The Role of Uncertainty in Aerospace Vehicle Analysis and Design

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Vision

Increase confidence and consistency in performance robustness assessments by improving methods and software tools for quantifying and managing uncertainty.

Effective uncertainty quantification (UQ) begins at the earliest phase in the design phase for which there are adequate models and continues tightly integrated to the analysis and design cycles as the refinement of the models and the fidelity of the tools increase. It is essential that uncertainty quantification strategies provide objective information to support the processes of identifying, analyzing and accommodating for the effects of uncertainty. Assessments of uncertainty should never render the results more difficult for engineers and decision makers to comprehend, but instead provide them with critical information to assist with resource utilization decisions and risk mitigation strategies. Success would be measured by the tools to enable engineers and decision makers to effectively balance critical project resources against system requirements while accounting for the impact of uncertainty.

Historical Perspective

The treatment of uncertainty quantification in physics-based models of aerospace systems has historically focused on their reliability analysis and is typically conducted in the final stages of the design cycle. Despite of its relatively late arrival to the analysis and design processes, this phase of application of UQ has proven to have significant impact on the final robustness and safety of the resulting vehicle. The conceptual design process is typically devoid of uncertainty quantification, and if done, it is usually only a crude assessment of global properties of the system. Of course, some of the primary challenges to performing UQ at the conceptual design stage are the lack of adequate representations of uncertainty, the lack of fidelity in the models representing the systems, and the need for an integrated multidisciplinary analysis environment. Although we feel that development of methods and tools to address conceptual level design and analysis may yield significant benefits, our background and experience is more aligned with application of UQ to the later design phases and will be the focus of the remainder of this document.

As mentioned, most UQ work is conducted after the design and models have reached a minimum level of fidelity and maturity. Most government and industry settings employ random sampling together with simulation to assess a given design for a set of assumed uncertainties. The approach, commonly referred to as Monte Carlo analysis, has been successfully applied to many systems and as such has the heritage necessary to serve, still today, as the baseline tool for the bulk of uncertainty quantification used within NASA and industry. Monte Carlo has at its core simplicity of implementation as it does not require rewriting source code and in fact can easily interface with most analysis codes. However, with this simplicity come fundamental limitations. One such limitation is the efficient identification of high-consequence low-probability events. This computational burden also makes it unsuitable for design, as most design procedures are based upon some type of search mechanism that requires repeated function evaluations, each requiring a separate Monte Carlo analysis. Furthermore, a significant limitation of Monte Carlo is that it requires the definition of probability density (joint or marginal) functions (PDF) for all uncertainties. A large amount of data is required to adequately prescribe a PDF-based uncertainty model. In many cases, sufficient data are not available and engineering judgment or expert opinion must be employed. This is particularly true for systems with limited or no heritage, cases in which UQ is performed utilizing models that have not been properly validated. Monte Carlo does not directly have the ability to access the impact of these modeling assumptions on the final UQ results. Yet another limitation of Monte Carlo is its inability to identify worst-case uncertainty combinations, to identify the dominant uncertain parameters and to evaluate the separation between the nominal operating conditions point and the failure event.

Another critical need within the aerospace community is for design tools and methods to synthesize robust aerospace vehicles in the presence of uncertainties. There has been some effort in this area, but many approaches simply cannot handle the computational burden associated with propagating uncertainties through computationally expensive models in an iterative design loop. This certainly poses a difficult challenge, and in no way are we suggesting that we can radically redefine the fundamental limitations, but over the past several years, we have developed various methods for efficiently propagating uncertainties though nonlinear models and feel that we are well-positioned to make significant progress in this area. The following is a list of some of the techniques that we have developed and successfully applied to robust control design problems: moment propagation methods for efficiently estimating statistical performance of closed-loop systems, formulation of reliability-based performance metrics and their use in control design, homothetic deformations for failure domain bounding together with conventional design optimization for robustness enhancement, and more recently adaptive response surface technology to alleviate some of the computational expense.

It is fully understood that models will always be uncertain and data too sparse to perfectly prescribe the physical phenomenon of interest, however a goal must be to explore and deploy alternate methods to conventional Monte Carlo analysis to better address its fundamental limitations.

New Tools in UQ

The primary analysis task in UQ is robustness and performance analysis. This task requires a set of analysis tools to efficiently assess robustness margins in the presence of parametric uncertainties. These uncertainties may be probabilistic in nature or may be defined by multidimensional bounded regions in the parameter space. Typical robustness and performance assessments may include computing: probability of failure, mean system response, standard deviations, or bounded regions for guaranteed safe operation similar to a flight envelope.

Recent work by the authors has resulted in a suite of efficient methods for uncertainty propagation and quantification of robustness for systems subject to parametric uncertainty. These unique tools can efficiently estimate the most likely behavior of the system, as well as the probability of rare, high-consequence events, as well as identify the most significant contributors to performance degradation (overall mission risk) caused by uncertainty. One such tool is failure set bounding as shown in Figure 1. The result in Figure 1 illustrates a system with two requirements (red and green lines), which separate the safe domain (white region) from the failure domain (gray region). This figure illustrates the maximal set, that for a given geometry and center, captures the maximal size object whose interior, by construction, resides in the safe domain. Techniques have been developed to use these maximal sets to compute upper bounds on failure probability, resulting in substantial computational saving over sampling-based methods. Maximal sets can also be used to determine the worst-case combinations of parameters, denoted by in Figure 1. Unlike Monte Carlo, the computational burden of failure set bounding is independent of failure probabilities, thereby making it ideally suited for aerospace applications.

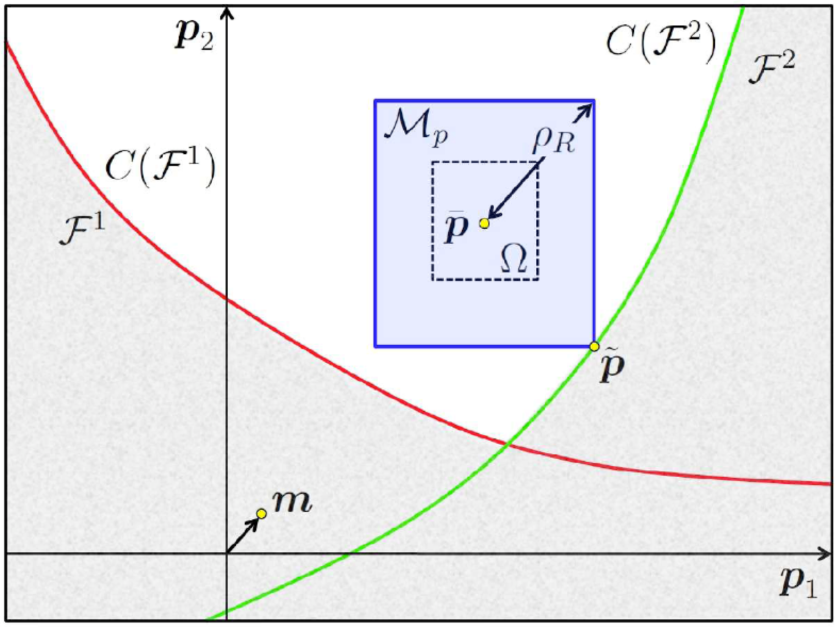

Figure 1: Failure Set Bounding in Two Dimensions

Design in the Presence of Uncertainty

Aerospace problems commonly involve complex, interdependent systems whose physics are multi-disciplinary in nature. Because of this complexity, the design of such systems, i.e., the process of prescribing the value of the system's design variables so that requirements are met, is usually performed using deterministic parameter realizations. Examples of these design variables are control gains, geometric properties, and those that characterize a particular structural material. Examples of their requirements are closed-loop stability, maximum allowable weight, and maximum allowable cost. The challenge faced by the designer is to find a design point that satisfies conflicting design objectives while being robust to uncertainties.

Since most current design practices do not directly accommodate for uncertainty it is quite possible that the performance degradation caused by it will render the design unacceptable. In such a case, the discipline expert faces the challenge of redesigning the system without having the knowledge, methods, and tools required to make the design sufficiently robust/reliable. This situation may lead to unnecessary design-analysis iterations and likely overly conservative solutions.

Ideally, a framework would be developed that integrates the design and analysis processes thereby eliminating the costs associated with the prolonged generation of a satisfactory design as well as those of operating an overly conservative solution, e.g., excessive weight, poor performance under nominal operating conditions, higher manufacturing costs, etc. Robust design tools will enable the discipline expert to systematically pursue engineering solutions with the optimal robust/reliability characteristics.

The integrated framework of the failure set bounding approach has been shown to facilitate robust design. This approach enables the discipline expert to pose the problems in a design-optimization setting from where engineering solutions with optimal robust/reliability characteristics can be systematically pursued. For a given uncertainty model, this would enable searching for the design points that (i) minimize the probability of violating the design requirements, (ii) minimize/maximize the expected value of a performance function, or (iii) minimize the variability in the system's performance for a range of uncertainties.

Figure 2 and Figure 3 present an example of robust design of a control system for the NASA Langley subscale Generic Transport Model (GTM). The colors displayed in these figures indicate the number of requirements that are violated at any parameter realization of pitch stiffness and roll damping uncertainty. The color green indicates zero requirements violations, progressing from yellow (one violation) to black (nine violations). Figure 2 shows a robustness analysis of the nominal controller for the GTM. Notice that the points labeled A and B indicate stable and unstable system responses respectively. Figure 3 presents a redesigned controller that robustly accommodate for the uncertainty. Notice that the redesigned controller has a much larger safe operating domain and that point B is now within the safe domain resulting in a controller with greatly improved robustness characteristics.

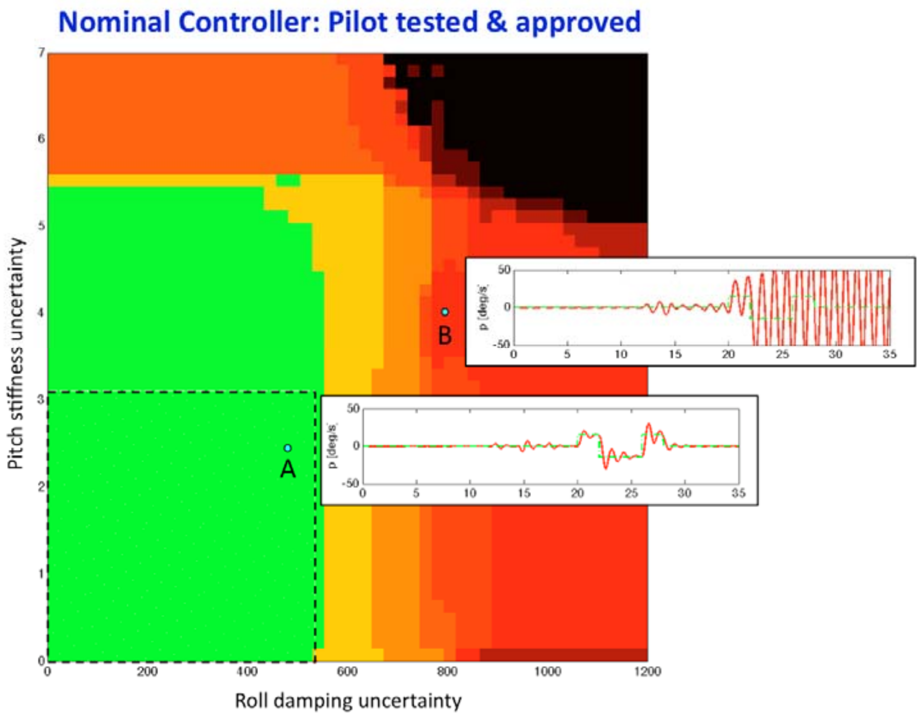

Figure 2: Before Robust Design

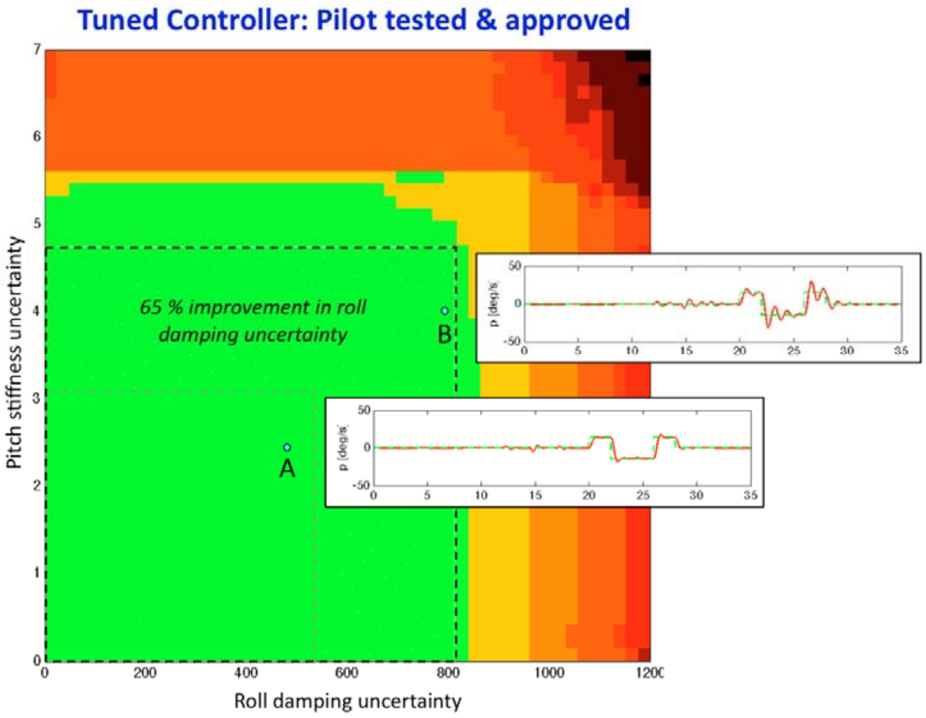

Figure 3: After Robust Design

The analysis and design of aerospace systems, where multiple uncertain parameters and various conflicting design requirements are present, are tasks of great practical importance that pose formidable challenges to the state of both the art and the practice. The aerospace industry as a whole will considerably benefit from the development of methods and tools that address these needs in a computationally efficient framework.

Relevant Recent Publications (2008-Present)

Journal Publications

1. Crespo L. G., Kenny S. P., Giesy D. P.; *A Computational Framework to Control Verification and Robustness Analysis*, NASA TP 2010-216189.
2. Kenny S. P.; Crespo L. G. and Giesy D. P.: Dimensionality Reduction for Uncertain Dynamic Systems, International Journal of Numerical Methods in Engineering, Special issue on Uncertainty Quantification and Prediction Science. Vol. 80, March 2009, pp. 767-788
3. Crespo, L.; Giesy, D. and Kenny, S.: Reliability-Based Analysis and Design via Failure Domain Bounding. Structural Safety, volume 31, 2009, 306-315.
4. Horta L. G., Kenny S. P., Crespo L. G., and Elliot K. B.: NASA Langley's Approach to the Sandias' Structural Dynamics Challenge Problem. Computer Methods in Applied Mechanics and Engineering. Volume 197, Issues 29-32, article 24, 1 May 2008, pages 2607-2620.
5. Crespo L. G., Giesy D. P., Kenny S. P.; *Robustness Analysis and Robust Design of Uncertain Systems*, AIAA Journal, Volume 46, Number 2 (February), 2008.

Conference Publications

1. Crespo L. G., Kenny S. P., Giesy D. P.; *Uncertainty Based Methods*, NASA Aviation Safety Conference, McLean, VA, November 17-19, 2009
2. Kenny S. P., Crespo L. G., Giesy D. P.; *Application of Dimensionality Reduction to a Large-Order Uncertain Dynamic System*, AIAA-2009-2300, AIAA Conference in Non-deterministic Approaches, Palm Springs, California, May 4-7, 2009.
3. Crespo L. G., Kenny S. P., Giesy D. P.; *Sampling-based Strategies for the Estimation of Probabilistic Sensitivities*, AIAA-2009-2283, AIAA Conference in Non-deterministic Approaches, Palm Springs, California, May 4-7, 2009.
4. Giesy D. P., Crespo L. G., Kenny S. P.; *Approximation of Failure Probability Using Conditional Sampling*, AIAA-2008-5946, AIAA/ISMO Multidisciplinary Analysis and Optimization Conference, British Columbia, Canada, September 10-12, 2008.
5. Crespo, L.; Kenny, S. and Giesy, D.: Figures of Merit for Control Verification. AIAA-2008-6339. Presented at the AIAA Guidance, Navigation and Control Conference and Exhibit, Honolulu, Hawaii, Aug. 18-21, 2008.
6. Crespo, L.; Kenny, S. and Giesy, D.: A Verification-driven Approach to Control Analysis and Tuning. AIAA-2008-6339. Presented at the AIAA Guidance, Navigation and Control Conference and Exhibit, Honolulu, Hawaii, Aug. 18-21, 2008.