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Adaptive Control in the Presence of Simultaneous Sensor Bias and Actuator Failures

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Abstract

The problem of simultaneously accommodating unknown sensor biases and unknown actuator failures in uncertain systems is considered in a direct model reference adaptive control (MRAC) setting for state tracking using state feedback. Sensor biases and actuator faults may be present at the outset or may occur at unknown instants of time during operation. A modified MRAC law is proposed, which combines sensor bias estimation with control gain adaptation for accommodation of sensor biases and actuator failures. This control law is shown to provide signal boundedness in the resulting system. For the case when an external asymptotically stable sensor bias estimator is available, an MRAC law is developed to accomplish asymptotic state tracking and signal boundedness. For a special case wherein biases are only present in the rate measurements and bias-free position measurements are available, an MRAC law is developed using a model-independent bias estimator, and is shown to provide asymptotic state tracking with signal boundedness.

1 Introduction

Loss of control due to anomaly and upsets has been known to be a major cause of aircraft accidents and incidents. Actuator and sensor faults have been implicated in several aircraft accidents. Some examples of accidents caused by actuator failures include: hydraulics/multiple actuator failures [1], rudder failure [2], and horizontal stabilizer failure [3]. Sensor failures have also been implicated in a number of accidents and incidents, for example, radio altimeter fault [4], angle-of-attack sensor fault [5], and airspeed sensor fault [6].

Direct model reference adaptive control (MRAC) methods offer an approach for maintaining stability and controllability in the presence of uncertainties and failures, without requiring explicit fault detection, isolation, and controller reconfiguration. Direct MRAC methods use controller gains that are adaptively adjusted to achieve a performance close to that of a reference model while maintaining system stability and close tracking of the reference model response. Direct MRAC methods for state- or output- tracking using state feedback have been well established. In particular, direct MRAC schemes that use state feedback for state-tracking (SFST) have the advantage of simplicity of implementation and can achieve effective state tracking in the presence of parameter uncertainties [7]. Such schemes have been extended to the case with actuator failures [8], [9], [10]; to plant-model mismatch due to damage or icing [11]; and to simultaneous plant-model mismatch and actuator failures [12].

In addition to actuator faults, sensor faults may also compromise safety. A common type of sensor fault is sensor bias, which can develop during operation in sensors such as rate gyros, accelerometers, altimeter, etc. One approach to deal with sensor faults is to use redundant sensor packages. However, common-mode failures can occur across all the sensors, and each sensor can typically develop a different unknown bias. If used directly in an MRAC law, such offsets in sensor measurements can have detrimental effects on closed-loop stability, which can no longer be theoretically guaranteed. Literature addressing accommodation of sensor faults (or simultaneous actuator and sensor faults) in an MRAC setting is relatively limited. In [13], a modified MRAC law was developed, which combines a bias estimator with control gain adaptation, to obtain signal boundedness and bounded tracking error. Further, for the case wherein an asymptotically stable sensor bias estimator is available, an MRAC control law was developed to accomplish asymptotic tracking and signal boundedness.

In this paper, the results of [13] are extended to the case with simultaneous actuator failures and sensor bias faults. Such faults (of unknown magnitude) may occur in unknown sensors and/or actuators at unknown instants of time. The problem formulation is presented in Section 2, and an MRAC law that incorporates adaptive sensor bias estimation to ensure signal boundedness is developed in Section 3. Section 4 considers the case when an external asymptotically stable sensor bias estimator is available, and Section 5 addresses a special case when sensor bias exists only in the rate measurements. Section 6 contains the concluding remarks.

2 Problem Formulation

Consider a linear time-invariant plant, subject to actuator failures and sensor biases, described by:

$$\begin{aligned} \dot{x}\,(t) &= Ax\,(t) + Bu\,(t) \\ y\,(t) &= x(t) + \beta \end{aligned} \tag{1}$$

where $A \in R^{n\times n}$ is the system matrix assumed to be unknown, $B \in R^{n\times m}$ is the input matrix, $x\,(t) \in R^n$ is the system state, and $u\,(t) \in R^m$ is the control input. $y\,(t) \in R^n$ is the available state measurement with an unknown constant bias $\beta \in R^n$. In practice, additive sensor noise and process noise are also present in the system. However, as is customary in the MRAC literature, sensor noise and process noise are not included in the analysis in order to facilitate analytical proofs of tracking stability and signal boundedness.

In addition, the actuators $u(t) \in R^m$ (e.g., control surfaces or engines in aircraft flight control) may fail during the operation. Actuator failures are modeled as

$$u_j(t) = \bar{u}_j,\ t \geq t_j,\ j \in \mathcal{J}_p \tag{2}$$

$$\mathcal{J}_p = \{j_1, j_2, \ldots, j_p\} \subseteq \{1, 2, \ldots, m\}$$

where the failure pattern $\mathcal{J}_p$, the failure value $\bar{u}_j$ (assumed to be constant), and the failure time of occurrence t_j are all unknown. For example, an aircraft control surface may be locked at some unknown fixed value due to hydraulics failure. Let $v(t) = [v_1, v_2, \ldots, v_m]^\intercal \in R^m$ be the applied(commanded) control input signal. In the presence of actuator failures, the *actual* input vector $u(t)$ to the system can be described as

$$u(t) = v(t) + \sigma(\bar{u} - v(t)) = (I - \sigma)v(t) + \sigma\bar{u} \tag{3}$$

where

$$\begin{aligned}
\bar{u} &= [\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_m]^\intercal \\
\sigma &= \text{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_m\} \\
\sigma_i &= 1 \text{ if the ith actuator fails, i.e., } u_i = \bar{u}_i \\
\sigma_i &= 0 \text{ otherwise.}
\end{aligned} \tag{4}$$

That is, σ is a diagonal matrix ("failure pattern matrix") whose entries are piecewise constant signals that take on the values of zero or one. The components of the applied input signal $v(t) = [v_1(t), v_2(t), \ldots, v_m(t)]^\intercal$ which correspond to the failed actuators, cannot affect the system dynamics. The actuator failures are uncertain in value, pattern and time of occurrence.

The objective is to design an adaptive feedback control law using the available measurement $y\,(t)$ with unknown bias β, such that closed-loop signal boundedness is ensured and the system state $x\,(t)$ tracks the state of a reference model described by

$$\dot{x}_m\,(t) = A_m x_m\,(t) + B_m r\,(t) \tag{5}$$

where $x_m \in R^n$ is the reference model state, $A_m \in R^{n\times n}$, $B_m \in R^{n\times m_r}$, and $r(t) \in R^{m_r}$ $(1 \le m_r \le m)$ is a bounded reference input used in system operation (e.g., pilot input in the case of aircraft).

This paper considers the single-reference-input case, i.e., r is a scalar $(m_r = 1)$ and $B_m \in R^n$. The actuators are assumed to be similar (e.g., segments of the same control surface), i.e., the columns (b_i) of the B matrix can differ only by an unknown scalar multiplier. It is also assumed that b_i are parallel to the reference model input matrix $B_m \in R^n$, i.e.,

$$b_i = B_m/\alpha_i,\ \ i = 1, \ldots, m \tag{6}$$

for some unknown (finite and non-zero) α_i's whose signs are assumed known. The objective is to design an adaptive control law that will ensure closed-loop signal boundedness and asymptotic state tracking (i.e., $\lim_{t\to\infty}(x(t) - x_m(t)) = 0$) despite system uncertainties, actuator failures, and sensor bias faults. The adaptive controller should synthesize the control signal $v(t)$ so as to ensure system stability and asymptotic tracking regardless of whether (or which) actuators have failed, or the failure values, in the presence of unknown biases in the sensor outputs. That is, $v(t)$ should be capable of compensating for the actuator failures and sensor bias faults automatically.

As in the case of actuator failures with no sensor bias [9], it is assumed that the system (A, B) and the reference model (A_m, b_m) satisfy the following SFST matching conditions (similar to those in [9], modified for actuator failure and sensor bias accommodation): For every failure pattern, there exist gains $K_1 \in R^{n\times m}$, and $k_2,\ k_3 \in R^m$, such that

$$A_m = A + B(I-\sigma)K_1^\intercal;\ B_m = B(I-\sigma)k_2;\ B\sigma\bar{u} = -B(I-\sigma)(K_1^\intercal\beta + k_3). \tag{7}$$

The reference model (A_m, B_m) is usually designed to capture the desired closed-loop response of the plant. For example, the reference model may be designed using optimal and robust control methods such as LQR, H_2, or H_∞ methods. For the adaptive control scheme, only A_m and B_m need to be known. Because A_m is a Hurwitz matrix, there exist positive definite matrices $P = P^\intercal$, $Q = Q^\intercal \in R^{n\times n}$, such that the following Lyapunov inequality holds:

$$A_m^\intercal P + PA_m \leq -Q. \tag{8}$$

Remark- Another type of actuator failure, **reduced effectiveness**, can occur due to reasons such as partial loss of a control surface, or control surface icing in aircraft. When such failures are present in addition to the failures defined in (2), the actual control input generated by the non-failed actuators is reduced by a factor between 0 and 1. Reduced actuator effectiveness can be readily incorporated in the unknown coefficients α_i's in Eq. (6).

3 MRAC Using Adaptive Sensor Bias Estimation

The sensor measurements available for feedback have unknown biases as in Eq. (1). This section considers a modified adaptive control law that includes sensor bias estimation similar to the approach considered in [13]. Let $\hat{\beta}\,(t)$ denote an estimate of the unknown sensor bias β. Using $\hat{\beta}$, define the 'corrected' state $\bar{x}(t) \in R^n$ as

$$\bar{x} = y - \hat{\beta}. \tag{9}$$

Therefore,

$$\bar{x} = x + \beta - \hat{\beta} = x + \tilde{\beta}. \tag{10}$$

where $\tilde{\beta} = \beta - \hat{\beta}$. Design an adaptive control law as

$$v = \hat{K}_1^\intercal y + \hat{k}_2 r + \hat{k}_3 \tag{11}$$

where $\hat{K}_1\,(t) \in R^{n\times m}$, and $\hat{k}_2,\ \hat{k}_3\,(t) \in R^m$ are the adaptive gains. Therefore, the closed-loop corrected-state equation is

$$\begin{aligned}
\dot{\bar{x}} &= Ax + B(I-\sigma)\left(\hat{K}_1^\intercal y + \hat{k}_2 r + \hat{k}_3\right) + \dot{\tilde{\beta}} + B\sigma\bar{u} \\
&= Ax + B(I-\sigma)\left(K_1^\intercal y + k_2 r + k_3\right) + B(I-\sigma)\left(\tilde{K}_1^\intercal y + \tilde{k}_2 r + \tilde{k}_3\right) + \dot{\tilde{\beta}} + B\sigma\bar{u} \\
&= \left(A + B(I-\sigma)K_1^\intercal\right)x + B(I-\sigma)\left(\tilde{K}_1^\intercal y + \tilde{k}_2 r + \tilde{k}_3\right) + B(I-\sigma)k_2 r \\
&\quad + B(I-\sigma)K_1^\intercal \beta + B(I-\sigma)k_3 + \dot{\tilde{\beta}} + B\sigma\bar{u}
\end{aligned} \tag{12}$$

where $\tilde{K}_1 = \hat{K}_1 - K_1$, $\tilde{k}_2 = \hat{k}_2 - k_2$, and $\tilde{k}_3 = \hat{k}_3 - k_3$. The matching conditions of (7) are assumed to be satisfied. Using (7) and (10) in (12), we get

$$\dot{\bar{x}} = A_m\bar{x} + B_m r + B(I-\sigma)\left(\tilde{K}_1^\intercal y + \tilde{k}_2 r + \tilde{k}_3\right) - A_m\tilde{\beta} + \dot{\tilde{\beta}}. \tag{13}$$

Define a measurable auxiliary error signal $\hat{e}(t) \in R^n$ as

$$\hat{e} = \bar{x} - x_m. \tag{14}$$

Therefore, from (10), we have

$$\hat{e} = x - x_m + \tilde{\beta} = e + \tilde{\beta} \tag{15}$$

where $e = x - x_m$ denotes the state tracking error. Differentiating (14) with respect to time, the closed-loop auxiliary error system can be expressed as

$$\dot{\hat{e}} = \dot{\hat{x}} - \dot{x}_m. \tag{16}$$

Substituting (13) and (5) into (16) yields

$$\begin{aligned}\dot{\hat{e}} &= A_m\hat{e} + B(I-\sigma)\left(\tilde{K}_1^{\intercal}y + \tilde{k}_2 r + \tilde{k}_3\right) - A_m\tilde{\beta} + \dot{\hat{\beta}} \\ &= A_m\hat{e} + \sum_{j\notin\mathcal{J}_p}^{m} b_j(\widetilde{K}_{1j}^{\intercal}y + \widetilde{k}_{2j}r + \widetilde{k}_{3j}) - A_m\tilde{\beta} + \dot{\hat{\beta}} \\ &= A_m\hat{e} + B_m\sum_{j\notin\mathcal{J}_p}^{m}(1/\alpha_j)(\widetilde{K}_{1j}^{\intercal}y + \widetilde{k}_{2j}r + \widetilde{k}_{3j}) - A_m\tilde{\beta} + \dot{\hat{\beta}} \end{aligned} \tag{17}$$

where the subscript j denotes the j^{th} column of $\tilde{K}_1$ and the j^{th} element of $\tilde{k}_2,\ \tilde{k}_3$ (similar notation for $K_1,\ \hat{K}_1,\ k_2,\ \hat{k}_2,\ k_3,\ \hat{k}_3$). The following theorem gives adaptive gain update and bias estimation laws that guarantee closed-loop signal boundedness as well as bounded tracking error.

Theorem 1: For the system given by (1), (3), (5); the adaptive controller (11), the gain adaptation laws

$$\begin{aligned}\dot{\hat{K}}_{1j} &= -sgn(\alpha_j)\Gamma_{1j}B_m^{\intercal}P\hat{e}y \\ \dot{\hat{k}}_{2j} &= -sgn(\alpha_j)\gamma_{2j}B_m^{\intercal}P\hat{e}r \\ \dot{\hat{k}}_{3j} &= -sgn(\alpha_j)\gamma_{3j}B_m^{\intercal}P\hat{e}\end{aligned} \tag{18}$$

for $j = 1, 2, \ldots, m$, where $\Gamma_{1j} \in R^{n\times n}$ is a constant symmetric positive definite matrix, $\gamma_{2j},\ \gamma_{3j}$, are constant positive scalars, and P was defined in (8); and the bias estimation law

$$\dot{\hat{\beta}} = \eta P^{-1}A_m^{\intercal}P\hat{e} \tag{19}$$

where $\eta \in R$ is a tunable positive constant gain, guarantee that all the closed-loop signals including the adaptive gains and bias estimate are bounded and the tracking error $e\,(t)$ is bounded.

Proof: Define

$$V = \hat{e}^{\intercal}P\hat{e} + \sum_{j\notin\mathcal{J}_p}^{m}\frac{1}{|\alpha_j|}(\tilde{K}_{1j}^{\intercal}\Gamma_{1j}^{-1}\tilde{K}_{1j} + \tilde{k}_{2j}^2\gamma_{2j}^{-1} + \tilde{k}_{3j}^2\gamma_{3j}^{-1}) + \frac{1}{\eta}\tilde{\beta}^{\intercal}P\tilde{\beta}. \tag{20}$$

Differentiating (20) with respect to time, and using (8), (17), and the gain update laws in (18), the following expression is obtained upon simplification:

$$\dot{V} \leq -\hat{e}^{\intercal}Q\hat{e} - 2\hat{e}^{\intercal}PA_m\tilde{\beta} + 2\hat{e}^{\intercal}P\dot{\hat{\beta}} + \frac{2}{\eta}\tilde{\beta}^{\intercal}P\dot{\hat{\beta}}. \tag{21}$$

Using the bias estimation law of (19) in (21), we get

$$\begin{aligned}\dot{V} &\leq -\hat{e}^\intercal Q\hat{e} - 2\hat{e}^\intercal PA_m\tilde{\beta} + 2\eta\hat{e}^\intercal A_m^\intercal P\hat{e} + 2\tilde{\beta}^\intercal A_m^\intercal P\hat{e} \\ &\leq -\hat{e}^\intercal Q\hat{e} - \eta\hat{e}^\intercal Q\hat{e} = -(1+\eta)\,\hat{e}^\intercal Q\hat{e}.\end{aligned}$$

Therefore, $\dot{V} \leq 0$, i.e., $V(t)$ is bounded for all t, and $\hat{e}(t)$, $\hat{\beta}(t)$, $y(t)$, $\hat{K}_1$, $\hat{k}_2$, $\hat{k}_3$ are all bounded and $\hat{e}(t) \in L^2$. From (17), (19) and closed-loop signal boundedness, we have $\dot{\hat{\beta}}$, $\dot{\hat{e}}(t) \in L^\infty$, therefore, using Barbalat's lemma [7], $\lim_{t\to\infty}\hat{e}(t) = 0$. That is, all signals and estimates are bounded, and $\lim_{t\to\infty}(\bar{x} - x_m) = 0$. ■

The adaptive control law in Theorem 1 guarantees stability (signal boundedness) and bounded tracking error. However, although $\hat{e} \to 0$ as $t \to \infty$, it cannot be concluded that $e(t) \to 0$ unless $\tilde{\beta}(t) \to 0$. If persistent excitation is present, $\tilde{\beta}(t)$ would approach 0 as $t \to \infty$, in which case, $e(t) \to 0$ as $t \to \infty$.

In an effort to accomplish asymptotic tracking without the need for persistent excitation, the next section addresses the case when a separate asymptotically stable bias estimator is available.

4 MRAC Using Asymptotic Bias Estimator

In the presence of actuator failures and sensor bias as defined in Section 2, suppose an external bias estimator is available, such that the estimation error dynamics is of the form

$$\dot{\tilde{\beta}} = A_\beta\tilde{\beta} \tag{22}$$

where $\hat{\beta}(t)$ is an estimate of β, and

$$\tilde{\beta}(t) = \beta - \hat{\beta}(t)$$

is the estimation error; $A_\beta \in R^{n\times n}$ is a known Hurwitz (asymptotically stable) matrix, which implies that $\lim_{t\to\infty}\tilde{\beta}(t) = 0$. Defining the adaptive control law as in (11) and proceeding as in Section 3, we obtain (17). The adaptive control law (11) along with the gain adaptation laws (18) guarantee signal boundedness and asymptotic tracking.

Theorem 2: For the system given by (1), (3), (5), (11) with a bias estimator that satisfies (22), and the gain adaptation laws (18) guarantee that all the closed-loop signals including adaptive gains are bounded and the tracking error $e(t) \to 0$ as $t \to \infty$.

Proof: Define

$$V = \hat{e}^\intercal P\hat{e} + \sum_{j\notin\mathcal{J}_p}^{m}\frac{1}{|\alpha_j|}(\tilde{K}_{1j}^\intercal\Gamma_{1j}^{-1}\tilde{K}_{1j} + \tilde{k}_{2j}^2\gamma_{2j}^{-1} + \tilde{k}_{3j}^2\gamma_{3j}^{-1}) + \tilde{\beta}^\intercal P_\beta\tilde{\beta} \tag{23}$$

where $P_\beta = P_\beta^\intercal \in R^{n\times n}$ is a positive definite solution of the Lyapunov inequality

$$A_\beta^\intercal P_\beta + P_\beta A_\beta \leq -Q_\beta \tag{24}$$

for some $Q_\beta = Q_\beta^\top > 0 \ \in R^{n\times n}$. Differentiating (23) with respect to time and using (8), (17), (18), (22), (24), the following expression is obtained upon simplification:

$$\begin{aligned}\dot{V} &\le -\hat{e}^\top Q\hat{e} - 2\hat{e}^\top P\left(A_m - A_\beta\right)\tilde{\beta} - \tilde{\beta}^\top Q_\beta\tilde{\beta} \\ &\le -z^\top \bar{Q} z\end{aligned}$$

where $z \in R^{2n}$ is defined as

$$z = \begin{bmatrix}\hat{e}^\top & \tilde{\beta}^\top\end{bmatrix}^\top$$

and $\bar{Q} \in R^{2n\times 2n}$ is defined as

$$\bar{Q} = \begin{bmatrix} Q & P\left(A_m - A_\beta\right) \\ \left(A_m - A_\beta\right)^\top P & Q_\beta\end{bmatrix}.$$

Since Q is positive definite, the matrix $\bar{Q}$ is positive definite iff

$$Q_\beta - \left(A_m - A_\beta\right)^\top PQ^{-1}P\left(A_m - A_\beta\right) > 0. \tag{25}$$

Q_β can be chosen such that (25) is satisfied; therefore, $\dot{V} \le 0$, i.e., $V(t)$ is bounded for all t, and $\hat{e}(t),\ \hat{\beta}(t),\ y(t),\ \hat{K}_1,\ \hat{k}_2,\ \hat{k}_3$ are all bounded and $\hat{e}(t),\ \tilde{\beta}(t) \in L^2$. Using arguments similar to those in the proof of Theorem 1, and noting from (22) that $\tilde{\beta}(t) \to 0$ as $t \to \infty$, it can be concluded that all signals including the adaptive gains are bounded, and $\lim_{t\to\infty} e(t) = 0$, i.e., $x(t) \to x_m(t)$ ■

5 A Special Case- Bias in Rate Measurements Only

This section considers a special case wherein sensor bias is present only in the rate measurements, but bias-free measurements of the corresponding position variables are available. For example, rate gyros, which measure angular velocities, can be prone to constant or slowly-varying biases, while bias-free angle (attitude) measurements can be available by using GPS data. Suppose the state vector $x(t)$ in (1) is composed of rate variables $\xi(t) \in R^{n_1}$, and the remaining states $x_2(t) \in R^{(n-n_1)}$, and that bias exists only in the measurements of $\xi(t)$. Then (1) can be written in the form:

$$\dot{x} = \frac{d}{dt}\begin{bmatrix}\xi \\ x_2\end{bmatrix} = [A]\begin{bmatrix}\xi \\ x_2\end{bmatrix} + \begin{bmatrix}B_1 \\ B_2\end{bmatrix}u \tag{26}$$

$$y = \begin{bmatrix}y_\xi \\ y_2\end{bmatrix} = \begin{bmatrix}\xi \\ x_2\end{bmatrix} + \begin{bmatrix}\beta \\ 0\end{bmatrix} \tag{27}$$

where $\beta \in R^{n_1}$ is a constant unknown bias in the rate measurements. (For example, for a longitudinal aircraft model in a wings-level cruise condition, having a bias error in the pitch rate gyro, the state variables can be represented by: $\xi = q,\ x_2 = [v, \alpha, \theta]^\top$, where $q,\ v,\ \alpha,\ \theta$ denote the pitch rate, airspeed, angle of attack, and pitch angle respectively). In general, some components of ρ may be included in x_2.

The position variable vector ρ corresponding to ξ is given by

$$\dot{\rho} = \xi = y_\xi - \beta. \tag{28}$$

It will be assumed that a bias-free measurement of $\rho(t)$ is available, i.e.,

$$y_\rho = \rho. \tag{29}$$

5.1 Model-Independent Observer

Using the bias-free position measurements, a non-model-based observer can be designed to estimate the velocity sensor bias as shown below.

From (28) and (29), and augmenting the equation

$$\dot{\beta} = 0$$

the following system is obtained.

$$\begin{bmatrix} \dot{\rho} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} 0 & -I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ \beta \end{bmatrix} + \begin{bmatrix} I \\ 0 \end{bmatrix} y_\xi \tag{30}$$

$$y_\rho = [I \quad 0] \begin{bmatrix} \rho \\ \beta \end{bmatrix}. \tag{31}$$

The observability of the above system can be readily verified, and an observer gain $L = [L_1^\intercal \ L_2^\intercal]^\intercal$ can be designed to yield an asymptotically stable state estimator:

$$\begin{bmatrix} \dot{\hat{\rho}} \\ \dot{\hat{\beta}} \end{bmatrix} = \begin{bmatrix} 0 & -I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\rho} \\ \hat{\beta} \end{bmatrix} + \begin{bmatrix} I \\ 0 \end{bmatrix} y_\xi + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} (\rho - \hat{\rho}). \tag{32}$$

The estimation error equation is:

$$\begin{bmatrix} \dot{\tilde{\rho}} \\ \dot{\tilde{\beta}} \end{bmatrix} = \begin{bmatrix} -L_1 & -I \\ -L_2 & 0 \end{bmatrix} \begin{bmatrix} \tilde{\rho} \\ \tilde{\beta} \end{bmatrix} := [A_f] \begin{bmatrix} \tilde{\rho} \\ \tilde{\beta} \end{bmatrix} \tag{33}$$

where $\tilde{\rho} = \rho - \hat{\rho}$ and A_f is a Hurwitz matrix. Therefore, given any symmetric positive definite matrix $Q_f \in R^{2n_1 \times 2n_1}$, there exists a symmetric positive definite matrix $P_f \in R^{2n_1 \times 2n_1}$ such that

$$A_f^\intercal P_f + P_f A_f \leq -Q_f. \tag{34}$$

5.2 Asymptotic State Tracking

Note that the system state variables are re-ordered as in (26); therefore the state variables of the reference model in (5) are also similarly re-ordered for consistency. The bias estimate $\hat{\beta}$ can be used to define the "corrected" state vector

$$\bar{x} = y - \begin{bmatrix} \hat{\beta} \\ 0 \end{bmatrix} = x + \begin{bmatrix} \beta \\ 0 \end{bmatrix} - \begin{bmatrix} \hat{\beta} \\ 0 \end{bmatrix} = x + \begin{bmatrix} \tilde{\beta} \\ 0 \end{bmatrix}. \tag{35}$$

As in Section 3, the auxiliary tracking error (which can be measured) is defined as

$$\hat{e} = \bar{x} - x_m = x - x_m + \begin{bmatrix} \tilde{\beta} \\ 0 \end{bmatrix} = e + \begin{bmatrix} \tilde{\beta} \\ 0 \end{bmatrix}. \tag{36}$$

Proceeding as in Section 3, a slightly modified version of (17) is obtained:

$$\dot{\hat{e}} = A_m e + B_m \sum_{j \notin \mathcal{J}_p}^{m} (1/\alpha_j)(\widetilde{K}_{1j}^\intercal y + \widetilde{k}_{2j} r + \widetilde{k}_{3j}) - A_m \mathcal{I} \tilde{\beta} + \mathcal{I} \dot{\tilde{\beta}} \tag{37}$$

where $\mathcal{I} = [I_{n_1} \; 0_{n_1 \times (n-n_1)}]^\intercal$ (I_l and $0_{l \times k}$ denote the identity matrix and the zero matrix of the subscript dimensions). The following result gives an adaptive control law that guarantees closed-loop signal boundedness and asymptotic tracking for systems with rate sensor bias and actuator failures.

Theorem 3: For the system given by (26), (5), (11), (32) and the gain adaptation laws (18), all closed-loop signals including the adaptive gains are bounded and the tracking error $e(t) \to 0$ as $t \to \infty$.

Proof: Define

$$V = \hat{e}^\intercal P \hat{e} + \sum_{j \notin \mathcal{J}_p}^{m} \frac{1}{|\alpha_j|} (\tilde{K}_{1j}^\intercal \Gamma_{1j}^{-1} \tilde{K}_{1j} + \tilde{k}_{2j}^2 \gamma_{2j}^{-1} + \tilde{k}_{3j}^2 \gamma_{3j}^{-1}) + \tilde{z}^\intercal P_f \tilde{z} \tag{38}$$

where $\tilde{z} = [\tilde{\rho}^\intercal \; \tilde{\beta}^\intercal]^\intercal$. Differentiating (38) with respect to time and using (8), (37), (33), (34), the following expression is obtained upon simplification after using the gain update laws (18):

$$\dot{V} \leq -\hat{e}^\intercal Q \hat{e} - 2\hat{e}^\intercal P [\mathcal{I} L_2 \;\; A_m \mathcal{I}] \tilde{z} - \tilde{z}^\intercal Q_f \tilde{z}$$

that is,

$$\dot{V} \leq -\chi^\intercal \bar{Q} \chi \tag{39}$$

where $\chi \in R^{n+2n_1}$ is defined as

$$\chi = \begin{bmatrix} \hat{e}^\intercal & \tilde{z}^\intercal \end{bmatrix}^\intercal$$

and $\bar{Q} \in R^{(n+2n_1)\times(n+2n_1)}$ is defined as

$$\bar{Q} = \begin{bmatrix} Q & P(\mathcal{I} L_2 \;\; A_m \mathcal{I}) \\ (\mathcal{I} L_2 \;\; A_m \mathcal{I})^\intercal P & Q_f \end{bmatrix}. \tag{40}$$

Since Q is positive definite, the matrix $\bar{Q}$ is positive definite iff

$$Q_f - [\mathcal{I} L_2 \;\; A_m \mathcal{I}]^\intercal P Q^{-1} P [\mathcal{I} L_2 \;\; A_m \mathcal{I}] > 0. \tag{41}$$

Q_f can be chosen such that (41) is satisfied; therefore, $\dot{V} \leq 0$, i.e., $V(t)$ is bounded for all t. Using arguments similar to those in the proof of Theorem 1, and noting from (33) that $\tilde{\beta}(t) \to 0$ as $t \to \infty$, it can be concluded that all signals including the adaptive gains are bounded, and $\lim_{t\to\infty} e(t) = 0$, i.e., $x(t) \to x_m(t)$. ■

6 Concluding Remarks

This paper addressed accommodation of simultaneous actuator failures and sensor bias faults in a model reference adaptive control (MRAC) setting. A modified MRAC law, which combines bias estimation with control gain adaptation, was developed and shown to provide bounded tracking error and signal boundedness. For the case when an external asymptotically stable sensor bias estimator is available, an MRAC law was developed to accomplish asymptotic state tracking and signal boundedness. For a special case wherein biases are present only in the rate measurements and bias-free position measurements are available, an MRAC law was developed using a model-independent bias estimator, and was shown to provide asymptotic state tracking with signal boundedness. This paper focused on analytical results. Application examples and simulation results will be included in future papers on this topic. Future work will also address extension to cases with multiple actuator groups and multiple reference inputs.

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14. ABSTRACT
The problem of simultaneously accommodating unknown sensor biases and unknown actuator failures in uncertain systems is considered in a direct model reference adaptive control (MRAC) setting for state tracking using state feedback. Sensor biases and actuator faults may be present at the outset or may occur at unknown instants of time during operation. A modified MRAC law is proposed, which combines sensor bias estimation with control gain adaptation for accommodation of sensor biases and actuator failures. This control law is shown to provide signal boundedness in the resulting system. For the case when an external asymptotically stable sensor bias estimator is available, an MRAC law is developed to accomplish asymptotic state tracking and signal boundedness. For a special case wherein biases are only present in the rate measurements and bias-free position measurements are available, an MRAC law is developed using a model-independent bias estimator, and is shown to provide asymptotic state tracking with signal boundedness.

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