Preprocessing Inconsistent Linear System for a Meaningful Least Squares Solution

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Abstract Mathematical models of many physical/statistical problems are systems of linear equations. Due to measurement and possible human errors/mistakes in modeling/data, as well as due to certain assumptions to reduce complexity, inconsistency (contradiction) is injected into the model, viz. the linear system. While any inconsistent system irrespective of the degree of inconsistency has always a least-squares solution, one needs to check whether an equation is too much inconsistent or, equivalently too much contradictory. Such an equation will affect/distort the least-squares solution to such an extent that renders it unacceptable/unfit to be used in a real-world application. We propose an algorithm which (i) prunes numerically redundant linear equations from the system as these do not add any new information to the model, (ii) detects contradictory linear equations along with their degree of contradiction (inconsistency index), (iii) removes those equations presumed to be too contradictory, and then (iv) obtain the minimum norm least-squares solution of the acceptably inconsistent reduced linear system. The algorithm presented in Matlab reduces the computational and storage complexities and also improves the accuracy of the solution. It also provides the necessary warning about the existence of too much contradiction in the model. In addition, we suggest a thorough relook into the mathematical modeling to determine the reason why unacceptable contradiction has occurred thus prompting us to make necessary corrections/modifications to the models – both mathematical and, if necessary, physical.