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THE *KEPLER* LIGHT CURVES OF V1504 CYGNI AND V344 LYRAE: A STUDY OF THE OUTBURST PROPERTIES

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ABSTRACT

We examine the *Kepler* light curves of V1504 Cyg and V344 Lyr, encompassing ~460 d at 1 min cadence. During this span each system exhibited ~40 outbursts, including four superoutbursts. We find that, in both systems, the normal outbursts lying between two superoutbursts increase in duration by a factor ~1.2−1.7, and then reset to a small value after the following superoutburst. In V344 Lyr the trend of quiescent intervals between normal outbursts is to increase to a local maximum about half way through the supercycle – the interval from one superoutburst to the next – and then to decrease back to a small value by the time of the next superoutburst, whereas for V1504 Cyg the quiescent intervals are relatively constant during the supercycle. Both of these trends are inconsistent with the Osaki's thermal-tidal model, which robustly predicts a secular increase in the quiescent intervals between normal outbursts during a supercycle. Also, most of the normal outbursts have an asymmetric, fast-rise/slower-decline shape, which would be consistent with outbursts triggered at large radii. The exponential rate of decay of the plateau phase of the superoutbursts is 8 d mag^{-1} for V1504 Cyg and 12 d mag^{-1} for V344 Lyr. This time scale gives a direct measure of the viscous time scale in the outer accretion disk given the expectation that the entire disk is in the hot, viscous state during superoutburst. The resulting constraint on the Shakura-Sunyaev parameter, $\alpha_{\rm hot} \simeq 0.1$, is consistent with the value inferred from the fast dwarf nova decays. By looking at the slow decay rate for superoutbursts, which occur in systems below the period gap, in combination with the slow decay rate in one long outburst above the period gap (in U Gem), we infer a steep dependence of the decay rate on orbital period for long outbursts. We argue that this relation implies a steep dependence of $\alpha_{\rm cold}$ on orbital period, which may be consistent with recent findings of Patterson, and is consistent with tidal torquing as being the dominant angular momentum transport mechanism in quiescent disks in interacting binary systems.

Subject headings: cataclysmic variables: dwarf novae

1. INTRODUCTION

Cataclysmic variables (CVs) are semi-detached interacting binaries in which a Roche lobe filling K or M secondary transfers matter to a white dwarf (WD). CVs evolve to shorter orbital periods and show a "gap" between $P_h = 2$ and 3 (where $P_h = P_{\rm orbital}/1$ hr) during which time the secondary star loses contact with its Roche lobe and mass transfer ceases. Thus the binary becomes fully detached. At $P_h = 2$ the secondary comes back into contact with its Roche lobe and mass transfer resumes. For $P_h < 2$ angular momentum loss from the binary is thought to be due solely to gravitational radiation. The CV subclass of dwarf novae (DNe) are characterized by their semi-periodic outbursts. SU UMa stars are DNe lying below the period gap that exhibit short, normal outbursts (NOs) and superoutbursts (SOs). We refer the time from one SO to the next as the supercycle. SOs show superhumps which are modulations in the light curve at periods slightly exceeding the orbital period. There are two further subdivisions within the SU UMa grouping: (i) the VW Hyi stars at long orbital periods, near $P_h = 2$, for which the decay rate is fairly constant during a SO, and (ii) the WZ Sge stars at short orbital periods, a little greater than $P_h = 1$, which have less frequent, larger amplitude SOs, for which the decay rate decreases during a SO. DNe outbursts are thought to be due to a limit cycle accretion disk instability (Lasota 2001) in which material is accumulated in quiescence and then dumped onto the WD during outburst. During short outbursts in longer period DNe, a few percent of the stored mass is accreted, and during long outbursts a significant fraction ~0.2 of the stored mass is accreted. For the SU UMa stars, a SO is thought to accrete $\gtrsim 0.7-0.8$ of the stored mass. Although the accretion disk is never in steady state during the limit cycle, it is close to steady state during SO, with the local rate of accretion inside the disk $|\dot M(r)|$ decreasing linearly from a maximum value at the disk inner edge to zero at the outer edge. The accretion disk modeling has traditionally been done within the Shakura & Sunyaev (1973, hereafter SS) formalism, using two values for the α viscosity parameter, $\alpha_{\rm hot}$ for gas in the hot, ionized disk, and $\alpha_{\rm cold}$ for gas in the quiescent disk.

There are two bright SU UMa stars in the *Kepler* field exhibiting a variety of temporal behavior that make them worthy of a detailed statistical study of their outbursts, V1504 Cyg (*Kepler* ID 7446357; $P_h = 1.67$) and V344 Lyr (*Kepler* ID 7659570; $P_h = 2.1$). These are members of the VW Hyi subdivision. To date the two light curves have amassed 459.8 d at 1−min cadence. Excluding gaps and bad data points, the light curves contain 637948 and 637915 entries, respectively. Previous studies of the *Kepler* data on SU UMa stars have found quiescent superhumps in V344 Lyr (Still et al. 2010), presented numerical models of the long term light curve of V344 Lyr (Cannizzo et al. 2010; hereafter C10), and studied superhumps, both positive and negative, in the long term V344 Lyr

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light curve (Wood et al. 2011).

Statistical studies of DNe have been useful in delineating the long-term properties of outbursts, e.g.., the outburst duration, recurrence times, and quiescent intervals, and placing physical constraints on models of outbursts (e.g., Campbell & Shapley 1940, Sterne, Campbell & Shapley 1940, Bath & van Paradijs 1983, van Paradijs 1983, Szkody & Mattei 1984, Cannizzo & Mattei 1992, 1998, Ak, et al. 2002, Simon 2004). Several interesting studies of the SU UMa stars have been carried out. For instance, van Paradijs (1983) compared the variation in outburst duration t_b for long and short outbursts above the $2-3$ hr CV period gap with those below, and found that long outbursts appear to be more significant in contrast to short outbursts as one progresses to shorter orbital period systems only because the durations of the short outbursts scale with orbital period, but those of the long outbursts are relatively constant. Therefore the ratio of outburst durations $t_b(\mathrm{SO})/t_b(\mathrm{NO})$ is large. This finding was amplified by Ak et al. (2002). In this work we seek to extend previous studies by analyzing the high fidelity *Kepler* light curves of two SU UMa stars to investigate the properties of the outbursts. In Section 2 we examine the outburst properties of the NOs and SOs, in Section 3 we discuss the results, in particular the scaling of the SO decay rate with orbital period, and in Section 4 we summarize our results.

2. OUTBURST PROPERTIES

2.1. *Normal Outbursts*

The relatively small number of total outbursts observed in the two systems to date does not yet allow one to amass detailed frequency histogram distributions, as was done for instance by Cannizzo & Mattei (1992) for the $\sim$700 outbursts observed up to that time in SS Cyg. However, the variations of the outburst duration t_b and intervening quiescent interval t_q within a supercycle contain information which can potentially guide theoretical models. For instance, in the thermal-tidal model for the supercycle in the SU UMa systems (Ichikawa & Osaki 1992, 1994; Ichikawa, Hirose, & Osaki 1993), one sees a monotonically increasing sequence of NO quiescent intervals leading up to a SO. Now with the exquisite *Kepler* data, this can be directly tested. The NOs of the SU UMa stars have been poorly characterized prior to *Kepler* due to their intrinsic faintness.

Figures 1 and 2 present the currently available long term light curves for V1504 Cyg and V344 Lyr. To obtain the apparent *Kepler* magnitude from the original data, electron flux per cadence, a correspondence[7] of $Kp = 12$ to 10^7 e^- cadence^{-1} was adopted. Each light curve encompasses four SOs; V1504 Cyg had 36 NOs while V344 Lyr had 33. Two systematic effects are worth noting: (i) There are some gaps in the data due to monthly data downloads and quarterly 90° spacecraft rolls. (ii) In addition, the long-term absolute flux calibration for *Kepler* can be problematic for several reasons (Haas et al. 2010, Koch et al. 2010): (1) Target masks do not contain all the flux from the target. The pixel mask is chosen to optimize signal-to-noise for each target. This is done for the mid-time of the quarter. As *Kepler* orbits the sun, however, targets drift across the mask as a result of differential velocity aberration. Varying amounts of target flux fall outside of the pixel aperture over the quarter, so the target flux tends to vary smoothly over the quarter due to this effect. (2) After each *Kepler* rotation, the target falls on a different CCD chip with a different PSF and the target is centered differently, e.g., perhaps on a pixel boundary rather than a pixel center. The 4 arcsec pixels undersample the PSF. This means that the pixel mask changes shape and size each quarter, with different amount of light losses. (3) With different shaped masks, the number and amount of contaminating sources in the mask will differ across the quarter gap.

[7] See http://keplergo.arc.nasa.gov/CalibrationSN.shtml.

Problematic long term flux calibration is evident in the elevated flux level seen in V1504 Cyg during $100 \lesssim t(\mathrm{d}) \lesssim 200$ (i.e, Q2 of observations). In addition there can also be a secular trending of the flux calibration within a quarter, which may account for the apparent $\sim$50 d $e-$folding decrease of the quiescent level following the first two SOs in V344 Lyr, both of which occurred just after a roll maneuver. In view of these considerations, we limit our attention in this study to timescales rather than amplitudes.

In a standard light curve plot of magnitude versus time, a straight line decay means that the flux $f = f_0 e^{-t/\tau}$, which we would therefore refer to as an exponential decay. A decay which deviates from exponential could be concave upward (slower-than-exponential), concave downward (faster-than-exponential), or some more complicated shape. The decays of the NOs are faster-than-exponential, which obviates theoretical studies that attempt to introduce a weak radial power law dependency into the SS α parameter in order to get precisely exponential decays (e.g., Ichikawa, Hirose, & Osaki 1993; Cannizzo 1994). We may quantify the deviation from exponentiality in the NO decays by dividing the decay into bins, each spanning a small range $\Delta Kp = 0.15 - 0.20$, and calculating the locally defined decay rate $t_d(\mathrm{d\ mag}^{-1})$ within each bin. Dividing the decay into a total of 10 bins for each star, we take $\Delta Kp = 0.15$ for V1504 Cyg and $\Delta Kp = 0.20$ for V344 Lyr. For demonstration purposes, the binning distribution for the decay of the first NO in V34 Lyr is shown in Figure 3. Within each bin we compute a frequency histogram distribution of t_d values for all NO decays. The results are shown in Figures 4 and 5. The small numbers in each panel indicate the magnitude range in Kp measured from the local maximum for each NO. Thus, the upper panels show the decay rates computed near maximum light, and as one progresses to lower panels one is sampling the fainter portions. For both systems one sees a shifting in the centroid of the frequency histogram distribution to lower t_d values as the decay proceeds, equivalent to saying that the decays are concave downward. In both systems, the parts of the decays fainter than about 1 mag below NO maximum asymptote to a centroid median of $\sim$0.6$-$0.7 d mag^{-1}. Thus the concave-downward aspect of the NO decays is due in large part to the rounded maxima; by the time the NO has faded to within $\Delta Kp \approx 1$ mag of quiescence, the rest of the decay is closely exponential.

Bailey (1975) discovered that the decay rate for NOs is correlated with orbital period (see his Figure 1 for eight well-studied systems), and derived the empirical relation $t_d = 0.38$ d mag^{-1} P_h. Smak (1984) was the first to show that this relation can be used to infer $\alpha_{\mathrm{hot}} \simeq 0.1$. Subsequent work confirmed and expanded on this finding (Cannizzo 2001a). Warner (1995b) looked at a larger sample than Bailey and deduced the fit $t_d = 0.53\,\mathrm{d\ mag}^{-1}\ P_h^{0.84}$. For the two systems of interest in this study, for which $P_h = 1.67$ (V1504 Cyg) and 2.1 (V344 Lyr), respectively, the predictions for the rates of decay would be 0.64 and 0.80 d mag^{-1} from the original Bai-

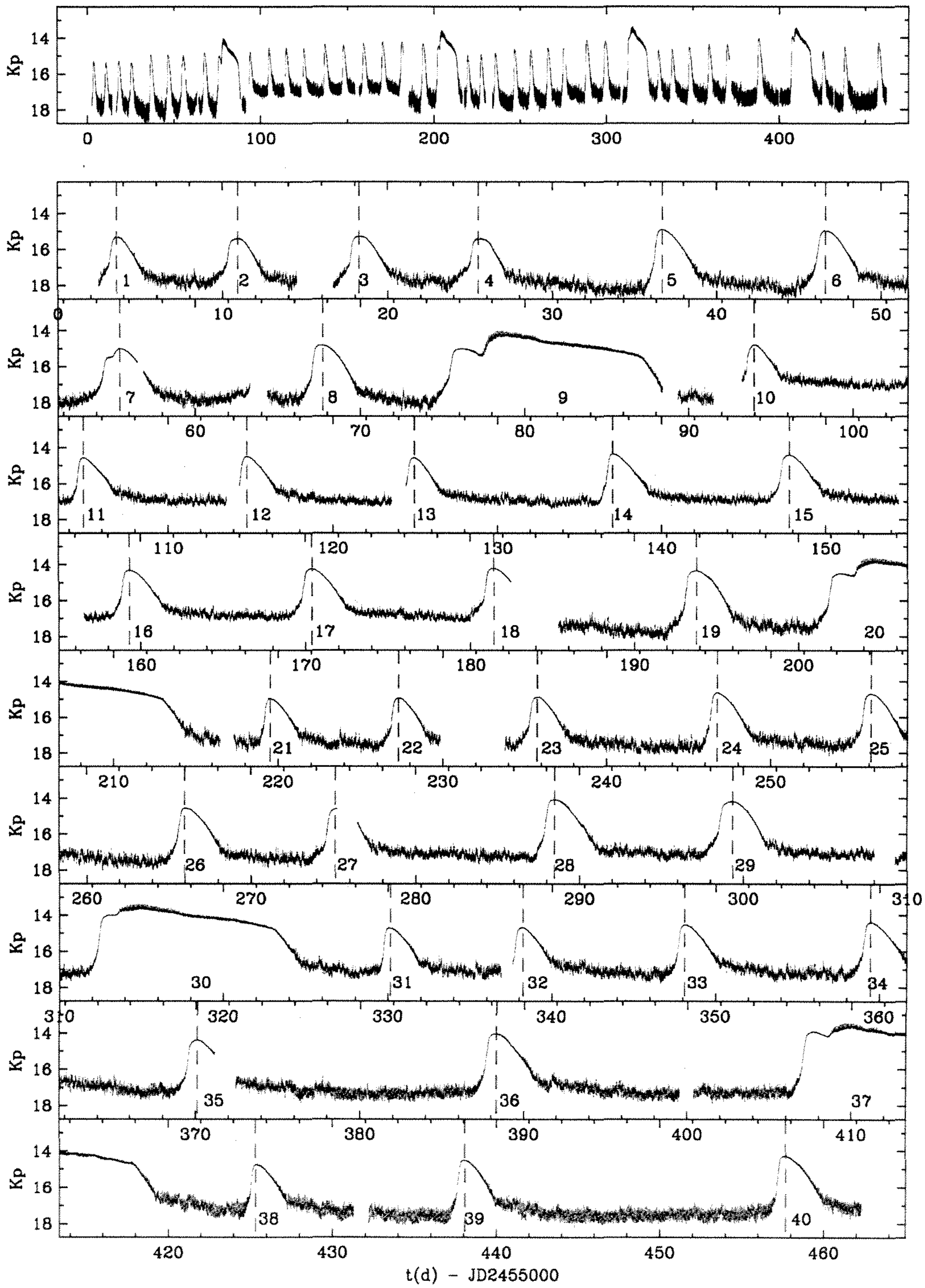

FIG. 1.— *Kepler* light curve for V1504 Cyg at 1 min cadence, spanning ~460 d. The light curve covers four superoutbursts and 36 normal outbursts. The vertical red lines indicate the local maxima for the normal outbursts in which coverage permits a reliable determination.

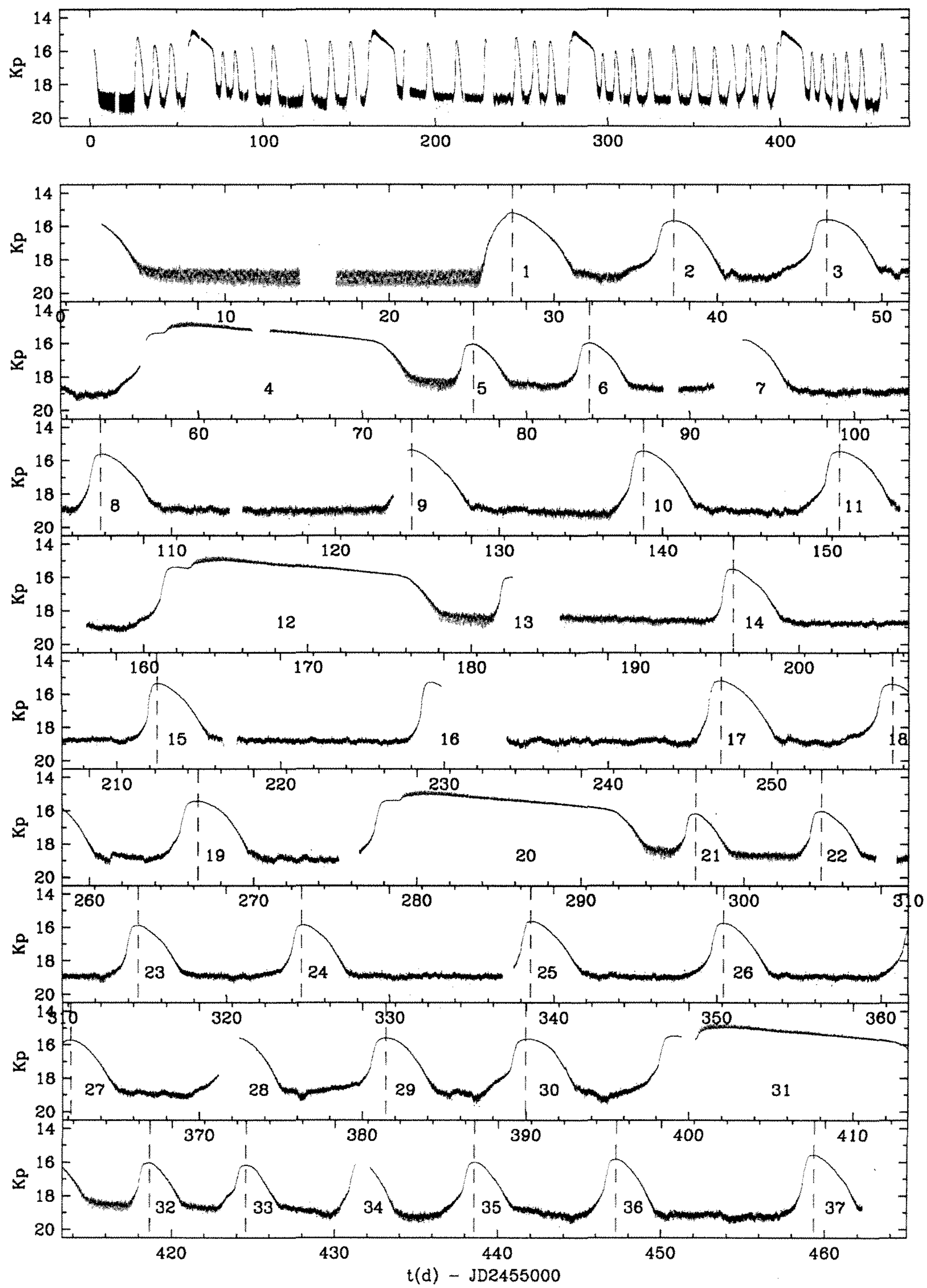

FIG. 2.— *Kepler* light curve for V344 Lyr at 1 min cadence, spanning ~460 d. The light curve encompasses four superoutbursts and 33 complete normal outbursts. The conventions are the same as in Figure 1.

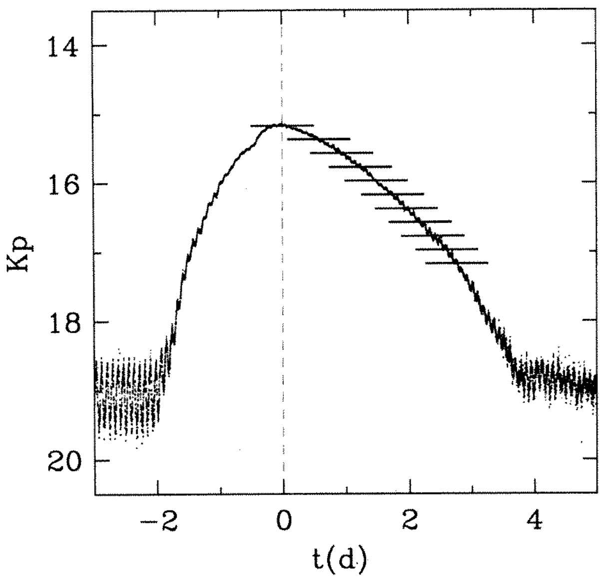

FIG. 3.— The distribution of decay bins for the first normal outburst in V344 Lyr.

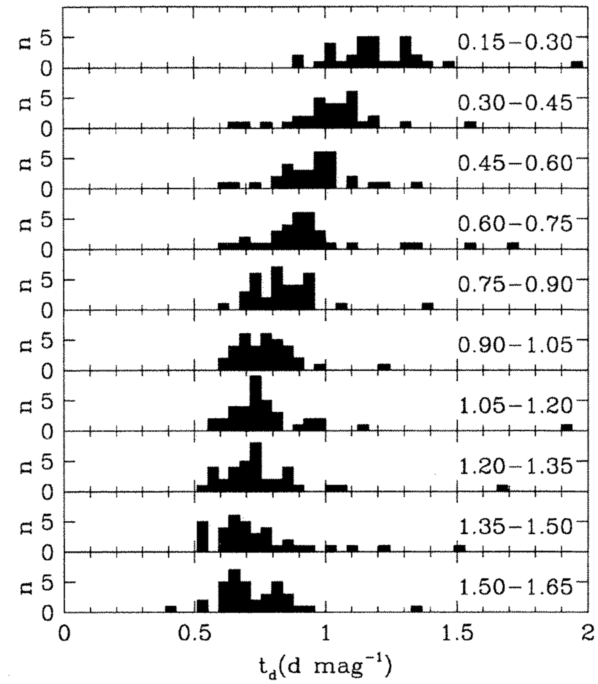

FIG. 4.— A histogram of the fast decay rates for the 36 normal outbursts in V1504 Cyg with well-defined maxima, as shown in Figure 1. The numbers in each panel give the magnitude range in ΔKp down from maximum corresponding to the interval within which the local decay rate t_d is measured, thus brighter portions of the decay are in the upper panels.

ley relation, and 0.82 and 0.99 d mag^{-1} from the Warner fit. The presently determined values $\sim$0.7 d mag^{-1} in V1504 Cyg and $\sim$0.6 d mag^{-1} in V344 Lyr for the faintest bins are closer to Bailey's original expression. If one averaged over the entire decay, the values would be larger, closer to Warner's expression. The precision of the current determinations are limited by the small number of total NOs in both systems. With time, better statistics will allow better determinations, as has been done for instance with SS Cyg (Cannizzo & Mattei 1998, see their Figure 6).

Figure 6 shows the outburst durations t_b for the two systems. Based on the magnitude range for outbursts in the two systems, we use a cut of $Kp = 16$ in V1504 Cyg and $Kp = 17.9$ in V344 Lyr to define outburst onset and termination. There is an obvious trend of increasing outburst width t_b between consecutive SOs: during a supercycle, t_b increases by $\sim$20−70%. Figure 7 shows the asymmetry parameter f_r, defined to be the ratio of the rise time to the total outburst duration, for the individual NOs depicted in Figures 1 and 2. Figure 8 shows the quiescent intervals t_q between successive outbursts. For most of the time, the quiescent intervals in V1504 Cyg are relatively constant. For the last two supercycles there is an increase in t_q following a SO. In contrast, for V344 Lyr the t_q values increase to a local maximum value about half way between successive SOs, and then decrease back to roughly their starting values by the time of the next SO.

At roughly the time of the third SO in V1504 Cyg, strong negative superhumps appear (Wood et al. 2011, in preparation) and persist through the end of the light curve. The presence of negative superhumps indicates that the mass is not accreting primarily at the rim of the disk. Although there is not an obvious change in f_r at this time, it is perhaps noteworthy that t_q increases. This may indicate a strong drop in the mass transfer rate from the secondary star, which would increase the recurrence time for outbursts. In the magnetic systems such as AM Her, which lack accretion disks, one has direct evidence for the instantaneous mass transfer rate from the secondary stars in the long term light curves, and one does often see intervals during which the mass transfer rate reduces to a low level (e.g., Hessman, Gänsicke, & Mattei 2000).

TABLE 1
SUMMARY OF PARAMETERS

Parameter	V1504 Cyg	V344 Lyr	Comment
NO: t_b(d)	1.1−3.0	2.4−4.9	NO duration
SO: t_b(d)	11.9	6.8	SO duration
NO: f_r	0.18−0.44	0.22−0.37	NO rise/duration
t_q(d)	5.3−17.6	2.4−21.4	Quiescent duration
NO: t_d(d mag^{-1})	0.7	0.6	NO decay rate
SO: t_d(d mag^{-1})	7.9	12	SO decay rate

2.2. *Superoutbursts*

Figure 9 shows a composite of the four SOs in V1504 Cyg, positioned in time so that $t = 4$ d corresponds to the crossing of $Kp = 16$ during SO onset. The rate of decay during SO is $\sim$8 d mag^{-1}. Figure 10 shows a composite of the four SOs in V344 Lyr, positioned in time so that $t = 3$ d corresponds to the crossing of $Kp = 18$ during SO onset. The rate of decay during SO is $\sim$12 d mag^{-1}. Within each system the SOs are quite similar in overall appearance and duration.

3. DISCUSSION

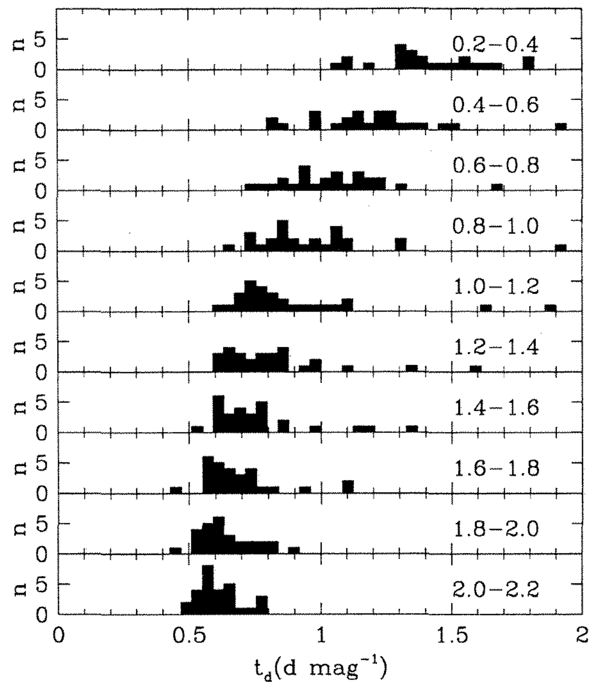

FIG. 5.— A histogram of the fast decay rates for the 28 normal outbursts in V344 Cyg with well-defined maxima and well-sampled decays, as shown in Figure 2. Conventions are the same as in Figure 4.

A summary of the parameters associated with the outbursts in the two systems is given in Table 1. The behavior of the sequences of outburst durations t_b and quiescent intervals t_q during a supercycle are quite different. The t_b sequence shows an increase between SOs in both systems. The t_q values remain relatively constant between SOs in V1504 Cyg, whereas in V344 Lyr they increase to a local maximum roughly half way between SOs before returning to their initial small value by the time the next SO begins. Thus the t_q values are roughly continuous across SOs. In both systems the variation of t_q values between SOs appears to be inconsistent with the prediction of the thermal-tidal instability model, which shows a monotonic increase in t_q between SOs (Ichikawa, Hirose, & Osaki 1993, see their Figure 1; Osaki 2005, see his Figure 3).

The asymmetry of the NOs is similar in the two systems, $f_r \simeq 0.2 - 0.3$ in V1504 Cyg and $f_r \simeq 0.25 - 0.35$ in V344 Lyr. According to theory, the degree of outburst asymmetry reveals whether the outburst is triggered at small or large radii in the disk (Smak 1984; Cannizzo, Wheeler, & Polidan 1986, hereafter CWP; Cannizzo 1998, see his Figure 6 and associated discussion). Outbursts triggered at large radii tend to have fast, asymmetric rise times ($f_r < 0.5$), while those triggered at small radii are more symmetric ($f_r \simeq 0.5$). Smak referred to these two types as Type A and Type B, respectively, whereas CWP used the terminology "outside-in" and "inside-out" outbursts. The low f_r values in both systems studied in this work are consistent with the Type A/outside-in outbursts. In is interesting that the one outburst which seems to be certifiably symmetric and therefore presumably inside-out – outburst 1 of V344 Lyr – occurred after an unusually long quiescence interval, which may have led to a more filled disk at the end of quiescence (compare this outburst with that shown in Figure 5b of Smak 1984 and Figure 3 of CWP). During this part of the V344 Lyr light curve there were strong negative superhumps (Wood et al. 2011, see their Figure 13), indicating a tilted accretion. This would have the effect of spreading the accretion stream impact over a large range in radii, thereby lengthening the time to build up to the next outburst.

The mean durations of SOs for V1504 Cyg and V344 Lyr are 11.9 d and 16.8 d, and the e−folding rates of the (slow) decay during SO are 7.9 d mag^{-1} and 12.0 mag^{-1}, respectively. Thus the SOs span 1.5 e−foldings in V1504 Cyg, and 1.4 e−foldings in V344 Lyr. Given that the SO accretion disks are close to steady state, the depletion fraction f_d in the stored disk mass during SO for the two systems,

$$f_d \simeq 1 - e^{-t_b(\mathrm{SO})/t_d(\mathrm{SO})}, \quad (1)$$

is ~0.78 and ~0.75, respectively. (These numbers probably over-estimate the depleted mass fraction because they encompass portions of the SO fainter than the e−folding plateaus.) These depletion fractions are larger than the $f_d \simeq 0.2$ inferred for long outbursts in DNe above the period gap (e.g., Cannizzo 1993), and stem from the larger $\alpha_{\mathrm{hot}}/\alpha_{\mathrm{cold}}$ ratios in the SU UMa systems. The ratio $t_d(\mathrm{SO})/t_d(\mathrm{NO}) \simeq 20$ is consistent with the previous interpretation of fast decays being due to the action of a cooling front within the disk, and slow decays being the result of gradual accretion onto the WD, and therefore governed by the viscous time scale in the outer disk. The viscous and thermal time scales can be written in terms of the local Keplerian rotation frequency Ω and the local disk height to radius ratio h/r (Pringle 1981). The viscous time scale

$$t_\nu = \frac{1}{\alpha_{\mathrm{hot}}\Omega}\left(\frac{r}{h}\right)^2 \quad (2)$$

and the thermal time scale

$$t_{\mathrm{th}} = \frac{1}{\alpha_{\mathrm{hot}}\Omega}\left(\frac{r}{h}\right) \quad (3)$$

differ by a factor h/r, therefore $h/r \approx 1/20$.

C10 present an equation for the rate of decay of a SO, taken from Warner (1995a, 1995b), which is based on the viscous time in the outer accretion disk

$$t_d \approx t_\nu = 17\ \mathrm{d}\ \alpha_{\mathrm{hot},-1}^{-4/5}\ P_h^{1/4}\ m_1^{1/6}, \quad (4)$$

where $\alpha_{\mathrm{hot},-1} = \alpha_{\mathrm{hot}}/0.1$ and $m_1 = M_1/M_\odot$. This e−folding time scale should approximate the decay rate, expressed in d mag^{-1}. Hence in a large sample of DNe the prediction is that the rate of decay during SO should vary as $P_h^{0.25}$. Our analysis shows, however, that in going from V1504 Cyg with $P_h = 1.67$ to V344 Lyr with $P_h = 2.1$, the SO decay rate increases from 7.9 d to 12 d. One does expect object-to-object variations in NO and SO properties which can be large compared to overall, secular trends with orbital period, but given the dependencies in Equation (4) and the inference from a large number of DNe that $\alpha_{\mathrm{hot}} \simeq 0.1$, there is no way to account for such a steep variation, $t_d \tilde{\propto} P_h^{1.8}$.

It is worth pausing for a moment and looking at how the SO slow decay rates for V1504 Cyg and V344 Lyr fit into the larger picture. Table 2 lists the slow decay rate of SOs for SU UMa stars taken from the literature, most of which are derived from VSNET data (see Kato et al. 2009, 2010). We only show

TABLE 2
SLOW DECAY OF SUPEROUTBURSTS IN SU UMA STARS

System	decay rate (d mag^{-1})	P_h	Reference
SW UMa	10.	1.36	1
WX Cet	10.	1.40	2
FL Tra	7.7	1.41	3
V1028 Cyg	8.3	1.45	4
HO Del	7.1	1.50	5
GO Com	6.7	1.58	6
NSV 4838	10.	1.63	7
VW CrB	10.3	1.70	8
EG Aqr	7.1	1.83	9
V503 Cyg	9.1	1.86	10

REFERENCES. — 1. Soejima et al. (2009), 2. Kato et al. (2001), 3. Imada et al. (2008b), 4. Baba et al. (2000), 5. Kato et al. (2003), 6. Imada et al. (2005), 7. Imada et al. (2009), 8. Nogami et al. (2004), 9. Imada et al. (2008a), 10. Corrado et al. (2000)

data for the VW Hyi subclass for which the slow decay rate is relatively constant over the course of the SO. In Figure 11 we plot the SO decay rates for the systems shown in Table 2, as well as V1504 Cyg, V344 Lyr, WZ Sge and U Gem. For WZ Sge we only plot the value for the initial, steep rate of decay, 4 d mag^{-1}. As noted in the Introduction, the SO decays in the WZ Sge stars exhibit a strong deviation from exponentiality presumably due to the accretion of virtually all the stored mass. As the disk mass decreases, one expects a concomitant increase in the viscous time, leading to a pronounced concave upward SO decay. Therefore only the very initial SO decay rate gives a valid representation for the viscous time within the disk corresponding to the stored mass. As regards DNe above the period gap, virtually none of the long outbursts ever observed exhibited enough dynamic range in their overall decay that one could reliably extract the rate of decay. Only for the unusually long, 1985 October outburst of U Gem could this determination be made (Cannizzo, Gehrels, and Mattei 2002), hence the single data point above the period gap in Figure 11. If one takes the overall $t_d - P_h$ trend between WZ Sge and U Gem as a true physical baseline, $t_d \mathrel{\tilde{\propto}} P_h^{1.6}$, the trend between V1504 Cyg and V344 Lyr in $\log t_d - \log P_h$ space, $t_d \mathrel{\tilde{\propto}} P_h^{1.8}$, is roughly the same, although the normalization is offset slightly. Given the vertical scatter in Figure 11 for the other SU UMa systems, this coincidence may be fortuitous.

The failure of Equation (4) to account for the difference between the SO slow decay rate in V1504 Cyg and V344 Lyr indicates that a more sophisticated expression may be required. One obvious deficiency is that there is no dependence on $\alpha_{\rm cold}$ which controls the available amount of fuel in quiescence to power a SO. The physics behind the viscosity in quiescence is unknown at present, and one could in principle have large variations in $\alpha_{\rm cold}$ between systems at similar orbital periods. Let us go back to the definition of the viscous time scale $t_\nu = (\alpha_{\rm hot}\Omega)^{-1}(r/h)^2$. Since the controlling time scale in this situation is the slowest one, this expression is to be evaluated at the outer disk radius r_d. Following Cannizzo (2001b), we consider the hot state of the disk by adopting a Rosseland mean opacity $\kappa = 2.8 \times 10^{24}$ g cm$^{-2}\rho T^{-3.5}$ and mean molecular weight $\mu = 0.67$. We can then write the viscous time as

$$t_\nu = 5.7\ {\rm d}\ m_1^{5/14} r_{d,10}^{13/14} \alpha_{{\rm hot},-1}^{-8/7} \left(\frac{\Sigma(r_d)}{2000\ {\rm g\ cm^{-2}}}\right)^{-3/7}, \quad (5)$$

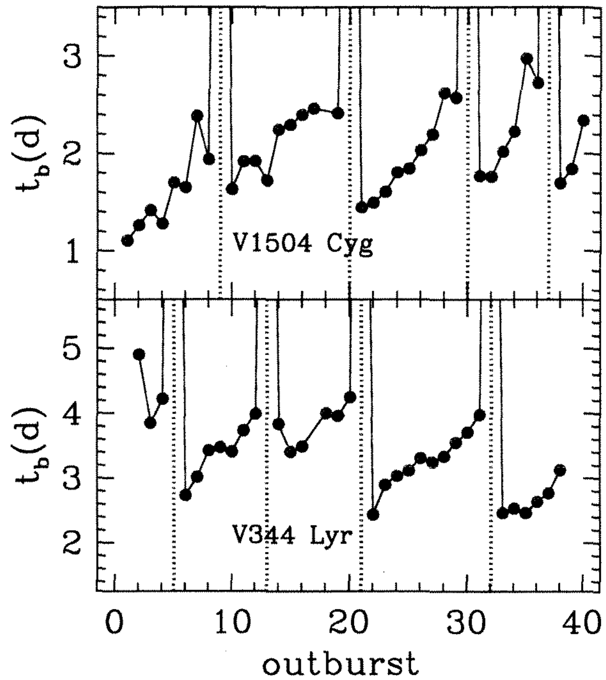

FIG. 6.— The values of the outburst durations t_b for the two systems, taking a cut line $Kp_{\rm cut} = 16$ for V1504 Cyg and $Kp_{\rm cut} = 17.9$ for V344 Lyr. The vertical dotted lines (*red*) indicate the superoutbursts. The values of t_b for the superoutbursts are t_b(d) = 12.0, 11.9, 11.8, and 11.9 for V1504 Cyg, and t_b(d) = 16.6, 17.1, 16.6, and 16.9 for V344 Lyr.

where $m_1 = M_{\rm WD}/1M_\odot$, $r_{d,10} = r_d/(10^{10}\ {\rm cm})$, and $\Sigma(r_d)$ is the surface density at the outer disk edge.

Looking at this expression, we see that the Warner (1995a, 1995b) equation already contained an implicit assumption about Σ, as it only contains $\alpha_{\rm hot}$ and P_h. To derive a more meaningful expression for $t_d \approx t_\nu$ in the context of SOs we need to take into account two additional pieces of information, namely that during a SO essentially all the mass accumulated during quiescence is accreted onto the WD, and thereby lost from the disk, and also that the accumulated mass depends on the critical surface density $\Sigma_{\rm max}$, which depends in turn on $\alpha_{\rm cold}$. This scaling is given by (Lasota 2001)

$$\Sigma_{\rm max} = 13.4\ {\rm g\ cm^{-2}}\ m_1^{-0.38} r_{10}^{1.14} \alpha_{\rm cold}^{-0.83}. \quad (6)$$

Since the attainment of $\Sigma_{\rm max}$ is the triggering mechanism for a DN outburst, the quiescence disk mass $\sim(\pi/2)r_d^2\Sigma_{\rm max}(r_d,\ \alpha_{\rm cold})$ at the time of SO onset can be written as

$$\Delta M_{\rm cold} \approx 4.2 \times 10^{21}\ {\rm g}\ m_1^{-0.38} r_{d,10}^{3.14} \alpha_{\rm cold}^{-0.83}. \quad (7)$$

Once the SO has started, the mass that was present in the non-steady state quiescent disk gets redistributed into a quasi-steady disk with $\dot M(r) \simeq$ constant, and therefore with a radial profile $\Sigma(r) \mathrel{\tilde{\propto}} r^{-3/4}$ (SS). Therefore the mass of the hot disk can be written as

$$\Delta M_{\rm hot} = \int 2\pi r dr \Sigma(r,\ \alpha_{\rm hot}). \quad (8)$$

We now set $\Sigma(r,\ \alpha_{\rm hot})$ equal to its standard functional form for a SS disk with gas pressure and free-free opacity, $\Sigma_{\rm hot} = \Sigma_0\ \alpha_{\rm hot}^{-4/5} r_{10}^{-3/4}$. This is the same form that entered into Equa-

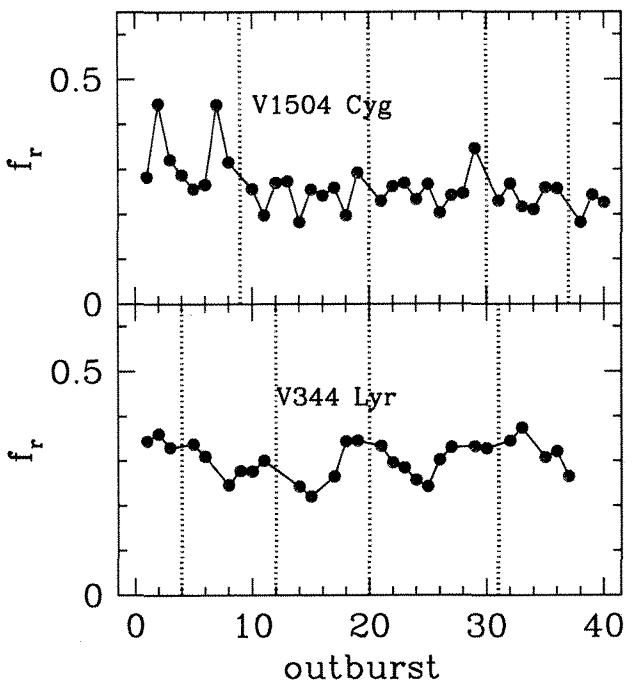

FIG. 7.— The values of the asymmetry parameter $f_r \equiv t_{\rm rise}/t_b$ for the normal outbursts in the two systems. Only values for which the times of outburst maxima can be reliably determined are shown. The same cut line values are used as in Figure 6.

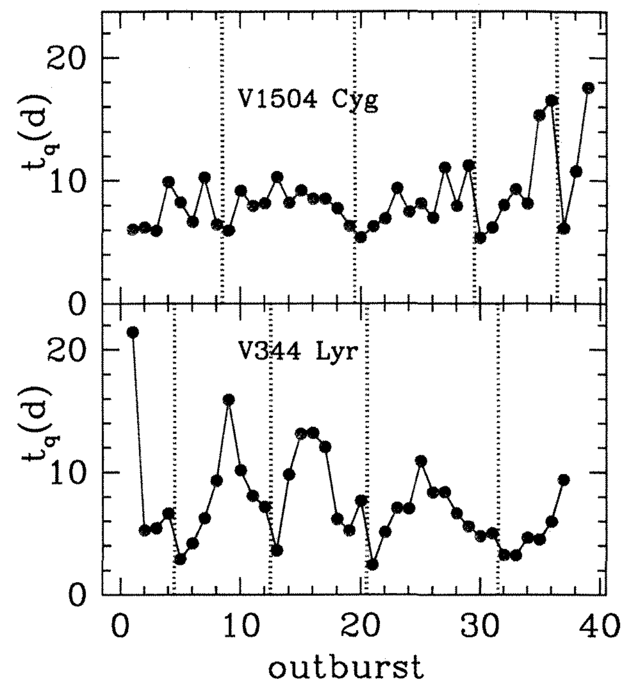

FIG. 8.— The values of the quiescence durations t_q between consecutive outbursts for the two systems. As with Figure 6, the vertical dotted lines (*red*) indicate the positions of the superoutbursts. The same cut line values are used as in Figure 6.

tion (5). Equating the disk mass at the end of the quiescent state with that at the beginning of the SO, i.e., $\Delta M_{\rm cold} = \Delta M_{\rm hot}$, gives

$$\Sigma_{\rm hot}(r_d) = \frac{5}{16}\Sigma_{\rm max}(r_d), \quad (9)$$

the relevant Σ value for Equation (5). Therefore we can rewrite Equation (5) as

$$t_\nu = 80.2 \text{ d } m_1^{0.52}\alpha_{\rm cold}^{0.36}\alpha_{{\rm hot},-1}^{-1.14} r_{d,10}^{0.44}. \quad (10)$$

This expression differs from Equation (5) in that it contains $\alpha_{\rm cold}$, reflecting the fact that the total amount of matter stored in quiescence enters into the viscous time scale in the resultant SO disk.

In extreme mass ratio systems such as the SU UMa stars, the Roche lobe of the primary asymptotes to ~0.82 of the binary separation (Eggleton 1983). Also, the relation between orbital separation and orbital period is $a_{10} = 3.53 P_h^{2/3}$, where a_{10} is the orbital separation in units of 10^{10} cm. If we further take the outer disk radius r_d to lie at 75% of the primary Roche lobe, we have $r_{d,10} = 2.19 P_h^{2/3}$. Since it is fairly certain that α in outburst is always close to 0.1, $\alpha_{\rm hot}$ been scaled to that value in the previous equations. On the other hand, the α value in quiescence is uncertain, and has been traditionally adjusted by trial and error for each system. For SU UMa stars, workers have typically had to adopt much smaller $\alpha_{\rm cold}$ values than for DNe above the period gap. Given this uncertainty, let us take $\alpha_{\rm cold}$ to be a power law in orbital period, $\alpha_{\rm cold} = \alpha_{c,0} P_h^{\psi}$. Making these substitutions into Equation (10) we obtain

$$t_\nu = 113 \text{ d } m_1^{0.52}\alpha_{c,0}^{0.36}\alpha_{{\rm hot},-1}^{-1.14} P_h^{0.29+0.36\psi}. \quad (11)$$

The dynamic variation associated with one magnitude (2.512) is 0.924 times that associated with one e−folding (2.718). Therefore if we associate the rate of SO decay, expressed in d mag^{-1}, with the viscous time at the outer edge, which is an e−folding time, then Equation (11) needs to be corrected by a factor 0.924.

By comparing the SO decay rates of V1504 Cyg and V344 Lyr, we infer a decay rate $t_d = 0.924 t_\nu = 12 \text{ d } (P_h/2.1)^{1.8} = 3.16 \text{ d } P_h^{1.8}$. Furthermore, adopting $m_1 = 0.6$ for the SU UMa systems gives $m_1^{0.52} = 0.767$, and therefore

$$80.2 \text{ d } \alpha_{c,0}^{0.36} P_h^{0.29+0.36\psi} = 3.16 \text{ d } P_h^{1.8}. \quad (12)$$

Solving for the constant and exponent in the $\alpha_{\rm cold}$ scaling gives $\alpha_{c,0} = 1.25 \times 10^{-4}$ and $\psi = 4.2$ – a steep dependence of $\alpha_{\rm cold}$ on orbital period. For V344 Lyr at $P_h = 2.1$ we obtain $\alpha_{\rm cold} \simeq 2.8 \times 10^{-3}$, which accounts for why C10, in their modeling of V344 Lyr, had to adopt the small value $\alpha_{\rm cold} = 0.0025$. The inference $\psi = 4.2$ is only valid, however, if the trend between V1504 Cyg and V344 Lyr were representative of a trend with P_h, which seems unlikely given the data for the other SU UMa systems. Based only on the SU UMa data, it would be more likely that $\alpha_{c,0}$ has a large variation between the two systems. As noted earlier however, if one takes the WZ Sge $\leftrightarrow$ U Gem interpolation to be physically meaningful, then the V1504 Cyg $\leftrightarrow$ V344 Lyr interpolation appears to be fortuitously coincident.

A steep dependence of $\alpha_{\rm cold}$ on P_h may have observational support from another line of argument. In a recent study of the aggregate properties of DNe, Patterson (2011, see his Figure

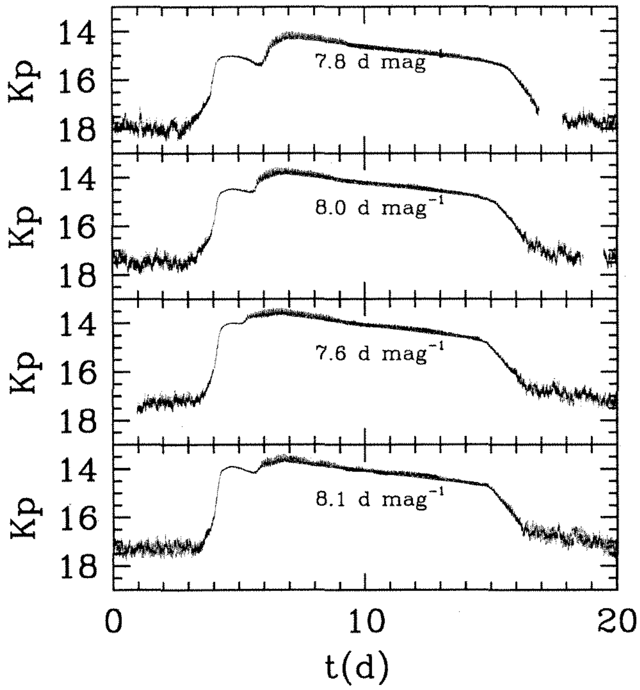

FIG. 9.— A composite of four superoutbursts in V1504 Cyg, positioned in time so that $t = 4$ d corresponds to the crossing of $Kp = 16$ during superoutburst onset. The line segments (*red*) indicate the linear least squares fit, for the range of data indicated. The rate of decay for the four superoutbursts are as shown in each panel. The mean value is 7.9 d mag^{-1}.

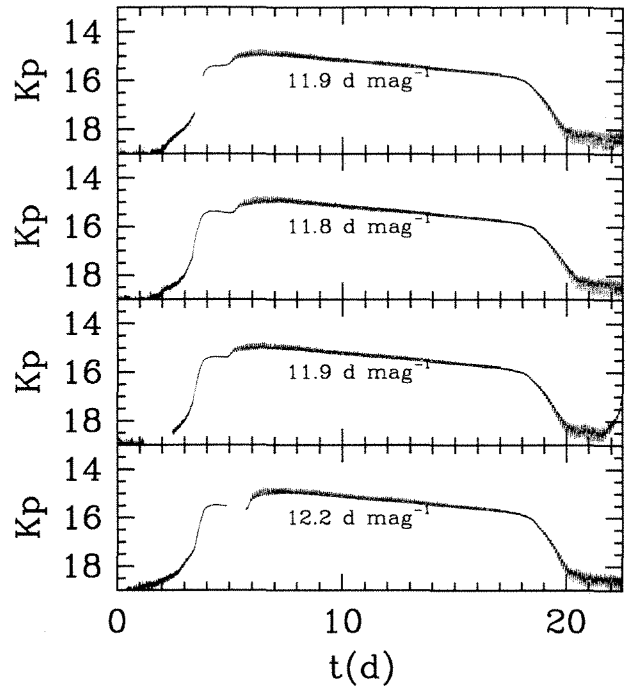

FIG. 10.— A composite of four superoutbursts in V344 Lyr, positioned in time so that $t = 3$ d corresponds to the crossing of $Kp = 18$ during superoutburst onset. The line segments (*red*) indicate the linear least squares fit, for the range of data indicated. The rate of decay for the four superoutbursts are as shown in each panel. The mean value is 12 d mag^{-1}.

11) finds that the recurrence time for SOs has a steep inverse relation with binary mass ratio $t_{\rm recur}({\rm SO}) \propto q^{-2.63\pm0.17}$, where $q = M_2/M_1$. Since the secondaries in cataclysmic variables are, by definition, in contact with their Roche lobes, this implies that M_2 scales with orbital period, and hence $t_{\rm recur}({\rm SO}) \propto P_h^{-2.63\pm0.17}$. Given that (1) SOs accrete virtually all their stored mass $\Delta M_{\rm cold}$, (2) the mass feeding rate into the outer disk from the secondary $\dot M_T$ is thought to be relatively constant at $\sim 10^{-10} M_\odot$ yr^{-1} for systems below the period gap (Kolb et al. 1998; Howell et al. 2001), and (3) the recurrence time (Cannizzo et al. 1988)

$$t_{\rm recur}({\rm SO}) \approx \frac{\Delta M_{\rm cold}}{|\dot M_T|} \tilde\propto r_d^{3.14} \alpha_{\rm cold}^{-0.83}, \qquad (13)$$

it is quite surprising that $t_{\rm recur}({\rm SO})$ apparently has a strong inverse relation with orbital period. As one progresses to shorter orbital period, one expects more frequent SOs since the disks are smaller and the critical mass is reached more quickly. We are seemingly drawn to the same conclusion as previously, that $\alpha_{\rm cold}$ must have a strong dependence on orbital period. Howell et al. (2001; see their Figure 2, panel (a)) indicates a rough dependence $d\log|\dot M_T|/d\log P_h \approx 1$ for the SU UMa stars. If we again set $\alpha_{\rm cold} \propto P_h^\psi$, substitute P_h for r_d in Equation (13), and solve for ψ, we obtain $\psi = 4.5$.

If the overall trend in t_d for long outbursts between WZ Sge and U Gem, which is roughly matched (coincidentally) by that between V1504 Cyg and V344 Lyr, is real, then two independent lines of reasoning imply a very steep dependence of $\alpha_{\rm cold}$ on orbital period. Why should one have $\alpha_{\rm hot} \simeq$ constant,

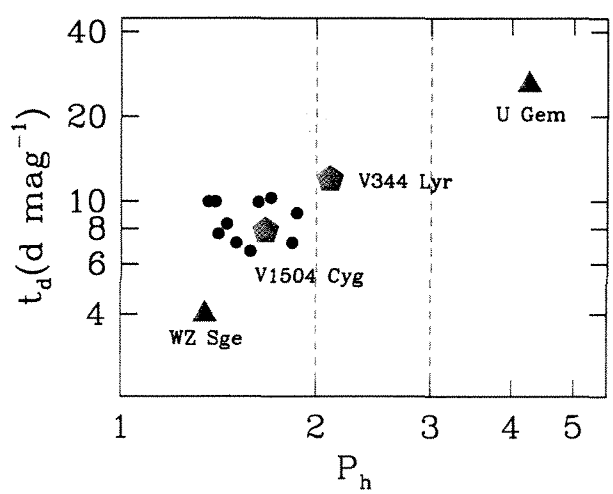

FIG. 11.— A comparison of superoutburst slow decay rates of other SU UMa stars, from Table 2, with V1504 Cyg and V344 Lyr. Also shown are WZ Sge (initial superoutburst decay rate only) and U Gem. The vertical dashed lines indicate the 2 – 3 hr orbital period gap for DNe.

while at the same time $\alpha_{\rm cold} \propto P_h^\psi$, where $\psi \simeq 4.2 - 4.5$? Current thinking on the physical mechanism for viscosity in accretion disks centers on the magnetorotational instability (MRI - Balbus & Hawley 1998), an instability based on the shearing amplification of a weak magnetic field. The nonlinear saturation limit of the MRI in ionized gas is thought to explain why $\alpha_{\rm hot} \rightarrow \sim 0.1$ (McKinney & Narayan 2007a, 2007b). However, as the disk cools to the quiescent state and the supply of free electrons drops precipitously due to the exponential dependence of partial ionization on temperature below $\sim 10^4$ K, the magnetic field no longer couples effectively to the gas, and fails to provide an effective viscosity. Menou (2000) tried to model the complete accretion disk limit cycle model for DNe using only the MRI, and failed. He discussed the failure of the MRI in quiescence, and predicted that if tidal torques (a completely different physical mechanism from the MRI) dominate the angular momentum transport in the quiescent accretion disks in the SU UMa stars, one would expect an anticorrelation between $t_{\rm recur}$(SO) and q. He presents data for six SU UMa stars showing an anticorrelation. (Patterson [2011] shows the anticorrelation in much greater detail.) Menou notes that theories of tidally induced spiral waves or shock waves indicate a strong correlation between effective torque and q (Papaloizou & Pringle 1977; Goldreich & Tremaine 1980). Our finding $\alpha_{\rm cold} \,\tilde{\propto}\, P_h^{4.2}$ seems to support the view that *tidal torquing is the dominant angular momentum transport mechanism operating in quiescent accretion disks in interacting binaries.*

4. CONCLUSION

We have analyzed long term *Kepler* light curves of V1504 Cyg, containing 40 outbursts, and V344 Lyr, containing 38 outbursts. Each system showed four superoutbursts. The findings are:

(1) The decays of the NO are faster-than-exponential, with most of the deviation occurring near maximum light. Near quiescence the decays are close to exponential, $\sim$0.7 d mag^{-1} for V1504 Cyg and $\sim$0.6 d mag^{-1} for V344 Cyg. These rates are in line with that expected from the Bailey relation. Our values are based on frequency histogram distributions of only $\sim$30$-$35 NOs and should improve as more outbursts are accumulated.

(2) The outburst durations t_b for the NOs increase by a factor $\sim$1.2$-$1.7 between consecutive superoutbursts.

(3) The quiescent intervals t_q between normal outbursts remain relatively constant for three of the four supercycles in V1504 Cyg, whereas in V344 Lyr the t_q values increase to a local maximum about half way between superoutbursts. There was also an anomalously long quiescence interval at the start of the V344 Lyr light curve, potentially the result of a tilted disk. Neither of the observed t_q trends are consistent with Osaki's thermal tidal instability model for SOs, in which one expects a monotonic increase in t_q values accompanying the monotonic build-up in disk mass and angular momentum leading to a SO.

(4) The asymmetric shape of the NOs is coincident with the triggering for the outbursts being outside-in.

(5) The inference of the steep relation $\alpha_{\rm cold} \,\tilde{\propto}\, P_h^4$ gives strength to the notion of tidal torquing as the dominant angular momentum transport mechanism operating in quiescent accretion disks in interacting binaries.

If one considers the slow decay rate of the long, viscous outbursts spanning DNe from WZ Sge to U Gem, one infers an overall variation $\tilde{\propto}\, P_h^{1.6}$. Given the scatter in the decay rate for the 10 well-studied SU UMa systems shown in Table 2, it may be fortuitous, given the general scatter in DN outburst properties at a given orbital period, that the decay rate between V1504 Cyg and V344 Lyr shows a similar law $\tilde{\propto}\, P_h^{1.8}$. These dependencies are much steeper than the $P_h^{0.25}$ expected from the previous theoretical expression, due to Warner (1995a, 1995b). By starting with the definition for the viscous time scale in the outer disk during superoutburst, and taking into account the scaling for material accumulated during quiescence, we derive a more physically motivated formula, and find that a steep dependence of the quiescent state value of α is required, $\alpha_{\rm cold} \,\tilde{\propto}\, P_h^{4.2}$. In simple terms, the dramatic increase in stored quiescent mass with decreasing orbital period brings about a decrease in the viscous time scale of matter in the superoutburst disk, which is formed out of redistributed gas from the quiescent state. Recent work by Patterson (2011) on the recurrence time scale in SU UMa stars for SOs also supports a steep dependence of $\alpha_{\rm cold}$ on P_h. The long outbursts in systems above the period gap and superoutbursts in systems below the gap both represent viscous decays, with the absence of transition waves in the disk. It is somewhat counterintuitive that the recurrence time for these long, viscous outbursts varies as $P_h^{-2.6}$, and yet their decay time varies as $P_h^{1.8}$. The steepness of the $\alpha_{\rm cold}(P_h)$ relation explains why accretion disk modelers have had to adopt such small $\alpha_{\rm cold}$ values for the SU UMa stars, compared to DNe above the period gap, and also probably accounts for the fact that the extreme mass ratio black hole transient systems undergoing limit cycle accretion disk outbursts, like A0620-00, have such long recurrence times for outbursts.

The *Kepler* data have already provided important constraints on the physics of accretion disks, in particular the Shakura-Sunyaev α parameter: C10 showed that an interpolation between $\alpha_{\rm cold}$ and $\alpha_{\rm hot}$ based on the degree of partial ionization of gas was needed to account for the shoulders in the SOs of V344 Lyr, and in this work we have shown that the scaling of SO decay rate with orbital period mandates a steep dependence of $\alpha_{\rm cold}$ on orbital period, enforcing the notion of tidal torquing as being the dominant viscosity mechanism in quiescent accretion disks.

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