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TITLE: *Tests of the Grobner Basis Solution for Lightning Ground Flash Fraction Retrieval*

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Abstract.

Satellite lightning imagers such as the NASA Tropical Rainfall Measuring Mission Lightning Imaging Sensor (TRMM/LIS) and the future GOES-R Geostationary Lightning Mapper (GLM) are designed to detect total lightning (ground flashes + cloud flashes). However, there is a desire to discriminate ground flashes from cloud flashes from the vantage point of space since this would enhance the overall information content of the satellite lightning data and likely improve its operational and scientific applications (e.g., in severe weather warning, lightning nitrogen oxides studies, and global electric circuit analyses). A Bayesian inversion method was previously introduced for retrieving the fraction of ground flashes in a set of flashes observed from a satellite lightning imager. The method employed a constrained mixed exponential distribution model to describe the lightning optical measurements. To obtain the optimum model parameters (one of which is the ground flash fraction), a scalar function was minimized by a numerical method. In order to improve this optimization, a Grobner basis solution was introduced to obtain analytic representations of the model parameters that serve as a refined initialization scheme to the numerical optimization. In this study, we test the efficacy of the Grobner basis initialization using actual lightning imager measurements and ground flash truth derived from the national lightning network.

Tests of the Grobner Basis Solution for Lightning Ground Flash Fraction Retrieval

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1. INTRODUCTION

When viewing lightning from space at optical wavelengths, the cloud multiple scattering medium obscures the view thereby preventing one from easily determining what flashes strike the ground. However, recent studies have made some progress in estimating the ***ground flash fraction,*** α in a set of N flashes observed from space [Koshak (2010), Koshak and Solakiewicz (2011), and Koshak (2011)]. Knowledge of α is important for better understanding:

- Severe Weather,
- Lightning Nitrogen Oxides Chemistry/Climate Studies, and
- Global Electric Circuit.

In the study by Koshak (2011), a Bayesian inversion method was introduced for retrieving the fraction of ground flashes in a set of flashes observed from a (low earth orbiting or geostationary) satellite lightning imager. This method has formed the basis of a Ground Flash Fraction Retrieval Algorithm (GoFFRA) that is being tested as part of GOES-R Geostationary Lightning Mapper (GLM) risk reduction.

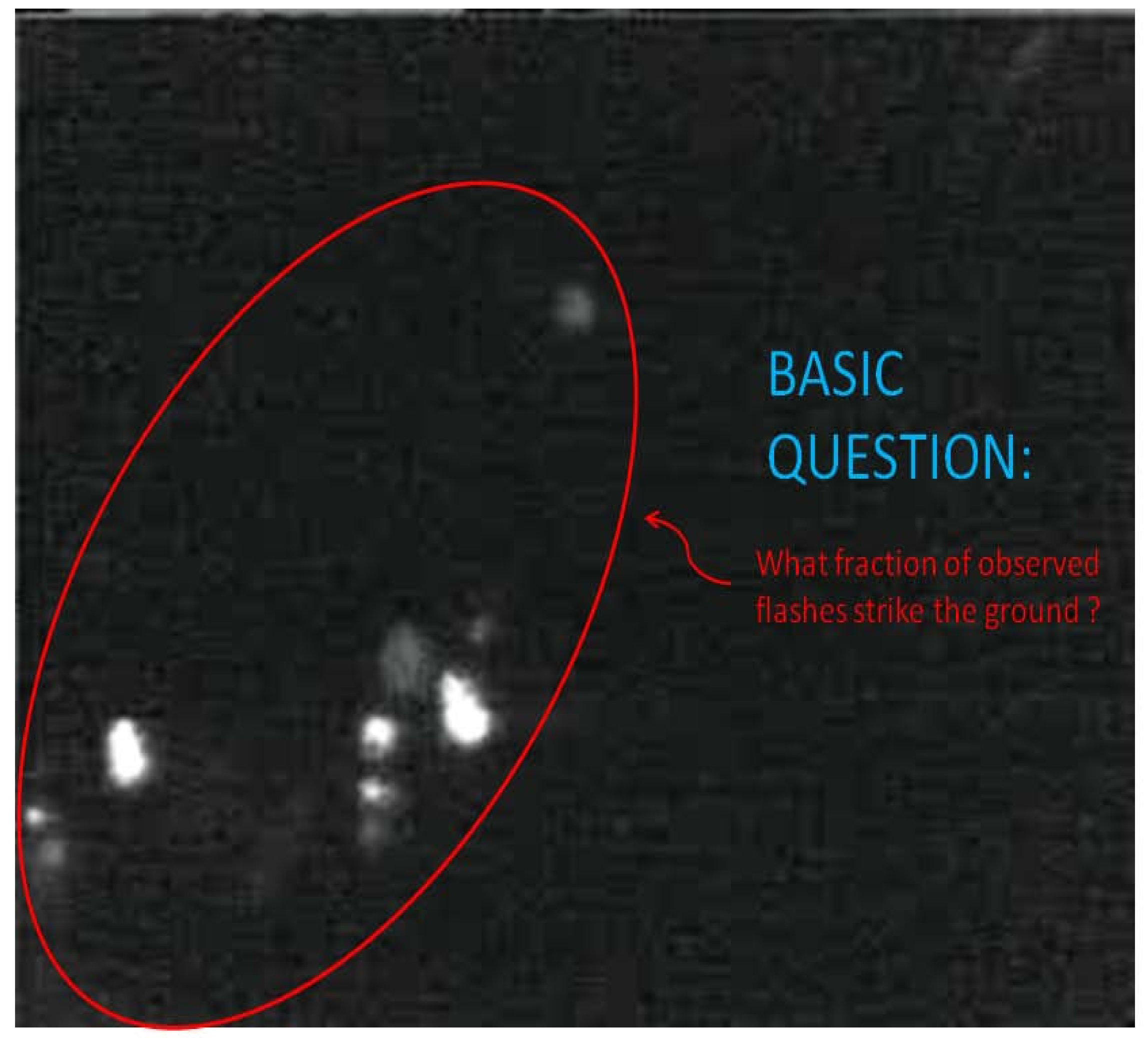


Figure 1. Example of lightning optical emissions as seen from space (STS-52 video).

2. THE BAYESIAN RETRIEVAL

Figure 2 highlights the mathematical attributes of the Bayesian retrieval process. The μ's are the population mean Maximum Group Areas (MGAs) decremented by the nadir footprint area of the space-based lightning imager employed. In summary, the distribution of y (nadir footprint decremented MGA) is assumed to follow a mixed exponential distribution; one finds the 3 parameters of this distribution that maximizes a Bayesian scalar function.

Bayesian Inversion

Bayes' Law:

$$P(\alpha,\mu_g,\mu_c|\mathbf{y}) = \frac{P(\mathbf{y}|\alpha,\mu_g,\mu_c)P(\alpha,\mu_g,\mu_c)}{P(\mathbf{y})},$$

Find parameters $\mathbf{v} = (\alpha,\mu_g,\mu_c)$ that maximize the probability on LHS.

This means one maximizes the following :

$$S(\mathbf{v}) \equiv \ln\left[P(\mathbf{y}|\mathbf{v})P(\mathbf{v})\right] = \ln\prod_{i=1}^{m} p(y_i|\mathbf{v}) + \ln P(\mathbf{v}) = \sum_{i=1}^{m}\ln\left[\frac{\alpha}{\mu_g}e^{-y_i/\mu_g} + \frac{(1-\alpha)}{\mu_c}e^{-y_i/\mu_c}\right] + \ln P(\mathbf{v}),$$

Formally :

$$\frac{\partial S(\mathbf{v})}{\partial \mathbf{v}} = \mathbf{0} \quad \Rightarrow \quad \mathbf{v} = \text{"Maximum A Posteriori (MAP) Solution"}$$

Practically :

Use Broyden-Fletcher-Goldfarb-Shannon variant of Davidon-Fletcher-Powell numerical method to minimize $-S(\mathbf{v})$. Also, $P(\mathbf{v})$ is simplified by assuming model parameter independence, with $P(\alpha)$ uniform, and $P(\mu_g)$ & $P(\mu_c)$ both normal distributions.

Original Initialization

$$\alpha = 0.5$$

$$\mu_g = \bar{y} + \sqrt{\frac{1}{2}\left(s^2 - \bar{y}^2\right)}$$

$$\mu_c = \bar{y} - \sqrt{\frac{1}{2}\left(s^2 - \bar{y}^2\right)}$$

Grobner Initialization

$$\alpha = \frac{\bar{y} - \mu_c}{\mu_g - \mu_c}$$

$$\mu_g = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$$

$$\mu_c = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

where :

$A = \bar{y}^2 - q,\quad B = r - \bar{y}q,\quad C = q^2 - \bar{y}r$

$q = \frac{1}{2}(\bar{y}^2 + s^2)$

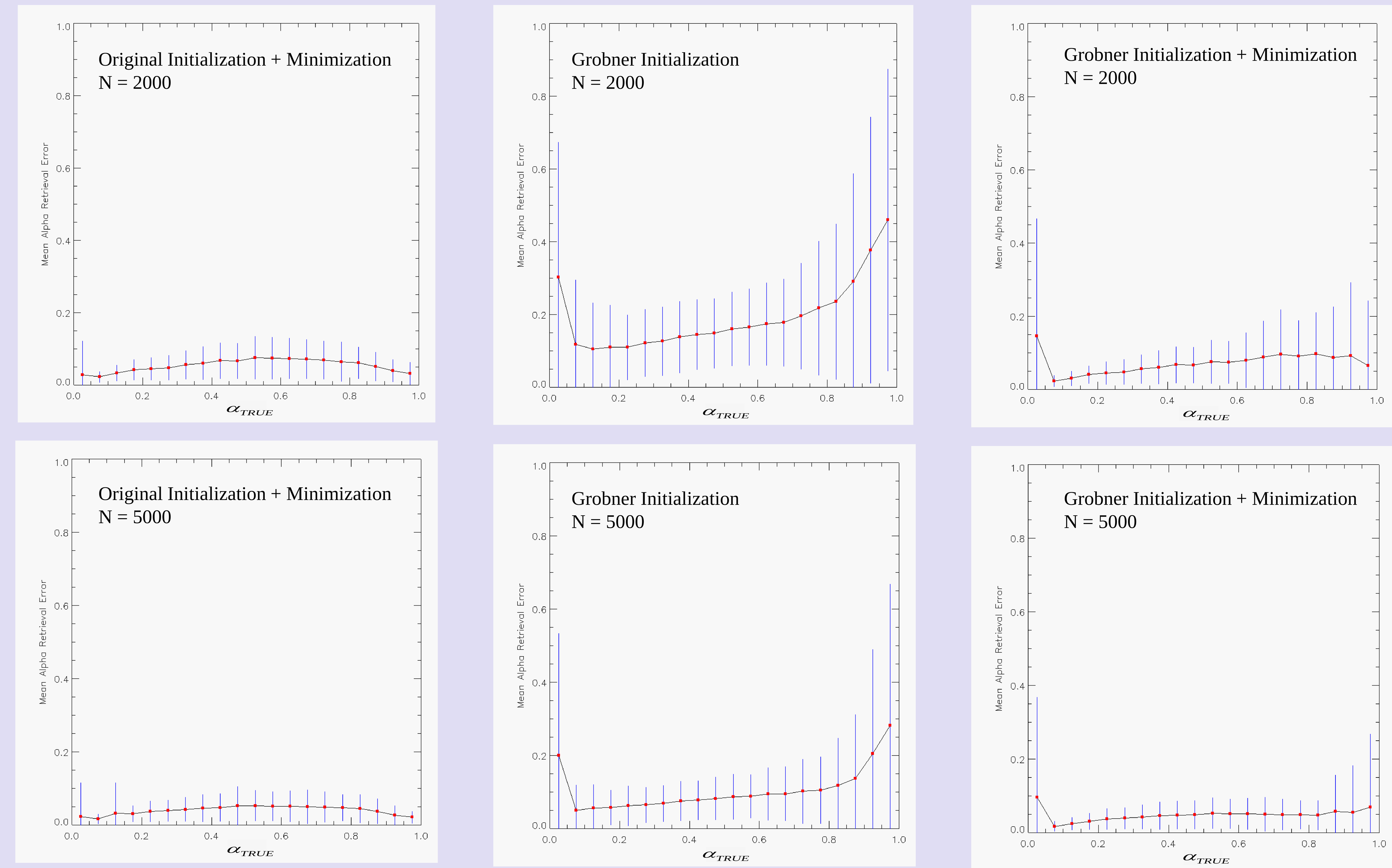
$r = \frac{1}{6}(\bar{y}^3 + 3\bar{y}s^2 + \gamma_1 s^3)$

$(\bar{y}, s^2, \gamma_1) =$ Sample (mean, variance, skewness)

Figure 2. Left slide summarizes the Bayesian retrieval approach. To obtain the optimum parameters, the minimization begins by using either the "Original Initialization" (top right panel [see Koshak (2011) for additional details], or the "Grobner Initialization" derived in this study (lower right panel).

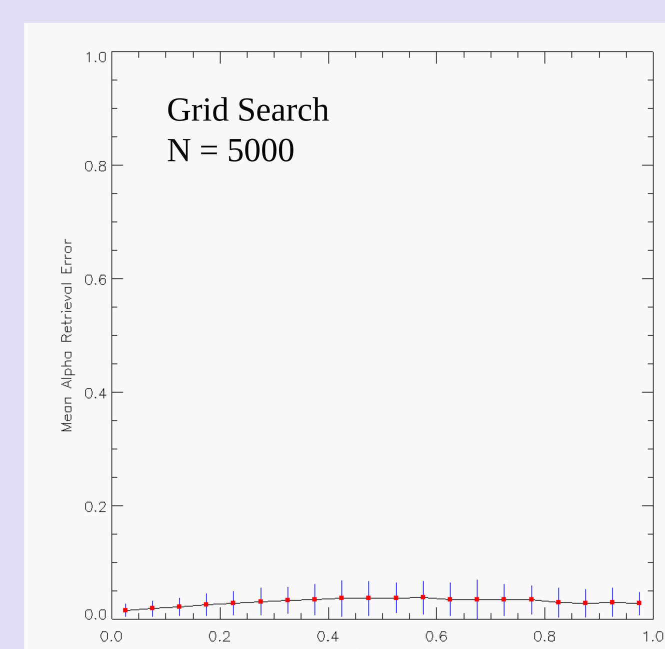
3. TEST OF THE GROBNER INITIALIZATION

To test the Grobner Initialization approach and compare it with the Original Initialization, we perform simulated retrievals. To simulate the observation vector **y** (i.e., the set of MGAs each decremented by the nadir footprint area), we generate the components of **y** from a mixed exponential distribution having parameters $(\alpha_{TRUE}, \mu_{gTRUE}, \mu_{cTRUE})$. These true values are compared with the Bayesian retrieved values to assess retrieval errors. The true values are varied over a wide range, and a total of 100 retrievals are performed in each ground flash fraction bin (0-0.05, 0.05-0.1, … , 0.95-1.0) . The mean retrieval error (and std. deviation) are plotted below.

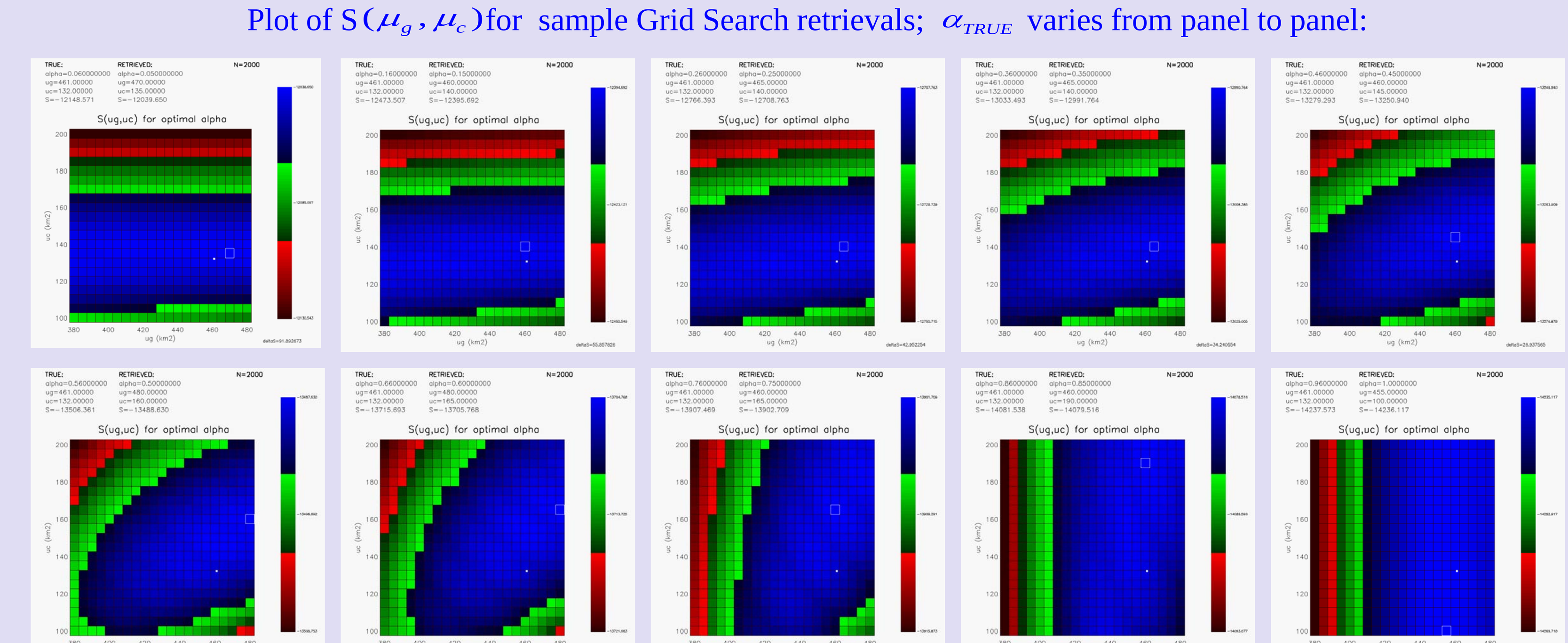


4. GRID SEARCH

The Grobner Initialization + Minimization does not improve upon the Original Initialization + Minimization. Moreover, we have found that a simple grid search over the 3 parameters (for finding the *maximum* in S) can beat out both of these methods as shown in the (below left) retrieval error plot. Each retrieval takes only tens of seconds on a Dell T3500 Linux machine (depending on grid size) and no normal prior constraints were needed. Plots of the S scalar can also be generated to provide more insight (see sequence of plots below right).



Plot of S(μ_g, μ_c) for sample Grid Search retrievals; α_{TRUE} varies from panel to panel:



5. REFERENCES

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