Probabilistic Models for Solar Particle Events

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Abstract: Probabilistic Models of Solar Particle Events (SPEs) are used in space mission design studies to provide a description of the worst-case radiation environment that the mission must be designed to tolerate. The models determine the worst-case environment using a description of the mission and a user-specified confidence level that the provided environment will not be exceeded. This poster will focus on completing the existing suite of models by developing models for peak flux and event-integrated fluence elemental spectra for the Z>2 elements. It will also discuss methods to take into account uncertainties in the data base and the uncertainties resulting from the limited number of solar particle events in the database.

These new probabilistic models are based on an extensive survey of SPE measurements of peak and event-integrated elemental differential energy spectra. Attempts are made to fit the measured spectra with eight different published models. The model giving the best fit to each spectrum is chosen and used to represent that spectrum for any energy in the energy range covered by the measurements. The set of all such spectral representations for each element is then used to determine the worst case spectrum as a function of confidence level. The spectral representation that best fits these worst case spectra is found and its dependence on confidence level is parameterized. This procedure creates probabilistic models for the peak and event-integrated spectra.

Introduction: Several methods have been used to develop probabilistic models. The earliest method still in use is the JPL Model [1,2]. More recently the Model for Emission of Solar Protons (ESP) [3,4] and the Probabilistic model for fluences and peak fluxes of solar energetic particles (Nymmik) [5,6] Models were developed. Most recently the Prediction of Solar particle Yields for CHaracterizing Integrated Circuits (PSYCHIC) Model [7] was developed.

These models for solar proton events that describe the peak flux, event-integrated fluence and mission-integrated fluence. In addition the PSYCHIC model describes the mission-integrated fluence for the Z>2 elemental spectra.

All models consist of an initial distribution that describes the cumulative distribution of event fluxes (fluences). All the models convolve the initial distribution with a Poisson distribution for the number of solar particle events expected during the planned mission duration.

In the case of the JPL model, the assumed initial distribution is a log-normal distribution given by

$$p(F) = \exp[-(F-\mu)^2/2\sigma^2]/\sqrt{(2\pi\sigma)}$$

Where F is $log_{10}(\phi)$ and ϕ is the integral particle flux (fluence). p is the probability density, μ is the mean of the distribution and σ is the standard deviation. This initial distribution fits only the upper half of the flux (fluence distribution well) but that is the most critical part of the distribution.

The solar cycle dependence is accounted for by dividing the cycles into 7 active years that are assumed to be identical and 4 inactive years that can be neglected.

For it's initial distribution the Nymmik model assumes a power law with an exponential cutoff,

$$f(\phi) = C\phi^{-1.32}/\exp(\phi/k)$$

where C and k are parameters to be determined by fitting to data.

The solar cycle dependence of the mean number of events per year, $<n>$, used to define the Poisson distribution is scaled from the monthly mean sunspot numbers predicted to occur during the mission.

The Nymmik model determines differential fluxes (or fluences), $\Phi(E)$, by assuming all SPE spectra can be described by power laws in magnetic rigidity

$$\Phi(E) = D\left(\frac{R}{239}\right)^{-\gamma}\left(\frac{dR}{dE}\right)$$

where D is the spectral coefficient and γ is the spectral index. For energies < 30 MeV/nuc, γ is given by $\gamma = \gamma_0(E/30)^{\alpha}$. The extreme values of $\Phi(E)$ are then chosen by Monte Carlo simulations of n events with n chosen from a Poisson distribution with a mean $<n>$. The parameters D, γ_0, and α are picked from log-normal distributions.

The Nymmik model is a representational model, based on formulas that are assumed ad-hoc to fit the data.

By contrast, the ESP model is based on extreme value theory [8]. This theory is used to derive the correct form for the cumulative initial distribution, $P(\phi)$, which turns out to be a truncated exponential distribution,

$$F(\phi) = (\phi_{min}^{-b} - \phi^{-b})/(\phi_{min}^{-b} - \phi_{max}^{-b})$$

where b and ϕ_{max} are determined by fitting the data. Like the JPL Model, the ESP model assumes that solar cycles can be divided into 7 active years and 4 inactive years. When convolved with the Poisson distribution for the number of events in T years, the result is

$$F_T(\phi) = \exp\{-\mu T[1 - F(\phi)]\}$$

Where μ is the assumed-to be constant rate at which events occur during solar-active years.

The method used in the PSYCHIC model differs from that in the ESP model. Once again maximum entropy theory is used, but this time with the constraints that are appropriate for mission-integrated fluence. The resulting cumulative probability, $F_{cum}(\Phi)$, is

$$F_{cum}(\Phi) = 1 - \frac{1}{\sigma\sqrt{2\pi}}\int_0^{\Phi}\frac{1}{\phi}\exp\{-\frac{1}{2\sigma^2}[\ln(\phi) - \mu']^2\}d\phi$$

where μ' and σ are related to the mean and the relative variance of the initial distribution.

Because the distributions used in ESP and PSCHIC are derived to fit the properties of the distributions they describe rather than assumed or developed ad-hoc. We will follow the examples of these models to extend the models to spectra for the peak fluxes and event-integrated fluences for protons and heavy ions.

The Experimental Data: The parameters b, ϕ_{max}, μ, μ' and σ must be determined from the experimental data. We propose to do this by fitting the data on the peak differential flux and the event-integrated differential fluence for all the measured SPEs in the 20th, 21st, 22nd and 23rd solar cycles. Cumulative fluences will then be determined for one-year intervals by summing the event-integrated fluences during each of the years.

Reliable space-based measurements of SPE proton spectra are available for solar cycles 20 - 23. The sources of data for solar cycle 20 are IMP-3, -4, -5, -7, and -8. Data for solar cycle 21 are available from IMP-8. GOES-5 and -7 provide data for solar cycle 22. IMP-8, GOES-7, -8 and -10 provide data for solar cycle 23. The proton data from solar cycle 20, 21 and 22 have been analyzed and are available to us from earlier work. We propose to extend this data set using data from the GOES spacecraft and the Goddard Medium Energy (GME) and CRNC/CRT instruments on IMP-8. These measurements will be cross-checked using data from the PET instrument on SAMPEX and the EPACT instrument on WIND.

There are also sufficient data available on the more abundant elements heavier than protons to measure their differential energy spectra. Data for solar cycles 21, 22 and the first part of 23 are available from IMP-8. During solar cycle 23 data are also available from the ACE spacecraft. These data can be checked against data from MAST on SAMPEX and EPACT on WIND. In addition, some data have begun to appear from the SOHO spacecraft. While these SOHO data suffer from saturation in large events, they will serve as a cross check on the ACE data after the end of the IMP-8 mission. We have analyzed the IMP-8 proton data from the CRNC/CRT instrument from solar cycles 21 and 22 with a time resolution of 30 minutes and the helium and heavy ion data with a time resolution 6 hours. We propose to extend this analysis to the end of the IMP-8 mission. This will provide sufficient time resolution to identify the peak fluxes as well as the event-integrated fluences so that spectra of the more abundant elements for these events can be constructed and fit. Our proposed strategy is to construct probabilistic models for the more abundant elements, e.g. He, O, Si, and Fe. The worst-case spectra for the remaining heavy nuclei will be obtained by scaling from one of these spectra using measured SPE composition data and other sources as described at the end of the next section.

Spectral Fitting: Each of the instruments we propose to use in this study has a natural energy binning determined by the design of the experimental hardware. To determine the measured spectra at a common set of energy points, we propose to fit spectral models to the measured peak elemental flux spectra and the event-integrated fluence spectra. Spectral fitting has been used in many previous investigations. Cohen et al.[47] and Mewaldt et al.[18] were successful in fitting the events of October and November 2003 with either the Ellsion-Ramaty Model[48] or the Band Function[49]. Tylka et al.[26] reports that the Ellsion-Ramaty Model successfully fits the spectra from the event-averaged spectra of the April 21, 2002 event while the Band Function fit the spectra of the August 24, 2002 event. Xapsos et al.[27] have proposed a model based on the Weibel distribution of the smallest values and used it successfully to fit many SPE spectra over a broad energy range.

To investigate our ability to fit the large number of spectra in this investigation, we have tested eight spectral models. These are the Ellsion-Ramaty Model, the Band function, the Lee-Ramaty Model[50,22], an exponential in rigidity[22], an exponential in energy[8], a power law in energy[8,23], the Weibel Distribution[27] and Mazur's Model[21]. These models were tested by fitting event-integrated or event-averaged spectra obtained from published papers. Figure 1 shows an example of such a fit that has a reduced χ^2 of 1.09. From these tests we found that the Ellison-Ramaty, Band and Weibel Distribution models provided good fits for most of the events.

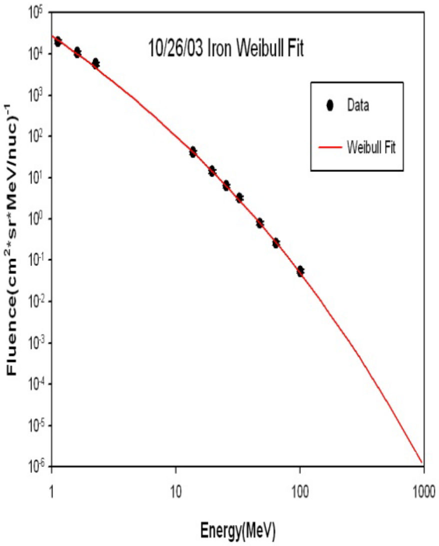

Figure 1: Weibel function fit to the event-integrated fluence spectrum of iron in the Oct 26 2003 SPE.

Using the spectral fits, we can find the differential fluxes (or fluencies) at a standard set of energies beginning at 1 MeV/nuc and extending to 100 MeV or higher depending on the availability of data. Using a power law in rigidity as shown by Tylka and Dietrich[51], we expect to be able to extend SPE proton fluence spectra to ~9 GeV.

We propose to explore the possibility that this approach could be extended to the spectra of heaver ions.

Figure 2 shows an example of fitting the initial distribution to the cumulative distribution of peak proton fluxes with energies >10 MeV. The resulting fit parameters are $N_{tot} = 27$; $b = 4.1$; $\phi_{max} = 1.3\text{x}10^5$ cm^{-1}s^{-1}sr^{-1} .

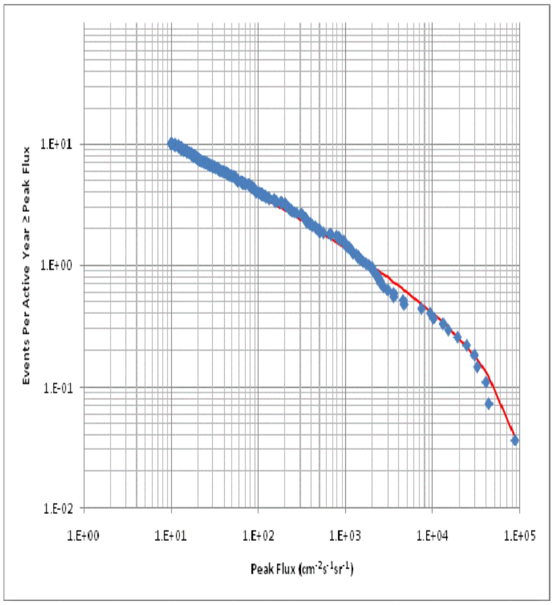

Figure 2: The IMAX measurements of helium are compared to the three models.

Conclusions: We have demonstrated a method for extending probabilistic models for solar energetic particle events to include worst-case models for peak flux and event-integrated fluence model spectra for both protons and heavy ions.

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