A slide with a white background and a grey curved border on the right and bottom. In the top left corner, there is a small thumbnail image of a person. The title "Experimental Designs" is at the top. Below it is a bulleted list. In the bottom left corner, there is a NASA logo, and in the bottom right corner, there is a small square icon with a stylized figure.

Experimental Designs

- ANOVA is very common with traditional designs of experiments involving 1 or more “factors,” with 2 or more “levels”
 - Factor
 - Level
- Factors can be “between” or “within”
 - A.k.a. Independent/Dependant Measures
 - A.k.a. Grouping/Repeated Factors

A slide with a white background and a grey curved border on the right and bottom. In the top left corner, there is a small thumbnail image of a person. The title "Today's Topic (SMAR-t): ANOVA" is at the top. Below it is a bulleted list. In the bottom left corner, there is a NASA logo, and in the bottom right corner, there is a small square icon with a stylized figure.

Today's Topic (SMAR-t): ANOVA

- Analysis of Variance (i.e. ANOVA)
 - Independent Measures ANOVA
 - Repeated Measures ANOVA
 - Mixed Factorials
 - Analysis of Covariance (ANCOVA)
 - Using Covariates

A slide with a white background and a grey curved border on the right and bottom. In the top left corner, there is a small thumbnail image of a person. The title "Types of Outcomes for ANOVA" is at the top. Below it is a bulleted list. In the bottom left corner, there is a NASA logo, and in the bottom right corner, there is a small square icon with a stylized figure.

Types of Outcomes for ANOVA

- Continuously scaled outcomes assumed to follow the normal distribution, or that can be transformed so that it does (i.e. “normalized”)
 - Examples: BMI, BP, BMD, Strength, Standardized Scores, Viral Loads, Force, Averages or Sums of Likert-Scaled items (scale scores), Optical Density, Volume, Response Time, Distance, etc.

Quick Review:
Gaussian Distribution Function

- A.K.A. The “Normal Distribution”
- A.K.A. The “Bell-Shaped Curve”
- Has known probabilities associated with it,
- Thus all Parametric Statistics are based on the Gaussian Distribution

$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2}$

Where \bar{x} = mean, and σ = standard deviation

Quick Review:
Gaussian Distribution Function

- About 68% of all scores fall within 1 SD unit from the mean.
- About 95% of all scores fall within 2 SD units from the mean.

Quick Review:
Gaussian Distribution Function

- About 68% of all scores fall within 1 SD unit from the mean.

Quick Review:
Gaussian Distribution Function

- About 68% of all scores fall within 1 SD unit from the mean.
- About 95% of all scores fall within 2 SD units from the mean.
- About 99% of all scores fall within 3 SD units from the mean.

Central Limit Theorem

- States that for any population with mean μ and standard deviation σ , the distribution of sample means with sample size n will approach a *normal distribution* with μ and SD of $\frac{\sigma}{\sqrt{n}}$ as n approaches infinity.
- REGARDLESS of the shape of the distribution in the population.
- By the time sample sizes hit around 30, sampling distribution of means is close to normal.

Demo of central limit theorem.

Thus...

- Since we know so much about the Normal Distribution
- And we know that sample summaries (means or otherwise) tend to follow that distribution
 - Even data collected from non-normal samples
 - Especially so with large sample size (big- n)
- We can usually apply our knowledge of the normal distribution to statistical comparisons, estimates, and probability
 - As long as we do some preliminary screening...

Moving to the t-test for comparing two samples

- Used for comparing two samples collected randomly from two populations
- Many other flavors of the t-test exist... but we'll start here.

Population 1

↓

Sample 1

X=0

X=2

X=4

Population 2

↓

Sample 2

X=4

X=6

X=8

$$\frac{d_1 + d_2}{n_1 + n_2}$$

$$\frac{d_1 + d_2}{d_1 + d_2}$$

Dissect the formula:

NASA

Dissect the formula: Denominator

The difference between two sample means

Divided by some measure of standard error of the differences

NASA

Dissect the formula: Numerator

The difference between two sample means

NASA

Dissect the formula: Question?

The difference between two sample means

Divided by some measure of standard error of the differences

Are the differences that I see between my two means unusual, given variability among other sample means of this size?

NASA

T-tests on the Computer:

- Software gives us t-score and a p-value
- Allowing us to test hypotheses that the two samples come from the same population (or not)
- And describe the magnitude of the differences (confidence intervals)
- Ex. $t = 4.87$, $p < .001$
 - H_{null} : Two samples are from same population
 - H_{alt} : Two samples are from different populations
- Reject the Null ($\alpha < .05$) & Report the magnitude of the differences





Virtues of the t-test

- EVERYONE seems to understand it!
- With CLT, it's easy to apply to lots of different data scenarios
- There are other versions that make it very flexible
 - Formula for "Repeated Measures" designs
 - Formula for problems associated with non-normality and/or variance heterogeneity



Hypothesis testing Scenario

- The "null" hypothesis for the t-test is that the two groups come from the same population
 - Thus will have similar means, given sd
- The "alternative" hypothesis is usually that they don't
 - Thus have "different" means, but similar sd
 - Can be directional
- We use the t-statistic in an attempt to Reject the null, supporting our claim of the alternative

Consequences of Hypothesis Testing & Alpha



Your decision is:	The Truth is:	
	H_0 Really is True (there's no effect)	H_0 is Actually False (there is an effect)
You Rejected H_0 Due to a Statistically Significant Result (Conclude the 2 groups must come from different populations)	Type I Error Probability = α 	Power Probability = $(1-\beta)$ 
You Accepted H_0 Due to a Non-Significant Result (Assume the 2 groups are come from same population)	Probability = $1 - \alpha$ 	Type II Error Probability = β 

If you have a “significant” result:

Your decision is:	The Truth is:	
	H_0 Really is True (there's no effect)	H_0 is Actually False (there is an effect)
You Rejected H_0 Due to a Statistically Significant Result (Conclude the 2 groups must come from different populations)	Wrong Conclusion 	Right Conclusion 

Given a significant t-score comparing means....

If you have a “non-significant” result:

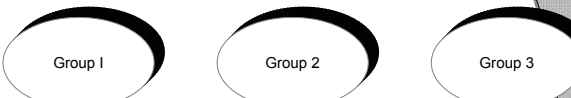
Your decision is:	The Truth is:	
	H_0 Really is True (there's no effect)	H_0 is Actually False (there is an effect)
You Accepted H_0 Due to a Non-Significant Result (Assume the 2 groups are come from same population)	Right Conclusion 	Wrong Conclusion 

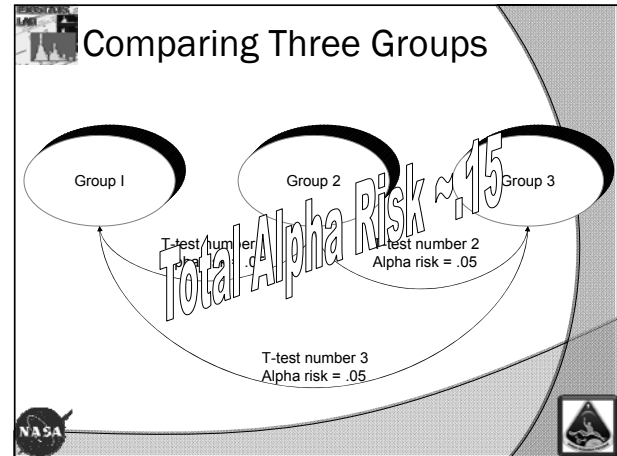
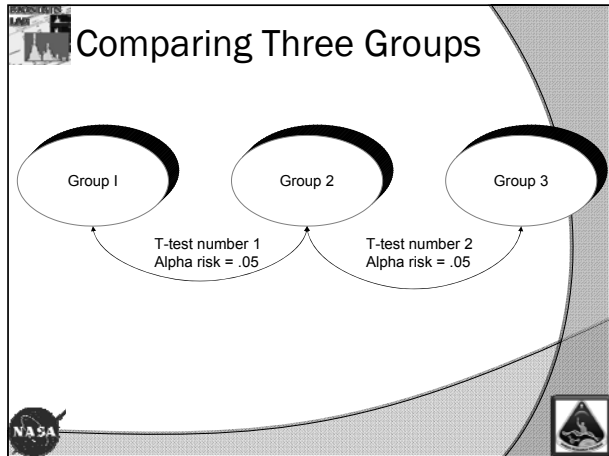
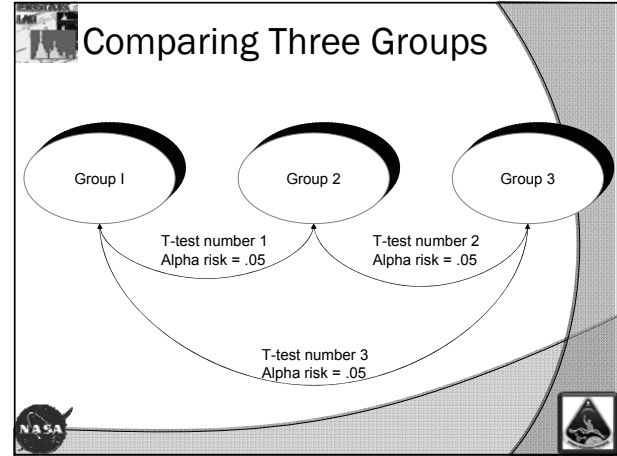
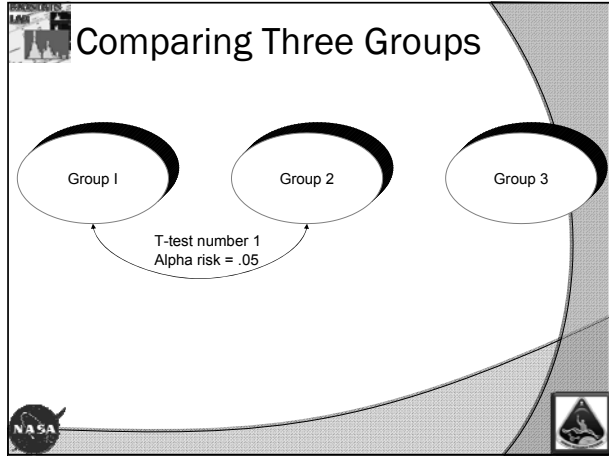
Given a non-significant t-score comparing means....

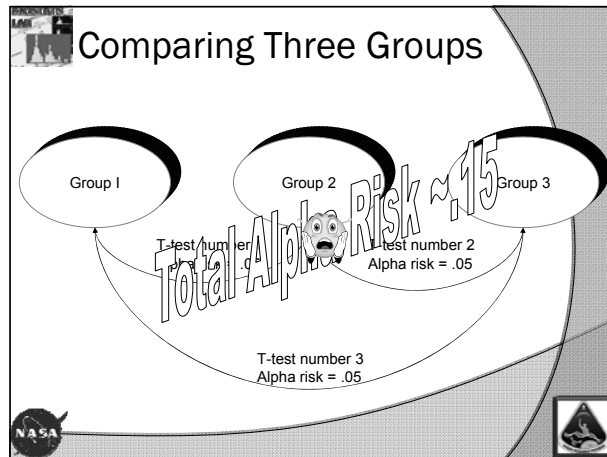
Limitations of t-tests

- Alpha risk is .05 for each t-test
 - Probability of falsely rejecting the null, and concluding that there is a difference, when it's really due to chance.
 - So comparing 3, 4, 5 or more groups is quite problematic!

Comparing Three Groups







- ### Assumptions Required of ANOVA
- ⊙ Data collected randomly from the population, with roughly equal n per cell
 - And sufficiently large n ($n > 30$, common r-o-t)
 - ⊙ Data measured on interval or ratio scale, and is normally distributed
 - ⊙ Homogeneity of variance across groups
 - ⊙ Sphericity for RM designs—variance of the differences between means for any pair of groups is equal to any other pair

- ### Analysis of Variance (ANOVA)
- ⊙ Can compare unlimited number of groups or occurrences, and still keep alpha risk = .05
 - ⊙ Able to take multiple grouping (or time) factors into account and determine their independent and combined effects
 - ⊙ Can examine “trends” in data, and can test specific (often complex) hypotheses
 - ⊙ The analytic focus is on variance, but the interpretation falls back to means—thus results become intuitive

- ### Assumption of Randomly Collected Data with Sufficiently Large n
- ⊙ Is our subject pool at NASA randomly selected from our inference-population?
 - Are those bedrest subjects representative of astronauts?
 - Are today's astronauts representative of future ones?
 - ⊙ Regarding n, How big is big enough?
 - Rule of Thumb... at least 30 per group
 - More is better
 - Cautions about overpowered studies...
 - But BALANCE is critical!!
 - Rule of thumb—smallest group should not be less than 1/3rd the size of the largest group.

Assumption of Interval or Ratio Scale & Normal

- The “bell-shaped” curve—assumption of all parametric statistics
- Studies show that ANOVA is robust to violations of this, but only if sample size is substantially large, and Homogeneity is met

GAUSSIAN or "NORMAL" DISTRIBUTION

More on Homogeneity of Variance

- If distributions are normal in one, then should be for all

Group 1 Group 2 Group 3

Assumption of Homogeneity of Variance Across Groups

- Variance on the dependant variable should be similar across groups
 - Why?
- Because we're examining VARIANCE in ANOVA, and so we need for variance in each group to be roughly similar before we can conclude that any differences that we find are attributable to *group* differences (not mere variability differences).
- Even in Means Comparisons (ex.t-tests), since Means are highly affected by variability, we need for variability to be similar in our groups so that differences that we find can be attributed to true group differences, and not merely by variability differences between our groups.

More on Homogeneity of Variance

- If distributions in 1 group is leptokurtotic (tall and skinny), then it should be for all other groups

Group 1 Group 2 Group 3

More on Homogeneity of Variance

- ◉ If distributions in 1 group is platykurtotic (short & fat) then it should be for all other groups

Group 1 Group 2 Group 3

What about skewed data?

- ◉ Positive or negative skews in the data can wreak havoc with statistical analysis
 - Thus always recommend thorough data screening
 - Identify outliers—data entry errors?
 - Consider data transformations if necessary
 - A great thing to google!

More on Homogeneity of Variance

- ◉ Any Miss-Match is a Problem
 - Because we might interpret a statistical differences to real group differences, when it's actually due to heterogeneity of variance
- ◉ ... Thankfully there are ways to test for this problem, and solutions are sometimes possible. SPSS will test this assumption for us (stay tuned)

Group 1 Group 2 Group 3

Common Transformations

Tabachnick, B.G., & Fidell, L.S. (1996). *Using Multivariate Statistics 2nd Ed.* New York: Harper-Collins.

Two General Types of ANOVA

- Independent Measures ANOVA (IM-ANOVA)
 - Data are collected from separate groups of subjects, and comparisons among *groups* are desired
 - Muscle Size by Treatment (controls vs. two intervention groups)
 - Blood Flow by Gender
- Repeated Measures ANOVA (RM-ANOVA)
 - Data are collected from the same group of subjects on multiple occasions/times, and comparisons of *occasions* are desired.
 - Longitudinal Studies
 - Outcomes measured at L-10, L-1, L+2, L+5 L+25, R+2...
 - Balance Scores Pre Bedrest, During and Post Bedrest
- Mixed Factorial
 - Mix of IM and RM factors in the same experiments
 - Gender (m,f) by Time (pre, post) effects

One-Way IM-ANOVA

- For comparing two or more populations
 - Where sample data have been collected

IM & RM Designs...

ANOVA: What's in a Name?

Analysis of Variance F-Ratio

- ANOVA is truly an analysis of a measure of *variability*, called “*variance*,” that measures and separates variability attributable to
 - Within-Groups Variability
 - Between-Groups Variability
- We Evaluate an “F-Ratio” Representing the Ratio of B/T over W/I:

$$F = \frac{\text{variability between groups}}{\text{variability within groups}} = \frac{\text{ID's + error + group differences}}{\text{ID's + error}}$$

Recall your Simple Algebra...

- If the same quantity exists in the Numerator and Denominator of a fraction, they “cancel each other out”
 - Leaving us with a number (F) that represents Group Differences!

The F-Ratio

$$F = \frac{\text{variability between groups}}{\text{variability within groups}} = \frac{\text{ID's + error + group differences}}{\text{ID's + error}}$$

Recall your Simple Algebra...

- If the same quantity exists in the Numerator and Denominator of a fraction, they “cancel each other out”

Assuming homogeneity of variance

Total Variability

Between-Groups Variability
Within-Groups Variability
Total Error

The F-Ratio

$$F = \frac{\text{variability between groups}}{\text{variability within groups}} = \frac{\text{ID's + error + group differences}}{\text{ID's + error}}$$

Analysis of Variance F-Ratio

- If F=1...
- As F increases...
- How do you know if F is “big enough” to be considered significant?
 - How do you know a t-test is significant??

The F-Ratio

$$F = \frac{\text{variability between groups}}{\text{variability within groups}} = \frac{\text{ID's + error + group differences}}{\text{ID's + error}}$$



Confidence Intervals with the F-test

- ◉ CI's for comparing two groups are straightforward and intuitive
- ◉ CI's for "Omnibus" differences are less so
 - Effect size calculations exist, but less intuitive interpretation..
- ◉ Stay tuned for discussions about post-hoc tests, and how they can sometimes help
- ◉ Plots will also be very informative



IM-ANOVA Summary Tables

- ◉ Purpose is to provide the necessary components of the F-test
 - Variability (SS) Sum of Squared Deviations from the Mean
 - Degrees of Freedom (df)
 - Mean Square (MS)
 - F-statistic (F)
 - Probability values associated with F
- ◉ Total, Between Groups, Within Groups



IM-ANOVA Summary Tables

- ◉ Purpose is to provide the necessary components of the F-test
 - Variability (SS)
 - Degrees of Freedom (df)
 - Mean Square (MS)
 - F-statistic (F)
 - Probability values associated with F
- ◉ Total, Between Groups, Within Groups





IM-ANOVA Summary Tables

- ◉ Purpose is to provide the necessary components of the F-test
 - Variability (SS) Like in a t-test, each F-test has df values for significance testing
 - Degrees of Freedom (df)
 - Mean Square (MS)
 - F-statistic (F)
 - Probability values associated with F
- ◉ Total, Between Groups, Within Groups





IM-ANOVA Summary Tables

- ◎ Purpose is to provide the necessary components of the F-test
 - Variability (SS)
 - Degrees of Freedom (df)
 - Mean Square (MS) MS is the Variance Statistic for ANOVA—calculated with SS & df
 - F-statistic (F)
 - Probability values associated with F
- ◎ Total, Between Groups, Within Groups



IM-ANOVA Summary Tables

- ◎ Purpose is to provide the necessary components of the F-test
 - Variability (SS)
 - Degrees of Freedom (df)
 - Mean Square (MS) ...and p values tell us the significance level of the ratio
 - F-statistic (F)
 - Probability values associated with F
- ◎ Total, Between Groups, Within Groups



IM-ANOVA Summary Tables

- ◎ Purpose is to provide the necessary components of the F-test
 - Variability (SS)
 - Degrees of Freedom (df)
 - Mean Square (MS)
 - F-statistic (F) The "F" statistic is another word for the F-ratio
 - Probability values associated with F
- ◎ Total, Between Groups, Within Groups






This is what it looks like...

	df	SS	MS	F	p
Between Groups	##	##	###		
Within Groups (error)	##	##	###	##	##






This is where it comes from (Independent Measures Designs)

$$df_{within} = N - k$$






This is where it comes from (Independent Measures Designs)

F-tables provide a p value for a given F-statistic, using $df_{between}$ (numerator) and df_{within} (denominator).

Example 1

- Compare Pain Ratings of Patients in Randomized Clinical Trial
 - Usual Care (control)
 - Pain Medication
 - Pain Medication + Caffeine
- Simplest of ANOVA Models, with ONE Independent Factor (Treatment Group)













Design: One-way ANOVA

- Compare Three Groups on Pain Assessment

UNIVERSAL PAIN ASSESSMENT TOOL

This pain assessment tool is intended to help patient care providers assess pain according to individual patient needs. Explain and use 0-10 Scale for patient self-assessment. Use the faces or behavioral observations to interpret expressed pain when patient cannot communicate his/her pain intensity.

	0	1	2	3	4	5	6	7	8	9	10
Verbal Descriptor Scale	NO PAIN		MILD PAIN		MODERATE PAIN		MODERATE PAIN		SEVERE PAIN		WORST PAIN POSSIBLE
WONG-BAKER FACIAL GRIMACE SCALE											
ACTIVITY TOLERANCE SCALE	After eating, NO PAIN		Can tolerate the honor CAN BE HONORED		Person can attend to school, work, INTERFERES WITH TASKS		Person can attend to the right functioning INTERFERES WITH CONCENTRATION		After sleep, INTERFERES WITH BASIC NEEDS		Person needs special care INTERFERES WITH BASIC NEEDS

The Data:

The screenshot shows the SPSS Data Editor with a data file named 'bl_anova.sav'. The data is entered in Data View. A 'Value Labels' dialog box is open, showing the mapping of values to labels for the 'group' variable.

Value	Label
1	Usual Care (control)
2	Drug A
3	Drug A + Caffeine

One-Way Point-n-Click:

The screenshot shows the SPSS Data Editor with the 'Analyze' menu open, navigating to 'One-Way ANOVA'. A 'One-Way ANOVA: Post Hoc: Multiple Comparisons' dialog box is also visible. Below, the SPSS Syntax Editor shows the generated command:

```

ONEWAY pain BY group
/STATISTICS=DESCRIPTIVES HOMOGENEITY BROWNFORSYTHE
/PLOT=MEANS
/MISSING=ANALYSIS
/POSTHOC=HOMGENEITY BONFERRONI (H ALPHA=0.05)
    
```

One-Way Point-n-Click:

The screenshot shows the SPSS Data Editor with the 'Analyze' menu open, navigating to 'One-Way ANOVA'. The 'One-Way ANOVA' dialog box is open, showing the dependent variable 'pain' and the factor 'group'.

Results

The screenshot shows the SPSS Output window displaying the results of a one-way ANOVA for the dependent variable 'Pain Score (up is bad)'. The output includes Descriptives, a Test of Homogeneity of Variances, and the ANOVA table.

Descriptives

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean			Minimum	Maximum
					Lower Bound	Upper Bound			
Usual Care (control)	88	6.65	1.713	.185	6.28	7.02	4	9	
Drug A	78	5.49	1.778	.201	5.09	5.89	3	8	
Drug A + Caffeine	89	5.53	1.803	.191	5.15	5.91	3	8	
Total	255	5.90	1.840	.116	5.67	6.13	3	9	

Test of Homogeneity of Variances

Levene Statistic	df1	df2	Sig.
1.04	2	250	.449

ANOVA

Source	Sum of Squares	df	Mean Square	Sig.
Corrected Total	104.000	250		

A callout box points to the Descriptives table with the text: "Means, SD, n, etc.. Note the unequal cell sizes, but still pretty close."

Results

SPSS Statistics Viewer

One-way ANOVA example for SMAR

[DataSet1] 2: vrjps1lectures1\SMAR1R_inova.sav

Descriptives

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean			Minimum	Maximum
					Lower Bound	Upper Bound			
Visual Care (control)	86	6.65	1.713	.185	6.28	7.02	4	9	
Drug A	79	5.49	1.779	.201	5.09	5.89	3	8	
Drug A + Caffein	89	5.53	1.803	.191	5.15	5.91	3	8	
Total	252	5.90	1.840	.116	5.67	6.13	3	9	

Test of Homogeneity of Variances

	Levene Statistic	df1	df2	Sig.
Pain Score (up is best)	1.64	2	250	.849

ANOVA

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	74.126	2	37.063	11.891	.000
Within Groups	779.202	250	3.117		
Total	853.328	252			

Robust Tests of Equality of Means

	Statistic*	df1	df2	Sig.
Brown-Forsythe	11.892	2	247.767	.000

* Asymptotically F distributed.

Multiple Comparisons

Dependent Variable: Pain Score (up is best)

SPSS Statistics Processor is ready. H: 8.4, W: 7.26

Levene's test of Homogeneity. The null for Levene's is that the variances are similar, so we do NOT want to reject this one!

Results

SPSS Statistics Viewer

Test of Homogeneity of Variances

	Levene Statistic	df1	df2	Sig.
Pain Score (up is best)	1.64	2	250	.849

Sum of Squares

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	74.126	2	37.063	11.891	.000
Within Groups	779.202	250	3.117		
Total	853.328	252			

Robust Tests of Equality of Means

	Statistic*	df1	df2	Sig.
Brown-Forsythe	11.892	2	247.767	.000

* Asymptotically F distributed.

Multiple Comparisons

Dependent Variable: Pain Score (up is best)

Had we failed homogeneity, the Brown-Forsythe is a robust test that we could rely on. Note that it adjusts the df-denominator to an extent needed to adjust for heterogeneity. On our case, no adjustment was needed, thus similar hypothesis-testing conclusions.

Results

SPSS Statistics Viewer

Test of Homogeneity of Variances

	Levene Statistic	df1	df2	Sig.
Pain Score (up is best)	1.64	2	250	.849

ANOVA

	Sum of Squares	df	Mean Square	F	Sig.
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Multiple Comparisons

Dependent Variable: Pain Score (up is best)

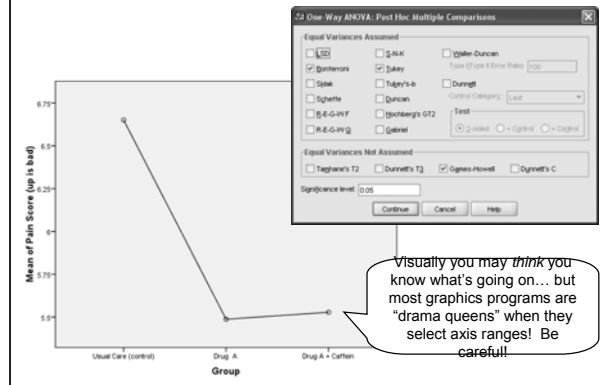
Here's the ANOVA summary table, with the F-test. The Null is that all three groups come from the same population, thus if rejected, we conclude the alternative

Satisfied?

- We rejected the Omnibus F-test, concluding that the three groups must be different... All done?
- Usually pairwise comparisons are of interest too
 - Post-Hoc (compares all pairs)
 - Different choices available
 - A-Priori Contrasts (hypothesis-specific subset of comparisons)
 - Different choices available

NASA

Recall our earlier analytic choices...:



If we had a-priori contrasts?

- We chose all possible pairs to compare post-hoc, adjusting for the number of comparisons
 - UC vs. Treatment A
 - UC vs. Treatment A + Caffeine
 - Treatment A vs. Treatment A+Caffeine
- If we had a more specific set of comparisons that we wanted to make *A-priori*, we could have more statistical power, at the expense of unnecessary comparisons.

Post-hoc Pairwise Comparisons

		(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
Tukey HSD	Usual Care (control)	Drug A	1.164 [*]	.276	0.000	.51	1.61	
		Drug A + Caffeine	1.123 [*]	.267	0.000	.49	1.75	
	Drug A	Usual Care (control)	-1.164 [*]	.276	.000	-1.81	-.51	
		Drug A + Caffeine	-.041	.274	0.988	-.69	.60	
	Drug A + Caffeine	Usual Care (control)	-1.123 [*]	.267	.000	-1.75	-.49	
		Drug A	.041	.274	.988	-.60	.69	
Bonferroni	Usual Care (control)	Drug A	1.164 [*]	.276	.000	.50	1.83	
		Drug A + Caffeine	1.123 [*]	.267	.000	.49	1.77	
	Drug A	Usual Care (control)	-1.164 [*]	.276	.000	-1.83	-.50	
		Drug A + Caffeine	-.041	.274	1.000	-.70	.62	
	Drug A + Caffeine	Usual Care (control)	-1.123 [*]	.267	.000	-1.77	-.48	
		Drug A	.041	.274	1.000	-.62	.70	
Games-Howell	Usual Care (control)	Drug A	1.164 [*]	.273	.000	.52	1.81	
		Drug A + Caffeine	1.123 [*]	.266	.000	.49	1.75	
	Drug A	Usual Care (control)	-1.164 [*]	.273	.000	-1.81	-.52	
		Drug A + Caffeine	-.041	.276	.988	-.70	.62	
	Drug A + Caffeine	Usual Care (control)	-1.123 [*]	.266	.000	-1.75	-.49	
		Drug A	.041	.278	.988	-.62	.70	

*. The mean difference is significant at the 0.05 level.

In this example?

- May make sense to compare Usual care to either of the novel Treatments, but not to compare the two novel treatments?
- "Simple" contrasts, with a reference category (usual care)
 - Usual Care vs. Treatment A
 - Usual Care vs. Treatment A + Caffeine

Back to our data...

SPSS Statistics Data Editor

One-Way ANOVA

Dependent List: pain

Factor(s): group

Contrast 1 of 2

Contrast	Usual Care (control)	Drug A	Drug A + Caffein
1	1	-1	0
2	1	0	-1

Contrast 2 of 2

Contrast	Usual Care (control)	Drug A	Drug A + Caffein
1	1	-1	0
2	1	0	-1

Cake anyone?

- You can't have your cake, and eat it too!
- Good Science dictates that you either HAVE a-priori contrasts, or you DON'T!
 - Contrasts are theory-driven, not something that you do "after the fact"
 - Post-hoc tests are more appropriate if you want all possible pairs of comparisons

Results of the Contrasts

Here's the results of our a-priori contrasts...


Contrast	Usual Care (control)	Drug A	Drug A + Caffein
1	1	-1	0
2	1	0	-1

Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)	
Pain Score (up is bad)	Assume equal variances	1.116	276	4.217	250	.000
	Does not assume equal variances	1.112	267	4.207	250	.000
	Assume equal variances	1.116	273	4.259	159.987	.000
	Does not assume equal variances	1.112	266	4.224	172.952	.000

i,j Group		Mean Difference (i-j)	Std. Error	Sig.	95% Confidence Interval	
Tukey HSD	Usual Care (control) Drug A	1.164	.276	.000	.51	1.81
	Usual Care (control) Drug A + Caffein	1.123*	.267	.000	.49	1.75
	Drug A Usual Care (control)	-1.164	.276	.000	-1.81	-.51
	Drug A + Caffein Usual Care (control)	-1.123*	.267	.000	-1.75	-.49

Next Time: Two-Factor ANOVAS




- What if you want to compare 2+ groups on MORE THAN one factor?
 - Effect of subjects' gender *and* Treatment on BMD?
 - Effect of Novel Treatment (vs. control) *and* Implementation Schedule (two types) on Countermeasure's Effectiveness?
 - Effect of Suit Pressure (3 settings) *and* Glove Design (2 types) on EVA performance?



SMAR Session 3




ANOVA

(CONTINUED)




Recap

- Analysis of Variance (ANOVA) examines variability between groups, relative to within groups, to determine whether there's evidence that the groups are not from the same population
- Analysis focuses on *variance*, but interpretation is about *mean's*
- One-way ANOVA compares more than two groups.
 - Similar to a t-test, but for 3, 4, 5+ groups



Recap

- ANOVA assumes
 - Random samples from the population
 - Sufficiently large enough n to detect effects, distributed evenly among groups
 - Similar variability among groups (Homogeneity of Variance)
- We should examine our data and test our assumptions
 - Sometimes we need to consider data transformations to meet these assumptions
 - Sometimes we need to rely on robust alternatives to the typical ANOVA statistic

Recap

- ANOVA results summarized in a ANOVA table, with an "Omnibus F-statistic" and p-value
 - Represents the ratio of between/within variability
 - If significant, reject the null hypothesis that the groups are from the same population
- Researchers typically follow-up a significant F-ratio with either
 - Post Hoc tests
 - A-Priori Contrasts

Today... Multifactorial ANOVA

- What if you want to compare 2+ groups on MORE THAN one factor?
 - Effect of subjects' gender *and* Treatment on BMD?
 - Effect of Novel Treatment (vs. control) *and* Implementation Schedule (two types) on Countermeasure's Effectiveness?
 - Effect of Suit Pressure (3 settings) *and* Glove Design (2 types) on EVA performance?
- Still working with completely Independent Measures Designs
 - Subjects in one "cell" are not also in any other "cell" of the design

Table Representation of Experimental Two-Factor Designs

3 x 2 design Study n=120	No Drug	Low Dose	High Dose
Drug A	n=20	n=20	n=20
Drug B	n=20	n=20	n=20

3 x 3 design Study n=180	No Drug	Low Dose	High Dose
No Therapy	n=20	n=20	n=20
Therapy A	n=20	n=20	n=20
Therapy B	n=20	n=20	n=20

2 x 2 design Study n=78	Control	Intervention
Males	n=20	n=18
Females	n=19	n=21

More Complicated Designs:

- ANOVA can handle 3, 4, 5, or even more factors!
 - "k" is the common notation for number of factors in an ANOVA design
- But be careful what you ask for... stay tuned!




2x2x2 design Study n=152	Placebo		Experimental Drug	
	Chronic Use	Acute Administration	Chronic Use	Acute Administration
Males	n=20	n=18	n=19	n=17
Females	n=19	n=21	n=18	n=20

Main Effects and Interactions

- Main Effects
 - One per factor... an F-statistic evaluating the impact of each factor in the model
 - Gender effect on performance (M/F diffs?)
 - Race/ethnicity effect on performance
- Interaction Effects
 - One per interaction... an F-statistic evaluating how two (or more) factors interact with one another to affect the outcome
 - Gender "by" Race/Ethnicity interactive effects on performance
 - More complex... often more interesting!

Interactions...



- Interaction effects are often the most interesting, but can be tricky to understand at first
- We describe them in terms of how many factors are involved
 - “Two-way” means two factors work together to explain the observed difference
 - “Three-way” means that three factors tell the story
 - “Four-way” means that you’d better have some pain relievers nearby!

The Usual Assumptions...




- Random Sample
- Roughly Equal n Per Cell
- Continuously Scaled outcome (Performance Gains) follows Normal Distribution
- Homogeneity of Variance

2 x 3 design Study n=116	Exercise Intensity		
	Low	Med	High
Couch Potatoes	n=20	n=19	n=18
Fit Individuals	n=18	n=19	n=22

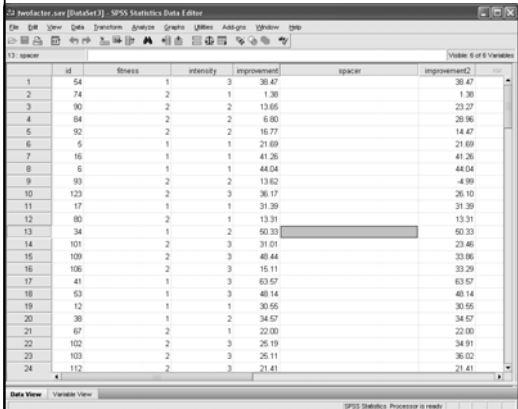



Two-Factor Example

- Compare Performance Gains Following Exercise Intervention by Subjects’ Initial Fitness Status
 - Subjects’ Current Fitness Level Upon Enrollment in Study
 - Couch Potatoes, Fit Individuals
 - Intensity of Exercise Program
 - Low, Medium, High
- 2 x 3 ANOVA

The Data



id	stress	intensity	improvement	spacet	improvement2
1	54	1	3	38.47	38.47
2	74	2	1	1.38	1.38
3	90	2	2	13.65	23.27
4	94	2	2	6.80	29.96
5	92	2	2	16.77	14.47
6	5	1	1	21.69	21.69
7	16	1	1	41.26	41.26
8	6	1	1	44.04	44.04
9	93	2	2	13.62	4.99
10	123	2	3	36.17	26.10
11	17	1	1	31.39	31.39
12	80	2	1	13.91	13.91
13	34	1	2	50.33	50.33
14	101	2	3	31.01	23.46
15	109	2	3	48.44	33.86
16	106	2	3	15.11	33.29
17	41	1	3	63.57	63.57
18	63	1	3	48.14	48.14
19	12	1	1	30.55	30.55
20	38	1	2	34.57	34.57
21	67	2	1	22.00	22.00
22	102	2	3	25.19	34.81
23	103	2	3	25.11	36.02
24	112	2	3	21.41	21.41

The Data

	id	sex	stress	intensity	improvement	spacer	improvement2
1	54	f	Couch Potatoes	High	38.47		38.47
2	74	f	Ft Individuals	Low	1.38		1.38
3	90	f	Ft Individuals	Med	13.65		23.27
4	84	f	Ft Individuals	Med	6.90		28.96
5	92	f	Ft Individuals	Med	16.77		14.47
6	5	f	Couch Potatoes	Low	21.69		21.69
7	16	f	Couch Potatoes	Low	41.26		41.26
8	6	f	Couch Potatoes	Low	44.04		44.04
9	93	f	Ft Individuals	Med	13.62		-4.99
10	123	f	Ft Individuals	High	36.17		26.10
11	17	f	Couch Potatoes	Low	31.39		31.39
12	80	f	Ft Individuals	Low	13.31		13.31
13	34	f	Couch Potatoes	Med	60.33		60.33
14	101	f	Ft Individuals	High	31.01		23.46
15	109	f	Ft Individuals	High	42.44		33.86
16	106	f	Ft Individuals	High	16.11		33.20
17	41	f	Couch Potatoes	High	63.57		63.57
18	53	f	Couch Potatoes	High	48.14		48.14
19	12	f	Couch Potatoes	Low	30.55		30.55
20	38	f	Couch Potatoes	Med	34.57		34.57
21	67	f	Ft Individuals	Low	22.00		22.00
22	102	f	Ft Individuals	High	25.19		34.91
23	103	f	Ft Individuals	High	25.11		36.02
24	112	f	Ft Individuals	High	21.41		21.41

The Analysis:

Univariate

Dependent Variable: improvement

Factor(s): sex, stress, intensity

Model: Contrast

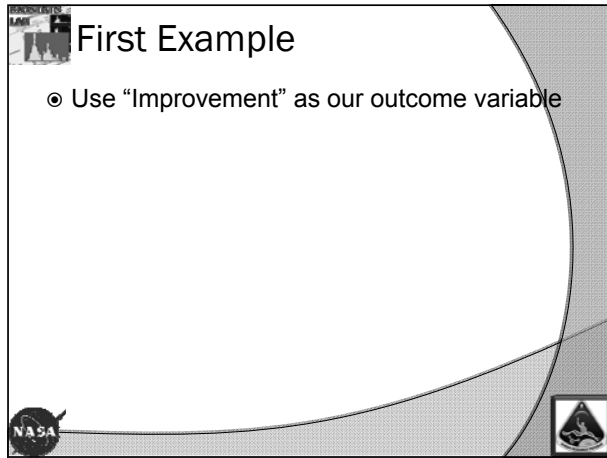
Display: Post hoc

Save: Save

Options: Options

First Example

- Use "Improvement" as our outcome variable



The Analysis:

Univariate: Options

Estimated Marginal Means

Factor(s) and Factor Interactions: (OVERALL), stress, intensity, stress*intensity

Display Means for: (empty)

Display:

- Descriptive statistics
- Homogeneity tests
- Estimates of effect size
- Spread vs. level plot
- Observed power
- Breakout plot
- Parameter estimates
- Lack of fit
- Contrast coefficient matrix
- General estimable function

Significance level: .05 Confidence intervals are 95.0%

Buttons: Continue, Cancel, Help

The Analysis: Post-Hoc Tests?

The screenshot shows the 'Univariate: Post-Hoc Multiple Comparisons for Observed Means' dialog box. The 'Factor(s)' list contains 'Intensity'. The 'Post-Hoc Tests for' section is empty. Under 'Equal Variances Assumed', the 'LSD' checkbox is selected. Under 'Equal Variances Not Assumed', the 'Bonferroni' checkbox is selected. The 'Test' section has '2-sided' selected. The 'Continue' button is highlighted.

The Analysis: Post-Hoc Tests?

The screenshot shows the 'Univariate: Profile Plots' dialog box. The 'Factor(s)' list contains 'Intensity'. The 'Horizontal Axis' is set to 'Improvement2'. The 'Separate Lines' checkbox is selected. The 'Continue' button is highlighted.

The Analysis: Post-Hoc Tests?

The screenshot shows the 'Univariate: Post-Hoc Multiple Comparisons for Observed Means' dialog box. A callout bubble with a thought bubble icon contains the text: "Let's skip this for now... we don't know if we'll need them just yet!". The dialog box settings are identical to the first screenshot.

The Analysis: Post-Hoc Tests?

The screenshot shows the 'Univariate: Profile Plots' dialog box. A callout bubble with a thought bubble icon contains the text: "Let's skip this for now... we don't know if we'll need them just yet!". The dialog box settings are identical to the second screenshot.

The Analysis (the easy way!)

The screenshot shows the SPSS Syntax Editor with the following syntax:

```

UNANOVA improvement2 BY fitness intensity
  /PLOT=PROFILE(fitness*intensity)
  /PRINT=HOMOGENEITY DESCRIPTIVE
  /DESIGN=fitness intensity stress*intensity.
    
```

The background shows a data view with columns: id, stress, intensity, improvement, spacer, improvement2.

Results...scrolling down.

The screenshot shows the SPSS Output window. A callout bubble points to the ANOVA Summary Table with the following text:

The ANOVA Summary Table:
 ->Interaction Effect?
 ->Main Effect for Fitness Level?
 ->Main Effect for Exercise Intensity?

The ANOVA Summary Table is as follows:

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	19132.455 ^a	5	3826.491	28.181	.000
Intercept	87033.082	1	87033.082	663.722	.000
fitness	8389.275	1	8389.275	63.977	.000
intensity	9564.422	2	4782.211	36.470	.000
fitness * intensity	1908.979	2	954.490	7.279	.001
Error	14424.170	110	131.129		
Total	122781.121	116			
Corrected Total	33556.625	115			

^a R Squared = .570 (Adjusted R Squared = .551)

Results?

The screenshot shows the SPSS Output window. A callout bubble points to the Descriptive Statistics table with the text: "Table of means & sd...".

Another callout bubble points to the Levene's Test of Equality of Error Variances table with the text: "Anyone remember what the Levene statistic tells us??".

The Descriptive Statistics table is as follows:

Dependent Variable: improvement2	Fitness Level of Subjects	Intensity of Exercise	Mean	Std. Deviation	N
Couch Potatoes	Low		28.1633	10.53231	15
	Med		36.9520	11.91881	18
	High		43.8276	10.42592	18
	Total		35.7240	12.38884	57
Fit Individuals	Low		8.4597	14.03810	18
	Med		11.6027	9.88446	19
	High		36.7344	11.47262	22
	Total		20.0149	17.52332	59
Total	Low		18.8300	15.71308	33
	Med		24.2779	14.81367	38
	High		39.4763	11.29954	40
	Total		27.7340	17.98206	116

The Levene's Test of Equality of Error Variances table is as follows:

Source	F	Sig.
Corrected Total	33556.625	115

Results...scrolling down.

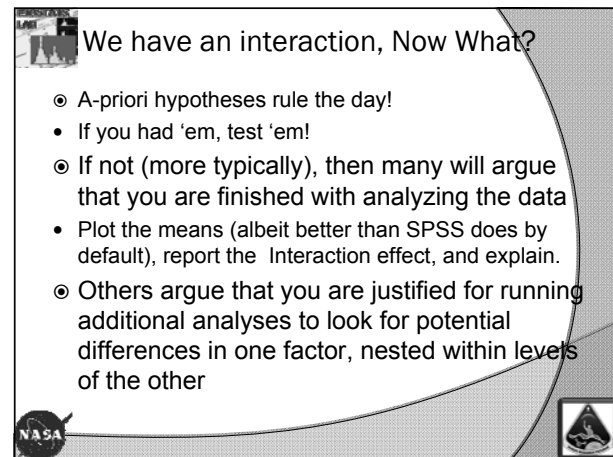
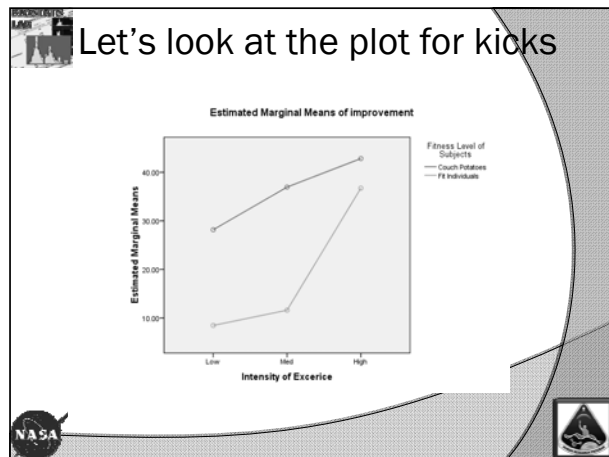
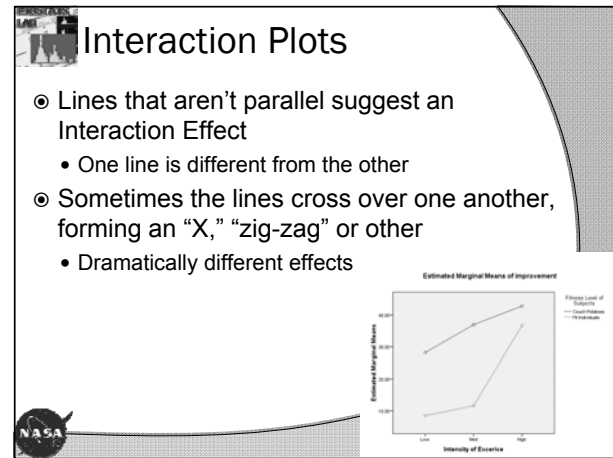
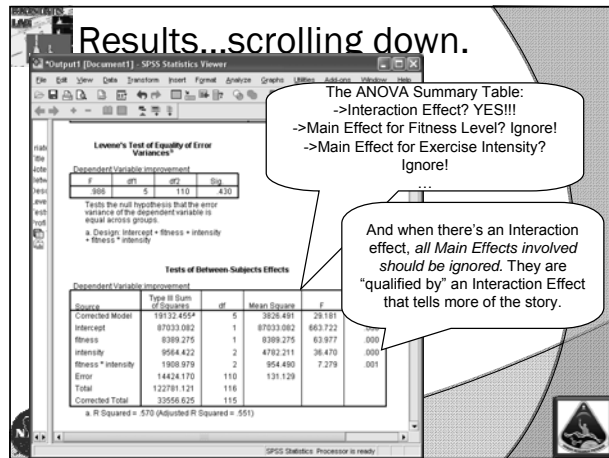
The screenshot shows the SPSS Output window. A callout bubble points to the ANOVA Summary Table with the following text:

The ANOVA Summary Table:
 ->Interaction Effect? YES!!!
 ->Main Effect for Fitness Level?
 ->Main Effect for Exercise Intensity?

The ANOVA Summary Table is as follows:

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	19132.455 ^a	5	3826.491	28.181	.000
Intercept	87033.082	1	87033.082	663.722	.000
fitness	8389.275	1	8389.275	63.977	.000
intensity	9564.422	2	4782.211	36.470	.000
fitness * intensity	1908.979	2	954.490	7.279	.001
Error	14424.170	110	131.129		
Total	122781.121	116			
Corrected Total	33556.625	115			

^a R Squared = .570 (Adjusted R Squared = .551)



Our Study?

- Let's assume we thought there would be a difference in effects of low, medium, high intensity by fitness levels
- Let's further assume that we did not have a-priori hypotheses begging for specific contrasts, but rather wanted to follow-up with whatever post-hoc tests we are justified at running

How not to do this with SPSS?

Software differs in how you ask for pairwise comparisons of one factor, within levels of the other. In SPSS, for example, you should NOT use the GUI as I show here... Anyone want to guess what this would do if we ran it as shown here??

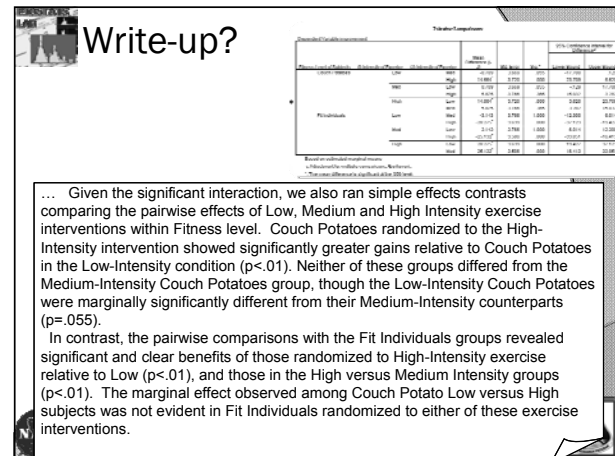
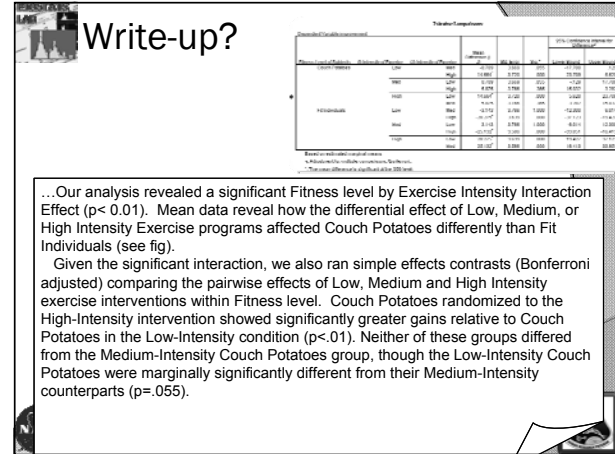
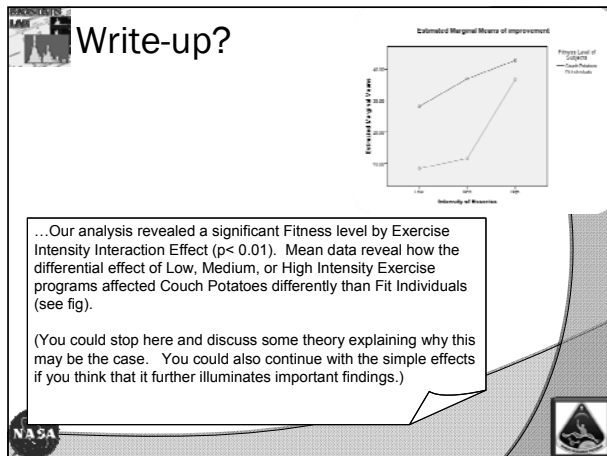
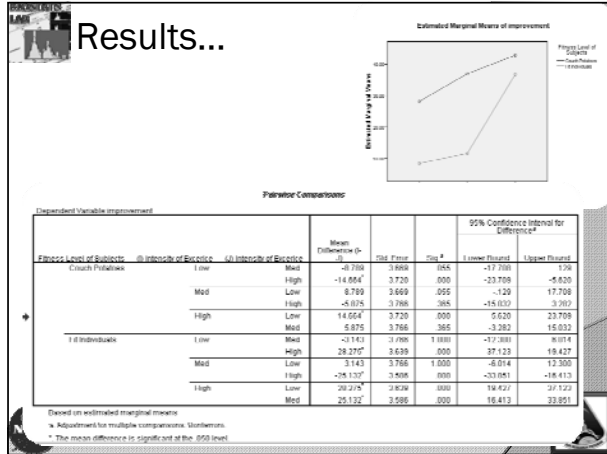
How not to do this with SPSS?

Software differs in how you ask for pairwise comparisons of one factor, within levels of the other. In SPSS, for example, you should NOT use the GUI as I show here... Anyone want to guess what this would do if we ran it as shown here??

Simple Effects

- To get pairwise comparisons of Intensity (Low, Medium, High) within levels of Fitness (Couch Potato, Fit Individuals), we need to use Syntax (code) as shown below

```
UNANOVA improvement BY Stress intensity
/PLOT=PROFILE(intensity*Stress)
/PRINT=HOMOGENEITY DESCRIPTIVE
/RESIDUALS=TABLES(intensity*Stress) COMPARE(intensity) ADJ(BONF)
/COCORR=Stress intensity Stress*intensity
```



Next?

- Let's try it again using "Improvement2" instead of "Improvement" as our observed results.
- This is for illustration purposes—pretend like these were our data instead of the earlier results...
 - The analysis set-up is the same.

Results...scrolling down

The ANOVA Summary Table:
->Interaction Effect
...
->Main Effect for Fitness Level
->Main Effect for Exercise Intensity

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	15486.877 ^a	5	3097.375	26.655	.000
Intercept	82686.199	1	82686.199	711.886	.000
fitness	9007.681	1	9007.681	84.403	.000
intensity	6140.708	2	3070.354	26.423	.000
fitness * intensity	372.211	2	186.106	1.602	.206
Error	12782.047	110	116.200		
Total	111956.008	116			
Corrected Total	28268.725	115			

Results?

Table of means & sd...

Dependent Variable	Fitness Level of Subjects	Intensity of Exercise	Mean	Std. Deviation	N
Couch Potatoes	Low		28.1633	10.53331	20
	Med		36.9526	11.91881	19
	High		42.8276	18.42592	18
Fit Individuals	Low		8.4597	14.03618	18
	Med		14.9237	5.98378	19
	High		29.2594	18.41089	22
Total	Low		18.2967	13.88077	69
	Med		18.0300	15.71306	38
	High		25.9382	14.52844	38
Total			35.7240	12.38884	67

What is the Levene test telling us this time?

Dependent Variable	F	df1	df2	Sig.
Improvement2	3.015	5	110	.014

Results...scrolling down

The ANOVA Summary Table:
->Interaction Effect NO
...
->Main Effect for Fitness Level YES!
->Main Effect for Exercise Intensity YES!

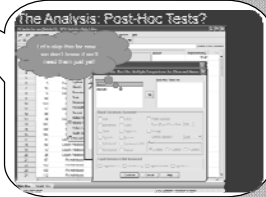
Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	15486.877 ^a	5	3097.375	26.655	.000
Intercept	82686.199	1	82686.199	711.886	.000
fitness	9007.681	1	9007.681	84.403	.000
intensity	6140.708	2	3070.354	26.423	.000
fitness * intensity	372.211	2	186.106	1.602	.206
Error	12782.047	110	116.200		
Total	111956.008	116			
Corrected Total	28268.725	115			

Interpreting Multi-Factorial ANOVA Results

- If you have an Interaction Effect, Start There!
 - All main effects involved in a significant interaction are qualified effects anyway—they don't tell the whole story
 - This was the case in our earlier example
- If you do not have Interaction Effects, Interpret whatever Main Effects you have
 - This is the case in our current example
 - Post-hoc or Contrast effects may be useful now?

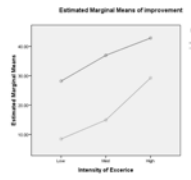
Remember when we skipped post-hoc tests?

- Now that we know we don't have an Interaction effect, it's time to consider post-hoc tests, if you desire
 - Compare All Intensity Levels Pairwise, collapsing across Fitness Level
- Why not contrast comparisons of the levels of one factor "within" the other?



Back to our Example...

- Main effect for Fitness Level
 - Couch Potatoes improved more than Fit Individuals
- Main effect for Intensity of the Exercise
 - Looks like increasing intensity produced greater benefits overall
 - Follow-up?




Tests of Between-Subjects Effects

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	15486.637 ^a	5	3097.325	26.655	.000
Intercept	82688.189	1	82688.189	711.666	.000
fitness	9507.651	1	9507.651	84.463	.000
intensity	6146.708	2	3073.354	28.423	.000
fitness * intensity	372.211	2	186.106	1.602	.206
Error	12792.047	110	116.200		
Total	111956.868	116			
Corrected Total	29288.725	115			

^a. R Squared = .548 (Adjusted R Squared = .527)

Remember when we skipped post-hoc tests?

- Now that we know we don't have an Interaction effect, it's time to consider post-hoc tests, if you desire
 - Compare All Intensity Levels Pairwise, collapsing across Fitness Level
- Why not contrast comparisons of the levels of one factor "within" the other?
 - No Evidence that the differences between Low, Med, High are any different in Couch Potatoes versus Fit People!



Ok to re-run this as a Oneway?

- We know that there's no interaction effect, so can we just run it as a oneway and get the pairwise comparisons that way?
 - Nope. That would ignore the variance structure in the data, *and* inflates Type I error
 - This was a multifactorial design from the start, so stick with your multifactorial analysis

Pairwise Comparisons

Dependent Variable: improvement		Multiple Comparisons					
		Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval		
(I) Intensity of Exercise	(J) Intensity of Exercise				Lower Bound	Upper Bound	
Tukey HSD	Low	Med	-7.1082 ^a	2.47302	.013	-12.9838	-1.2327
		High	-16.5345 ^a	2.44191	.000	-22.3362	-10.7329
	Med	Low	7.1082 ^a	2.47302	.013	-1.2327	12.9838
		High	-9.4263 ^b	2.44191	.001	-15.2278	-3.6247
	High	Low	16.5345 ^a	2.44191	.000	10.7329	22.3362
		Med	9.4263 ^b	2.44191	.001	3.6247	15.2278
Bonferroni	Low	Med	-7.1082 ^a	2.47302	.015	-13.1205	-1.0960
		High	-16.5345 ^a	2.44191	.000	-22.4712	-10.5979
	Med	Low	7.1082 ^a	2.47302	.015	1.0960	13.1205
		High	-9.4263 ^b	2.44191	.001	-15.3629	-3.4897
	High	Low	16.5345 ^a	2.44191	.000	10.5979	22.4712
		Med	9.4263 ^b	2.44191	.001	3.4897	15.3629

Based on observed means.
The error term is Mean Square(Error) = 116.200.
*. The mean difference is significant at the .05 level.

Back to our analysis:

- Generate posthoc tests for the *Intensity* factor (GUI or Syntax, your preference)
- And remember that we're now, essentially, averaging across levels of Subject Fitness (the Couch Potatoes and Fit Individuals), but we are doing so in a multivariate context

```

16 UNIANOVA improvement BY fitness intensity
17   /METHOD=SSTYPE(3)
18   /INTERCEPT=INCLUDE
19   /POSTHOC=intensity(TUKEY BONFERRONI)
20   /POSTHOC=intensity(fitness)
21   /PRINT=HOMOGENEITY DESCRIPTIVE
22   /CRITERIA=ALPHA(.05)
23   /DESIGN=intensity fitness*intensity
  
```

BTW... if you ran as a Oneway?

- Similar conclusions, but note that Low vs. Med is no longer significant??

Dependent Variable: improvement		Multiple Comparisons					
		Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval		
(I) Intensity of Exercise	(J) Intensity of Exercise				Lower Bound	Upper Bound	
Tukey HSD	Low	Med	-7.10824	3.29540	.080	-14.8835	-.4711
		High	-16.53454 ^a	3.22432	.000	-24.1933	-8.8768
	Med	Low	7.10824	3.29540	.080	-.4711	14.6835
		High	-9.42630 ^b	3.22432	.012	-17.0840	-1.7686
	High	Low	16.53454 ^a	3.22432	.000	8.8768	24.1923
		Med	9.42630 ^b	3.22432	.012	1.7686	17.0840
Games-Howell	Low	Med	-7.10824	3.47171	.108	-15.4128	1.1963
		High	-16.53454 ^a	3.21084	.000	-24.2226	-8.8465
	Med	Low	7.10824	3.47171	.108	-1.1963	15.4128
		High	-9.42630 ^b	3.06684	.008	-18.7493	-2.1033
	High	Low	16.53454 ^a	3.21084	.000	8.8465	24.2226
		Med	9.42630 ^b	3.06684	.008	2.1033	16.7493


*. The mean difference is significant at the 0.05 level.



Next time

- Quick Discussion of why, in general, we advise against 3-factor, 4-factor...k-factorial models
- Move into Repeated Measures designs
 - Pre, Post1, Post2, Post3








SMAR Session 4




ANOVA

(CONTINUED)






Recap—Independent Measures ANOVA

- IM Analysis of Variance (ANOVA) examines variability between groups, relative to within groups, to determine whether there's evidence that the groups are not from the same population
- Analysis focuses on *variance*, but interpretation is about *mean's*
- One-way ANOVA compares more than two groups, defined by a single Factor
- Multi-Factorial considers additional factors





Recap—Independent Measures ANOVA

- With Multiple Factors:
 - Main Effects
 - Interaction Effects
- IM ANOVA assumes
 - Random samples from the population
 - Sufficiently large enough n to detect effects, distributed evenly among groups
 - Similar distributions of variability among groups (Homogeneity of Variance)



Recap—Independent Measures ANOVA

- General Strategy is to Interpret Significant Interactions if you have them
 - Main Effects only tell part of the story
 - Simple Effects can help further
- If no Interactions, Interpret Main Effects
 - Post-Hoc or Contrasts Available for Pairwise Comparisons



Today: Repeated Measures ANOVA

- With RM ANOVA, we consider measuring the SAME subjects at different times, or under different conditions, to see if something changes over time, or between the different conditions
 - Ex. Does performance decrease in response to time spent in microgravity?
 - Ex. Does bone mass decrease as subjects age?
 - Ex. Compare Subjects' Ratings of 'XYZ' when taking Placebo, versus Drug A, versus Drug B, with adequate washout periods between drugs
 - Ex. Compare PRE to Post1 and Post2...

Repeated Measures ANOVA

- Same people... no "individual differences" in the F-ratio
- More powerful statistics
- The F-Ratio represents:

$$F = \frac{\text{Variability among times (or conditions)}}{\text{Variability within the sample}} = \frac{\text{error} + \text{time (or condition) differences}}{\text{error}}$$

Repeated Measures Designs

- Only one sample
- Differences based on time, or condition
- Using the SAME subjects time after time
- Measuring the SAME outcome each time
- Looking for changes...

Repeated Measures ANOVA

- Same people... no "individual differences" in the F-ratio
- More powerful statistics
- The F-Ratio represents:

$$F = \frac{\text{Variability among times (or conditions)}}{\text{Variability within the sample}} = \frac{\cancel{\text{error}} + \text{time (or condition) differences}}{\cancel{\text{error}}}$$

Assumptions for RM ANOVA

- Same as for IM-ANOVA RE Ordinal or Continuously Scaled Outcomes following the normal distribution
- Random sampling from the population with sufficient n
 - Except now only one sample...
- Homogeneity of Variance Does Apply in *purely* RM models. (only 1 group!)
- Instead, Assumption of Sphericity
 - Assume that the covariance among pairs of repeated observations are equal.

This is where it comes from (Repeated Measures Designs)

SS_{total} = Same as IM Anova
 $SS_{between}$ = Same as IM Anova
 SS_{within} = Same as IM Anova

$SS_{b/t\ subjects} = \sum \frac{(\text{each person's total across treatments})^2}{k} - \frac{(\sum X)^2}{N}$

$SS_{error} = SS_{within} - SS_{b/t\ subjects}$

$df_{total} = n - 1$
 $df_{between} = k - 1$
 $df_{within} = n - k$
 $df_{b/t\ subjects} = n - 1$
 $df_{error} = (n - k) - (n - 1)$

k = number of times measured

RM-ANOVA Summary Tables

- Same Concept as IM Table, but now
 - Instead of "Between Groups" effects, we have "Between Treatments" effects
 - And also "Within Treatments"
 - Consist of subject differences (among subjects)
 - And error
- One Group measured several times, thus we partition "within group" variability into that which is due to individual differences, and error.

This is where it comes from (Repeated Measures Designs)

$$MS_{between} = \frac{SS_{between}}{df_{between}} \quad F = \frac{MS_{between}}{MS_{error}}$$

$$MS_{error} = \frac{SS_{error}}{df_{error}}$$

F-tables provide a p value for a given F-statistic, using $df_{between}$ (numerator) and df_{error} (denominator).

Example

- Compare Performance Pre, During, and Post Bed-Rest
 - Pre (time zero)
 - Three Weeks Into Bedrest
 - Six Weeks Into Bedrest (end of bedrest)
 - Three Weeks FOLLOWING Bedrest (week 9)
- Same Subjects measured 4 times
- Equal Interval between time periods

In This Example...

- Subjects (n=34) were measured on some validated performance scale four times, with equal intervals between time periods
 - Pre, Week 3, Week 6, Week 9.
 - Bedrest STOPPED at the end of Week 6
 - (Dataset created for instructional purposes)
- Prior Research has shown that these data tend to follow the normal distribution

How to Organize RM Data

- Wide Versus Long Format
 - Wide = one row per subject, with multiple columns containing the multiple repeated observations
 - Long = as many rows per subject as needed, where each row contains an observation
- The Choice of Format Depends on What Software You Will Be Using
 - SPSS needs Wide
 - Stata needs Long
 - Both can convert, so for data management, use what you are comfortable with

Examples of Wide Dataset

Wide format, with 1 row per subject, and as many columns as necessary to capture all of the repeated observations

Examples of Long Dataset

Long format, with as many rows per subject as needed to capture all of the repeated observations (same data as prior slide)

Subject	Day	Sex	Age	Week	Recovery	
1	1	male	9	6	5	4
2	2	female	5	3	3	7
3	3	female	9	3	6	7
4	4	male	8	7	6	6
5	5	male	6	7	3	6
6	6	female	9	7	4	9
7	7	female	8	7	5	8
8	8	male	9	3	3	8
9	9	male	6	3	3	8
10	10	male	5	4	3	8
11	11	male	5	6	4	8
12	12	female	4	3	3	8
13	13	male	5	3	3	5
14	14	female	6	5	4	4
15	15	female	6	3	3	8
16	16	male	7	4	5	4
17	17	female	6	5	4	4
18	18	female	5	7	4	6
19	19	male	7	4	3	8
20	20	male	8	4	4	6
21	21	male	5	7	4	4
22	22	female	6	4	3	4
23	23	female	8	6	5	5
24	24	male	5	3	6	7
25	25	female	6	4	6	4
26	26	male	9	6	5	9

Using SPSS...

Note the occasional missing observation...

id	male	female	week3	week5	recovery3	
1	1	male	9	6	5	4
2	2	female	5	3	3	7
3	3	female	9	3	6	7
4	4	male	8	7	6	6
5	5	male	6	7	3	6
6	6	female	9	7	4	9
7	7	female	8	7	5	8
8	8	male	9	3	3	8
9	9	male	6	3	3	8
10	10	male	5	4	3	8
11	11	male	5	6	4	8
12	12	female	4	3	3	8
13	13	male	5	3	3	5
14	14	female	6	5	4	4
15	15	female	6	3	3	8
16	16	male	7	4	5	4
17	17	female	6	5	4	4
18	18	female	5	7	4	6
19	19	male	7	4	3	8
20	20	male	8	4	4	6
21	21	male	5	7	4	4
22	22	female	6	4	3	4
23	23	female	8	6	5	5
24	24	male	5	3	6	7
25	25	female	6	4	6	4
26	26	male	9	6	5	9

Using SPSS...

SPSS requires Wide format for Repeated Measures Designs.

This gets tricky when there are two repeated measures factors... stay tuned!

Using SPSS...

Subjects with ANY missing observation will be completely ignored in a purely repeated measures (fixed) ANOVA.

This can be a big problem with small n! We have n=34 here, but 4 are missing at least one observation... so Study n=30

SPSS Statistics Data Editor - Repeated Measures Define Factor(s) dialog box. The dialog box is open, showing the 'Within-Subject Factor Name' field with the text 'factor' and the 'Number of Levels' field with the value '4'. Two callout boxes are present: one pointing to the 'Within-Subject Factor Name' field with the text 'Give your factor a name that is meaningful to you', and another pointing to the 'Number of Levels' field with the text '...and enter the number of repeated observations here'.

id	mal	5	7	4	4	
1	1	1.0				
2	2					
3	3					
4	4					
5	5					
6	6					
7	7					
8	8					
9	9					
10	10					
11	11					
12	12					
13	13					
14	14					
15	15					
16	16					
17	17					
18	18	female	5	7	4	4
19	19	male	7	4		
20	20	male	8	4		
21	21	male	5	7	4	4
22	22	female	6	4	3	4
23	23	female	8	6	5	5
24	24	male	5	3	6	7
25	25	female	6	4	6	4
26	26	male	9	6	5	9

SPSS Statistics Data Editor - Repeated Measures Define Factor(s) dialog box. The dialog box is open, showing the 'Within-Subject Factor Name' field with the text 'factor' and the 'Number of Levels' field with the value '4'. The 'Define' button is highlighted.

id	mal	5	7	4	4	
1	1	1.0				
2	2					
3	3					
4	4					
5	5					
6	6					
7	7					
8	8					
9	9					
10	10					
11	11					
12	12					
13	13					
14	14					
15	15					
16	16					
17	17					
18	18	female	5	7	4	4
19	19	male	7	4		
20	20	male	8	4		
21	21	male	5	7	4	4
22	22	female	6	4	3	4
23	23	female	8	6	5	5
24	24	male	5	3	6	7
25	25	female	6	4	6	4
26	26	male	9	6	5	9

SPSS Statistics Data Editor - Repeated Measures Define Factor(s) dialog box. The dialog box is open, showing the 'Within-Subject Factor Name' field with the text 'factor' and the 'Number of Levels' field with the value '4'. The 'Add' button is highlighted. A callout box points to the 'Add' button with the text 'Then click on "Add" to enter that factor into your model statement'.

id	mal	5	7	4	4	
1	1	1.0				
2	2					
3	3					
4	4					
5	5					
6	6					
7	7					
8	8					
9	9					
10	10					
11	11					
12	12					
13	13					
14	14					
15	15					
16	16					
17	17					
18	18	female	5	7	4	4
19	19	male	7	4		
20	20	male	8	4		
21	21	male	5	7	4	4
22	22	female	6	4	3	4
23	23	female	8	6	5	5
24	24	male	5	3	6	7
25	25	female	6	4	6	4
26	26	male	9	6	5	9

SPSS Statistics Data Editor - Repeated Measures Define Factor(s) dialog box. The dialog box is open, showing the 'Within-Subject Factor Name' field with the text 'factor' and the 'Number of Levels' field with the value '4'. The 'Define' button is highlighted. A callout box points to the 'Define' button with the text 'Now you're ready to "Define" your model'.

id	mal	5	7	4	4	
1	1	1.0				
2	2					
3	3					
4	4					
5	5					
6	6					
7	7					
8	8					
9	9					
10	10					
11	11					
12	12					
13	13					
14	14					
15	15					
16	16					
17	17					
18	18	female	5	7	4	4
19	19	male	7	4		
20	20	male	8	4		
21	21	male	5	7	4	4
22	22	female	6	4	3	4
23	23	female	8	6	5	5
24	24	male	5	3	6	7
25	25	female	6	4	6	4
26	26	male	9	6	5	9

Contrasts (k-1) ...

Factors: RepeatedMeasures

Change Contrast:

Contrast: Polynomial

Buttons: Change, Contrast, Deviation, Deviate, Difference, Invariant, Help

Buttons: Model..., Contrast..., Plot..., Post hoc..., Save..., Options...

Buttons: OK, Paste, Speed, Cancel, Help

id	mal	week1	week2	week3		
1	1	0				
2	2	2				
3	3					
4	4					
5	5					
6	6					
7	7					
8	8					
9	9					
10	10					
11	11					
12	12					
13	13					
14	14					
15	15					
16	16					
17	17					
18	18					
19	19					
20	20					
21	21	male	5	7	4	4
22	22	female	6	4	3	4
23	23	female	8	6	5	5
24	24	male	6	3	6	7
25	25	female	6	4	6	4
26	26	male	9	6	5	9

The easy way...with Syntax!

```

DATASET ACTIVATE DataSet1.
GLM mal week3 week6 recovery9
  /ANSF(Actor) = 4 Polynomial
  /METHOD=SSTYPE(3)
  /PLOT=PROFILE(mal)
  /PRINT=DESCRIPTIVE
  /CRITERIA=ALPHA(0.05)
  /ANSD=GRH(mal)
  .
  
```

id	mal	pr0	week3	week6	recovery9	
1	1	male	9	6	5	4
2	2	female	6	3	3	7
3	3					
4	4					
5	5					
6	6					
7	7					
8	8					
9	9					
10	10					
11	11					
12	12					
13	13					
14	14					
15	15					
16	16	male	7	4	5	4
17	17	female	6	6	4	4
18	18	female	5	7	4	6
19	19	male	7	4	3	8
20	20	male	8	4	4	6
21	21	male	5	7	4	4
22	22	female	6	4	3	4

Always nice to get a graph...

Factors: time

Horizontal Axis: time

Separate Lines: Separate Lines

Separate Plots: Separate Plots

Buttons: Add, Change, Remove, Continue, Cancel, Help

id	mal	week1	week2	week3
1	1	0		
2	2	2		
3	3			
4	4			
5	5			
6	6			
7	7			
8	8			
9	9			
10	10			
11	11			
12	12			
13	13			
14	14			
15	15			
16	16			
17	17			
18	18			
19	19			
20	20			
21	21			
22	22			
23	23			
24	24			
25	25			
26	26			

Interpreting the Output...

Always good to double check the W/ Subjects Levels to be sure that you brought them in the correct order

Note our descriptives... with n=30

Measure	Mean	Std. Deviation	N
pr0	6.77	1.569	30
week3	4.87	1.137	30
week6	4.13	1.137	30
recovery9	6.47	1.717	30

Effect	Value	F	Hypothesis df	Error df	Sig.	
Time	Pillai's Trace	748	26.914 ^a	3.000	27.000	.000
	Wilk's Lambda	251	26.914 ^a	3.000	27.000	.000
	Hotelling's Trace	3.990	26.914 ^a	3.000	27.000	.000
	Roy's Largest Root	3.990	26.914 ^a	3.000	27.000	.000

^a Exact statistic
^b Decision Interval

Interpreting the Output...

Multivariate Tests^a

Effect	Value	F	Hypothesis df	Error df	Sig.	
Time	Pillai's Trace	749	26.914 ^a	3,000	27,000	.000
	Wilks' Lambda	251	26.914 ^a	3,000	27,000	.000
	Hotelling's Trace	2,990	26.914 ^a	3,000	27,000	.000
	Roy's Largest Root	2,990	26.914 ^a	3,000	27,000	.000

Mauchly's Test of Sphericity^a

Measure	Mauchly's W	Approx. Chi-Square	df	Sig.	Greenhouse-Geisser	Huynh-Feldt	Lower-bound
Time	.769	7.270	5	.202	.842	.929	.333

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.
 a. May be used to adjust the degrees of freedom for the averaged tests of significance. Corrected tests are displayed in the Tests of Within-Subjects Effects table.
 b. Design: Intercept
 Within Subjects Design: time

Tests of Within-Subjects Effects

Measure	Type III Sum of Squares	df	Mean Square	F	Sig.
Time	187.225	3	62.408	2.116	.121

Interpreting the Output...

Mauchly's Test of Sphericity^a

Measure	Mauchly's W	Approx. Chi-Square	df	Sig.	Greenhouse-Geisser	Huynh-Feldt	Lower-bound
Time	.769	7.270	5	.022	.842	.929	.333

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.
 a. May be used to adjust the degrees of freedom for the averaged tests of significance. Corrected tests are displayed in the Tests of Within-Subjects Effects table.
 b. Design: Intercept
 Within Subjects Design: time

Tests of Within-Subjects Effects

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	
Time	Sphericity Assumed	153.035	3	51.011	23.703	.000
	Greenhouse-Geisser	153.025	2.527	60.563	23.703	.000
	Huynh-Feldt	153.025	2.768	54.891	23.703	.000
	Lower-bound	153.025	1.000	153.025	23.703	.000
Error(time)	Sphericity Assumed	187.23	87	2.15		
	Greenhouse-Geisser	187.225	73.275	2.555		
	Huynh-Feldt	187.225	80.846	2.616		
	Lower-bound	187.225	29.000	8.456		

Interpreting the Output...

Mauchly's Test of Sphericity^a

Measure	Mauchly's W	Approx. Chi-Square	df	Sig.	Greenhouse-Geisser	Huynh-Feldt	Lower-bound
Time	.769	7.270	5	.202	.842	.929	.333

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.
 a. May be used to adjust the degrees of freedom for the averaged tests of significance. Corrected tests are displayed in the Tests of Within-Subjects Effects table.
 b. Design: Intercept
 Within Subjects Design: time

Tests of Within-Subjects Effects

Measure	Type III Sum of Squares	df	Mean Square	F	Sig.
Time	187.225	3	62.408	2.116	.121

Mauchly's test tells us whether we met the assumption of sphericity. If significant, we violated this assumption and need to adjust our F-statistic accordingly (stay tuned!)

Let's look at our graph...

- Performance started high, dropped during bedrest, and seems to have returned by recovery.
- Different Options on how to follow-up with our significant ANOVA telling us that "things changed"

Estimated Marginal Means of MEASURE_1

Time	Estimated Marginal Mean
1	0.8
2	-0.5
3	-0.8
4	0.5

What next?

- Like with the Oneway IM-ANOVA, we probably had a more in-depth research question in mind, other than “did things change?” Did we want to:
 - “Characterize the nature of the change?”
 - “Compare “Pre-” levels to all “Post- levels” and report on our findings?”
 - “Compare everything to everything else and hope that something, *anything*, is significant so that we can get a paper outta this study???”

Back to our Output...

If the goals of our research were to “characterize the nature of the changes,” one good option is to run Polynomial Contrasts and interpret...

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Time	147.41	1	147.41	59.092	.000
	2.535	1	2.535	1.494	.231
Error(Dtime)	65.680	28	2.346		
	72.342	28	2.584		
	49.215	28	1.758		

What next?

- Like with the Oneway IM-ANOVA, we probably had a more in-depth research question in mind, other than “did things change?” Did we want to:
 - “Characterize the nature of the change?”
 - “Compare “Pre-” levels to all “Post- levels” and report on our findings?”
 - ~~“Compare everything to everything else and hope that something, *anything*, is significant so that we can get a paper outta this study???”~~

Alternatively...

If our goal was to compare Pre- to all Post- Observations, with different syntax (or GUI clicks) we could test that too...

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Time	132.300	1	132.300	39.876	.000
	208.033	1	208.033	76.399	.000
	2.700	1	2.700	.592	.448
Error(Dtime)	96.700	28	3.454		
	78.967	28	2.820		
	132.300	28	4.725		

Contrasts with RM Factors

- Contrasts are powerful specific comparisons that can be run with Repeated Measures Factors
- They operate like “Post-Hoc” tests, but are called “Contrasts”
- With k levels of a Repeated Measures Factor, you can make $k-1$ contrast comparisons
 - So with 4 measures here, we can make 3 special contrast comparisons
- “Simple” and “Polynomial” are commonly used, but there are others too.

Additional Contrasts

- Custom—users can also specify their own set of $(k-1)$ contrasts per specific hypotheses
- Users can also perform Simple Effect Contrasts, like we demonstrated with IM-ANOVA, as long as an appropriate correction for the multiple comparisons are made...

```

GLM pre0 week3 week6 recovery0
  /VSFACTOR=time 4 Polynomial
  /METHOD=SSTYPE03
  /PLOT=PROFILE(SDme)
  /NAME=ANO=TABLE(SDme) compare(time) adj(bonf)
  /PRINT=DESCRIPTIVE
  /CRITERIA=ALPHA(.05)
  /MODEL=SDme
  
```

“Canned” Contrasts





- Polynomials – test for increasingly complex polynomial equations (linear, quadratic, cubic, etc.)
 - Useful to describe the trend, or nature of the changes
- Simple—compares all levels to a reference level
 - Common when there is a meaningful “pre” value
- Difference—compares each level (except the first) to the mean of all prior levels
- Helmert—compares each level (except the last) to the mean of all subsequent levels
- Repeated—compares each level (except the last) to the next subsequent level

Next Time





- We’ll discuss “doubly-repeated measures” designs, and run through an example or two.
- We’ll run mixed-factorial designs, where we use a combination of RM and IM factors.
- We’ll talk about including covariates in our models, and how that can be useful.

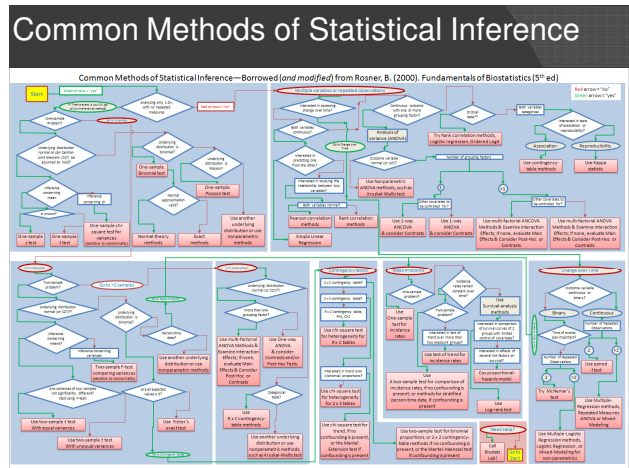
SMAR HYPOTHESIS TESTING

Truth Table

The Truth is:		
	H ₀ Really is True (there's no effect)	H ₀ is Actually False (there is an effect)
You Rejected H ₀ Due to a Statistically Significant Result	Wrong Conclusion 	Right Conclusion 
You Accepted H ₀ Due to a Non-Significant Result	Right Conclusion 	Wrong Conclusion 

Alpha, Beta Type I & II Errors & Power

The Truth is:		
	H ₀ Really is True (there's no effect)	H ₀ is Actually False (there is an effect)
You Rejected H ₀ Due to a Statistically Significant Result	Type I Error Probability = α 	Power Probability = $(1 - \beta)$ 
You Accepted H ₀ Due to a Non-Significant Result	Probability = $1 - \alpha$ 	Type II Error Probability = β 



Terms, Definitions & Other Stuff

Terms, Definitions, and other stuff to help you interpret the Methods of Statistical Inference Chart

Variable: In experimental design and analysis, there are two different functional uses for variables. One is to describe the independent variable, the other is to describe the dependent variable. The independent variable is the variable that the experimenter manipulates, and the dependent variable is the variable that the experimenter measures. The independent variable is often referred to as the "treatment" or "exposure" variable, and the dependent variable is often referred to as the "outcome" variable. Variables are often measured on a continuous and discrete scale.

Continuous: A type of outcome or predictor variable that can take on a full range of values between a lowest and highest bound (e.g., length, weight, height, IQ, etc.).

Discrete: A type of variable that only assumes a countable number of values or levels. These variables can be categorical (e.g., gender, eye color, etc.) or numerical (e.g., number of children, etc.).

Normality: A type of distribution that is symmetric and bell-shaped. It is often used as a reference distribution for many statistical tests.

Normality tests: A set of statistical tests used to determine if a distribution is normal. These include the Shapiro-Wilk test, the Kolmogorov-Smirnov test, and the Anderson-Darling test.

Power: The probability of correctly rejecting the null hypothesis when it is false. It is the complement of the Type II error rate.

Significance level: The probability of rejecting the null hypothesis when it is true. It is denoted by the Greek letter alpha (α).

Effect size: A measure of the magnitude of the effect being studied. It is often denoted by the Greek letter d.

Confidence interval: A range of values that is likely to contain the true population parameter. It is often used to estimate the magnitude of an effect.

Standard deviation: A measure of the spread or variability of a distribution. It is the square root of the variance.

Variance: A measure of the spread or variability of a distribution. It is the square of the standard deviation.

Mean: The average value of a distribution. It is often used as a measure of central tendency.

Median: The middle value of a distribution. It is often used as a measure of central tendency.

Mode: The most frequent value of a distribution. It is often used as a measure of central tendency.

Skewness: A measure of the asymmetry of a distribution. It is often used to describe the shape of a distribution.

Kurtosis: A measure of the "tailedness" of a distribution. It is often used to describe the shape of a distribution.

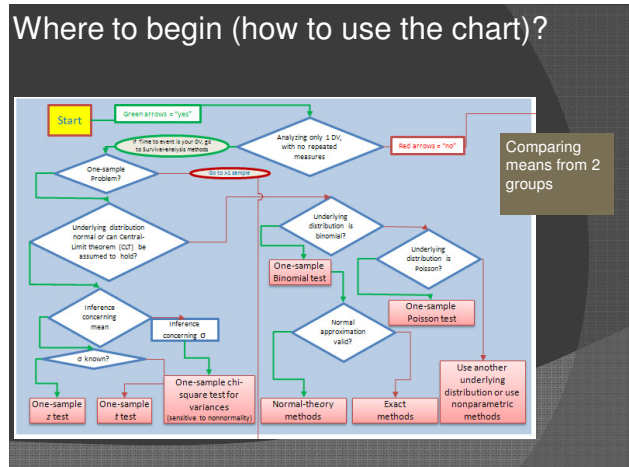
Outliers: Data points that are significantly different from the rest of the data. They can be caused by measurement error or by a true effect.

Assumptions: The conditions that must be met for a statistical test to be valid. These include normality, independence, and equal variance.

Robustness: The ability of a statistical test to perform well even when the assumptions are violated. Robust tests are often preferred to non-robust tests.

Nonparametric methods: Statistical methods that do not rely on the assumptions of normality, independence, and equal variance. These include the Mann-Whitney U test, the Wilcoxon signed-rank test, and the Kruskal-Wallis test.

Parametric methods: Statistical methods that rely on the assumptions of normality, independence, and equal variance. These include the t-test, the ANOVA, and the regression analysis.



Where to begin (how to use the chart)?

Comparing means from 2 groups(cont.)

Our data are scaled from a low number to a high one, and in the population, it tends to follow the bell-shape curve.

(What's missing from the flow chart?)

Another Example

Want to know if BMD changes during bedrest are affected by an intervention, so we have 2 groups (Control, Intervention) and we measured their BMD Pre-bedrest, then again at 30, 60 and 90 days.

Want to know if the change over time is less (better) in the Intervention group relative to controls.

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Next Time

- Meet again at noon, Thursday, Sept. 24th
- Begin reviewing Hypothesis Testing using ANOVA, Regression, or Other topic per today
- PPT Slides & "Screenshots" from Statistical Software
 - Promise... no hand calculations & minimal formulae!
 - Promise... fun & applied, with enough "meat" to get you started and keep you statistically-safe
 - Or at least enough to know when it's time to call us!!