

Similarity Rules for Scaling Solar Sail Systems

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Abstract

Future science missions will require solar sails on the order of 200 m² (or larger). However, ground demonstrations and flight demonstrations must be conducted at significantly smaller sizes, due to limitations of ground-based facilities and cost and availability of flight opportunities. For this reason, the ability to understand the process of scalability, as it applies to solar sail system models and test data, is crucial to the advancement of this technology. This paper will approach the problem of scaling in solar sail models by developing a set of scaling laws or similarity criteria that will provide constraints in the sail design process. These scaling laws establish functional relationships between design parameters of a prototype and model sail that are created at different geometric sizes. This work is applied to a specific solar sail configuration and results in three (four) similarity criteria for static (dynamic) sail models. Further, it is demonstrated that even in the context of unique sail material requirements and gravitational load of earth-bound experiments, it is possible to develop appropriate scaled sail experiments. In the longer term, these scaling laws can be used in the design of scaled experimental tests for solar sails and in analyzing the results from such tests.

Introduction

The NASA In-Space Propulsion (ISP) program funded the development of solar sail propulsion technology area through the near term verification and development of solar sail system level technology using ground-based testing and the development of subsystems, operations tools, as well as analytical and computational models. Solar sails can potentially provide low-cost propulsion and operate without the use propellant allowing access to non-Keplerian orbits through constant thrust. Solar sails consist of a gossamer structure, a membrane-based, large, lightweight structure. The primary objective of the sail is to convert solar pressure emitted into thrust on a spacecraft. This solar pressure is extremely small however, on the order of 9 Newtons per square kilometer at one AU. Therefore, the fundamental challenge in solar sail design is to create a sail area of significant size, with small enough system mass

to achieve reasonable accelerations of the spacecraft. NASA's solar sail propulsion technology area, managed by the In Space Propulsion Projects office at Marshall Space Flight Center, funded applied research and technology development based on the results of peer reviewed proposals for ground-based testing and design and analysis tools needed in order to advance solar sail's Technology Readiness Level (TRL). These activities include a focus on developing advanced modeling and simulation techniques for solar sail systems, and the development of a complete solar sail system via a pair of ground-based demonstrations that tested four-quadrant sail configurations as reported by Garbe et al. [1]. The objective of this work is to push the technology readiness level (TRL) of this type of propulsion system to a maximum level to prepare for flight demonstration. Flight demonstration will then lead to a system ready to perform a specific science mission. The science missions envisioned will require sails on the order 200 m^2 (or larger). However, ground demonstrations and flight demonstrations must be conducted at significantly smaller sizes, due to limitations of ground-based facilities and cost and availability of flight opportunities. For this reason, the ability to understand the process of scalability, as it applies to solar sail system models and test data, is crucial to the advancement of this technology. This paper will address issues of scaling in solar sail systems by developing a set of similarity or similitude functions that will guide the scaling process. The primary goal of these similarity functions (process invariants) that collectively form a set of scaling rules or guidelines is to establish valid relationships between models and experiments that are performed at different orders of scale. In the longer term, such an effort will help guide the size and properties of an experiment or test sail that would need to be flown to accurately represent a large, mission-level sail. Note that this paper will review a preliminary development of this set of similarity rules, based on simple mechanics models for the sail system. Further work in this area will need to be conducted to completely achieve the stated goals.

Background – Solar Sail Scaling

The ability to scale solar sail systems has been investigated by a number of researchers. For example, Holland et al. [2] observed a set of geometric scaling properties for inflatable structures while Greschik et al. [3] have investigated a specific geometric scaling technique based on constant thickness. Issues including geometric and dynamic scaling properties for solar sails are observed in a general sense by [4], and include pragmatic issues in sail packing and deployment. A general procedure for developing similarity rules, and applications to a variety of examples are presented in [5], [6] and [7]. This report adds to the current literature an effort to enumerate the

dimensionless parameters associated with the governing equations of a sail system, and to observe the degree to which experimental test results on a sail can be scaled.

Procedure

This paper will present the rules or criteria that should be considered in testing and analysis of solar sail system models. These rules, called similarity criteria, are derived by way of direct derivation from the governing equations of motion and boundary conditions for a sail system. The governing equations will be cast into dimensionless form and similarity criteria and characteristic parameters will be defined from these equations. This analysis will consider all of the similarity criteria that arise from the equations of motion considered, all phenomena that is included in the governing equations of motion employed here will be covered by the similarity criteria.

The motivation for this effort is to provide guidelines for the process of designing scale models for testing and evaluation of an actual sail system. Due to a number of issues, the testing will be performed on test sails that are many times smaller than a mission-size or actual sail (for example, a mission sail may have a defining dimensional size of 100 m, while the test articles may be on the order of 10 m). However, it is desirable that the response of the test sail will accurately predict or characterize the response of the mission-size sail. Therefore, the primary purpose of this work is to define the criteria, known as similarity criteria, which can be used to guide the proper design of scaled sail models for analysis and testing. This work will also be used to give a basis for the degree of scaling that can be practically applied to the results of a given test article. This work will be used to demonstrate some basic key functional relations of sail behavior based on sail size. Finally, the work will provide some simple guidelines that can be used in optimal sail design and will provide the starting point for one approach to sensitivity analysis.

The remainder of this paper will proceed as follows. The similarity criteria that apply to the sail will be first derived, followed by the criteria for the boom. The sail and boom models will then be combined through appropriate boundary conditions, resulting in a complete list of similarity criteria and corresponding characteristic terms. Once derived, the use of these similarity criteria will be discussed and demonstrated. Several examples will illustrate the use of similarity criteria in guiding the design of scaled models. An analysis of the extent of scaling that can be reasonably applied will be performed using the following process.

Sail Model

Consider a membrane model of the sail. This model assumes an isotropic sail material of constant thickness, h and pressure loading, q . This membrane model is correct when bending stiffness is negligible with respect to extensional stiffness and the deflection is on the order of material thickness (Von Karmen plate or Foppl membrane equations). As a result, wrinkling is not considered. The equations of motion for this membrane model of the sail are provided by Timoshenko [8] and Mierovitch [9] as,

$$F_{xxxx} + 2F_{xxyy} + F_{yyyy} = E((w_{xy})^2 - w_{xx}w_{yy}) \quad (1)$$

and

$$\frac{q}{h} + F_{yy}w_{xx} + F_{xx}w_{yy} - 2F_{xy}w_{xy} = \rho w_{tt} \quad (2)$$

where Eq. 1 defines compatibility while Eq. 2 defines Newtonian equilibrium and F is the stress function, w the sail deflection, E the sail material modulus, and ρ the sail density. The stress function is defined in terms of the in-plane loads as;

$$F_{yy} = N_x/h, \quad F_{xx} = N_y/h, \quad F_{yx} = -N_{xy}/h \quad (3)$$

with the N 's representing the stress resultants (in-plane loadings per length). Using the techniques found in [4], the governing equations are cast into a dimensionless form using the transformations,

$$\begin{aligned} x &= ax' \\ y &= ay' \end{aligned} \quad (4)$$

$$w = \sqrt[3]{\frac{q_c a^4}{hE}} w' \quad (5)$$

$$F = \sqrt[3]{\frac{Eq_c^2 a^8}{h^2}} F' \quad (6)$$

$$t = \sqrt[6]{\frac{\rho^3 h^2 a^4}{E q_c^2}} t' \quad (7)$$

$$q = q_c q' \quad (7b)$$

where the unprimed terms are dimensional and the primed terms are dimensionless. Note that this collection of parameters defined in Equations 4-7 is not unique. This selection identifies geometry (Eq. 4), deflection (Eq. 5), force (Eq. 6) and time (Eq. 7) as the primary parameters of interest in comparing sails design of various scales. The equations of motion can now be written in a dimensionless form,

$$F'_{x'x'x'x'} + 2F'_{x'x'y'y'} + F'_{y'y'y'y'} = (w'_{x'y'})^2 - w'_{x'x'} w'_{y'y'} \quad (8)$$

$$q' + F'_{y'y'} w'_{x'x'} + F'_{x'x'} w'_{y'y'} - 2F'_{x'y'} w'_{x'y'} = w'_{t't'} \quad (9)$$

These dimensional relations permit the deduction of the first sail dimensionless parameter for a four-quadrant sail (Fig. 1), as,

$$P_1 = \frac{b}{a} \quad (10)$$

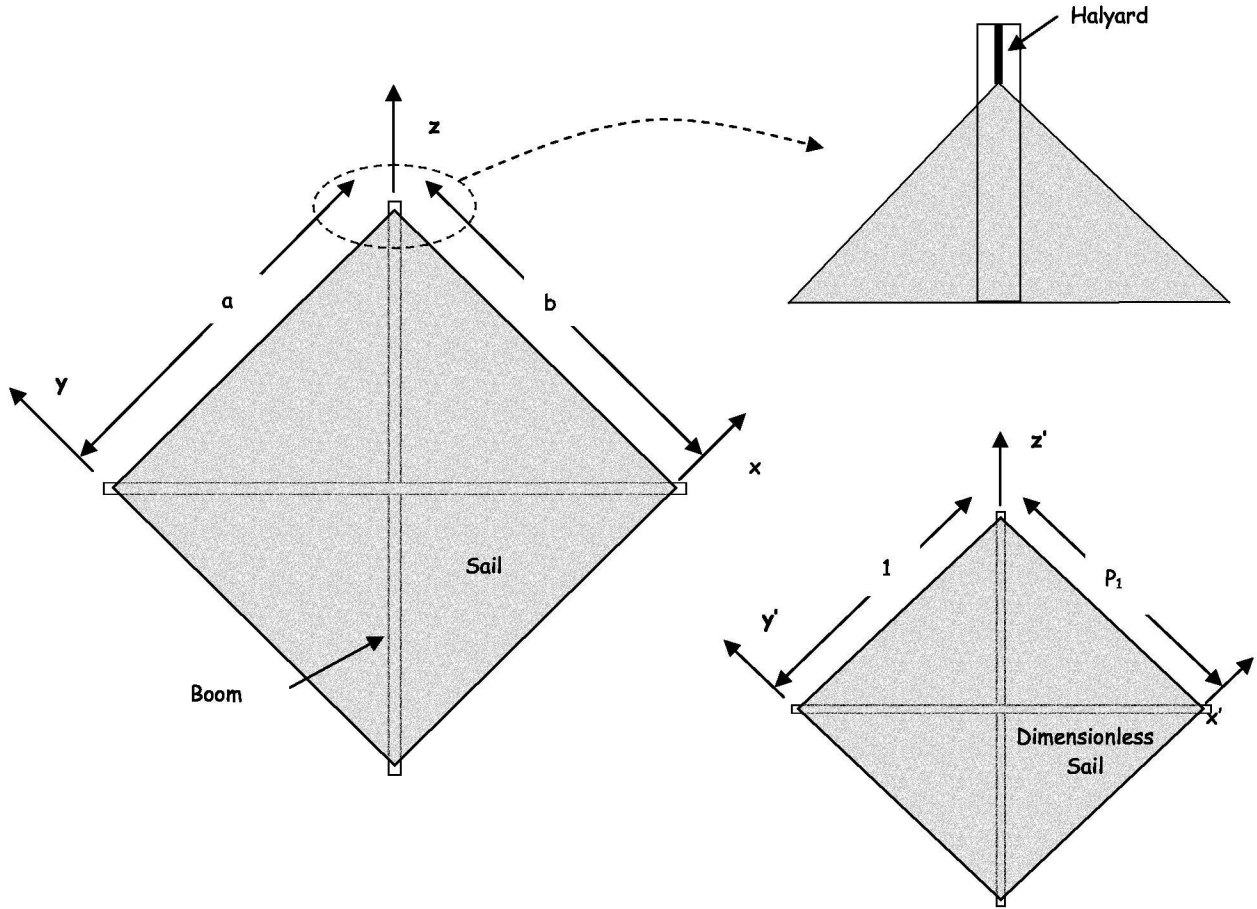


Figure 1: Four-Quadrant Sail Configuration

Equation (3) also serves to define the in-plane boundary conditions. On each boundary, two of three stress resultants provided in Eq. 3 would be specified. Again, following the procedure found in [5], the boundary conditions can be cast in a dimensionless form based using the transformations,

$$\begin{aligned}
 N_x &= N_{c,x} N_x' \\
 N_y &= N_{c,y} N_y' \\
 N_{xy} &= N_{c,xy} N_{xy}'
 \end{aligned}
 \tag{11}$$

(the N_c 's being characteristic boundary loads) to get,

$$\begin{aligned}
F'_{y'y'} &= P_2 N'_{x'} \\
F'_{x'x'} &= P_3 N'_{y'} \\
F'_{x'y'} &= P_3 N'_{xy'}
\end{aligned}
\tag{12}$$

where it is understood that Equations (12) apply on each boundary and

$$P_2 = \frac{N_{c,x}}{\sqrt[3]{Eh q_c^2 a^2}}, \tag{13}$$

$$P_3 = \frac{N_{c,y}}{\sqrt[3]{Eh q_c^2 a^2}}, \tag{14}$$

$$P_4 = \frac{N_{c,xy}}{\sqrt[3]{Eh q_c^2 a^2}}. \tag{15}$$

are additional dimensionless parameters. The conditions of similarity between a model and prototype can now be defined as,

$$P_{i,p} = P_{i,m}, \quad i = 1 \dots 4, \tag{16}$$

with the subscript i referring to the four criteria (Eqs. 10, 13, 14 and 15), the subscript p identifying the prototype and the subscript m identifying the model. Equating the dimensionless parameters of the model and prototype leads to the criteria,

$$\frac{b_m}{a_m} = \frac{b_p}{a_p} \tag{17}$$

$$\frac{N_{c,x,m}}{\sqrt[3]{E_m h_m q_{c,m}^2 a_m^2}} = \frac{N_{c,x,p}}{\sqrt[3]{E_p h_p q_{c,p}^2 a_p^2}}, \tag{18}$$

$$\frac{N_{c,y,m}}{\sqrt[3]{E_m h_m q_{c,m}^2 a_m^2}} = \frac{N_{c,y,p}}{\sqrt[3]{E_p h_p q_{c,p}^2 a_p^2}}, \quad (19)$$

$$\frac{N_{c,xy,m}}{\sqrt[3]{E_m h_m q_{c,m}^2 a_m^2}} = \frac{N_{c,xy,p}}{\sqrt[3]{E_p h_p q_{c,p}^2 a_p^2}}. \quad (20)$$

Alternatively, these relationships can be expressed as,

$$\frac{b_m}{b_p} = \frac{a_m}{a_p}. \quad (21)$$

$$\frac{N_{c,x,m}}{N_{c,x,p}} = \frac{N_{c,y,m}}{N_{c,y,p}} = \frac{N_{c,xy,m}}{N_{c,xy,p}} = \left(\frac{E_m}{E_p}\right)^{1/3} \left(\frac{h_m}{h_p}\right)^{1/3} \left(\frac{q_{c,m}}{q_{c,p}}\right)^{2/3} \left(\frac{a_m}{a_p}\right)^{2/3}. \quad (22)$$

One form of connection between the sail material and support boom are halyard cables connecting the sail corners to the support booms (figure 1). This form of connection will be considered in this paper. In practice, it may be more intuitive to consider tensile forces in the halyards rather than characteristic boundary loads, N .

The mapping between these boundary stresses (stress per sail-thickness) and halyard loading is determined by considering equilibrium of stresses on one of the two corner (boundary) elements. Figure 2 shows this corner element with loading from the halyard and boundary stresses on x and y faces.

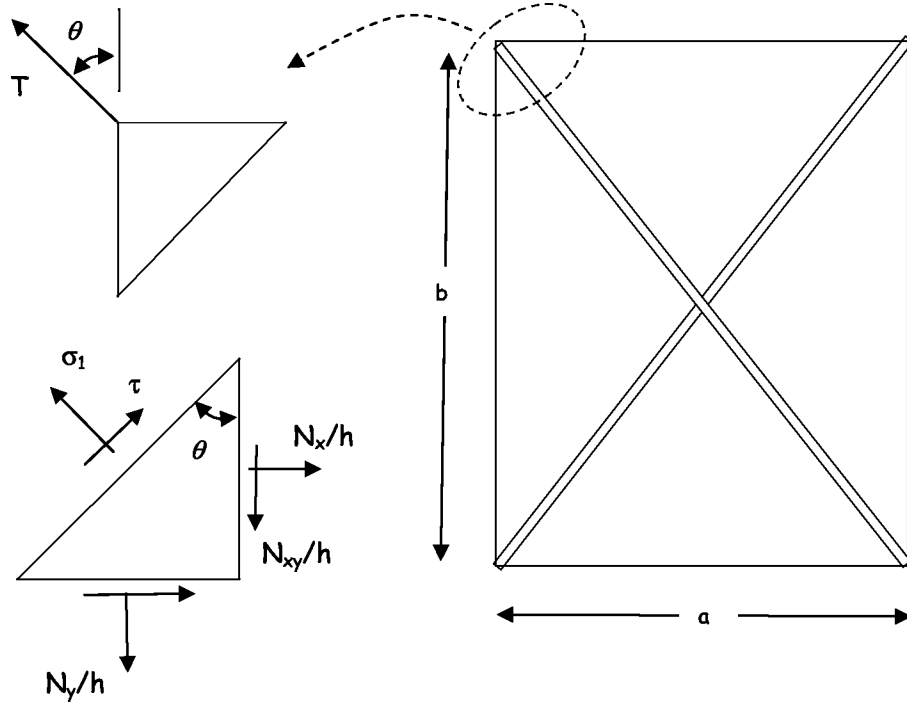


Figure 2: Stresses on a sail corner element due to halyard tension

The stress distribution resulting from the concentrated halyard load is defined by σ_x, σ_y, τ , (Timoshenko and Goodier [8]) as,

$$\sigma_x, \sigma_y, \tau \propto \frac{T}{ah} \quad (23)$$

where T is the tension in the halyard. Since geometric criteria (Eq. 17) will require the loading angle to be similar in prototype and model scale, the boundary characteristic term can be related to the halyard tension as,

$$N_{x,c}, N_{y,c}, N_{xy,c} \propto \frac{T}{a} \quad (24)$$

or

$$\frac{T_m}{T_p} = \left(\frac{E_m}{E_p} \right)^{1/3} \left(\frac{h_m}{h_p} \right)^{1/3} \left(\frac{q_{c,m}}{q_{c,p}} \right)^{2/3} \left(\frac{a_m}{a_p} \right)^{5/3} \quad (25)$$

Summary of Sail Material Criteria

Table I: Summary of Sail Model Criteria

| |
|---|
| $\frac{b_m}{b_p} = \frac{a_m}{a_p}$ |
| $\frac{N_{c,x,m}}{N_{c,x,p}} = \frac{N_{c,y,m}}{N_{c,y,p}} = \frac{N_{c,xy,m}}{N_{c,xy,p}} = \sqrt[3]{\frac{E_m}{E_p} \frac{h_m}{h_p} \left(\frac{q_{c,m}}{q_{c,p}}\right)^2 \left(\frac{a_m}{a_p}\right)^2}$ <p>(For distributed load along the sides of the sail)</p> |
| $\frac{T_m}{T_p} = \left(\frac{E_m}{E_p}\right)^{1/3} \left(\frac{h_m}{h_p}\right)^{1/3} \left(\frac{q_{c,m}}{q_{c,p}}\right)^{2/3} \left(\frac{a_m}{a_p}\right)^{5/3}$ <p>(For point load at the corner of the sail)</p> |

Collecting the results of the sail model yields a set of criteria for design sails of similarity. Table I summarizes these similarity criteria for the sail model.

Boom Model

The boom that supports the sail membrane is considered next. The boom will be initially considered independent of the sail and then combined with the sail criteria. The equation of motion for a general, uniform boom of length L that considers both flexural and axial rigidity is given as (see Meirovitch [9] for example)

$$YIv_{zzz} - Qv_{zz} = \gamma Av_{\tau\tau} \quad (26)$$

where z is the axial coordinate, Y is the Young's modulus of the boom, I is its area moment of inertia, v is the lateral deflection of the boom, γ is the boom density, A is the boom cross-sectional area, L is the boom length, and Q is the axial load on the boom. The sail also creates a point lateral load at the end of the boom of value V . Using the techniques found in [5], Eq. (26) is cast into a dimensionless form using the transformations,

$$z = Lz', \quad (27)$$

$$v = \frac{VL^3}{YI} v', \quad (28)$$

$$\tau = \sqrt{\frac{\gamma AL^4}{YI}} \tau'. \quad (29)$$

and the dimensionless parameter definition,

$$P_6 = \frac{QL^2}{YI}. \quad (30)$$

to get

$$v'_{z'z'z'} - P_6 v'_{z'z'} = v'_{\tau\tau}. \quad (31)$$

The boundary conditions for the boom consist of zero slope and deflection at the boom base, zero bending moment applied at the boom tip, and a known shear force V at the boom tip. This produces the non-zero boundary condition at the boom tip,

$$YIv_{zzz} = V. \quad (32)$$

Substituting Eqs. (27-29) into Eq. (32) produces the dimensionless boundary condition,

$$v'_{z'z'z'} = 1. \quad (35)$$

The conditions of similarity between a model and prototype boom can now be defined as,

$$P_{6,p} = P_{6,m} \quad (36)$$

or

$$\frac{Q_m}{Q_p} = \frac{Y_m I_m}{Y_p I_p} \left(\frac{L_p}{L_m} \right)^2 \quad (37)$$

Summary of Boom Model Criteria

Table II summarizes the similarity criteria that result from the boom model.

Table II: Summary of Boom Model Criteria

$$\frac{Q_m}{Q_p} = \frac{Y_m I_m}{Y_p I_p} \left(\frac{L_p}{L_m} \right)^2$$

Combining Sail and Boom Models

The boom and sail parameters are interconnected resulting in the following additional set of criteria for the system. The sail and boom must have common time and length scales. The former requires,

$$\sqrt{\frac{\rho^3 h^2 a^4}{Eq_c^2}} t' = \sqrt{\frac{\gamma AL^4}{YI}} \tau' \quad (38)$$

and the latter requires,

$$L = a \sqrt{1 + P_1^2} . \quad (39)$$

These two relationships yield an additional similarity criterion,

$$t' = \sqrt{P_7 (1 + P_1^2)} \tau' \quad (40)$$

with

$$P_7 = \left(\frac{\gamma}{\rho} \right) \left(\frac{A}{YI} \right) \left(\frac{Eq_c^2 a^4}{h^2} \right)^{\frac{1}{3}} . \quad (41)$$

which allows Eq. (31) to be rewritten as,

$$v'_{z'z'z'} - P_6 v'_{z'z'} = P_7 (1 + P_1^2) v'_{r'r}. \quad (42)$$

Similarly, the connectivity conditions between the sail corners and the boom tips will be selected as,

$$Q = T \quad (43)$$

for equal sail and boom in plane loads,

$$V = T w_z \quad (44)$$

for equal sail and boom transverse loads, and

$$v = w \quad (45)$$

for equal sail and boom transverse displacements. Equations (43), (44) and (45) lead to the respective results

$$\frac{T_m}{T_p} = \frac{Y_m I_m}{Y_p I_p} \left(\frac{a_p}{a_m} \right)^2, \quad (46)$$

$$w'_{z'} = P_8 \sqrt{1 + P_1^2}, \quad (47)$$

with

$$P_8 = \frac{V}{T} \sqrt[3]{\frac{Eh}{q_c a}}, \quad (48)$$

and

$$w'_{z'} = P_9 \sqrt{(1 + P_1^2)^3} v', \quad (49)$$

with

$$P_9 = \frac{V}{YI} \sqrt[3]{\frac{Eha^5}{q_c}}. \quad (50)$$

The additional conditions of similarity between a model and prototype that result for the combined sail and boom model can now be defined as,

$$P_{i,p} = P_{i,m}, i = 7, 8, 9 \quad (51)$$

which lead to the respective similarity criteria

$$\left(\frac{q_{c,m}}{q_{c,p}} \right) = \left(\frac{\rho_m}{\rho_p} \right)^{3/2} \left(\frac{\gamma_p}{\gamma_m} \right)^{3/2} \left(\frac{A_p}{A_m} \right)^{3/2} \left(\frac{Y_m I_m}{Y_p I_p} \right)^{3/2} \left(\frac{E_p}{E_m} \right)^{1/2} \left(\frac{h_m}{h_p} \right) \left(\frac{a_p}{a_m} \right)^2, \quad (52)$$

$$\frac{V_m}{V_p} = \left(\frac{q_{c,m}}{q_{c,p}} \right) \left(\frac{a_m}{a_p} \right)^2 \quad (53)$$

and

$$\frac{V_m}{V_p} = \left(\frac{Y_m I_m}{Y_p I_p} \right) \left(\frac{E_p}{E_m} \right)^{1/3} \left(\frac{h_p}{h_m} \right)^{1/3} \left(\frac{a_p}{a_m} \right)^{1/3} \left(\frac{q_{c,m}}{q_{c,p}} \right)^{1/3}. \quad (54)$$

Combining Eqs. (53) and (54) yields,

$$\left(\frac{q_{c,m}}{q_{c,p}} \right) = \left(\frac{Y_m I_m}{Y_p I_p} \right)^{3/2} \left(\frac{E_p}{E_m} \right)^{1/2} \left(\frac{h_p}{h_m} \right)^{1/2} \left(\frac{a_p}{a_m} \right)^{1/2} \quad (55)$$

Combining Eqs. (52) and (55) yields,

$$\left(\frac{A_m}{A_p}\right)^{3/2} = \left(\frac{\rho_m}{\rho_p}\right)^{3/2} \left(\frac{\gamma_p}{\gamma_m}\right)^{3/2} \left(\frac{h_m}{h_p}\right)^{3/2} \left(\frac{a_m}{a_p}\right)^{7/2}. \quad (56)$$

Summary of Sail/Boom System Model Criteria

Table III summarizes the similarity criteria that result from the sail system model.

Table III: Summary of Sail/Boom System Model Criteria

| | |
|----|--|
| C1 | $\frac{b_m}{b_p} = \frac{a_m}{a_p}$ |
| C2 | $\frac{T_m}{T_p} = \left(\frac{E_m}{E_p}\right)^{1/3} \left(\frac{h_m}{h_p}\right)^{1/3} \left(\frac{q_{c,m}}{q_{c,p}}\right)^{2/3} \left(\frac{a_m}{a_p}\right)^{5/3}$ |
| C3 | $\frac{T_m}{T_p} = \frac{Y_m I_m}{Y_p I_p} \left(\frac{a_p}{a_m}\right)^2$ |
| C4 | $\left(\frac{q_{c,m}}{q_{c,p}}\right) = \left(\frac{Y_m I_m}{Y_p I_p}\right)^{3/2} \left(\frac{E_p}{E_m}\right)^{1/2} \left(\frac{h_p}{h_m}\right)^{1/2} \left(\frac{a_p}{a_m}\right)^{11/2}$ |
| C5 | $\left(\frac{A_m}{A_p}\right)^{3/2} = \left(\frac{\rho_m}{\rho_p}\right)^{3/2} \left(\frac{\gamma_p}{\gamma_m}\right)^{3/2} \left(\frac{h_m}{h_p}\right)^{3/2} \left(\frac{a_m}{a_p}\right)^{7/2}$ |

Results:

Sail model design procedure.

Use of the similarity criteria developed above will now be demonstrated. First, use of the criteria to design a model sail are demonstrated for a static problem. The criteria, C1, C2, C3 and C4 from Table III apply. Due to availability of materials, it is assumed that the sail and boom material are fixed giving values for E , h , ρ and Y , γ for the prototype and model. Further, assume that the sail size will be defined by a . Then, C1 determines b and C2, C3, and C4 are considered to be three additional constraints. It can be shown that only two of these three constraints are independent. Thus, only two of the three remaining parameters, T_m , $q_{c,m}$ and I_m must be specified. For the dynamic case, the additional criteria, C5 applies and can be used to solve for the cross-sectional property of the beam, A_m (Table III).

Examples of Sail Scaling

The use of scaling laws for sail testing and prediction are demonstrated through a numerical example. In this example, a series of three model sails are specified based on a prototype and the scaling laws defined in this paper. Each model represents a sail of larger size (20 meter, 30 meter, 40 meters) that is designed to be similar to a prototype sail of 10 meters. The prototype and model sails are based on the four-quadrant sail configuration as shown in Fig. 1. Following the design procedure given heretofore, the three model sails are constructed as shown in Table 4. A prediction of the sail behavior, the predicted value of deflection at the sail centroid is provided as $w(pred)$ in Table IV based on Eq. 5. For comparison, each model sail is also modeled numerically using the commercial FEA package, ANSYS. For comparison, the predicted billow at the centroid of the sail is measured from the FEA model ($w(FEA)$) and compared with the predicted value ($w(predict)$) as shown in Table IV. Similarly, the predicted deflection at the end of the boom is measured from the FEA model ($v(FEA)$) and compared with the predicted value ($v(predict)$).

Table IV: Sail Scaling Example Data

| Sail system parameter | Prototype | Model 1 | Model 2 | Model 3 |
|-----------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| Specified parameters | | | | |
| a | 10 m | 20 m | 30 m | 40 m |
| E | 3.9e9 Pa | 3.9e9 Pa | 3.9e9 Pa | 3.9e9 Pa |
| h | 5.58e-6 m | 5.58e-6 m | 5.58e-6 m | 5.58e-6 m |
| ρ | | | ? kg/m ³ | ? kg/m ³ |
| Y | 7.854e-5 Pa | 7.854e-5 Pa | 7.854e-5 Pa | 7.854e-5 Pa |
| q | 9.0e-6 Pa | 9.0e-6 Pa | 9.0e-6 Pa | 9.0e-6 Pa |
| Calculated parameters | | | | |
| b | 10 m | 20 m | 30 m | 40 m |
| I | 7.854E-5 m ⁴ | 9.974E-5 m ⁴ | 4.411E-5 m ⁴ | 1.267E-5 m ⁴ |
| T | 1 N | 3.18 N | 6.24 N | 10.08 N |
| Predicted responses | | | | |
| $w(predict)$ | 2.47e-4 m | 6.23e-4 m | 10.7e-4 m | 15.7e-4 m |
| $w(FEA)$ | 2.47e-4 m | 6.26e-4 m | 10.8e-4 m | 15.8e-4m |
| % error | | 0.5% | 0.6% | 0.6% |
| $v(predict)$ | 1.70 e-9 m | 4.28 e-9 m | 7.36 e-9 m | 10.8 e-9 m |
| $v(FEA)$ | 1.70 e-9 m | 4.27 e-9 m | 7.35 e-9 m | 10.8 e-9 m |
| % error | | 0.3 % | 0.2 % | 0.4 % |

It can be seen from table 4 that there is very good agreement between the scaling rules developed in this paper and the numerical predictions. Other comparisons (not shown for the sake of brevity) involving sail only and boom

only systems yielded similar levels of agreement. Sample deflection plots of the numerical model are shown in Figs. 3-5 below.

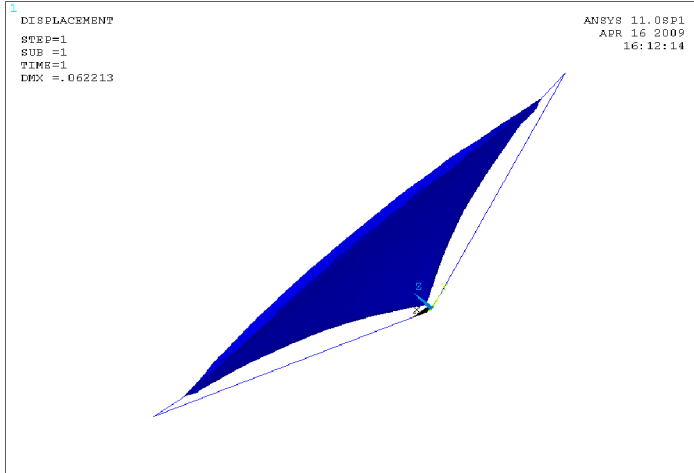


Fig. 3: 3-D view of Single Quadrant numerical Sail Model

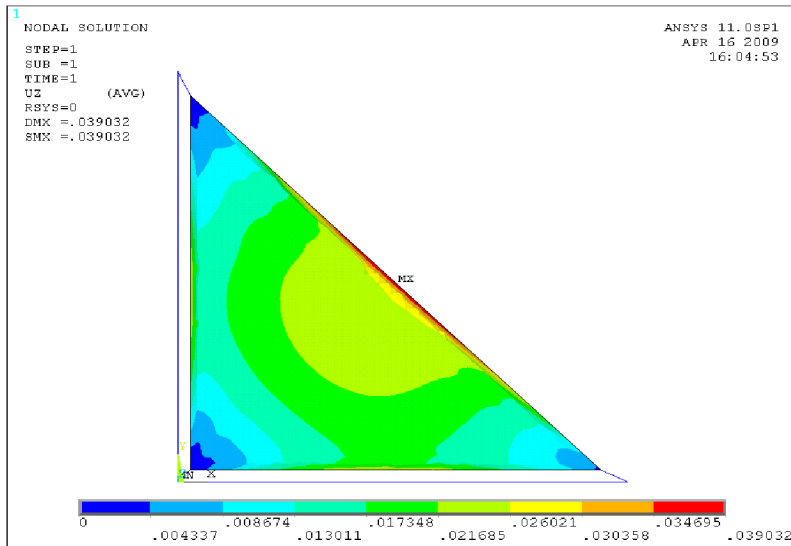


Fig. 4: Deflection plot of numerical Sail Model with deflection gradient (single Quadrant)

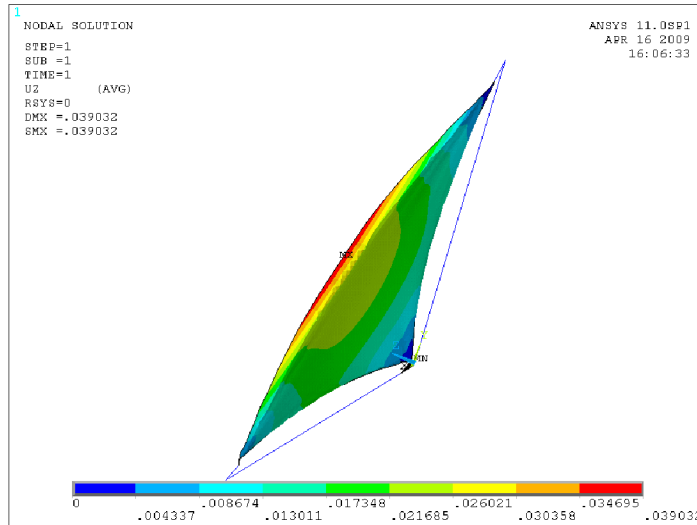


Fig. 5: 3-D view of deflection plot of numerical Sail Model with deflection gradient (single Quadrant)

Discussion and Conclusions

This paper has developed a set of fundamental scaling laws applicable to a class of solar sail systems. These scaling laws define the criteria that must be satisfied in designing a sail system that demonstrates similarity to a prototype sail. These scaling laws can be used in the design of scaled experimental test for solar sails and in analyzing the results from such tests. These similarity criteria are derived directly from the governing equations of motion and boundary conditions for a solar sail system model and result in five unique similarity criteria for the boom-sail system as well as a corresponding set of dimensionless numbers. Taken together, these criteria can guide the design of appropriately scaled models.

The results and application of these scaling laws were provided. It was shown that when considering the static sail case, three scaling laws must be considered to achieve similarity with a prototype sail. A set of three sufficient design parameters were demonstrated for a typical sail scenario. When considering the dynamic sail case, four scaling laws must be satisfied requiring a minimum of four design parameters. The sufficiency of these design parameters in maintaining similarity among sails of various sizes was demonstrated. Three model sails were

designed using the proposed scaling laws based on a prototype sail. The predicted response of the sails was demonstrated to be consistent with a corresponding finite-element analysis model of each sail.

It was demonstrated that for a fairly common situation in which many of the design parameters are defined by other constraints on material and facilities, it is possible and reasonable to create scaled sail models around the restrictions of an earth-bound experiment. In particular, the example demonstrates that unique sail properties and gravitational loads may be accommodated for in the static design.

The scaling laws developed in this paper were derived based on specific assumptions of the nature of the sail/boom system. In particular, the resulting criteria were based on a specific model of the connection between the sail and boom, in this case through a halyard of infinitesimal length with the ability to provide pretension. It should be noted that alternative scale laws could be developed following the procedures demonstrated in this paper for sail models that depart from the assumptions made in this paper.

References

- [1] Garbe, G., Montgomery, E., Heaton, A., Van Sant, J., and Campbell, B., "NASA's Integrated Development of Solar Sail Propulsion," AAS 04-103, *14th AAS/AIAA Space Flight Mechanics Meeting*, 2004
- [2] Holland, D.B., Tinker, M.L., Virgin, L.N., and K.N. Slade, "Geometric Scaling Properties of Inflatable Structures for Use in Space Solar Power Generation," *43rd AIAA Structures, Structural Dynamics, and Materials Conference*, AIAA-2002-1264, April 2002.
- [3] Greschik, G., Mikulas, M.M., and R.E. Freeland, "The Nodal Concept of Deployment and the Scale Model Testing of its Application to a Membrane Antenna," *40th AIAA Structures, Structural Dynamics, and Materials Conference*, AIAA-99-1523, April 1999.
- [4] Murphy, D.M and T.W. Murphey, "Scalable Solar Sail Subsystem Design Considerations," *43rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, AIAA-2002-1703, April 2002.
- [5] Staicu, C. I., *Restricted and General Dimensional Analysis*, Abacus Press, Tunbridge Wells, Kent, 1982.

- [6] Taylor, E. S., *Dimensional Analysis for Engineers*, Clarendon Press, Oxford, 1974.
- [7] Timoshenko, S. and S. Woinowsky-Krieger, *Theory of Plates and Shells*, 2nd ed., McGraw-Hill, New York, 1959.
- [8] Timoshenko, S. and J. N. Goodier, *Theory of Elasticity*, second Ed., McGraw-Hill, New York, 1951.
- [9] Meirovitch, L., *Principles and Techniques of Vibrations* Prentice Hall, New Jersey, 1997.