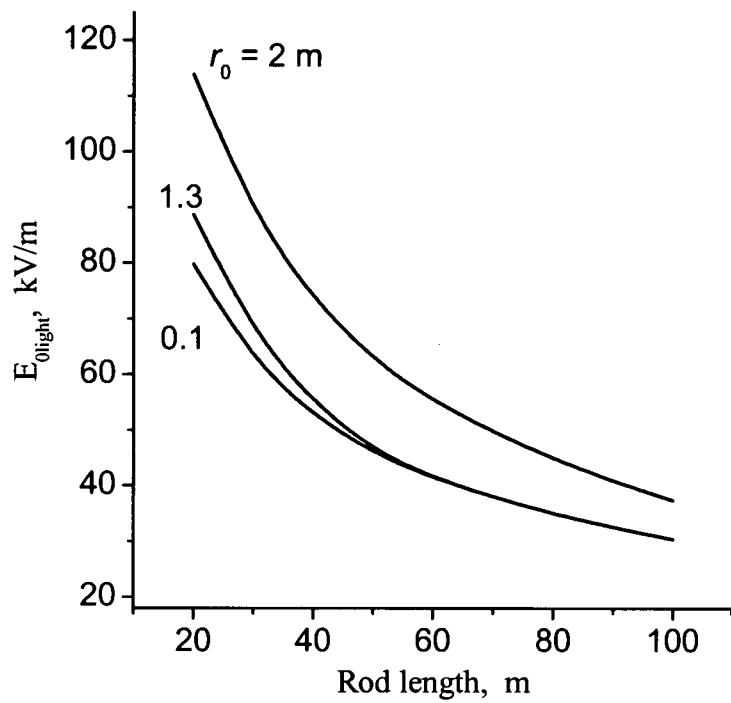


Fig. 7

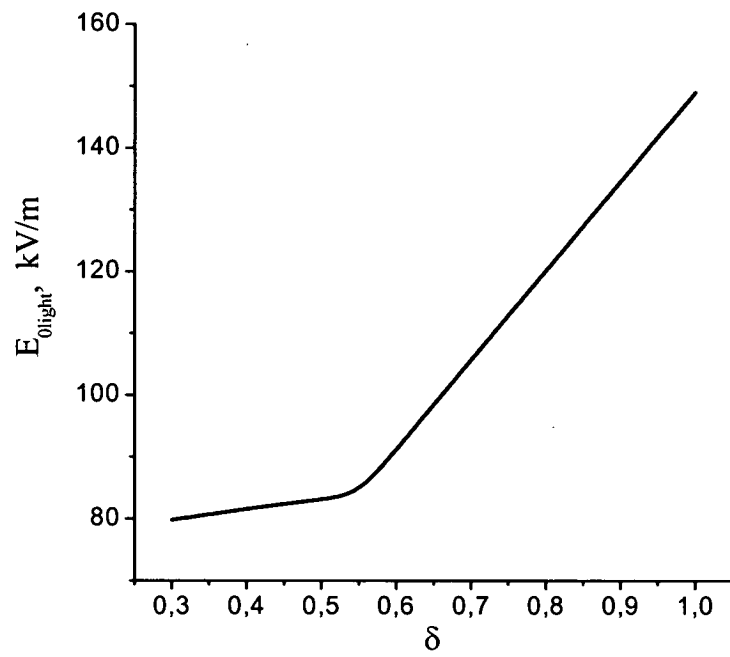


Fig. 8