Demonstrating the Safety and Reliability of a New System or Spacecraft: Incorporating Analyses and Reviews of the Design and Processing in Determining the Number of Tests to be Conducted

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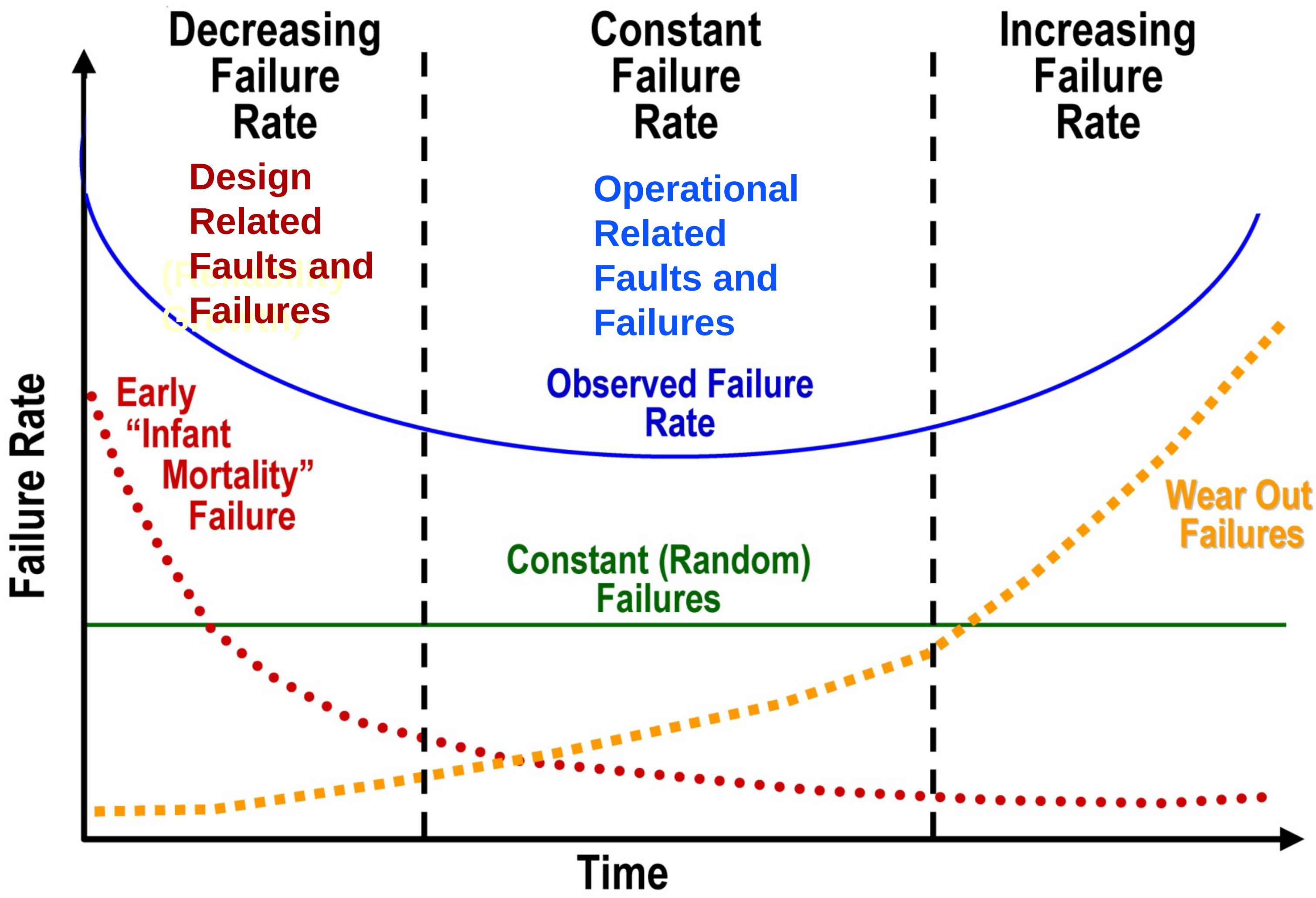
Presentation Outline

- Demonstrating Design Safety/Reliability
- Failure Rate Bathtub Curve
- Reliability-Growth-Based Testing Requirements
- Treatment of Uncertainties
- Benefits and Costs of Testing Strategies
- Summary
- Supplemental Slides
- Annotated References

Demonstrating Design Safety and Reliability

- Design safety and reliability is the probability that a new system has no failure-causing faults
- Design tests focus on detecting existing failure-causing faults
- Design tests can be partial tests or complete system tests and can consist of test flights
- In addition to design-related failures, random failures can occur
- The random failure contribution is generally associated with the steady-state operation of the mature system

The Failure-Rate Bathtub Curve

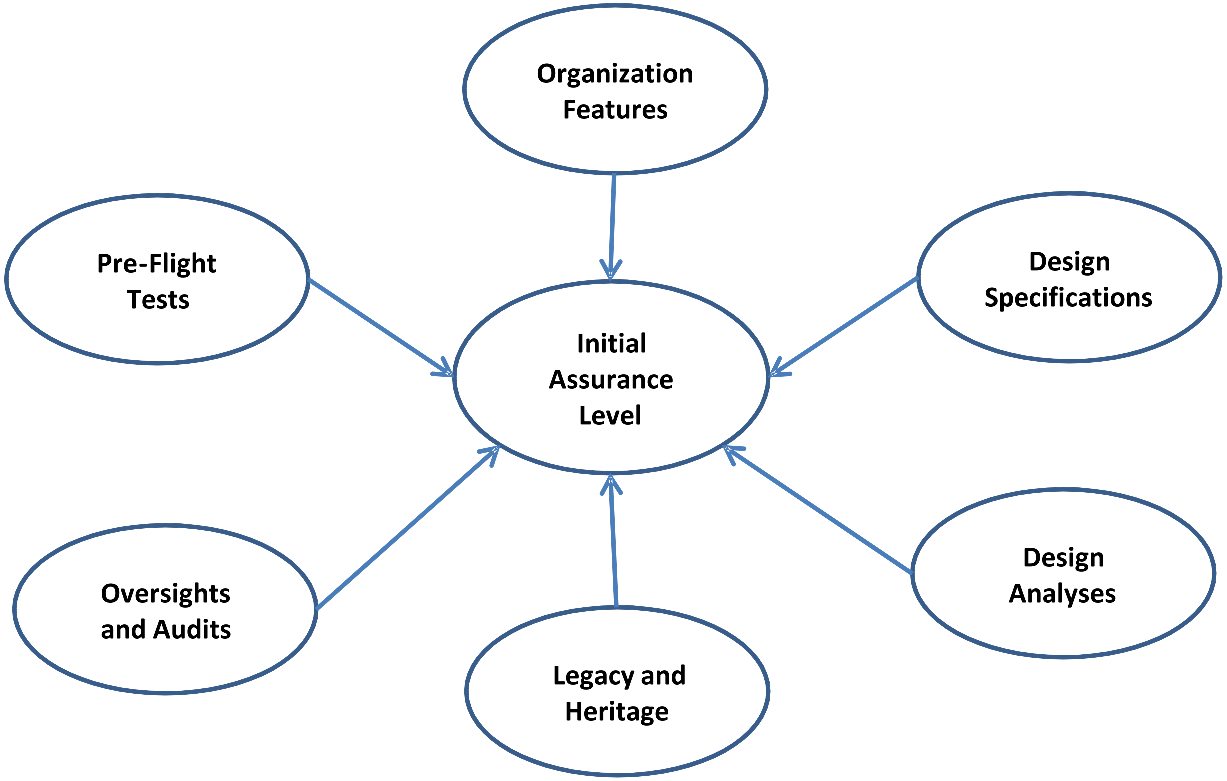


Reliability-Growth Principles Provide Required Numbers of Failure-Free Tests

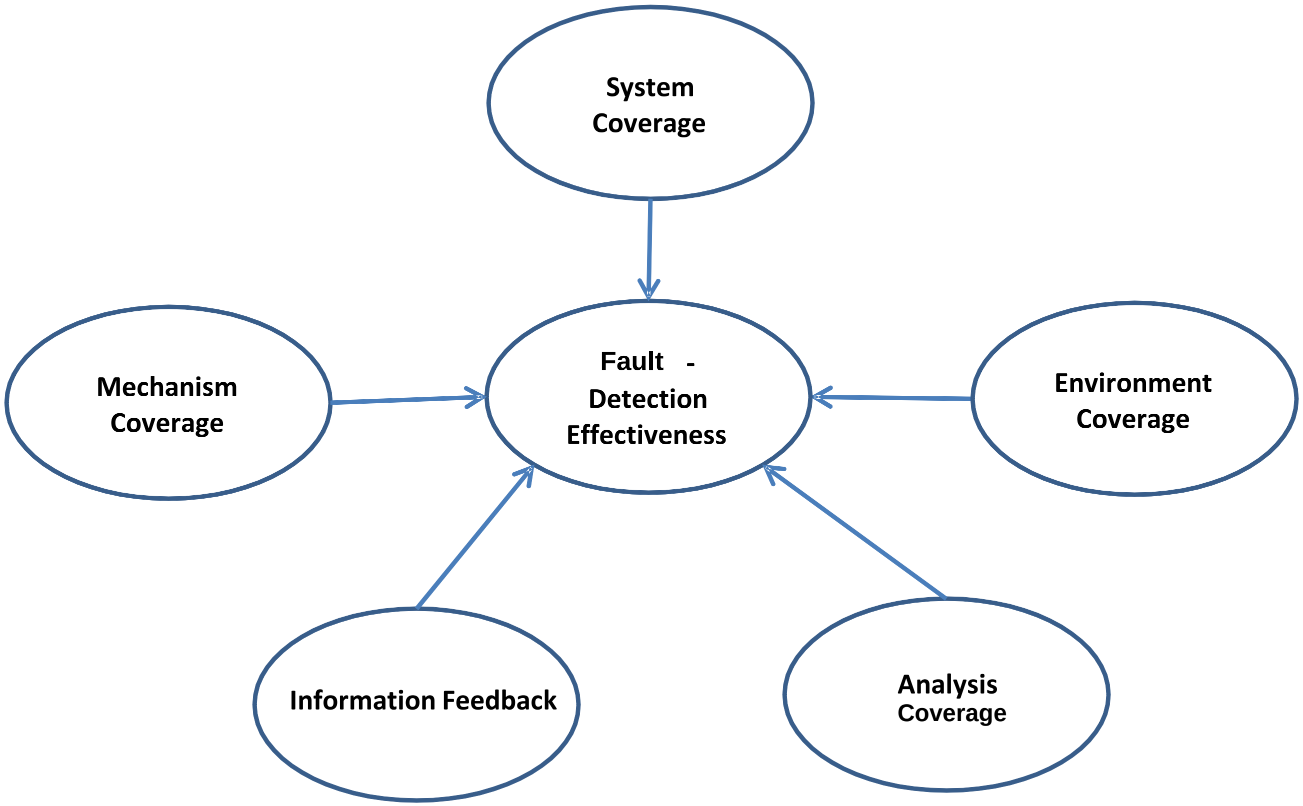
- Design safety and reliability is demonstrated by conducting sufficient tests without failure
- Based on reliability-growth principles, the required number of tests depends on three major factors:*
  - Initial System Assurance Level
  - Fault-Detection Effectiveness
  - Corrective Action Effectiveness
- Failures are handled by including corrective action effectiveness in the test requirements
- Binomial testing requirements are not applicable since failure correction and test feedback are not considered

*See the references for further background

Factors Determining Initial Assurance Level



Factors Determining Fault Detection Effectiveness

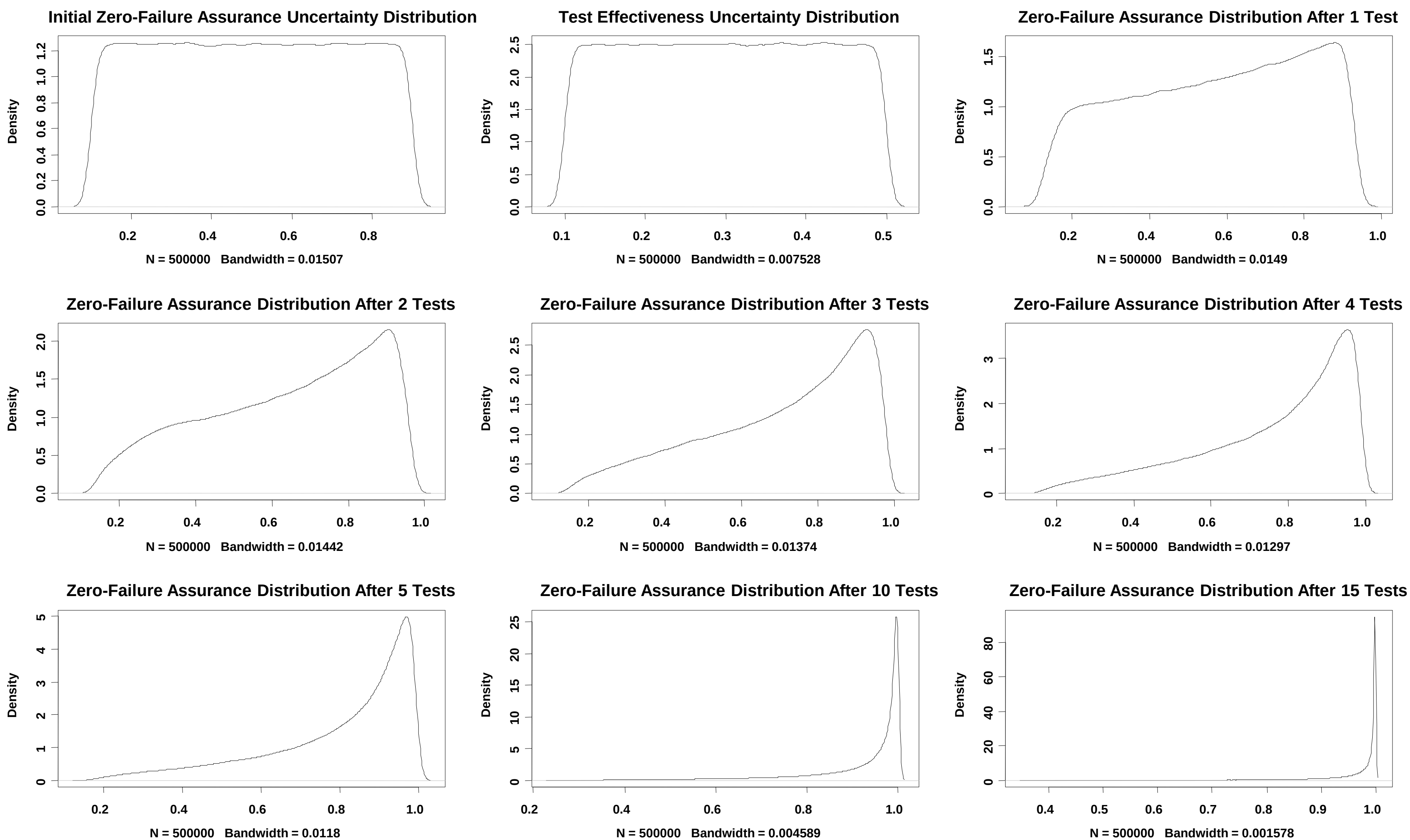


Required Reliability-Growth-Based Failure-Free Tests*

- The three tables in the supplemental slides give required failure-free tests to demonstrate a given system reliability
- The first table shows the value of having a high initial assurance with much fewer demonstration tests needed
- The second table shows that inapplicable binomial testing requirements are generally much higher than reliability-growth-based testing requirements
- The third table shows the effect of increased fault detection effectiveness in decreasing required numbers of tests
- To include uncertainties, lower bounds on the initial assurance level and fault detection effectiveness are used

*See the supplemental slides for the formulas and the references for further background.

Uncertainty Evaluations for Large Uncertainties on the Initial Assurance Level and Fault Detection Effectiveness



Evaluating the Benefits and Costs of Subsystem and System Test Strategies

	System Reliability with Subsystem Tests	System Failure Probability with Subsystem Tests	System Reliability with Subsystem and System Tests	System Failure Probability with Subsystem and System Tests	Total Subsystem Test Cost	Total System Test Cost	Total Subsystem Plus System Test Cost
Before Tests	49.00%	51.00%	90.09%	9.91%	120	300	420
After Tests	90.09%	9.91%	98.64%	1.36%			

	Subsystem 1	Subsystem 2	Subsystem 3	Subsystem 4	System
PreTest Reliability	70.00%	70.00%	100.00%	100.00%	90.09%
Number of Tests	3	3	1	1	3
Test Effectiveness	0.5	0.5	0.5	0.5	0.5
Post Test Reliability	94.92%	94.92%	100.00%	100.00%	98.64%
Post Test Failure Probability	5.08%	5.08%	0.00%	0.00%	1.36%
Cost per Test	20	20			100
Total Test Cost	60	60	0	0	300

Determining Reliability-Growth-Based Testing Requirements in Practice

- The initial assurance level and fault detection effectiveness are assessed
- The determining factors are assessed and are combined
- Grading criteria are defined for each factor
- Historical values are incorporated
- Uncertainties are treated
- Robust test requirements are determined
- Required tests are based on applicable reliability factors and not on inapplicable binomial lot sampling tables

Initial Failure History from 1960 to 2000 for Space Launch Vehicles for the First Five Launches*

Launch Number	1	2	3	4	5
Attempts	41	40	38	36	34
Failures	13	10	6	6	7
Mean Failure Rate	0.32	0.25	0.16	0.17	0.21
Standard Deviation	0.471	0.439	0.370	0.378	0.410
Bayesian Mean	0.33	0.26	0.18	0.18	0.22
95% Bayesian Interval	(0.20, 0.47)	(0.14, 0.40)	(0.08, 0.31)	(0.08, 0.32)	(0.10, 0.37)

***Seth D. Guikema and M. Elisabeth Pate-Cornell, "Probability of Infancy Problems for Space Launch Vehicles", Reliability Engineering and System Safety 87, March 2005, pp.303-314**

Summary

- **Design Safety/Reliability is Associated with the Probability of No Failure-Causing Faults Existing in a Design**
- **Confidence in the Non-Existence of Failure-Causing Faults is Increased by Performing Tests with No Failure**
- **Reliability-Growth Testing Requirements Are Based on Initial Assurance and Fault Detection Probability**
- **Using Binomial Tables Generally Gives Too Many Required Tests Compared to Reliability-Growth Requirements**
- **Reliability-Growth Testing Requirements are Based on Reliability Principles and Factors and Should Be Used**

Supplemental Slides

Reliability-Growth-Based Failure-Free Tests to Demonstrate a Given Design Reliability Versus Initial Assurance Level

Detection Effectivene	25%																		
	Initial System Assurance Level																		
Initial Assurance	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%
Failure-Free Tests	Probability That the System is Failure-Free After Conducting a Given Number of Failure-Free Tests																		
1	6.557%	12.903%	19.048%	25.000%	30.769%	36.364%	41.791%	47.059%	52.174%	57.143%	61.972%	66.667%	71.233%	75.676%	80.000%	84.211%	88.312%	92.308%	96.203%
2	8.556%	16.495%	23.881%	30.769%	37.209%	43.243%	48.908%	54.237%	59.259%	64.000%	68.482%	72.727%	76.753%	80.576%	84.211%	87.671%	90.970%	94.118%	97.125%
3	11.092%	20.847%	29.493%	37.209%	44.138%	50.394%	56.070%	61.244%	65.979%	70.330%	74.340%	78.049%	81.489%	84.688%	87.671%	90.459%	93.071%	95.522%	97.828%
4	14.262%	25.990%	35.804%	44.138%	51.303%	57.528%	62.988%	67.815%	72.113%	75.964%	79.436%	82.581%	85.443%	88.059%	90.459%	92.670%	94.712%	96.604%	98.362%
5	18.153%	31.890%	42.649%	51.303%	58.414%	64.362%	69.410%	73.749%	77.517%	80.821%	83.741%	86.341%	88.670%	90.769%	92.670%	94.400%	95.981%	97.431%	98.766%
6	22.823%	38.435%	49.787%	58.414%	65.192%	70.657%	75.158%	78.929%	82.134%	84.891%	87.289%	89.393%	91.255%	92.913%	94.400%	95.740%	96.955%	98.061%	99.072%
7	28.279%	45.427%	56.934%	65.192%	71.406%	76.251%	80.135%	83.318%	85.974%	88.224%	90.154%	91.828%	93.294%	94.589%	95.740%	96.771%	97.699%	98.539%	99.302%
8	34.457%	52.603%	63.804%	71.406%	76.903%	81.064%	84.322%	86.944%	89.098%	90.900%	92.429%	93.743%	94.885%	95.886%	96.771%	97.558%	98.264%	98.900%	99.476%
9	41.210%	59.674%	70.152%	76.903%	81.616%	85.092%	87.762%	89.877%	91.594%	93.016%	94.212%	95.233%	96.114%	96.882%	97.558%	98.157%	98.692%	99.173%	99.606%
10	48.310%	66.365%	75.809%	81.616%	85.548%	88.386%	90.532%	92.211%	93.560%	94.669%	95.595%	96.382%	97.057%	97.643%	98.157%	98.612%	99.016%	99.378%	99.704%
11	55.479%	72.458%	80.689%	85.548%	88.754%	91.029%	92.727%	94.042%	95.091%	95.948%	96.660%	97.261%	97.776%	98.222%	98.612%	98.955%	99.260%	99.533%	99.778%
12	62.428%	77.816%	84.782%	88.754%	91.322%	93.118%	94.444%	95.464%	96.273%	96.930%	97.474%	97.932%	98.323%	98.661%	98.955%	99.214%	99.444%	99.649%	99.834%
13	68.900%	82.385%	88.135%	91.322%	93.347%	94.748%	95.774%	96.559%	97.178%	97.679%	98.093%	98.441%	98.737%	98.992%	99.214%	99.410%	99.583%	99.737%	99.875%
14	74.708%	86.180%	90.829%	93.347%	94.926%	96.008%	96.797%	97.397%	97.869%	98.249%	98.563%	98.826%	99.050%	99.242%	99.410%	99.557%	99.687%	99.802%	99.906%
15	79.751%	89.264%	92.960%	94.926%	96.145%	96.976%	97.578%	98.035%	98.393%	98.681%	98.918%	99.117%	99.286%	99.431%	99.557%	99.667%	99.765%	99.852%	99.930%
16	84.003%	91.726%	94.626%	96.145%	97.081%	97.715%	98.173%	98.519%	98.790%	99.008%	99.187%	99.336%	99.463%	99.572%	99.667%	99.750%	99.823%	99.889%	99.947%
17	87.503%	93.663%	95.914%	97.081%	97.795%	98.276%	98.623%	98.885%	99.090%	99.254%	99.389%	99.501%	99.597%	99.679%	99.750%	99.812%	99.868%	99.917%	99.960%
18	90.325%	95.171%	96.904%	97.795%	98.337%	98.702%	98.964%	99.161%	99.316%	99.439%	99.541%	99.626%	99.697%	99.759%	99.812%	99.859%	99.901%	99.937%	99.970%
19	92.564%	96.334%	97.660%	98.337%	98.747%	99.023%	99.221%	99.370%	99.486%	99.579%	99.655%	99.719%	99.773%	99.819%	99.859%	99.894%	99.925%	99.953%	99.978%
20	94.317%	97.225%	98.235%	98.747%	99.058%	99.265%	99.415%	99.527%	99.614%	99.684%	99.741%	99.789%	99.830%	99.864%	99.894%	99.921%	99.944%	99.965%	99.983%
21	95.676%	97.904%	98.670%	99.058%	99.292%	99.448%	99.560%	99.645%	99.710%	99.763%	99.806%	99.842%	99.872%	99.898%	99.921%	99.941%	99.958%	99.974%	99.987%
22	96.722%	98.420%	98.999%	99.292%	99.468%	99.586%	99.670%	99.733%	99.782%	99.822%	99.854%	99.881%	99.904%	99.924%	99.941%	99.955%	99.969%	99.980%	99.991%
23	97.521%	98.810%	99.248%	99.468%	99.600%	99.689%	99.752%	99.800%	99.837%	99.866%	99.891%	99.911%	99.928%	99.943%	99.955%	99.967%	99.976%	99.985%	99.993%
24	98.129%	99.105%	99.435%	99.600%	99.700%	99.766%	99.814%	99.850%	99.878%	99.900%	99.918%	99.933%	99.946%	99.957%	99.967%	99.975%	99.982%	99.989%	99.995%
25	98.590%	99.327%	99.575%	99.700%	99.775%	99.825%	99.860%	99.887%	99.908%	99.925%	99.938%	99.950%	99.959%	99.968%	99.975%	99.981%	99.987%	99.992%	99.996%
26	98.939%	99.495%	99.681%	99.775%	99.831%	99.868%	99.895%	99.915%	99.931%	99.944%	99.954%	99.962%	99.970%	99.976%	99.981%	99.986%	99.990%	99.994%	99.997%
27	99.202%	99.620%	99.761%	99.831%	99.873%	99.901%	99.921%	99.937%	99.948%	99.958%	99.965%	99.972%	99.977%	99.982%	99.986%	99.989%	99.993%	99.995%	99.998%
28	99.400%	99.715%	99.820%	99.873%	99.905%	99.926%	99.941%	99.952%	99.961%	99.968%	99.974%	99.979%	99.983%	99.986%	99.989%	99.992%	99.994%	99.996%	99.998%
29	99.550%	99.786%	99.865%	99.905%	99.929%	99.944%	99.956%	99.964%	99.971%	99.976%	99.981%	99.984%	99.987%	99.990%	99.992%	99.994%	99.996%	99.997%	99.999%
30	99.662%	99.840%	99.899%	99.929%	99.946%	99.958%	99.967%	99.973%	99.978%	99.982%	99.985%	99.988%	99.990%	99.992%	99.994%	99.996%	99.997%	99.998%	99.999%

Binomial-Based Test Requirements Are Generally Much Too Large Compared to Reliability-Growth-Based Test Requirements

Number of Tests	Binomial (50%)	Initial Assurance Level: Test Effectiveness=25%						
		5%	10%	25%	50%	75%	90%	95%
1	25.000%	6.557%	12.903%	30.769%	57.143%	80.000%	92.308%	96.203%
2	50.000%	8.556%	16.495%	37.209%	64.000%	84.211%	94.118%	97.125%
3	62.996%	11.092%	20.847%	44.138%	70.330%	87.671%	95.522%	97.828%
4	70.711%	14.262%	25.990%	51.303%	75.964%	90.459%	96.604%	98.362%
5	75.786%	18.153%	31.890%	58.414%	80.821%	92.670%	97.431%	98.766%
6	79.370%	22.823%	38.435%	65.192%	84.891%	94.400%	98.061%	99.072%
7	82.034%	28.279%	45.427%	71.406%	88.224%	95.740%	98.539%	99.302%
8	84.090%	34.457%	52.603%	76.903%	90.900%	96.771%	98.900%	99.476%
9	85.724%	41.210%	59.674%	81.616%	93.016%	97.558%	99.173%	99.606%
10	87.055%	48.310%	66.365%	85.548%	94.669%	98.157%	99.378%	99.704%
11	88.159%	55.479%	72.458%	88.754%	95.948%	98.612%	99.533%	99.778%
12	89.090%	62.428%	77.816%	91.322%	96.930%	98.955%	99.649%	99.834%
13	89.885%	68.900%	82.385%	93.347%	97.679%	99.214%	99.737%	99.875%
14	90.572%	74.708%	86.180%	94.926%	98.249%	99.410%	99.802%	99.906%
15	91.172%	79.751%	89.264%	96.145%	98.681%	99.557%	99.852%	99.930%
16	91.700%	84.003%	91.726%	97.081%	99.008%	99.667%	99.889%	99.947%
17	92.169%	87.503%	93.663%	97.795%	99.254%	99.750%	99.917%	99.960%
18	92.587%	90.325%	95.171%	98.337%	99.439%	99.812%	99.937%	99.970%
19	92.964%	92.564%	96.334%	98.747%	99.579%	99.859%	99.953%	99.978%
20	93.303%	94.317%	97.225%	99.058%	99.684%	99.894%	99.965%	99.983%
21	93.612%	95.676%	97.904%	99.292%	99.763%	99.921%	99.974%	99.987%
22	93.893%	96.722%	98.420%	99.468%	99.822%	99.941%	99.980%	99.991%
23	94.151%	97.521%	98.810%	99.600%	99.866%	99.955%	99.985%	99.993%
24	94.387%	98.129%	99.105%	99.700%	99.900%	99.967%	99.989%	99.995%
25	94.606%	98.590%	99.327%	99.775%	99.925%	99.975%	99.992%	99.996%
26	94.808%	98.939%	99.495%	99.831%	99.944%	99.981%	99.994%	99.997%
27	94.995%	99.202%	99.620%	99.873%	99.958%	99.986%	99.995%	99.998%
28	95.170%	99.400%	99.715%	99.905%	99.968%	99.989%	99.996%	99.998%
29	95.332%	99.550%	99.786%	99.929%	99.976%	99.992%	99.997%	99.999%
30	95.484%	99.662%	99.840%	99.946%	99.982%	99.994%	99.998%	99.999%

Reliability-Growth-Based Failure-Free Tests to Demonstrate a Given Design Reliability Versus Fault Detection Effectiveness

Probability That a System is Free of Failures After a Given Number of Failure-Free Tests Have Been Conducted With Each Test Having a Given Detection Effectiveness For a Given Initial System Assurance Level

Initial System Assurance Level: 30%

Alternative Detection Effectiveness Values (Detection Effectiveness Value) / Number of Failure-Free Tests Conducted	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%
	Probability the System is Failure-Free After Conducting a Given Number of Failure-Free Tests																		
1	31.0881%	32.2581%	33.5196%	34.8837%	36.3636%	37.9747%	39.7351%	41.6667%	43.7956%	46.1538%	48.7805%	51.7241%	55.0459%	58.8235%	63.1579%	68.1818%	74.0741%	81.0811%	89.5522%
2	32.1975%	34.6021%	37.2324%	40.1070%	43.2432%	46.6563%	50.3567%	54.3478%	58.6224%	63.1579%	67.9117%	72.8155%	77.7706%	82.6446%	87.2727%	91.4634%	95.0119%	97.7199%	99.4200%
3	33.3273%	37.0233%	41.1022%	45.5650%	50.3937%	55.5453%	60.9462%	66.4894%	72.0353%	77.4194%	82.4657%	87.0070%	90.9056%	94.0734%	96.4824%	98.1675%	99.2187%	99.7672%	99.9708%
4	34.4766%	39.5116%	45.0854%	51.1317%	57.5281%	64.0930%	70.5958%	76.7813%	82.4053%	87.2727%	91.2674%	94.3634%	96.6170%	98.1451%	99.0968%	99.6281%	99.8820%	99.9767%	99.9985%
5	35.6444%	42.0555%	49.1325%	56.6705%	64.3620%	71.8307%	78.6946%	84.6425%	89.4908%	93.2039%	95.8721%	97.6664%	98.7893%	99.4362%	99.7727%	99.9254%	99.9823%	99.9977%	99.9999%
6	36.8295%	44.6423%	53.1910%	62.0475%	70.6572%	78.4613%	85.0356%	90.1824%	93.9330%	96.4824%	98.0993%	99.0533%	99.5729%	99.8302%	99.9431%	99.9851%	99.9973%	99.9998%	100.0000%
7	38.0307%	47.2584%	57.2078%	67.1440%	76.2508%	83.8814%	89.7356%	93.8686%	96.5695%	98.2097%	99.1356%	99.6192%	99.8501%	99.9490%	99.9858%	99.9970%	99.9996%	100.0000%	100.0000%
8	39.2468%	49.8897%	61.1317%	71.8665%	81.0638%	88.1436%	93.0795%	96.2287%	98.0836%	99.0968%	99.6092%	99.8473%	99.9475%	99.9847%	99.9964%	99.9994%	99.9999%	100.0000%	100.0000%
9	40.4763%	52.5215%	64.9165%	76.1513%	85.0921%	91.3944%	95.3900%	97.7026%	98.9368%	99.5463%	99.8238%	99.9389%	99.9816%	99.9954%	99.9991%	99.9999%	100.0000%	100.0000%	100.0000%
10	41.7179%	55.1395%	68.5225%	79.9655%	88.3862%	93.8165%	96.9544%	98.6088%	99.4125%	99.7727%	99.9206%	99.9755%	99.9936%	99.9986%	99.9998%	100.0000%	100.0000%	100.0000%	100.0000%
11	42.9701%	57.7292%	71.9183%	83.3034%	91.0292%	95.5897%	97.9990%	99.1606%	99.6760%	99.8862%	99.9643%	99.9902%	99.9977%	99.9996%	99.9999%	100.0000%	100.0000%	100.0000%	100.0000%
12	44.2313%	60.2772%	75.0809%	86.1812%	93.1175%	96.8714%	98.6902%	99.4947%	99.8215%	99.9431%	99.9839%	99.9961%	99.9992%	99.9999%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%
13	45.5001%	62.7706%	77.9963%	88.6308%	94.7478%	97.7892%	99.1447%	99.6962%	99.9018%	99.9715%	99.9928%	99.9984%	99.9997%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%
14	46.7747%	65.1979%	80.6584%	90.6930%	96.0084%	98.4421%	99.4424%	99.8175%	99.9459%	99.9858%	99.9967%	99.9994%	99.9999%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%
15	48.0535%	67.5487%	83.0684%	92.4132%	96.9761%	98.9044%	99.6368%	99.8904%	99.9703%	99.9929%	99.9985%	99.9997%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%
16	49.3349%	69.8143%	85.2331%	93.8370%	97.7148%	99.2305%	99.7636%	99.9342%	99.9836%	99.9964%	99.9993%	99.9999%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%
17	50.6172%	71.9873%	87.1638%	95.0081%	98.2763%	99.4601%	99.8462%	99.9605%	99.9910%	99.9982%	99.9997%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%
18	51.8986%	74.0620%	88.8751%	95.9662%	98.7016%	99.6215%	99.9000%	99.9763%	99.9951%	99.9991%	99.9999%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%
19	53.1776%	76.0341%	90.3833%	96.7467%	99.0230%	99.7347%	99.9350%	99.9858%	99.9973%	99.9996%	99.9999%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%
20	54.4524%	77.9011%	91.7062%	97.3803%	99.2655%	99.8142%	99.9577%	99.9915%	99.9985%	99.9998%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%
21	55.7214%	79.6615%	92.8615%	97.8932%	99.4481%	99.8698%	99.9725%	99.9949%	99.9992%	99.9999%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%
22	56.9829%	81.3154%	93.8666%	98.3074%	99.5855%	99.9089%	99.9821%	99.9969%	99.9995%	99.9999%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%
23	58.2355%	82.8636%	94.7382%	98.6414%	99.6888%	99.9362%	99.9884%	99.9982%	99.9998%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%
24	59.4775%	84.3084%	95.4919%	98.9101%	99.7664%	99.9553%	99.9925%	99.9989%	99.9999%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%
25	60.7075%	85.6524%	96.1420%	99.1262%	99.8247%	99.9687%	99.9951%	99.9993%	99.9999%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%
26	61.9241%	86.8992%	96.7016%	99.2997%	99.8685%	99.9781%	99.9968%	99.9996%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%
27	63.1259%	88.0528%	97.1824%	99.4390%	99.9013%	99.9847%	99.9979%	99.9998%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%
28	64.3116%	89.1175%	97.5949%	99.5507%	99.9260%	99.9893%	99.9987%	99.9999%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%
29	65.4800%	90.0980%	97.9483%	99.6402%	99.9445%	99.9925%	99.9991%	99.9999%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%
30	66.6301%	90.9990%	98.2506%	99.7120%	99.9583%	99.9947%	99.9994%	99.9999%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%

Reliability Distribution After a Given Number of Failure-Free Tests for the Large Uncertainty Case

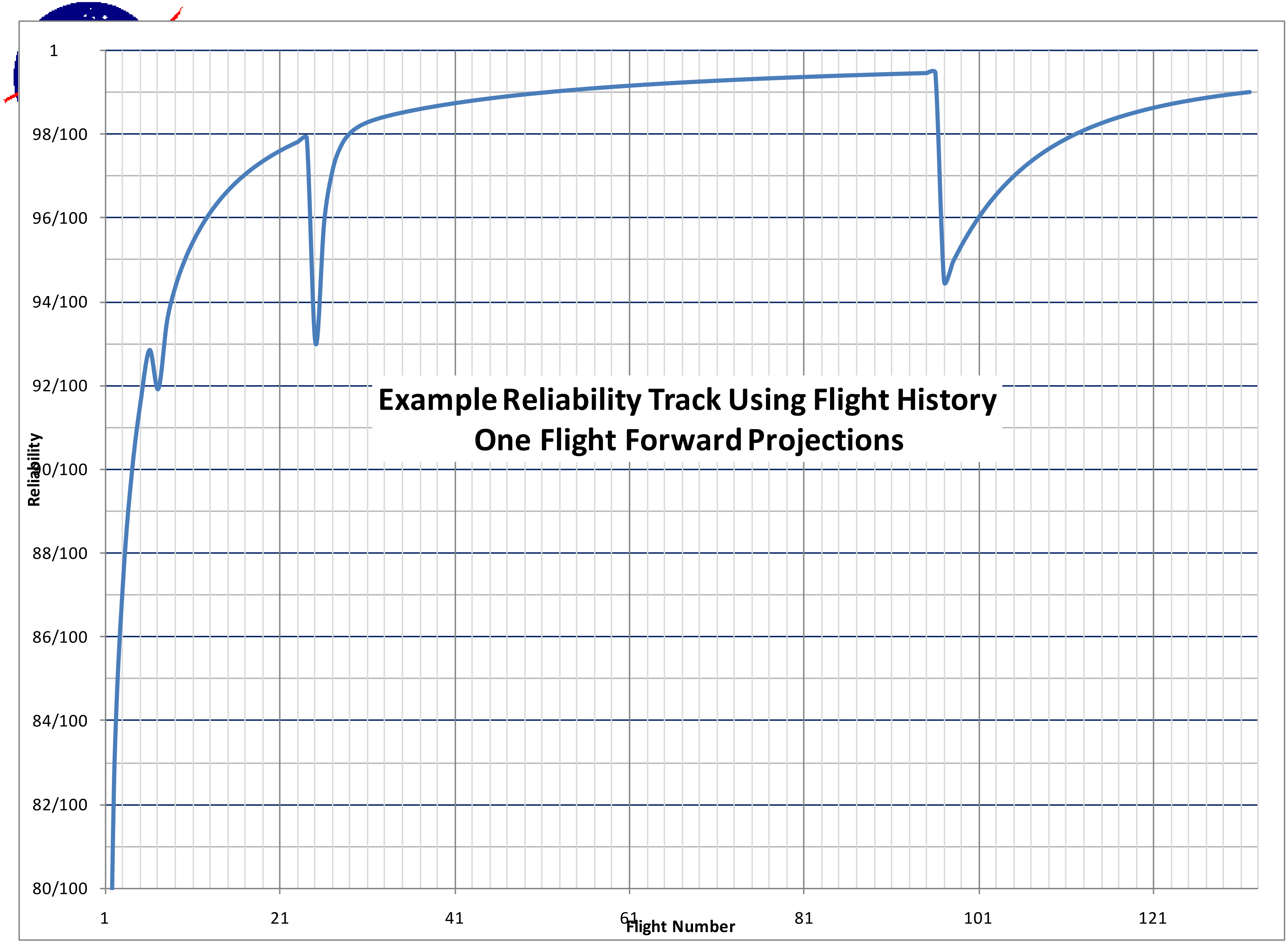
			Reliability Distribution Characteristics After a Given Number of Tests					
	1 Test	2 Tests	3 Tests	4 Tests	5 Tests	10 Tests	15 Tests	20 Tests
1%	14.075509%	16.811989%	19.535827%	22.401186%	25.350151%	41.706695%	58.565376%	73.192121%
2.50%	16.223551%	20.168430%	23.989777%	27.843530%	31.741291%	51.959003%	69.602750%	82.386888%
5%	18.972912%	24.312700%	29.469920%	34.532614%	39.569888%	62.489285%	79.163676%	89.124958%
50%	59.173562%	67.731535%	75.264301%	81.472889%	86.370113%	97.417663%	99.544091%	99.922318%
95%	89.990637%	93.176128%	95.683576%	97.403847%	98.485325%	99.917874%	99.996223%	99.999837%
97.50%	91.522460%	94.582432%	96.760481%	98.130984%	98.951206%	99.950266%	99.997926%	99.999918%
99%	92.754334%	95.688253%	97.549978%	98.653284%	99.271237%	99.969511%	99.998842%	99.999957%
Mean	57.238016%	64.074271%	70.213039%	75.477049%	79.833896%	91.788449%	96.173147%	98.105062%

Handling Failures That Occur

- A failure occurrence can be handled by discounting the failure in the reliability estimation
- The discount factor is one minus the corrective action effectiveness (the ineffectiveness)
- A failure occurrence can alternatively be handled by restarting the reliability at the value before the failure multiplied by the corrective action effectiveness
- Both alternatives give similar results
- With the restart alternative the reliability-growth test tables can be entered with the restarted reliability

Using a Dynamic Reliability Model to Monitor the Dynamic Reliability Growth of a Spacecraft

- **The following slide shows an example evaluation of the dynamic tracking of a hypothetical flight history**
- **The evaluations incorporate fault removal and include the random operational contribution**
- **The values are Kalman-Filter-predicted next-flight reliability based on the past history up to the flight**
- **The spikes on the curves are the predicted reliability after failure fixes have been made and have been included**
- **Such monitoring evaluations are important for tracking real-time reliability growth to update analyses and actions**


Example Reliability Track Using Flight History
One Flight Forward Projections
Reliability
1
98/100
96/100
94/100
92/100
90/100
88/100
86/100
84/100
82/100
80/100
1
21
41
61
81
101
121
Flight Number

Reliability-Growth-Based Model for the Probability of Zero Failure-Causing Faults Existing After a Given Number of Successful Tests

$P(0) =$ initial probability of no failure causing faults in the design (initial assurance level)

$N =$ number of failure free tests or flights conducted

$p =$ fault detection effectiveness (conditional probability of detecting a failure -causing fault)

$P(0/N) =$ probability of no failure causing faults in the design after N failure free tests

Using Bayes theorem

$$P(0/N) = \frac{P(0)}{P(0) + (1 - P(0))(1 - p)^N}$$

$P(0/N)$ Is also termed the design reliability and is calculated in the previous tables

the probability of k faults existing

Extensions of the Formula for No Failure-Causing Faults Existing (1)

The formula previously was for a single fault existing. For multiple faults existing the formula becomes

$$P(0/N) = \frac{P(0)}{P(0) + \sum_{k} P(k)P(0/k,N)}$$

where

$P(k) =$ the probability of k faults existing

$P(0/k,N) =$ the probability of missing all k faults in N tests

The sum in the denominator is over k faults existing with k greater than or equal to 1.

Extensions of the Formula for No Failure-Causing Faults Existing (2)

For a standard test which is less likely to miss all faults if more than one exists we have

$$P(0/k,N) \le P(0/1,N)$$

Hence,

$$P(0/N) \ge \frac{P(0)}{P(0) + (\sum_k P(k))(P(0/1,N))}$$

$$= \frac{P(0)}{P(0) + (1 - P(0))(1-p)^N}$$

which is the formula in the main body.

Consequently if multiple faults exist the formula for one fault gives a lower bound on the reliability, i.e., on $P(0/N)$ and the reliability can be somewhat higher

Extensions of the Formula for No Failure-Causing Faults Existing (3)

If $P(k)$ follows a Poisson then it is straightforward to show that

$$P(0/N) = \exp(-\Lambda(1-p)^{N})$$

where

$\Lambda =$ the expected number of faults existing

If the tests are repetitive or are correlated with a portion of the conditions repeated then N is replaced by the effective number of non-overlapping tests conducted. In the case of overlapping tests where N is replaced by log N then the model is similar to the Duane model where now the parameters are expressed in terms of the expected number of faults existing and the fault detection coverage.

Annotated References (1)

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Annotated References (2)

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