# Application of Temperature Sensitivities during Iterative Strain–Gage Balance Calibration Analysis

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**A new method is discussed that may be used to correct wind tunnel strain–gage balance load predictions for the influence of residual temperature effects at the location of the strain–gages. The method was designed for the iterative analysis technique that is used in the aerospace testing community to predict balance loads from strain–gage outputs during a wind tunnel test. The new method implicitly applies temperature corrections to the gage outputs during the load iteration process. Therefore, it can use uncorrected gage outputs directly as input for the load calculations. The new method is applied in several steps. First, balance calibration data is analyzed in the usual manner assuming that the balance temperature was kept constant during the calibration. Then, the temperature difference relative to the calibration temperature is introduced as a new independent variable for each strain–gage output. Therefore, sensors must exist near the strain–gages so that the required temperature differences can be measured during the wind tunnel test. In addition, the format of the regression coefficient matrix needs to be extended so that it can support the new independent variables. In the next step, the extended regression coefficient matrix of the original calibration data is modified by using the manufacturer specified temperature sensitivity of each strain–gage as the regression coefficient of the corresponding temperature difference variable. Finally, the modified regression coefficient matrix is converted to a data reduction matrix that the iterative analysis technique needs for the calculation of balance loads. Original calibration data and modified check load data of NASA's MC60D balance are used to illustrate the new method.**

## Nomenclature

| | |
|---|---|
| AF | = axial force component |
| $\mathbf{C_1}, \mathbf{C_2}$ | = parts of data reduction matrix that are used with the primary load iteration equation |
| $F$ | = balance load |
| $\mathbf{F}_\xi$ | = vector of predicted balance loads for load iteration step $\xi$ |
| $\mathbf{H}$ | = load dependent matrix that is used in the load iteration equation |
| $k$ | = strain–gage index |
| $l$ | = total number of strain–gage outputs |
| $n$ | = total number of balance load components |
| N1 | = forward normal force component |
| N2 | = aft normal force component |
| $R$ | = electrical output of a strain–gage |
| $R1,\ R2,\ ...$ | = electrical outputs of a strain–gages |
| RM | = rolling moment |
| S1 | = forward side force component |
| S2 | = aft side force component |
| $T$ | = temperature at strain–gage location |
| T1d, T2d, ... | = temperature difference at strain–gage location as dependent variable |

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T1i, T2i, ... = temperature difference at strain–gage location as independent variable
$T_\circ$ = uniform calibration temperature of a strain–gage balance

$\alpha_0,\ \alpha_1,\ ...$ = regression model coefficients of fitted gage outputs
$\mathbf{\Delta R}$ = vector of strain–gage output differences that is used for iterative load calculation
$\Delta T$ = difference between temperature at strain–gage and calibration temperature of balance
$\xi$ = load iteration step index

## I. Introduction

Different techniques are used in the aerospace testing community to predict strain–gage balance loads from measured gage outputs during a wind tunnel test. The load prediction is usually based on the result of a multivariate regression analysis of strain–gage balance calibration data. The iterative analysis technique, for example, first fits strain–gage outputs of a balance as a function of the loads that were applied during the calibration. Then, the regression coefficients are converted to data reduction matrix coefficients so that loads can be predicted from measured gage outputs using a load iteration scheme (see Ref. [1] for a detailed description of the iterative analysis technique).

By design, the iterative analysis technique makes the assumption that the number of independent variables (i.e., balance loads) must equal the number of dependent variables (i.e., strain–gage outputs). This limitation made it difficult in the past to include corrections for temperature effects directly in the regression model of a strain–gage output.

In principle, corrections for temperature effects may be estimated by using the temperature sensitivity of a strain–gage in combination with a new independent variable that is equal to the difference between the temperature of the balance during a wind tunnel test and the calibration temperature of the balance. Recently, an extension of the iterative analysis technique was introduced that allows an analyst to use more independent than dependent variables for the analysis of balance calibration data (for a description of the extension of the iterative analysis technique see Ref. [2]). This improvement makes it now possible to directly include temperature corrections during the load iteration process.

In general, temperature effects on the gage outputs may be included in the balance load calculation process using one of two processing approaches. The first approach corrects the gage outputs for temperature effects before the load iteration takes place. This approach has traditionally been used in the aerospace testing community. The second approach, i.e., the new approach, applies the temperature correction during the load iteration process by extending and editing the original regression model of the gage outputs. This leads to a new data reduction matrix that may be used with the load iteration scheme of the iterative analysis technique to predict the balance loads.

In the next section of the paper the two approaches are explained in more detail. First, the theoretical background of the two approaches is reviewed. Then, a realistic simulation of a temperature sensitive balance check load data set is discussed to illustrate basic elements of the newly introduced second approach.

## II. Inclusion of Temperature Sensitivity in Iterative Analysis Technique

### A. General Remarks

Sometimes the accuracy of iterative strain–gage balance load predictions may be improved by including a residual temperature correction in the regression model of the gage outputs. This residual correction ignores temperature gradients that may exist inside of the balance. Therefore, it only corrects for the difference between the uniform balance calibration temperature and the temperature of the balance during the wind tunnel test.

The temperature correction of the gage outputs may be a simple linear correction that uses (i) the temperature sensitivity and (ii) the temperature difference at the strain–gage as input. The first input, i.e., the temperature sensitivity of a strain–gage, is defined as the derivative of the electrical output of a strain–gage with respect to the temperature at or near the gage location. Then, we can write

$$\textit{temperature sensitivity} \quad \Longrightarrow \quad \frac{\partial\, R}{\partial\, T}(k)$$

where $R$ is the electrical output of the gage, $T$ is the temperature at or near the gage, and $k$ is the gage index. A numerical value of the derivative may be obtained by performing a sensitivity experiment for each strain–gage. Alternatively, the derivative may be supplied by the manufacturer of the strain–gage.

Now, a simple way needs to be found so that the temperature sensitivity can be included in the regression model of the gage outputs that the iterative analysis technique uses. Therefore, the regression model of the gage outputs at a constant calibration temperature $T_\circ$ needs to be revisited. It is given by the following equation (for more detail see Ref. [1], Eq. (3.1.3), or, Ref. [3], Eq. (13)):

$$R(k,T_\circ) \;=\; \underbrace{\alpha_0(k,T_\circ)}_{intercept} \;+\; \underbrace{\alpha_1(k,T_\circ)\cdot F(1) \;+\; \alpha_2(k,T_\circ)\cdot F(2) \;+\; \cdots}_{linear\ and\ non-linear\ terms} \quad ; \quad 1 \;\leq\; k \;\leq\; l \tag{1}$$

The original set of independent variables and regression coefficients used in Eq. (1) can be summarized as follows:

$$original\ set\ of\ independent\ variables \;\Longrightarrow\; F(1),\; F(2),\; \cdots,\; F(n)$$

$$original\ set\ of\ regression\ coefficients \;\Longrightarrow\; \alpha_0(k,T_\circ),\; \alpha_1(k,T_\circ),\; \cdots$$

A linear temperature correction of the gage outputs may be included in Eq. (1) by simply adding the product of the temperature sensitivity and the temperature difference relative to the calibration temperature to the left hand side of Eq. (1). Then, we get:

$$R(k,T) \;=\; R(k,T_\circ) \;+\; \underbrace{\left[\frac{\partial\, R}{\partial\, T}(k)\right]}_{temp.\ sensitivity} \cdot\; \Delta T(k) \quad ; \quad 1 \;\leq\; k \;\leq\; l \tag{2a}$$

where

$$\Delta T(k) \;=\; T(k) \;-\; T_\circ \tag{2b}$$

At this point a user of the iterative analysis technique may use one of two processing approaches in order to include temperature corrections. The first approach explicitly applies the temperature correction to the gage outputs before the load iteration process. The second approach implicitly applies the temperature correction to the gage outputs during the load iteration process. The two processing approaches are explained in more detail in the following sections.

### B. Explicit Application of Temperature Corrections

This first processing approach is traditionally applied in the aerospace testing community. It corrects the measured gage outputs for the difference between the calibration temperature and the temperature of the balance during the wind tunnel test before the load iteration begins. The required relationship between the gage output at the calibration temperature $T_\circ$ and the measured gage output at the balance temperature $T$ during the wind tunnel test is obtained after rearranging Eq. (2$a$). Then, we get

$$\underbrace{R(k,T_\circ)}_{corr.\ value} \;=\; \underbrace{R(k,T)}_{meas.\ value} \;-\; \underbrace{\left[\frac{\partial\, R}{\partial\, T}(k)\right]\cdot\; \Delta T(k)}_{temp.\ corr.} \quad ; \quad 1 \;\leq\; k \;\leq\; l \tag{3}$$

where the temperature difference $\Delta T$ is given by Eq. (2$b$). It is important to point out that the corrected gage output $R(k,T_\circ)$ and not the measured gage output $R(k,T)$ must be used as input for the balance load calculation because the original data reduction matrix is only valid for the calibration temperature $T_\circ$. The application of Eq. (3) also has the advantage that no change of the original data reduction matrix is required. The gage outputs are simply transformed back to the electrical output $R(k,T_\circ)$ that the balance would have had if the temperature during the wind tunnel test would have matched the original calibration temperature.

### C. Implicit Application of Temperature Corrections

This second processing approach implicitly applies the temperature correction to the gage outputs during the load iteration process. It has the advantage that the temperature effects are not treated separately from

the load calculation. Instead, they are directly included in the load iteration process. The second approach, however, has the disadvantage that the original regression analysis result needs to be modified so that the temperature difference $\Delta T(k)$ (see Eq. (2*b*)) may be used as an independent variable for the load iteration process. This modification may be better understood by reviewing the regression model of the gage outputs that the iterative analysis technique uses. After using the right hand side of Eq. (1) to replace the gage output $R(k, T_\circ)$ in Eq. (2*a*), we get the following expression:

$$R(k,T) = \underbrace{\alpha_0(k,T_\circ) + \alpha_1(k,T_\circ)\cdot F(1) + \alpha_2(k,T_\circ)\cdot F(2) + \cdots}_{\text{regression model of data recorded at calibration temperature } T_\circ} + \underbrace{\left[\frac{\partial R}{\partial T}(k)\right]\cdot \Delta T(k)}_{\text{linear correction}} \quad (4a)$$

Equation (4*a*) can also be interpreted in a different way. We can write:

$$R(k,T) = \underbrace{\alpha_0(k,T_\circ)}_{coef.} + \underbrace{\alpha_1(k,T_\circ)}_{coef.}\cdot \overbrace{F(1)}^{ind.\ var.} + \underbrace{\alpha_2(k,T_\circ)}_{coef.}\cdot \overbrace{F(2)}^{ind.\ var.} + \cdots + \underbrace{\left[\frac{\partial R}{\partial T}(k)\right]}_{coef.}\cdot \overbrace{\Delta T(k)}^{ind.\ var.} \quad (4b)$$

Consequently, the extended set of independent variables and regression coefficients used in Eq. (4*b*) can be summarized as follows:

$$\textit{extended set of independent variables} \implies F(1),\ F(2),\ \cdots,\ F(n),\ \Delta T(k)$$

$$\textit{extended set of regression coefficients} \implies \alpha_0(k,T_\circ),\ \alpha_1(k,T_\circ),\ \cdots,\ \frac{\partial R}{\partial T}(k)$$

The regresssion coefficients $\alpha_0(k,T_\circ), \alpha_1(k,T_\circ)$, ... of Eq. (4*b*) are the result of the regression analysis of the original balance calibration data that was recorded at calibration temperature $T_\circ$. The additional regression coefficient $\partial R/\partial T$ is the manufacturer specified temperature sensitivity of the gage.

It remains to include the additional independent variable $\Delta T(k)$ in the load iteration process that the iterative analysis technique uses. This additional independent variable needs to be added to the original data reduction matrix using an approach that was first introduced in Ref. [2]. At this point only basic steps of the preparation of the new data reduction matrix are summarized. They will be explained in more detail using the data example that is described in the next section.

The flowchart shown in Fig. 1 summarizes the most important steps that are needed for the preparation of the extended data reduction matrix that takes residual temperature effects into account. First, the balance is calibrated in the usual manner at a constant calibration temperature $T_\circ$ (Step 1 in Fig. 1). Then, the global regression solution of the original balance calibration data is found. This initial regression analysis solution is given by the regression coefficients $\alpha_0(k,T_\circ), \alpha_1(k,T_\circ)$, ... that are used in Eqs. (1) and (4*b*) above. The coefficients of all strain–gages of the balance are assembled in the regression coefficient matrix after the original regression analysis has been completed (Step 2 in Fig. 1). Now, the temperature sensitivities of the balance gages are determined in a separate sensitivity experiment (Step 3 in Fig. 1). Alternatively, the sensitivities may have been supplied by the manufacturer of the strain–gages. In the next step, the original regression coefficient matrix is modified (Step 4 in Fig. 1). Therefore, the format of the regression coefficient matrix needs to be extended so that it supports the additional independent variable set. For a six–component balance, for example, this means that the regression coefficient matrix is transformed from the original $6 \times 97$ format to the new $12 \times 337$ format because each strain–gage output is accompanied by a corresponding temperature difference measurement. Then, the temperature sensitivity of each strain–gage is entered as the linear coefficient of the corresponding temperature difference variable (i.e., of the new independent variable). In addition, using ideas presented in Ref. [2], a value of "1.0" is used as the coefficient of the new independent variable in the regression model of the matching new dependent variable. Finally, the modified regression coefficient matrix in the new $12 \times 337$ format is transformed to the corresponding data reduction matrix that contains matrices needed for the load iteration process.

Data from the calibration of a six–component force balance and a realistic simulation of check load data are used in the next section of the paper to illustrate the five steps that were discussed above.

## III. Discussion of Example

Manual calibration data of NASA's MC60D balance was selected to illustrate the newly introduced implicit processing approach that was discussed in the previous section. The MC60 balance was manufactured and calibrated by Triumph/Force Measurement Systems in 2008. Table 1 below summarizes important features of the balance and the calibration data set that was used for the present study.

**Table 1:** Balance and calibration data set characteristics of the MC60D balance.

| CHARACTERISTIC | DESCRIPTION |
|---|---|
| BALANCE TYPE | force balance |
| DIAMETER | 2.0 [in] |
| EXCITATION VOLTAGE | 6.0 [V] |
| CALIBRATION DATE | December 2008 |
| CALIBRATION METHOD | manual calibration |
| NUMBER OF CALIBRATION POINTS | 175 |

Table 2 lists load capacities of the balance:

**Table 2:** Load capacities of the MC60D balance.

| | N1, lbs | N2, lbs | S1, lbs | S2, lbs | RM, in–lbs | AF, lbs |
|---|---|---|---|---|---|---|
| CAPACITY | 2500 | 2500 | 1250 | 1250 | 5000 | 700 |

The balance was calibrated using a manual calibration approach. A total of 175 calibration points were recorded in 17 load series. The temperature was kept constant during the entire calibration. For the present study it was assumed that the calibration data was recorded at a nominal temperature of 70 [degF].

In the next step, the original balance calibration data was analyzed using the iterative analysis technique. Afterwards, the regression coefficient matrix in extended format (six gage outputs plus six temperature differences) was sent to a text file for further processing. Figure 2a shows the regression coefficient matrix in the extended format before its modification. The matrix is in the extended $12 \times 337$ format (only 50 of 337 rows are shown). The green color highlights the regression coefficients of the first gage output (R1) that are the result of the analysis of the original calibration data.

Figure 2b shows the regression coefficient matrix after it was modified. The modified regression coefficients are highlighted in the blue box. Two modifications may be seen by comparing Fig. 2a with Fig. 2b. Modification 1: The red numbers are simulated temperature sensitivities of the six balance gages that were entered in the original matrix. These simulated sensitivities are much larger than the true temperature sensitivities of the six gages of the MC60D balance. They were only artificially increased for the purpose of this study to better illustrate the application of the new method. Modification 2: A value of "1.0" has to be used as the coefficient of a new dependent variable that matches a corresponding independent variable (see, e.g., the coefficient of the independent variable T1d).

Finally, the modified regression coefficient matrix (Fig. 2b) was converted to the corresponding extended data reduction matrix. It is assumed that this data reduction matrix supports the following load iteration equation (from Ref. [3], p. 12, Table 4):

$$\mathbf{F}_{\xi} \;=\; \underbrace{\left[\,\mathbf{C_1^{-1}}\,\right]}_{\textbf{matrix}} \cdot\, \mathbf{\Delta R} \;-\; \underbrace{\left[\,\mathbf{C_1^{-1}C_2}\,\right]}_{\textbf{matrix}} \cdot\, \mathbf{H}(\mathbf{F}_{\xi-1}) \tag{5}$$

Equation (5) is the load iteration equation that is recommended in Ref. [1]. It is also derived in great detail in Ref. [3]. The equation parts $\mathbf{C_1^{-1}}$ and $\mathbf{C_1^{-1}C_2}$ are contained in the data reduction matrix (see also Fig. 3). The conversion from regression coefficient matrix to data reduction matrix is a well defined mathematical operation that is described in Ref. [1] and Ref. [3]. Figure 3 shows the converted data reduction matrix that was obtained for the modified regression coefficient matrix that is shown in Fig. 2b. The blue data block shows terms of the data reduction matrix that are related to the temperature sensitivities and the temperature differences T1i, ..., T6i that were introduced as new set of independent variables.

The modified data reduction matrix shown in Fig. 3 was tested using a set of simulated check loads of the MC60D balance. The original check loads remained unchanged. The gage outputs of the check loads, however, were modified by adding a gage output change to the original gage output. The output change was computed by using the simulated temperature sensitivities depicted in Fig. 2b and an assumed temperature difference between the actual and calibration temperature at the location of the strain-gage as input. The temperature differences were randomly assigned using values between -20 degF and +40 degF. First, the check loads were processed using the gage outputs of the check load points and assuming that all temperature differences of the check loads were zero. The corresponding check load residuals are shown in Fig. 4a. Then, the check loads were processed using both the gage outputs and the actual values of the temperature differences as input. The corresponding check load residuals are shown in Fig. 4b. Comparing the check load residuals of the six balance loads components without and with temperature correction we see that the residuals are noticable smaller when the temperature differences were used. This result illustrates that the implicit use of temperature corrections in an extended data reduction matrix works as intended.

## IV. Summary and Conclusions

A new method was discussed that may be used to include residual temperature corrections of strain–gage outputs in balance load calculations. The new method was specifically developed for the iterative analysis technique. It has the advantage that it leads to a data reduction matrix that is also a function of the temperature of the balance during a wind tunnel test. Therefore, the application of residual temperature corrections no longer needs to be separated from the load iteration process.

The new method implicitly applies the temperature correction during the load iteration by processing an extended data reduction matrix that uses the temperature difference between the balance temperature during the wind tunnel test and the balance temperature during the calibration as a new input variable. Manufacturer specified temperature sensitivities of strain–gages are directly used as regression coefficients of the related temperature difference variables. Calibration data and simulated check load data of NASA's MC60D six–component strain–gage balance were used to successfully test the proposed new method. The analysis of check load residuals, i.e., of the difference between applied and predicted check loads, showed that the direct inclusion of temperature sensitivities of the strain–gages in the data reduction matrix kept temperature dependent load prediction errors to a minimum.

The new method applies residual temperature corrections during the load iteration process as the temperature sensitivies are hidden in the data reduction matrix coefficients. Therefore, gage outputs may directly be used as input for the load calculation that are not corrected for residual temperature effects. It is important, however, to point out that the new approach takes advantage of unique capabilities of NASA's BALFIT software (see Ref. [4] for a basic description of BALFIT). Since 2010 BALFIT can process a data reduction matrix with an extended set of independent variables (see Ref. [2]). In addition, the inclusion of temperature sensitivities in the data reduction matrix has recently been automated. Therefore, as far as BALFIT's user is concerned, the effort required to prepare a data reduction without temperature correction is essentially identical to the effort required to prepare a data reduction matrix with temperature correction. Temperature sensitivities only have to be specified in BALFIT's calibration data input file whenever a temperature dependent data reduction matrix has to be generated. Then, BALFIT executes steps 2, 4, and 5 shown in Fig. 1 without requiring any user interaction. Most existing balance calibration data analysis software packages do not have these capabilities. In those cases, the residual temperature correction has to be applied explicitly to the gage outputs before the load iteration starts (see Eq. (3)) as the unmodified data reduction matrix is only valid for the original calibration temperature of the balance.

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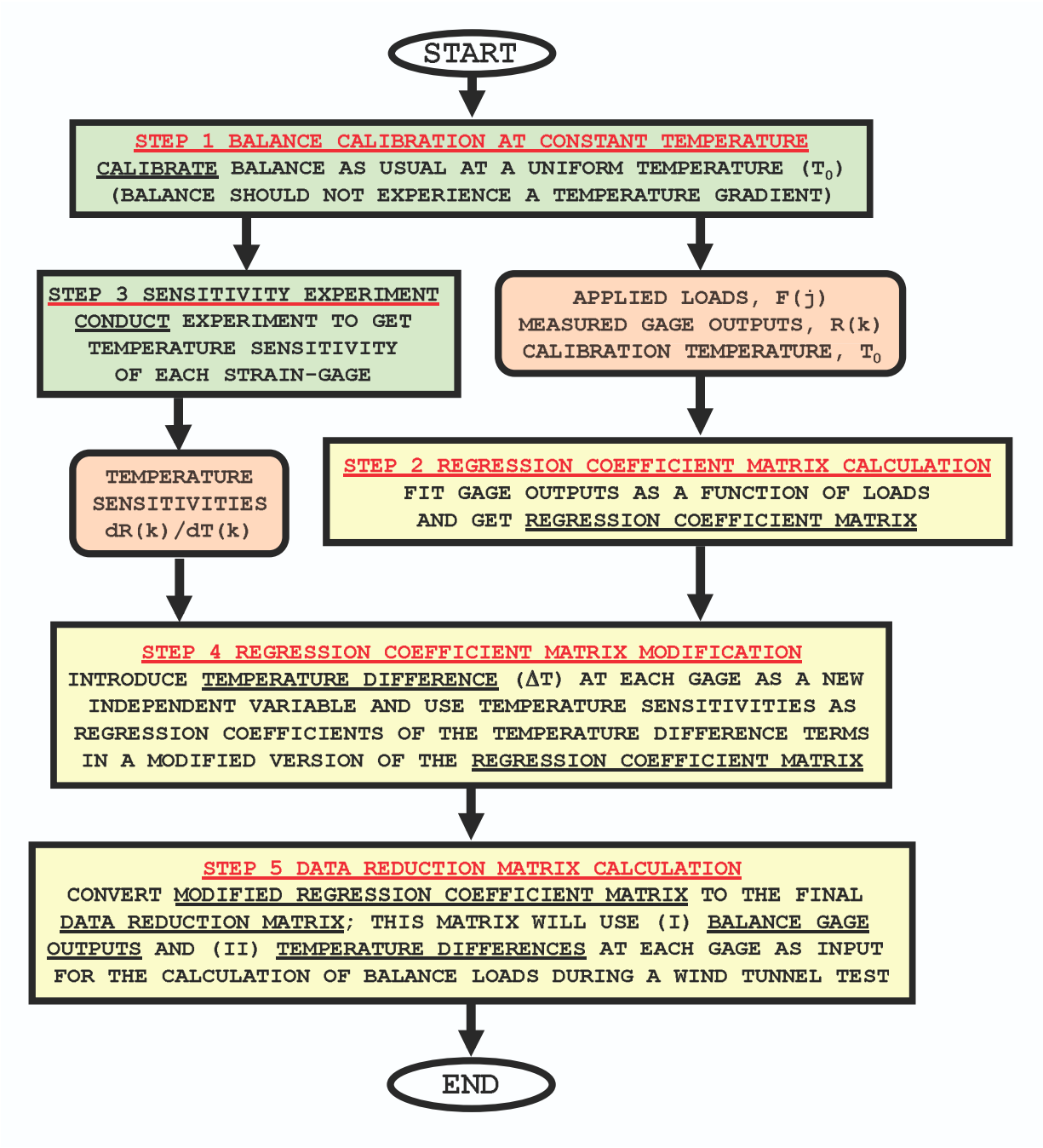
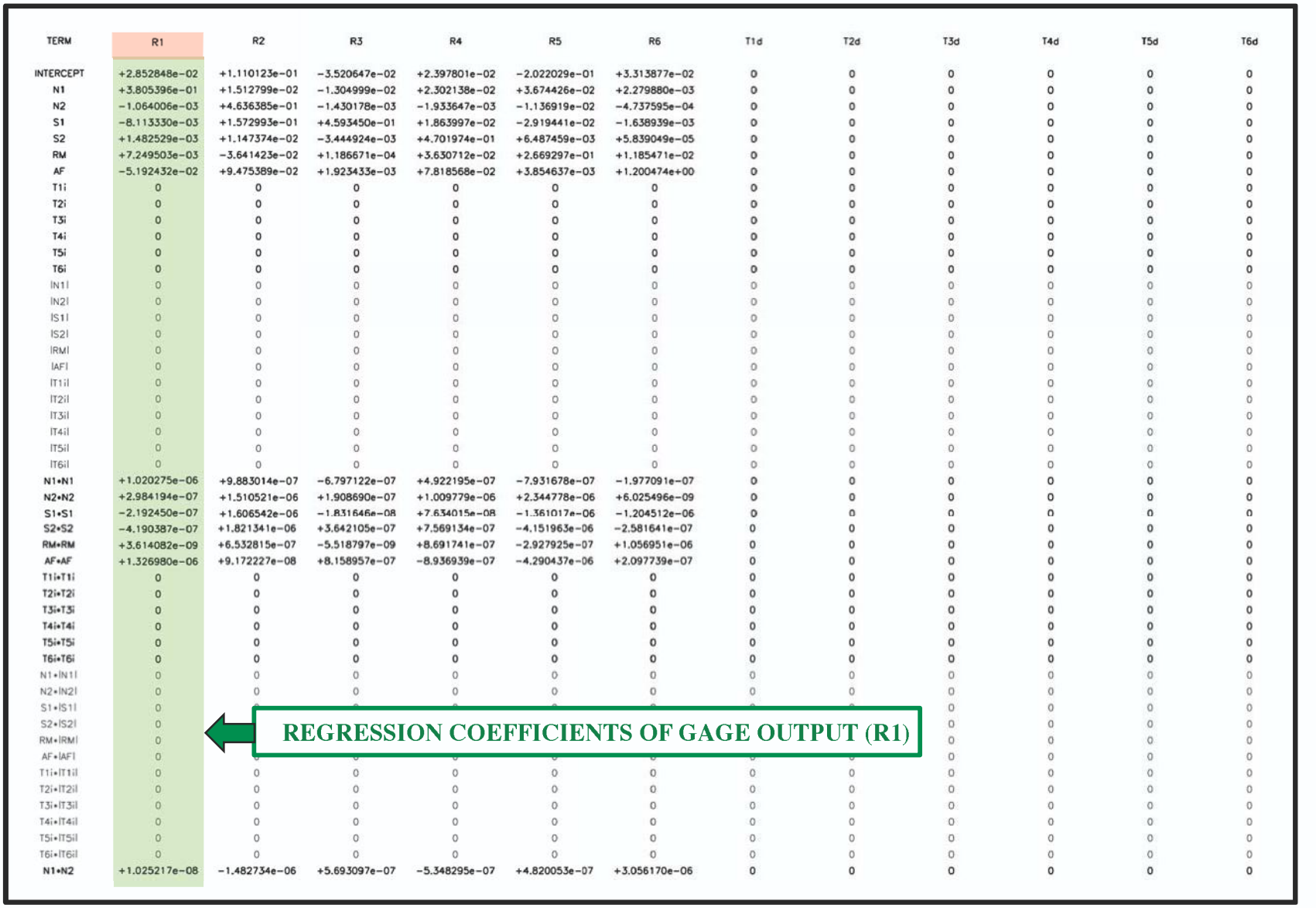


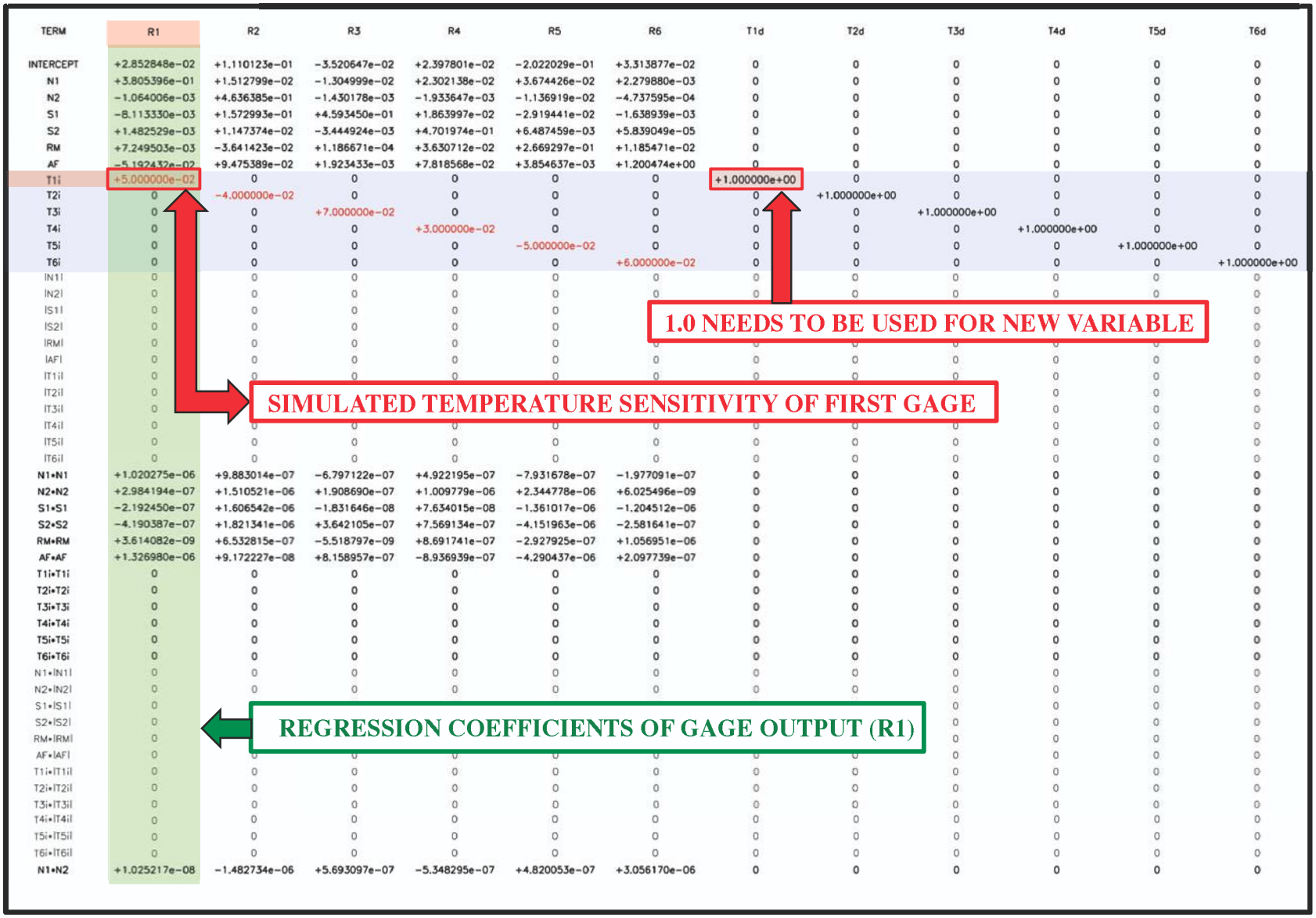

**Fig. 1** Preparation of a data reduction matrix that uses temperature sensitivities.



| TERM | R1 | R2 | R3 | R4 | R5 | R6 | T1d | T2d | T3d | T4d | T5d | T6d |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| INTERCEPT | +2.852848e-02 | +1.110123e-01 | −3.520647e-02 | +2.397801e-02 | −2.022029e-01 | +3.313877e-02 | 0 | 0 | 0 | 0 | 0 | 0 |
| N1 | +3.805396e-01 | +1.512799e-02 | −1.304999e-02 | +2.302138e-02 | +3.674426e-02 | +2.279880e-03 | 0 | 0 | 0 | 0 | 0 | 0 |
| N2 | −1.064006e-03 | +4.636385e-01 | −1.430178e-03 | −1.933647e-03 | −1.136919e-02 | −4.737595e-04 | 0 | 0 | 0 | 0 | 0 | 0 |
| S1 | −8.113330e-03 | +1.572993e-01 | +4.593450e-01 | +1.863997e-02 | −2.919441e-02 | −1.638939e-03 | 0 | 0 | 0 | 0 | 0 | 0 |
| S2 | +1.482529e-03 | +1.147374e-02 | −3.444924e-03 | +4.701974e-01 | +6.487459e-03 | +5.839049e-05 | 0 | 0 | 0 | 0 | 0 | 0 |
| RM | +7.249503e-03 | −3.641423e-02 | +1.186671e-04 | +3.630712e-02 | +2.669297e-01 | +1.185471e-02 | 0 | 0 | 0 | 0 | 0 | 0 |
| AF | −5.192432e-02 | +9.475389e-02 | +1.923433e-03 | +7.818568e-02 | +3.854637e-03 | +1.200474e+00 | 0 | 0 | 0 | 0 | 0 | 0 |
| T1i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T2i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T3i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T4i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T5i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T6i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IN1I | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IN2I | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IS1I | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IS2I | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IRMI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IAFI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IT1iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IT2iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IT3iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IT4iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IT5iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IT6iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N1•N1 | +1.020275e-06 | +9.883014e-07 | −6.797122e-07 | +4.922195e-07 | −7.931678e-07 | −1.977091e-07 | 0 | 0 | 0 | 0 | 0 | 0 |
| N2•N2 | +2.984194e-07 | +1.510521e-06 | +1.908690e-07 | +1.009779e-06 | +2.344778e-06 | +6.025496e-09 | 0 | 0 | 0 | 0 | 0 | 0 |
| S1•S1 | −2.192450e-07 | +1.606542e-06 | −1.831646e-08 | +7.634015e-08 | −1.361017e-06 | −1.204512e-06 | 0 | 0 | 0 | 0 | 0 | 0 |
| S2•S2 | −4.190387e-07 | +1.821341e-06 | +3.642105e-07 | +7.569134e-07 | −4.151963e-06 | −2.581641e-07 | 0 | 0 | 0 | 0 | 0 | 0 |
| RM•RM | +3.614082e-09 | +6.532815e-07 | −5.518797e-09 | +8.691741e-07 | −2.927925e-07 | +1.056951e-06 | 0 | 0 | 0 | 0 | 0 | 0 |
| AF•AF | +1.326980e-06 | +9.172227e-08 | +8.158957e-07 | −8.936939e-07 | −4.290437e-06 | +2.097739e-07 | 0 | 0 | 0 | 0 | 0 | 0 |
| T1i•T1i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T2i•T2i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T3i•T3i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T4i•T4i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T5i•T5i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T6i•T6i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N1•IN1I | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N2•IN2I | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| S1•IS1I | 0 | | | | | | | | 0 | 0 | 0 | 0 |
| S2•IS2I | 0 | | | | | | | | 0 | 0 | 0 | 0 |
| RM•IRMI | 0 | | | | | | | | 0 | 0 | 0 | 0 |
| AF•IAFI | 0 | | | | | | | | 0 | 0 | 0 | 0 |
| T1i•IT1iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T2i•IT2iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T3i•IT3iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T4i•IT4iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T5i•IT5iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T6i•IT6iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N1•N2 | +1.025217e-08 | −1.482734e-06 | +5.693097e-07 | −5.348295e-07 | +4.820053e-07 | +3.056170e-06 | 0 | 0 | 0 | 0 | 0 | 0 |

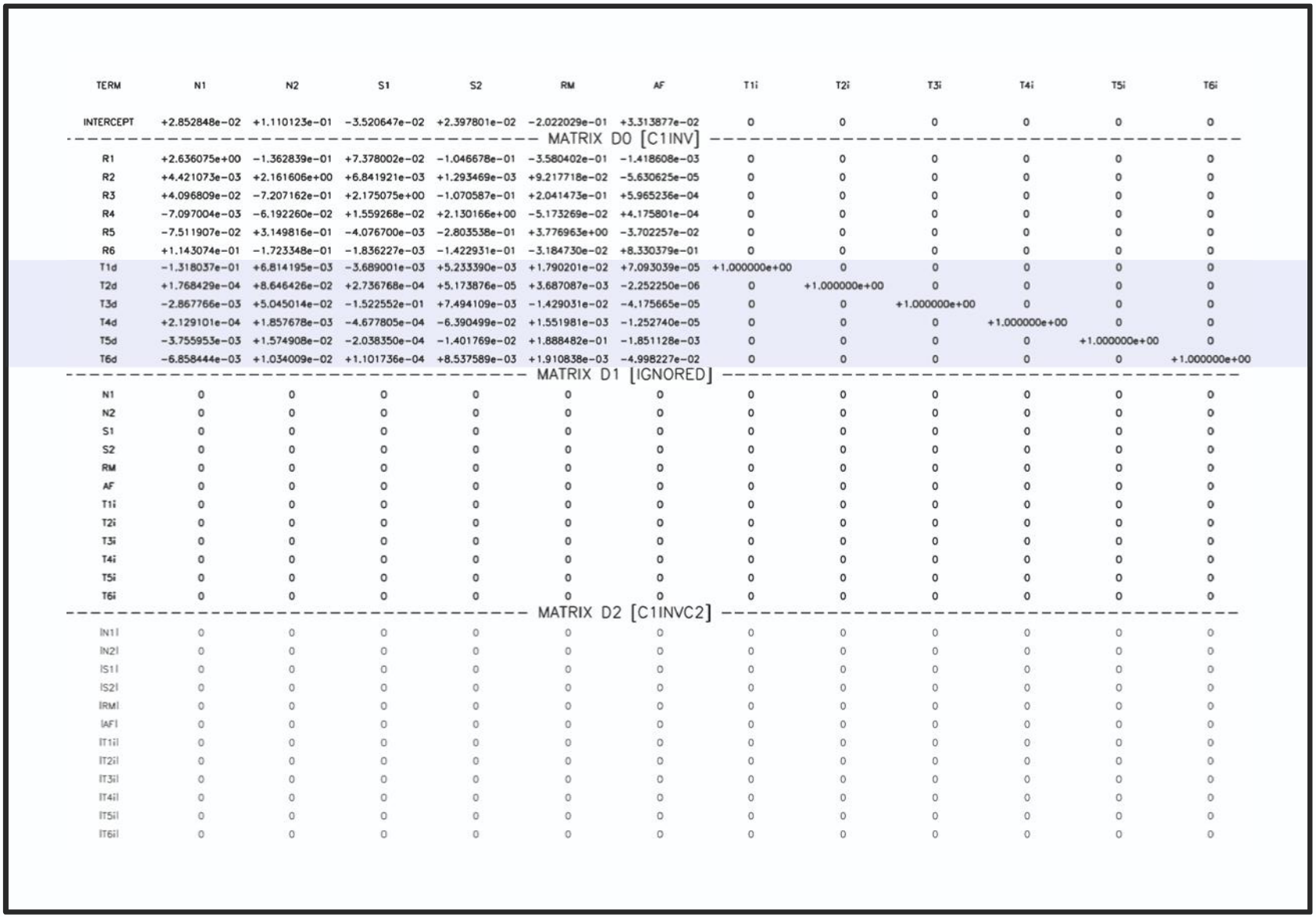


**Fig. 2a** Regression coefficient matrix before modification (50 of 337 rows shown).




| TERM | R1 | R2 | R3 | R4 | R5 | R6 | T1d | T2d | T3d | T4d | T5d | T6d |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| INTERCEPT | +2.852848e-02 | +1.110123e-01 | −3.520647e-02 | +2.397801e-02 | −2.022029e-01 | +3.313877e-02 | 0 | 0 | 0 | 0 | 0 | 0 |
| N1 | +3.805396e-01 | +1.512799e-02 | −1.304999e-02 | +2.302138e-02 | +3.674426e-02 | +2.279880e-03 | 0 | 0 | 0 | 0 | 0 | 0 |
| N2 | −1.064006e-03 | +4.636385e-01 | −1.430178e-03 | −1.933647e-03 | −1.136919e-02 | −4.737595e-04 | 0 | 0 | 0 | 0 | 0 | 0 |
| S1 | −8.113330e-03 | +1.572993e-01 | +4.593450e-01 | +1.863997e-02 | −2.919441e-02 | −1.638939e-03 | 0 | 0 | 0 | 0 | 0 | 0 |
| S2 | +1.482529e-03 | +1.147374e-02 | −3.444924e-03 | +4.701974e-01 | +6.487459e-03 | +5.839049e-05 | 0 | 0 | 0 | 0 | 0 | 0 |
| RM | +7.249503e-03 | −3.641423e-02 | +1.186671e-04 | +3.630712e-02 | +2.669297e-01 | +1.185471e-02 | 0 | 0 | 0 | 0 | 0 | 0 |
| AF | −5.192432e-02 | +9.475389e-02 | +1.923433e-03 | +7.818568e-02 | +3.854637e-03 | +1.200474e+00 | 0 | 0 | 0 | 0 | 0 | 0 |
| T1i | +5.000000e-02 | 0 | 0 | 0 | 0 | 0 | +1.000000e+00 | 0 | 0 | 0 | 0 | 0 |
| T2i | 0 | −4.000000e-02 | 0 | 0 | 0 | 0 | 0 | +1.000000e+00 | 0 | 0 | 0 | 0 |
| T3i | 0 | 0 | +7.000000e-02 | 0 | 0 | 0 | 0 | 0 | +1.000000e+00 | 0 | 0 | 0 |
| T4i | 0 | 0 | 0 | +3.000000e-02 | 0 | 0 | 0 | 0 | 0 | +1.000000e+00 | 0 | 0 |
| T5i | 0 | 0 | 0 | 0 | −5.000000e-02 | 0 | 0 | 0 | 0 | 0 | +1.000000e+00 | 0 |
| T6i | 0 | 0 | 0 | 0 | 0 | +6.000000e-02 | 0 | 0 | 0 | 0 | 0 | +1.000000e+00 |
| IN1I | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IN2I | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IS1I | 0 | 0 | 0 | 0 | 0 | | | | | | | 0 |
| IS2I | 0 | 0 | 0 | 0 | 0 | | | | | | | 0 |
| IRMI | 0 | 0 | 0 | 0 | 0 | | | | | | | 0 |
| IAFI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IT1iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IT2iI | 0 | | | | | | | | | 0 | 0 | 0 |
| IT3iI | 0 | | | | | | | | | 0 | 0 | 0 |
| IT4iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IT5iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IT6iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N1•N1 | +1.020275e-06 | +9.883014e-07 | −6.797122e-07 | +4.922195e-07 | −7.931678e-07 | −1.977091e-07 | 0 | 0 | 0 | 0 | 0 | 0 |
| N2•N2 | +2.984194e-07 | +1.510521e-06 | +1.908690e-07 | +1.009779e-06 | +2.344778e-06 | +6.025496e-09 | 0 | 0 | 0 | 0 | 0 | 0 |
| S1•S1 | −2.192450e-07 | +1.606542e-06 | −1.831646e-08 | +7.634015e-08 | −1.361017e-06 | −1.204512e-06 | 0 | 0 | 0 | 0 | 0 | 0 |
| S2•S2 | −4.190387e-07 | +1.821341e-06 | +3.642105e-07 | +7.569134e-07 | −4.151963e-06 | −2.581641e-07 | 0 | 0 | 0 | 0 | 0 | 0 |
| RM•RM | +3.614082e-09 | +6.532815e-07 | −5.518797e-09 | +8.691741e-07 | −2.927925e-07 | +1.056951e-06 | 0 | 0 | 0 | 0 | 0 | 0 |
| AF•AF | +1.326980e-06 | +9.172227e-08 | +8.158957e-07 | −8.936939e-07 | −4.290437e-06 | +2.097739e-07 | 0 | 0 | 0 | 0 | 0 | 0 |
| T1i•T1i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T2i•T2i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T3i•T3i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T4i•T4i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T5i•T5i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T6i•T6i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N1•IN1I | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N2•IN2I | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| S1•IS1I | 0 | | | | | | | | 0 | 0 | 0 | 0 |
| S2•IS2I | 0 | | | | | | | | 0 | 0 | 0 | 0 |
| RM•IRMI | 0 | | | | | | | | 0 | 0 | 0 | 0 |
| AF•IAFI | 0 | | | | | | | | 0 | 0 | 0 | 0 |
| T1i•IT1iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T2i•IT2iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T3i•IT3iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T4i•IT4iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T5i•IT5iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T6i•IT6iI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N1•N2 | +1.025217e-08 | −1.482734e-06 | +5.693097e-07 | −5.348295e-07 | +4.820053e-07 | +3.056170e-06 | 0 | 0 | 0 | 0 | 0 | 0 |

**Fig. 2b** Regression coefficient matrix after modification (50 of 337 rows shown).



| TERM | N1 | N2 | S1 | S2 | RM | AF | T1i | T2i | T3i | T4i | T5i | T6i |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| INTERCEPT | +2.852848e-02 | +1.110123e-01 | −3.520647e-02 | +2.397801e-02 | −2.022029e-01 | +3.313877e-02 | 0 | 0 | 0 | 0 | 0 | 0 |
| MATRIX D0 [C1INV] | | | | | | | | | | | | |
| R1 | +2.636075e+00 | −1.362839e-01 | +7.378002e-02 | −1.046678e-01 | −3.580402e-01 | −1.418608e-03 | 0 | 0 | 0 | 0 | 0 | 0 |
| R2 | +4.421073e-03 | +2.161606e+00 | +6.841921e-03 | +1.293469e-03 | +9.217718e-02 | −5.630625e-05 | 0 | 0 | 0 | 0 | 0 | 0 |
| R3 | +4.096809e-02 | −7.207162e-01 | +2.175075e+00 | −1.070587e-01 | +2.041473e-01 | +5.965236e-04 | 0 | 0 | 0 | 0 | 0 | 0 |
| R4 | −7.097004e-03 | −6.192260e-02 | +1.559268e-02 | +2.130166e+00 | −5.173269e-02 | +4.175801e-04 | 0 | 0 | 0 | 0 | 0 | 0 |
| R5 | −7.511907e-02 | +3.149816e-01 | −4.076700e-03 | −2.803538e-01 | +3.776963e+00 | −3.702257e-02 | 0 | 0 | 0 | 0 | 0 | 0 |
| R6 | +1.143074e-01 | −1.723348e-01 | −1.836227e-03 | −1.422931e-01 | −3.184730e-02 | +8.330379e-01 | 0 | 0 | 0 | 0 | 0 | 0 |
| T1d | −1.318037e-01 | +6.814195e-03 | −3.689001e-03 | +5.233390e-03 | +1.790201e-02 | +7.093039e-05 | +1.000000e+00 | 0 | 0 | 0 | 0 | 0 |
| T2d | +1.768429e-04 | +8.646426e-02 | +2.736768e-04 | +5.173876e-05 | +3.687087e-03 | −2.252250e-06 | 0 | +1.000000e+00 | 0 | 0 | 0 | 0 |
| T3d | −2.867766e-03 | +5.045014e-02 | −1.522552e-01 | +7.494109e-03 | −1.429031e-02 | −4.175665e-05 | 0 | 0 | +1.000000e+00 | 0 | 0 | 0 |
| T4d | +2.129101e-04 | +1.857678e-03 | −4.677805e-04 | −6.390499e-02 | +1.551981e-03 | −1.252740e-05 | 0 | 0 | 0 | +1.000000e+00 | 0 | 0 |
| T5d | −3.755953e-03 | +1.574908e-02 | −2.038350e-04 | −1.401769e-02 | +1.888482e-01 | −1.851128e-03 | 0 | 0 | 0 | 0 | +1.000000e+00 | 0 |
| T6d | −6.858444e-03 | +1.034009e-02 | +1.101736e-04 | +8.537589e-03 | +1.910838e-03 | −4.998227e-02 | 0 | 0 | 0 | 0 | 0 | +1.000000e+00 |
| MATRIX D1 [IGNORED] | | | | | | | | | | | | |
| N1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| S1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| S2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| RM | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| AF | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T1i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T2i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T3i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T4i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T5i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T6i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| MATRIX D2 [C1INVC2] | | | | | | | | | | | | |
| IN1I | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IN2I | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IS1I | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IS2I | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IRMI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IAFI | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IT1II | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IT2II | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IT3II | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IT4II | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IT5II | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| IT6II | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

**Fig. 3** Data reduction matrix of modified regression coefficient matrix (37 of 349 rows shown).

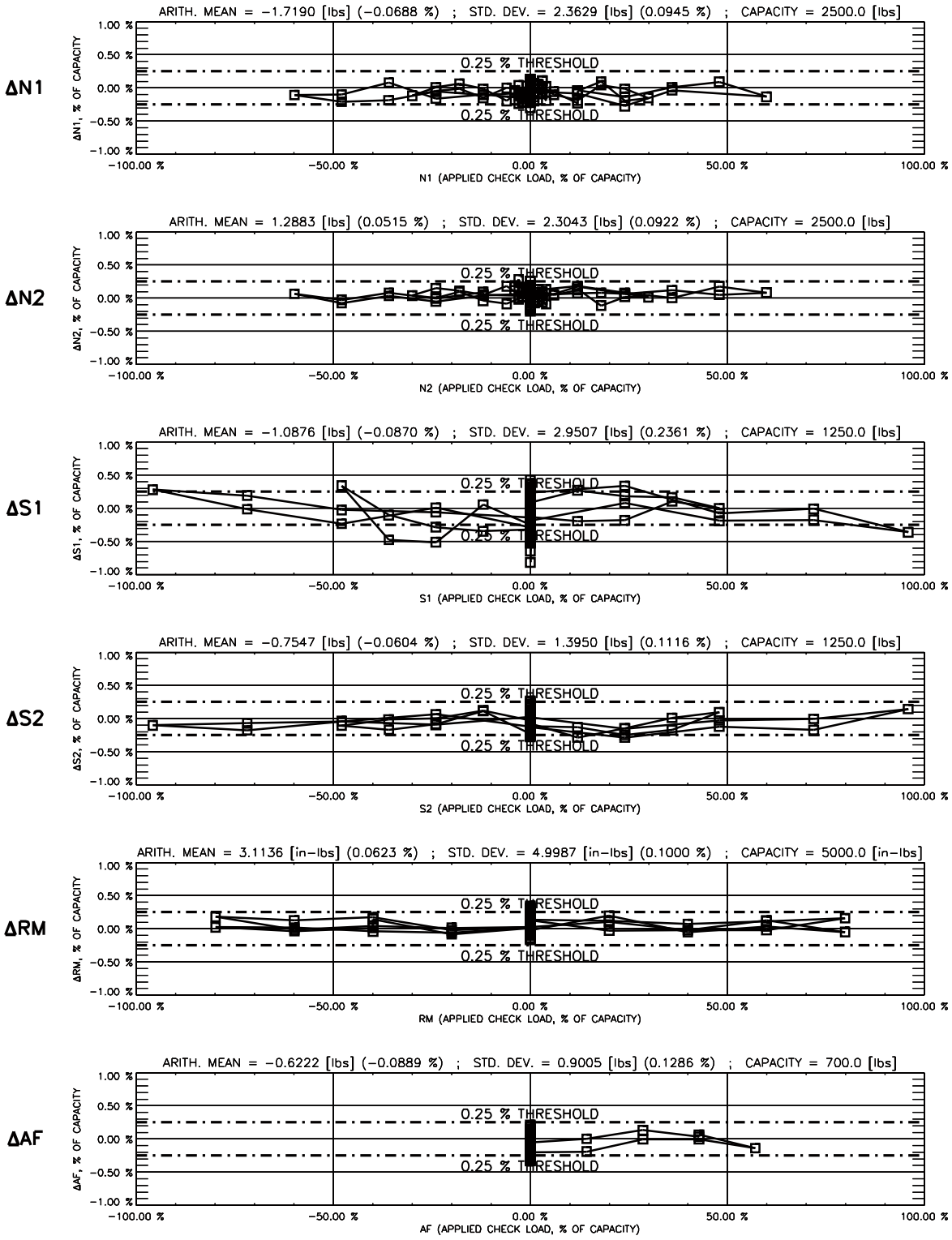



**Fig. 4a** Check load residuals without temperature correction.

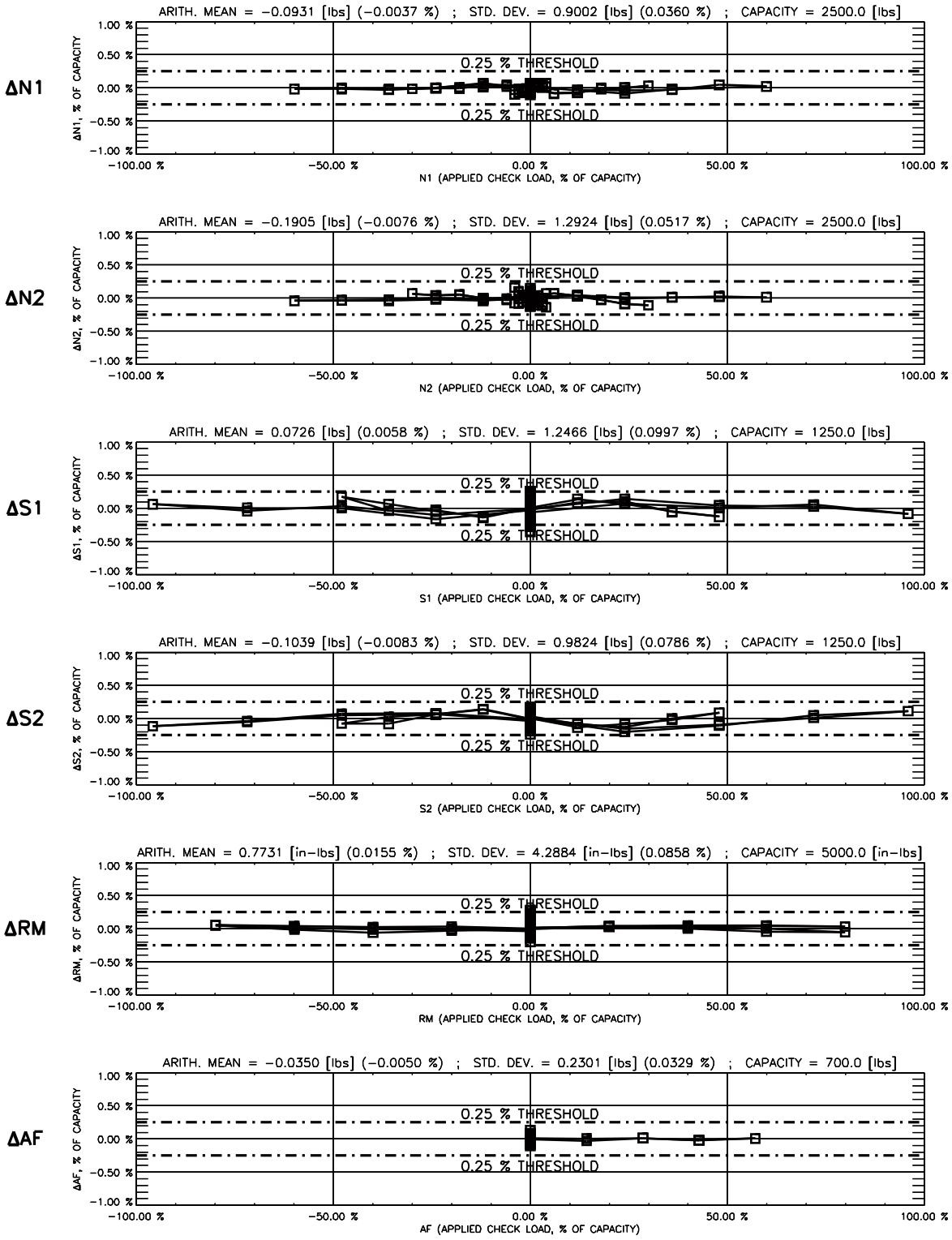



**Fig. 4b** Check load residuals with temperature correction.