



## **The memory inherent in a non-square** $2^{2n+1}$ -QAM can be exploited to obtain coding gains.

NASA's Jet Propulsion Laboratory, Pasadena, California

It has been shown that a non-square (NS)  $2^{2n+1}$ -ary (where *n* is a positive integer) quadrature amplitude modula-tion  $[(NS)2^{2n+1}-QAM]$  has inherent memory that can be exploited to obtain coding gains. Moreover, it should not be necessary to build new hardware to realize these gains.

The present scheme is a product of theoretical calculations directed toward reducing the computational complexity of decoding coded  $2^{2n+1}$ -QAM. In the general case of  $2^{2n+1}$ -QAM, the signal constellation is not square and it is impossible to have independent in-phase (I) and quadrature-phase (Q) mapping and demapping. However, independent I and Q mapping and demapping are desirable for reducing the complexity of computing the log likelihood ratio (LLR) between a bit and a received symbol (such computations are essential operations in iterative decoding). This is because in modulation schemes that include independent I and Q mapping and demapping, each bit of a signal point is involved in only one-dimensional mapping and demapping. As a result, the computation of the LLR is equivalent to that of a one-dimensional pulse amplitude modulation (PAM) system. Therefore, it is desirable to find a signal constellation that enables independent I and Q mapping and demapping for  $2^{2n+1}$ -QAM.

Careful labeling of each symbol in the constellation reveals the memory inherent in the corresponding modulation scheme. To obtain the signal constellation or  $(NS)2^{2n+1}$ -QAM, one starts with the square constellation for (NS)2<sup>2n+2</sup>-QAM

with independent I-dimension and Q-dimension Gray-code (GC) mapping. That is, each dimension of the square  $2^{2n+2}$ -QAM is an independent PAM with GC mapping. There are  $2^{2n+2}$  signal points in the square  $2^{2n+2}$ -QAM constellation. The GC label of each signal point can be obtained by concatenating its I-dimension GC label with its Q-dimension GC label. If one then deletes every other point in each dimension, then there remain only half of the points in each row and half of the points in each column. The distance between the remaining points in the rows and columns are the same. The total number of remaining points is  $2^{2n+1}$ and these points form the nonsquare constellation for  $(NS)2^{2n+1}$ -QAM.

The labels of the remaining  $2^{2n+1}$ points are the same as in the square  $2^{2n+2}$ -QAM constellation. This means that 2n+2bits are used to label each of the  $2^{2n+1}$ points in the  $(NS)2^{2n+1}$ -QAM constellation: n+1 bits for the I-dimension labeling, and n+1 bits for the Q-dimension labeling. However, each point in an  $(NS)2^{2n+1}$ -QAM constellation represents only 2n+1 bits of information. Hence, the 2n+2 bits used for labeling an (NS) $2^{2n+1}$ -QAM signal point are not independent.

Close examination has shown that for any signal point in the  $(NS)2^{2n+1}$ -QAM constellation, the last bit of its  $2^{n+2}$  labeling bits can be viewed as a parity-check bit of the other 2n+1 bits. Therefore, each  $(NS)2^{2n+1}$ -QAM symbol can be generated by first encoding the corresponding 2n+1bits with a (2n+2, 2n+1) single-paritycheck (SPC) block encoder and then using the 2n+2 encoded bits to select one of the 2n+1 points on the (NS) $2^{2n+1}$ -QAM.



A Non-Square 2<sup>2n+1</sup>-Quadrature Amplitude Modulator would be concatenated with a forward-errorcorrecting (FEC) encoder. The FEC output bits would be permuted by a random interleaver ( $\pi$ ) before each successive group of 2*n*+1 bits was mapped into an (NS)2<sup>2*n*+1</sup>-QAM symbol.

The I-dimension position and the Q-dimension position of a signal point on an  $(NS)\hat{2}^{2n+1}$ -QAM constellation can be independently determined by the first n+1encoded bits and the remaining n+1 encoded bits, respectively. The (2n+2, 2n+1)SPC block code can be generated as a recursive, systematic, terminated convolutional code with two states. In other words, the decomposition of  $(NS)2^{2n+1}$ -QAM into a block encoder and a memoryless modulator leads to a showing that  $(NS)2^{2n+1}$ -QAM is, by itself a form of coded modulation.

When concatenated with a forwarderror-correcting (FEC) code (see figure), this decomposition can be applied to obtain joint iterative demodulation and decoding algorithms that exploit the inherent memory of  $(NS)2^{2n+1}$ -QAM so as to achieve better coding gains. In addition, because of the independent I and Q mapping of  $(NS)2^{2n+1}$ -QAM, the decoding complexity can be reduced to that of one-dimensional PAM. Moreover, because the signal constellation of  $(NS)2^{2n+1}$ -QAM is a subset of the square  $2^{2n+2}$ -QAM constellation, it should be possible, in practice, to implement  $(NS)2^{2n+1}$ -QAM by use of  $2^{2n+2}$ -QAM equipment already in existence.

Results of some computational simulations have shown that with iterative demodulation and decoding according to this scheme, coded (NS)8-QAM performs 0.5 dB better than does coded standard 8-QAM and 0.7 dB better than does coded 8-ary phase-shift keying (8 PSK) when the FEC code is the (15,11) Hamming code concatenated with a rate-1 accumulator code. Other simulation results show that coded (NS)32-QAM performs 0.25 dB better than does coded standard 32-QAM.

This work was done by Lifang Li, Dariush Divsalar, and Samuel Dolinar of Caltech for NASA's Jet Propulsion Laboratory. Further information is contained in a TSP (see page 1).

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