A Globally Stable Lyapunov Pointing and Rate Controller for the Magnetospheric MultiScale Mission (MMS)

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The Magnetospheric MultiScale Mission (MMS) is scheduled to launch in late 2014. Its primary goal is to discover the fundamental plasma physics processes of reconnection in the Earth's magnetosphere. Each of the four MMS spacecraft is spin-stabilized at a nominal rate of 3 RPM. Traditional spin-stabilized spacecraft have used a number of separate modes to control nutation, spin rate, and precession. To reduce the number of modes and simplify operations, the Delta-H control mode is designed to accomplish nutation control, spin rate control, and precession control simultaneously. A nonlinear design technique, Lyapunov's method, is used to design the Delta-H control mode. A global spin rate controller selected as the baseline controller for MMS, proved to be insufficient due to an ambiguity in the attitude. Lyapunov's design method was used to solve this ambiguity, resulting in a controller that meets the design goals. Simulation results show the advantage of the pointing and rate controller for maneuvers larger than 90 deg and provide insight into the performance of this controller.

Nomenclature

${}^{N}\underline{\omega}^{B}$	=	angular velocity of the body, B, with respect to the inertial frame, N
ω_0	=	desired spin rate magnitude, constant
$[J]$	=	3x3 Inertia matrix
${}^{B}\underline{\tau}$	=	applied external torque to spacecraft in the body frame, B
${}^{B}\underline{H}$	=	angular momentum of spacecraft in body frame, B
${}^{B}\underline{H}_D$	=	desired spacecraft angular momentum in body frame, B
${}^{B}\underline{H}_e$	=	angular momentum error in the body frame, B
H_0	=	desired spacecraft angular momentum magnitude
$[I]$	=	3x3 identity matrix
$\bar{\lambda}$	=	maximum eigenvalue of $[J]$
${}^{N}\hat{s}$	=	desired major axis pointing direction in inertial space, N, fixed in inertial space
${}^{B}\hat{s}$	=	instantaneous unit vector representing the desired major axis pointing direction in inertial space resolved in the body frame, B
$k_{1:3}$	=	scalar positive constant
α_3	=	scalar positive constant
${}^{B}\hat{p}$	=	desired body pointing vector, fixed in body frame, B
${}^{B}\underline{e}$	=	controller error vector in the body frame, B
${}^{B}\underline{\tau}$	=	external applied torque in the body frame, B

I. Introduction

THE Magnetospheric MultiScale Mission (MMS), being designed and built by NASA's Goddard Space Flight Center (GSFC) in Greenbelt, MD, is scheduled to launch in late 2014. The primary goal of MMS is to discover the fundamental plasma physics processes of reconnection in the Earth's magnetosphere. Each of

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the four MMS spacecraft will be spin-stabilized at a nominal spin rate of 3 RPM and have the same complement of in-situ science instruments. Science instruments are placed at the end of two 15 m long axial coilable booms, two 5 m long radial booms, and four 60 m long axial wire booms. All spacecraft will be launched together on the Atlas V launch vehicle. The attitude control system (ACS) for this spin-stabilized spacecraft is not as simple as traditional spin-stabilized spacecraft. MMS's Delta-H control mode is the primary attitude mode and is responsible for simultaneous nutation control, spin rate control, and precession control; it's design is the main topic of this paper. Section II gives an overview of the attitude control system (ACS). Section III delves into the baseline Delta-H control design, presents a correction to the baseline controller, and discusses thruster-based implementation. Section IV shows simulation results comparing the baseline and augmented controllers and give the reader a better feel for the Delta-H mode performance. A conclusion is presented at the end of the paper.

II. Attitude Control System

A. ACS Design

The design philosophy is to have the ACS as simple as possible. There are two main control modes, one for attitude maneuvers (Delta-H mode) and one for orbit adjustment maneuvers (Delta-V). There is a Sun Acquisition mode, which is only used in the event of an off-nominal separation. A complete open-loop mode is offered for thruster checkout and spin-up during boom deployments. No control actions are performed in Science mode, the default start-up mode. Figure 1 is the ACS control mode transition diagram showing all the ACS control modes.

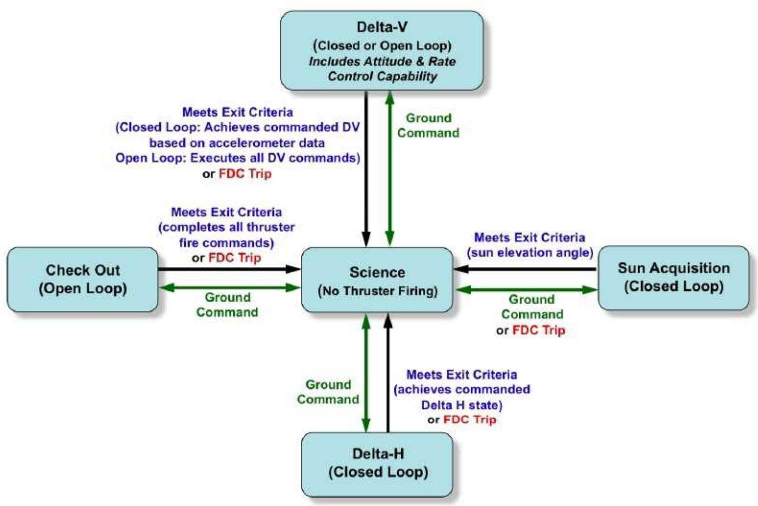

Figure 1. MMS ACS Control Mode Diagram

B. ACS Components

There are three ACS sensors on each MMS spacecraft. A digital sun sensor (DSS) is used as the primary Sun Acquisition attitude sensor, as well as for attitude and rate verification during Checkout mode operations. There is a 4-headed star sensor that provides up to 4 attitude solutions to the Delta-H and Delta-V control modes. An accelerometer measures translation during Delta-V maneuvers. There is no angular velocity sensor; an angular velocity is derived from a Kalman Filter.

MMS has twelve thrusters used for attitude and orbit control. Eight 4 lb thrusters are directed radially, while four 1 lb thrusters are directed axially. Refer to Fig. 2 for a thruster placement diagram.

- - INSERT Figure 2. Thruster placement on MMS --

C. Attitude Maneuver Operations Concept

After sun acquisition, the ACS is required to maintain the spacecraft's spin rate at 3 RPM while maneuvering up to 180 degrees to attain mission attitude. At mission attitude, the ACS must adjust the spacecraft spin rate from 3 to 7 RPM to accommodate the boom deployment sequence. At the conclusion of the deployment sequence, the spacecraft spin rate is back to 3 RPM. Then, during science operations, the ACS must precess the spacecraft up to two degrees every two weeks, maintain a constant spin rate, and minimize the core body nutation. Traditional spin-stabilized spacecraft have used a number of separate control modes to control nutation, spin rate and precession. To reduce the number of modes and simplify operations, we seek a controller which can perform all three simultaneously. The remainder of this paper details the MMS Delta-H controller design.

III. Delta-H Controller Design

Spacecraft attitude dynamics are governed by Euler's Equations:

$$[J]^N\dot{\underline{\omega}}^B + {}^N\underline{\omega}^B \times [J]^N\underline{\omega}^B = {}^B\underline{\tau} \quad , \tag{1}$$

where ${}^N\underline{\omega}^B$ is the spacecraft angular velocity, $[J]$ is the spacecraft inertia matrix, and ${}^B\underline{\tau}$ is the sum of external torques on the spacecraft. Theoretically, if ${}^N\underline{\omega}^B$ is small then Eq. 1 can be linearized. Looking at the attitude maneuver operations concept for MMS, there is a requirement to: maneuver 2 deg, spin up the spacecraft from 3 to 7 RPM, and maneuver up to 180 deg. Equation 1 can be linearized for the first and second maneuver. For the first maneuver, linearizing about a constant spin rate, and neglecting higher order terms, leads to a decoupling of the spin and transverse dynamics. For the second maneuver, assuming small transverse rates leads to a single integrator plant for the spin dynamics. The third maneuver cannot be linearized. Since we are seeking one controller to perform all attitude maneuvers, a nonlinear controller will be designed. Many options exist for nonlinear controllers. A Lyapunov design approach is chosen.

Conceptually, we want the Lyapunov function to be proportional to an energy-like term. If we ensure the change in the Lyapunov function is consistently decreasing, we can claim the controller drives the system to a minimum energy state. There are varying levels of nonlinear stability and the most desired is Global Exponential Stability (GES). GES means that from any initial state, the system will reach the single stable equilibrium point exponentially fast. See Ref. 1 for more details on nonlinear design strategies.

A. Global Spin Rate Controller

Reference 2 presents a globally stable Lyapunov controller for spin stabilized spacecraft. The Lyapunov based controller is designed for simultaneous spin, nutation and precession control. Let a candidate Lyapunov function be a continuously differentiable function defined in a domain $D \subset \mathbb{R}^3$ that contains the origin (as presented in Ref. 2):

$$V(\underline{x}) = \frac{1}{2}\underbrace{\left({}^B\underline{H} - H_0\,{}^B\hat{s}\right)^T}_{{}^B\underline{H}_e}\left({}^B\underline{H} - \underbrace{H_0\,{}^B\hat{s}}_{{}^B\underline{H}_D}\right) + \frac{1}{2}\,{}^B\underline{H}^T\underbrace{\left[\bar{\lambda}[J]^{-1} - [I]\right]}_{[K_\beta]}{}^B\underline{H} \tag{2}$$

where,

${}^B\underline{H} = [J]^N\underline{\omega}^B$ is the spacecraft angular momentum vector, ${}^N\underline{\omega}^B$ is the spacecraft angular velocity, ${}^B\underline{H}_D$ is the desired angular momentum, ${}^B\underline{H}_e$ is the error between the current and desired angular momentum, $[I]$ is the identity matrix, $[J]$ is a 3x3 inertia matrix, $\bar{\lambda}$ is the maximum eigenvalue of $[J]$, H_0 is a constant representing the desired angular momentum magnitude, $[K_\beta]$ is a symmetric positive definite gain matrix, and ${}^B\hat{s}$ is an instantaneous unit vector representing the desired major axis pointing direction in inertial space resolved in the body frame. Because ${}^N\hat{s}$, the desired major axis pointing direction in inertial space resolved in the inertial frame, N, is fixed in inertial space, ${}^B\hat{s}$ varies by:

$${}^B\dot{\hat{s}} = -{}^N\underline{\omega}^B \times {}^B\hat{s} \quad . \tag{3}$$

If the derivative of the Lyapunov function is negative then the Lyapunov function will tend towards zero, i.e., a minimum energy state. Therefore, to prove Lyapunov stability, we must show that V approaches zero.

Taking the derivative of Eq. 2 yields:

$$\dot{V}(\underline{x}) = {}^B\underline{H}_e^T \, {}^B\dot{\underline{H}}_e + {}^B\underline{H}^T [K_\beta] \, {}^B\dot{\underline{H}} \quad , \tag{4}$$

Substituting Euler's equation yields:

$$\dot{V}(\underline{x}) = {}^B\underline{H}_e^T \left({}^B\underline{\tau} - {}^N\underline{\omega}^B \times {}^B\underline{H}_e^T \right) + {}^B\underline{H}^T [K_\beta] \left({}^B\underline{\tau} - {}^N\underline{\omega}^B \times {}^B\underline{H}^T \right) \quad . \tag{5}$$

Simplification leads to:

$$\dot{V} = \left({}^B\underline{H}_e + {}^B\underline{H}[K_\beta] \right)^T {}^B\underline{\tau} \tag{6}$$

Further substitution and simplification follows:

$$= \left({}^B\underline{H} - {}^B\underline{H}_D + {}^B\underline{H}[K_\beta] \right)^T {}^B\underline{\tau} \tag{7}$$

Substituting for $[K_\beta]$:

$$= \left({}^B\underline{H} - {}^B\underline{H}_D + {}^B\underline{H}\left[\bar{\lambda}[J]^{-1} - [I] \right] \right)^T {}^B\underline{\tau} \tag{8}$$

Expanding:

$$= \left(-{}^B\underline{H}_D + {}^B\underline{H} \bar{\lambda} [J]^{-1} \right)^T {}^B\underline{\tau} \tag{9}$$

Substituting for ${}^B\underline{H}_D$ and ${}^B\underline{H}$ leads to:

$$= \left(-H_0 \, {}^B\hat{s} + [J] \, {}^N\underline{\omega}^B \bar{\lambda} [J]^{-1} \right)^T {}^B\underline{\tau} \quad . \tag{10}$$

Recall ${}^B\hat{s}$ is a unit vector representing the major axis; therefore $H_0 = \bar{\lambda}\,\omega_0$. Substitution and simplification yields:

$$= \bar{\lambda} \left({}^N\underline{\omega}^B - \omega_0 \, {}^B\hat{s} \right)^T {}^B\underline{\tau} \tag{11}$$

$$\dot{V} = \bar{\lambda} \left({}^N\underline{\omega}_e^B \right)^T {}^B\underline{\tau} \tag{12}$$

If the external torque applied to the spacecraft is ${}^B\underline{\tau} = -[K_c] \, {}^N\underline{\omega}_e^B$,i.e., in the opposite direction of the rate error, and $[K_c]$ is a 3x3 symmetric positive definite gain matrix, then:

$$\dot{V} = -\bar{\lambda} \left({}^N\underline{\omega}_e^B \right)^T [K_c] \, {}^N\underline{\omega}_e^B \tag{13}$$

Equation 13 is negative semi-definite; it must be negative definite to ensure GES by Lyapunov's Theorem. Therefore, we seek bounds on Eq. 13 such that we can guarantee it to be negative definite. From the definition of the induced 2-norm and the fact that the eigenvalues of a symmetric matrix are real, we get the following bound:

$$\dot{V} \leq -\bar{\lambda}\, \lambda_{min}([K_c]) \left\| {}^N\underline{\omega}_e^B \right\|_2^2, \quad \lambda_{min}([K_c]) > 0 \tag{14}$$

Defining the term $\alpha_3 = \bar{\lambda}\, \lambda_{min}([K_c])$ and substituting leads to:

$$\dot{V} \leq -\alpha_3 \left\| {}^N\underline{\omega}_e^B \right\|_2^2, \quad \forall \alpha_3 > 0, \forall {}^N\underline{\omega}_e^B \in \mathbb{R}^3 \quad . \tag{15}$$

Equation 15 is negative definite for all values of ${}^N\underline{\omega}^B$, which implies this controller is GES by Lyapunov's Theorem.

If the torque applied is aligned with the rate error, $\dot{V}$ is negative, and the final state will be V = 0. At V=0, the following occurs:

$${}^B\underline{H} = H_0 {}^B\hat{s} = \bar{\lambda}\,\omega_0 {}^B\hat{s} \quad . \tag{16}$$

Equation 16 shows that the desired momentum is achieved. In addition:

$${}^B\underline{H}^T\left[\bar{\lambda}[J]^{-1} - [I]\right]{}^B\underline{H} = \bar{\lambda}^2\omega_0^2\, {}^B\hat{s}^T\left[\bar{\lambda}[J]^{-1} - [I]\right]{}^B\hat{s} \quad . \tag{17}$$

Equation 17 shows that a spin about a major axis is achieved.

However, an ambiguity exists in the direction of major axis. If the major axis is the z-axis, and you want to point it at ${}^N\hat{s}$, then the controller can end up pointing the body at either +z or -z. This manifests itself in the controller development. Recall the rate error is given by ${}^N\underline{\omega}_e^B = {}^N\underline{\omega}^B - \omega_0 {}^B\hat{s}$, which indeed is driven to zero. However, inspection of the rate error and recalling Eq. 3 reveals that ${}^B\hat{s}$ is kinematically coupled with the spacecraft's attitude, i.e., ${}^N\underline{\omega}_e^B = {}^N\underline{\omega}^B - \omega_0 {}^BC^N {}^N\hat{s}$, where ${}^BC^N$ is the direction cosine matrix from the inertial frame to the body frame. Since the problem formulation for this controller did not include an attitude term in the Lyapunov function, it cannot guarantee convergence of the attitude. The controller still remains GES in rate-space (i.e., the rate error is driven to 0 from any initial condition), however it is not global with respect to attitude-space. Because MMS is concerned with attitude (pointing) as well as rate, an additional term must be added to Eq. 2 to account for the ambiguity.

B. Global Pointing and Spin Rate Controller

Augment the Lyapunov function, Eq. 2, with an attitude-dependent term, while maintaining the same conditions as Eq. 2:

$$V = \overbrace{\frac{k_1}{2}\left({}^B\underline{H} - H_0 {}^B\hat{s}\right)^T\left({}^B\underline{H} - H_0 {}^B\hat{s}\right)}^{Term 1} + \overbrace{\frac{k_2}{2}\left({}^B\underline{H} - H_0 {}^B\hat{p}\right)^T\left({}^B\underline{H} - H_0 {}^B\hat{p}\right)}^{Term 2} + \cdots$$
$$\cdots \underbrace{\frac{k_1 + k_2}{2} {}^B\underline{H}^T\left[\bar{\lambda}[J]^{-1} - [I]\right]{}^B\underline{H}}_{Term 3} \quad , \tag{18}$$

where k_1, and k_2 are scalar gains whose sum must equal 1, and ${}^B\hat{p}$ is the desired body pointing vector (fixed in body-space).

Term 1and Term 3 in Eq. 18 are components of the controller in Section III.A. Term 2 is the additional term added to resolve the attitude ambiguity. By adding Term 2, which is proportional to energy with respect to the attitude, we have selected a Lyapanov function that may guarantee rate control, nutation control, and precession control as well as guarantee the rate and major axis are pointing in the desired direction.

Taking the derivative of Eq. 18 and simplifying (similar to the procedure taken in Section III.A.) yields:

$$\dot{V} = \bar{\lambda}(k_1 + k_2)\left({}^N\underline{\omega}^B - \omega_0 \underbrace{\frac{k_1 {}^B\hat{s} + k_2 {}^B\hat{p}}{k_1 + k_2}}_{target\ vector}\right)^T {}^B\underline{\tau} = k_3 {}^B\underline{e}^T {}^B\underline{\tau} \quad , \tag{19}$$

where the target vector is a weighted sum of the body pointing direction and desired spin direction, $k_3=\bar{\lambda}(k_1+k_2)$, and ${}^B\underline{e}$ is the difference between the current spacecraft angular velocity and the weighted target vector. If we let ${}^B\underline{\tau}=-[K_c]{}^B\underline{e}$, where $[K_c]$ is a 3x3 symmetric positive definite gain matrix, then:

$$\dot{V}=-k_3{}^B\underline{e}^T[K_c]{}^B\underline{e}, \quad \forall\, {}^B\underline{e}\in\mathbb{R}^3 \tag{20}$$

From the definition of the induced 2-norm and the fact that the eigenvalues of a symmetric matrix are real, we can bound Eq. 20:

$$\dot{V}\le -k_3\lambda_{min}([K_c])\left\|{}^B\underline{e}\right\|_2^2, \quad \lambda_{min}([K_c])>0 \tag{21}$$

Defining the term $\alpha_3=k_3\lambda_{min}([K_c])$ and substituting leads to:

$$\dot{V}\le -\alpha_3\left\|{}^B\underline{e}\right\|_2^2, \quad \forall\,\alpha_3>0,\ \forall\, {}^B\underline{e}\in\mathbb{R}^3 \tag{22}$$

Equation 22 is negative definite for all values of ${}^B\underline{e}$, which implies this controller is GES by Lyapunov's Theorem. If the torque applied is aligned with the error vector, $\dot{V}$ is negative, and the final state will be V = 0. From Eq. 18, when V = 0, the desired momentum, spin about a major axis, and the desired pointing direction is achieved.

Looking at the error vector, ${}^B\hat{\underline{e}}$, reveals that the controller is essentially a proportional+derivative (PD) controller, since the error vector is composed of a term proportional to attitude and a term proportional to the rate. Even though a nonlinear design technique is used, the end result is a PD controller, a common result of nonlinear control design.

IV. Simulation Results

A. Advantage of Global Pointing and Rate Controller

Simulation results reveal the attitude ambiguity in the rate controller. When we perform a maneuver greater than 90 degrees, the spacecrafts is shown to despin and spin up in the opposite direction. Comparing the rate controller with the pointing and rate controller, shows that adding the attitude dependency resolves the ambiguity in the rate controller.

– INSERT Figure x. Spacecraft rate for two controllers. –

B. Performance

The Delta-H controller performs well. Figure xx shows a 60 deg precession maneuver on the unit body sphere. Notice that the angular momentum vector goes along a straight line from the initial to the desired state. Also notice that the body z-axis spirals towards the final state. The spiraling action is indicative of induced nutation from the execution of the precession maneuver. Once the final state is reached, the angular momentum and the body z-axis align and come to rest, meaning that the controller was able to control both precession and nutation.

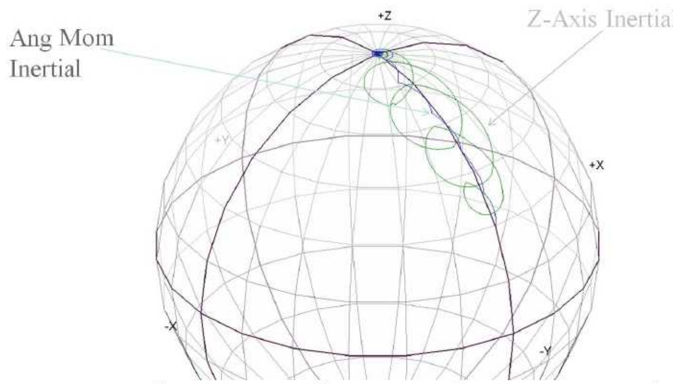

Figure xx.60 deg maneuver showing angular momentum on body sphere

Equation 18 gives two scalar gains, k_1 and k_2. Notice that if k_1 =1 and k_2 =0, then the controller collapses to the global rate controller, and all that it entails. The ratio of these gains represents the weighting between pointing vs. nutation. The target vector is a linear combination of the major axis pointing direction, ${}^{B}\hat{s}$, and the body pointing direction, ${}^{B}\hat{p}$. Weighting more heavily on ${}^{B}\hat{s}$ results in better pointing accuracy and weighting more on ${}^{B}\hat{p}$ results in better nutation control. Therefore, these gains should be selected appropriately based on the desired goal of a class of maneuvers.

V. Conclusions

Traditional spin-stabilized spacecraft have used a number of separate control modes to control nutation, spin rate and precession. To reduce the number of modes, and simplify operations, a controller to perform all three was designed. The Delta-H control mode accomplishes nutation control, spin rate control, and precession control simultaneously. In designing the controller following Lyapunov's technique, we found that a global Lyapunov rate controller was insufficient for our application because it did not guarantee global stability of the attitude. Therefore, a pointing and rate controller was developed using Lyapunov re-design that proved to guarantee the desired objective. Simulation results show the benefit of the pointing and rate controller for maneuvers greater than 90 deg. Simulation also shows the general performance characteristics of this controller and provides some insight into selecting the controller gains.

Although theoretically complex, this controller is essentially a PD controller. This GES controller should be used as a baseline for future spinner missions, as the design is theoretically sounds, and its implementation is quite simple. Future work involves significant flight software testing in preparation for flight in 2014.

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