

# Passively Shunted Piezoelectric Damping of Centrifugally-Loaded Plates

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Researchers at NASA Glenn Research Center have been investigating shunted piezoelectric circuits as potential damping treatments for turbomachinery rotor blades. This effort seeks to determine the effects of centrifugal loading on passively-shunted piezoelectric-damped plates. Passive shunt circuit parameters are optimized for the plate's third bending mode. Tests are performed both non-spinning and in the Dynamic Spin Facility to verify the analysis, and to determine the effectiveness of the damping under centrifugal loading. Results show that a resistive shunt circuit will reduce resonant vibration for this configuration. However, a tuned shunt circuit will be required to achieve the desired damping level. The analysis and testing address several issues with passive shunt circuit implementation in a rotating system, including piezoelectric material integrity under centrifugal loading, shunt circuit implementation, and tip mode damping.

## Nomenclature

$3B$	=	third bending mode
$C$	=	piezoelectric patch capacitance (nF)
$k_{31}$	=	piezoelectric material coupling coefficient
$L$	=	shunt inductance (H)
$R$	=	shunt resistance ( $\Omega$ )
$\eta$	=	loss factor

## I. Introduction

AS part of the Fundamental Aeronautics program, researchers at NASA Glenn Research Center (GRC) are developing new damping technologies to alleviate excessive vibratory stresses that lead to high cycle fatigue failures in aircraft engine components. Newer high-performance turbomachinery blade designs manufactured as integrally bladed disks (blisks), or without shrouds, have led to decreased blade damping and higher vibratory stresses<sup>1</sup>. This increases design and maintenance costs, but also can cause component failure. Designing damping treatments for rotating blades in an extreme engine environment is difficult because temperatures and centrifugal accelerations can be very high. Since a damping treatment must not affect the aerodynamics of the blade, it must be internal to the blade, out of the air stream, or applied to the blade surface in a thin layer.

Several damping methods have been investigated by NASA researchers and contractors at GRC for use in aircraft engines, including viscoelastic damping (Kosmatka<sup>2</sup>), impact damping (Duffy<sup>3,4</sup>), plasma sprayed damping

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coatings (Zhu<sup>5</sup>), and high-damping high-temperature shape memory alloy materials (Duffy<sup>6</sup>). The current investigation explores the ability of piezoelectric materials to damp vibrations of rotating turbomachinery blades. Choi<sup>7</sup> is studying the use of actively controlled piezoelectric materials to reduce multiple blade modes. This work describes the use of passively-shunted piezoelectric-damped plates in a simulated engine environment.

Many researchers have investigated piezoelectric materials in parallel with electric circuits that absorb the vibrational energy of components. A survey of smart structures state-of-the-art can be found in Chopra<sup>8</sup>. Hagood and von Flotow<sup>9</sup> detailed a method to determine the effective damping of a shunted piezoelectric material. Lesieutre<sup>10</sup> described the different types of shunt circuits and how they affect behavior. For the turbomachinery application, Hilbert, Pearson, and Crawley<sup>11</sup> acquired a patent for shunted piezoelectric damping of blades, where piezoelectric patches were placed below the blade platform. Livet<sup>12</sup> examined negative capacitance shunted piezoelectric materials on beams for the turbomachinery blade application. Cross and Fleeter<sup>13</sup> tested shunted piezoelectric damping of stator vanes. Piezoelectric networks have also been studied for reduction of mistuned blade vibration levels through blade coupling (Yu and Wang<sup>14</sup>). These efforts have included non-spinning testing only, and typically used a powered shunt circuit utilizing synthetic inductors or negative capacitance.

This effort seeks to determine the effects of centrifugal loading on purely passively-shunted piezoelectric-damped plates. Hagood and von Flotow's methods are used to determine the passive shunt circuit parameters optimized for particular plate modes. Tests are performed both non-spinning and under centrifugal loading to verify the analysis, and to determine the effectiveness of the damping under centrifugal loading. Unlike the Hilbert patent, the targeted mode has high resonant stresses out toward the tip of the plate (tip mode), and the piezoelectric materials are therefore placed in this area rather than at the blade platform. The analysis and testing address several issues with passive shunt circuit implementation in a rotating system, including mean stress due to centrifugal loading, shunt circuit implementation, and tip mode damping.

## II. Configuration

The target application for this work is a titanium-alloy cold-side compressor or fan blade, which operates at temperatures ranging up to about 600°F. The technology could also be applied to a carbon-fiber composite fan blade. Due to centrifugal loading, mean strains can range from zero at the blade tip, to on the order of  $10^{-3}$  within the blade. Vibratory strain amplitudes can reach  $10^{-3}$ . Frequencies are on the order of 100 to 10,000 Hz.

The blade structure loss factor  $\eta$  is typically on the order of  $10^{-3}$  or less for titanium alloy blades, depending on the vibration mode, but  $10^{-2}$  or more would be desirable for resonant stress reduction. This can be accomplished by incorporating a high damping material within or on the surface of the blade. However, if the damping material is placed on the blade surface, it needs to be very thin so that it does not adversely affect the aerodynamics. Thus a thickness of 0.005 inches or less would be preferred.

Fundamental modes (e.g. first bending or first torsion) are more easily damped with traditional methods such as platform dampers. There is interest in developing new damping techniques for higher-order modes where resonant stresses occur near the blade tips. The focus of this effort is on these higher-order modes.

Centrifugal loading also causes blade stiffening, leading to changes in resonant frequency for some modes with rotational speed. The piezoelectric shunt circuit design will need to provide damping over the desired speed range.

Keeping in mind these requirements, a configuration was chosen to demonstrate the effectiveness of passive shunted piezoelectric materials for damping centrifugally-loaded plates. This test configuration addresses the following concerns with piezoelectric damping implementation:

- Centrifugal loading
  - Effect of mean strain due to centrifugal loading on the piezoelectric material.
  - Surface-mounted patch integrity under centrifugal loading.
- Target modes
  - Modes with high resonant stress levels toward the blade tip (e.g. third bending mode).
- Shunt circuit implementation
  - Passive circuit requiring no external power source.
  - Small circuit size.
  - Circuit integrity under centrifugal loading.
  - Effect of the resonant frequency shift from 0 rpm to maximum operating speed.

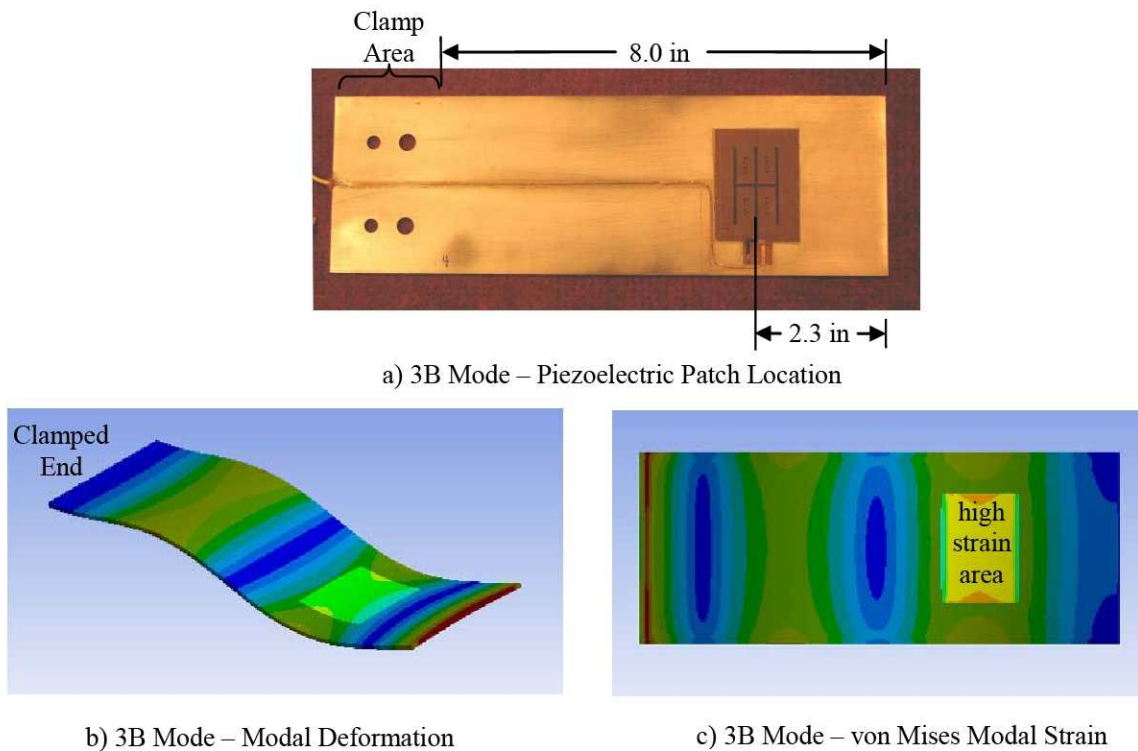
This test does not deal with the issue of a realistic piezoelectric patch implementation. In a real blade, the patch will likely need to be placed inside or recessed into the blade. In addition, the test will include a piezoelectric material thickness that is larger than the desired thickness for surface implementation. This test is also at room

temperature, using an off-the-shelf piezoelectric material PZT-5A that has a Curie temperature of 600°F, enabling it to function to less than half that value. Efforts are currently under way at NASA GRC to address these difficult concerns, including the development of new high-temperature piezoelectric materials.

### A. Test Articles

To simulate a fan blade type structure, the test article configuration is a simple rectangular plate fabricated from titanium alloy 6Al-4V. The plate extends 8.0 inches beyond the clamp area, is 3.2 inches wide and 0.078 inches thick. Figure 1 shows the location of the piezoelectric patch, as well as the plate third bending (3B) modal deformation and modal strain at 0 rpm. The plate dimensions were chosen to lower the first three bending mode frequencies below the test facility excitation frequency bandwidth of 1000 Hz, to provide adequate separation between the torsional and bending modes over the operational speed range, and to provide modest strain levels in the plate and patch. Figure 2 shows the resonance frequencies as a function of rotor speed. A second article configuration to be tested in future consists of a tapered plate, which is specifically designed to reduce two-stripe (chordwise bending) mode below 1000 Hz.

A single Midé PZT-5A qp10w<sup>15</sup> piezoelectric patch is bonded with Vishay M-Bond AE-10 adhesive to one side of each plate. The patch is centered at 2.3 inches from the tip, in a high modal strain area for the third bending mode. The patches are 1.5 inches long, 2.0 inches wide, and 0.015 inches thick. The piezoelectric material itself within each patch has dimensions of 1.31 inches long, 1.81 inches wide, and 0.010 inches thick. The capacitance of the patches applied to the two test plates was measured to be 63.8 nF and 64.5 nF.



**Figure 1. ANSYS Workbench Results – 3B Configuration at 0 rpm**

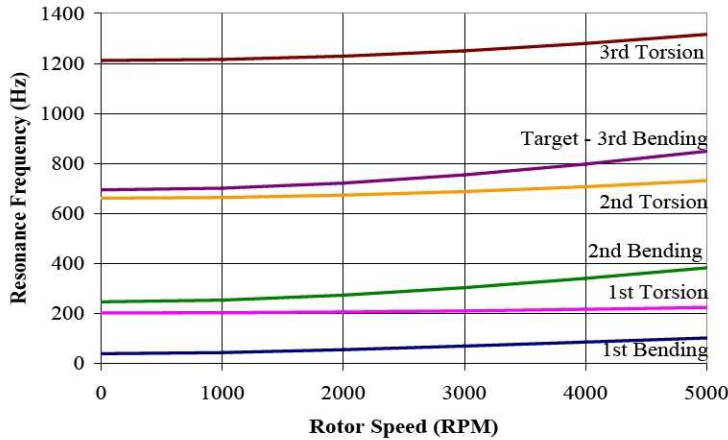


Figure 2. Plate Resonance Frequencies

### B. Shunt Circuit Design

Testing was performed with a passive shunt circuit, which will be the easiest to implement in an engine. There are two types of purely passive circuits that were investigated:

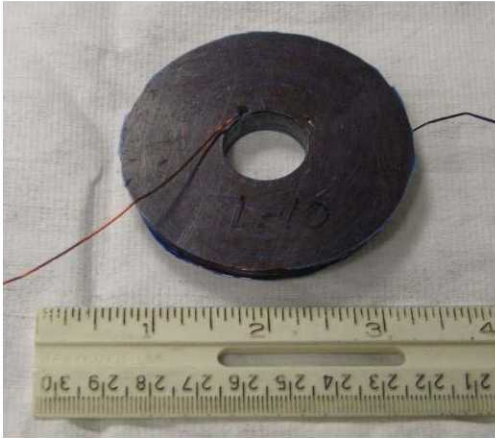
- Resistive circuit (R circuit)
  - The circuit dissipates energy through the resistor (R).
  - It can damp out a large frequency range, but with lower damping than the tuned circuit.
- Tuned resonant circuit (RL circuit)
  - The circuit is tuned to the target vibration frequency through the addition of an inductor (L), and energy is dissipated through the resistor.
  - The RL circuit results in higher damping at the target frequency than the R circuit, with reduced damping off the target frequency.
  - The inductor can be a simple coiled wire, which is fully passive.

Both circuit types were tested on a vibration tester (shaker), and the R circuit was also tested under centrifugal loading. Table 1 lists the shunt circuit configurations used for analysis and testing. The circuit components were chosen to maximize damping based on the Hagood and von Flotow loss factor equations, and based on shaker results. One plate was analyzed and tested on a shaker, non-spinning, with the piezoelectric patch in the open circuit condition, short circuit condition, with a resistive shunt circuit, and with a tuned resonant circuit. A pair of plates was then tested in the Dynamic Spin Facility (spin rig) in the open circuit condition and with the optimal resistor.

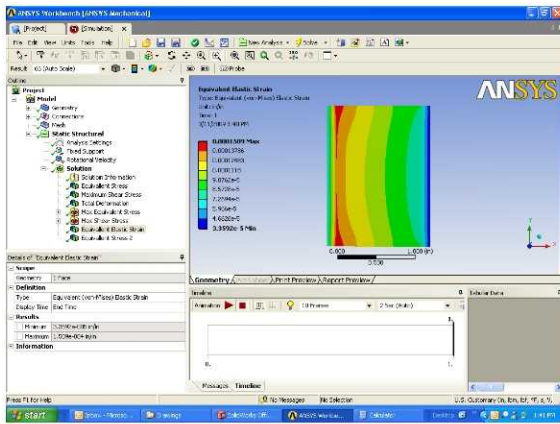
Table 1. Shunt Circuit Configurations

Test	Shunt Circuit Configuration	Shunt Resistance	Shunt Inductance
Shaker	3B-O-S	Open circuit	---
	3B-S-S	Short circuit	---
	3B-R-S	0-22 k $\Omega$	---
	3B-RL-S	0-22 k $\Omega$	0.38H, 0.69H, 1.27H
Spin Rig	3B-O-C	Open circuit	---
	3B-R-C	4.12 k $\Omega$	---

The inductors are open-core wound inductors manufactured in-house. The inductor shown in Fig. 3, 0.69 H, is quite large, since the target frequency of 700-800 Hz was chosen to be below the excitation bandwidth of 1000 Hz. It was made from 34 gauge wire, providing a resistance of 510  $\Omega$ . It has a 0.75-inch inner diameter, a 2.6-inch outer diameter, and a 0.30-inch length. These dimensions could be made smaller with a tighter packing factor. For engine implementation, the size would be significantly reduced as the optimal inductance decreases with increasing resonance frequency. At 2000 Hz, the required inductance is 0.1 H, and at 5000 Hz the required inductance is only 0.015 H.



**Figure 3. 0.69 H Inductor**



**Figure 4. Mean Strain in Piezoelectric Patch Due to Centrifugal Loading at 4000 rpm – ANSYS Workbench Results**

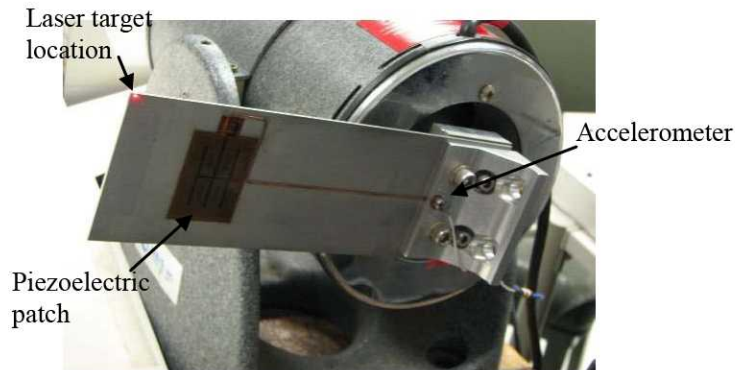
### III. Analysis

A modal analysis was performed to predict frequencies and damping levels under centrifugal loading. The Rayleigh-Ritz method was used with assumed mode shapes for a rotating cantilever plate. This analysis was performed using Mathematica software, which solved the eigenvalue problem to give resonant frequency and loss factor for each plate mode. The piezoelectric patch was assumed to have a complex modulus based on the Hagood and von Flotow analysis for the piezoelectric material in parallel with a shunt circuit. The reduced order modal analysis was found to agree well with ANSYS frequency predictions for this very thin plate.

A finite element analysis was performed in ANSYS Workbench to determine the mean centrifugal load on the piezoelectric patch. Figure 4 shows the von Mises strain level at the patch surface at the maximum rotor speed of 4000 rpm. The maximum strain of about  $150 \times 10^{-6}$  is well below the strain limit of  $500 \times 10^{-6}$  for the piezoelectric material. A key reason for placing the patch closer to the tip, besides targeting a tip mode, is that the centrifugal stress will be much lower at the tip than close to the base.

### IV. Test Facilities

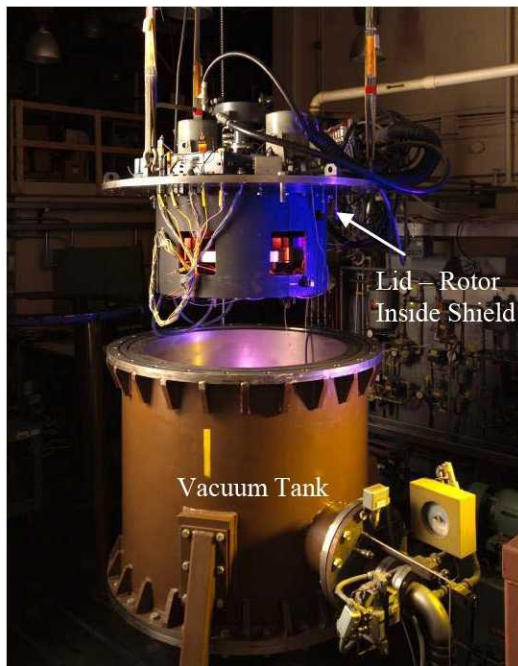
Non-spinning, room-temperature vibration tests are performed in the Adaptive Structures Laboratory at NASA GRC. A cantilevered test plate is clamped to an MB Dynamics PM100 100-lb shaker as shown in Fig. 5. A Columbia model 6062-HT accelerometer is placed on the clamp to measure the excitation level. A Polytec OFV-505 laser vibrometer with an OFV-5000 controller is used to measure the velocity of the corner of the plate tip. An HP 3566A analyzer is used in the swept-sine mode to generate a signal to the shaker. The analyzer measures the signal to the shaker, as well as the accelerometer and laser



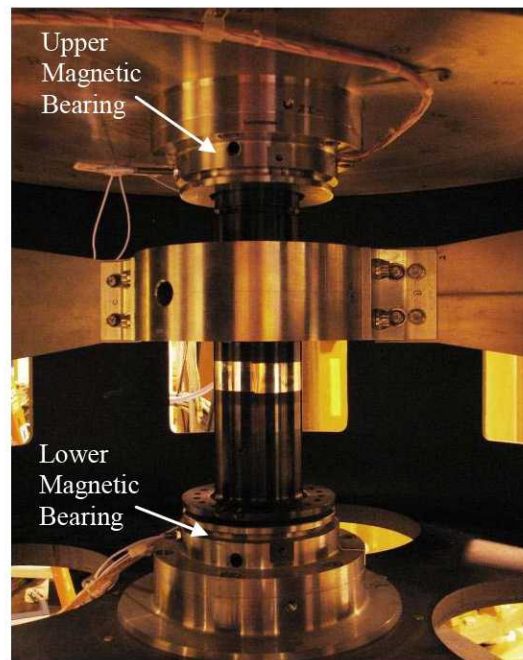
**Figure 5. Plate on Shaker**

vibrometer signals. The damping is calculated from the transfer function of the tip velocity to the clamp acceleration, as well as from the tip velocity frequency response.

Spin testing is performed in the Dynamic Spin Facility at NASA GRC. Figure 6 shows a photograph of the spin facility, and Fig. 7 shows a pair of tapered plates attached to a hub on the vertical rotor. The rotor is attached to the lid, which is lowered into the vacuum tank. Testing is done in a 0.01 psia vacuum at room temperature. The rotor is fully levitated on three conventional active magnetic bearings. An axial magnetic bearing supports the weight of the rotor system. The two other magnetic bearings support the rotor radially, and also provide a radial excitation to the shaft, transmitting vibration to the plates. With this excitation, the base of the plate sees approximately a one-g excitation level with a bandwidth of about 1000 Hz. A non-contacting stress measurement system (NSMS), with IFOSYS laser displacement probes, measure the plate tip deflection while the rotor spins. Figure 8 shows the laser displacement probes. Hood Technology Corporation's Acquire Blade Data software acquires the NSMS data, and Hood's Analyze Blade Vibration software calculates the tip deflection and damping.



**Figure 6. Dynamic Spin Facility**



**Figure 7. Rotor with Tapered Test Plates**



**Figure 8. Laser Displacement Probes**

For this test, the magnetic bearing control system provided an engine order excitation frequency – the excitation frequency was an integer order of the rotation speed. This is more typical of the type of excitation seen by a rotor blade, and also makes the NSMS software easier to implement. The excitation is provided to the rotor in a rotating frame, maintaining a constant angle with the plate faces.

Two identical plate configurations are tested at the same time, and are oriented directly opposite each other on the rotor. The shunt resistors are placed on the disk, directly inboard of the plate clamps. The inductors will be placed in a housing below the lower magnetic bearing in future tests. Testing was performed at speeds of zero rpm to 4000 rpm, and the facility has the capability of spinning at up to 20,000 rpm.

## V. Results

### A. Non-Spinning Results

A plate was tested on the shaker as shown in Fig. 5. Figure 9 gives the results for the R-circuit case (3B-R-S). As predicted, there was optimal damping at a resistance of approximately 4 k $\Omega$ , and a resistance of 4.12 k $\Omega$  was chosen for the spin test. Table 2 details results from the shaker test. Included in the table are the resonance frequency and loss factor measurements based on the transfer function of the tip velocity to the clamp accelerometer, as well as based on the tip velocity frequency response. The transfer function data is more accurate, and is used to compare with model predictions. However, the spin test data will be based on tip measurements only, and the tip velocity damping calculation is included to compare with that data.

The desired loss factor of 0.01 was not achieved with the R circuit. In order to reach that goal, a tuned RL circuit will be required. Figure 10 shows the plate loss factor predicted by the reduced order model as a function of shunt inductance for a spinning resonance frequency of 760 Hz. This analysis shows a range of approximately 0.52 H to 1.0 H yields a loss factor of 0.01 or more.

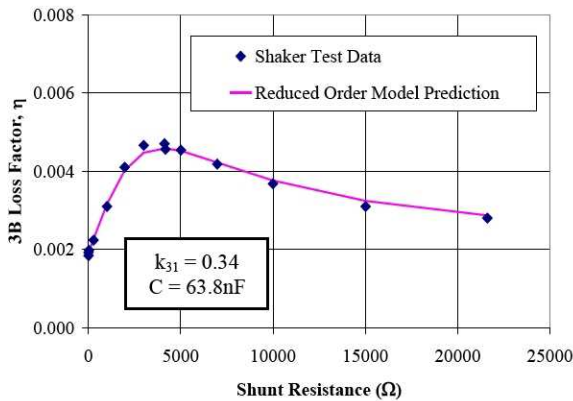


Figure 9. Shaker Test Results – 3B-R-S

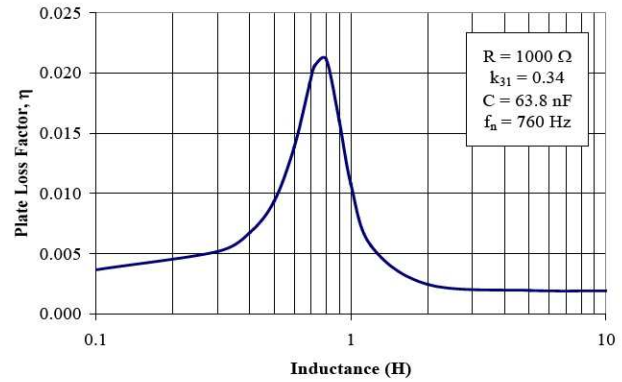


Figure 10. Plate Loss Factor Reduced Order Model Prediction – 3B-RL-S

Table 2 – Shaker Test Results

Configuration	Transfer Function			Tip Velocity Measurement		
	3B Frequency (Hz)	3B Loss Factor $\eta$	3B Loss Factor Increase $\Delta\eta$	3B Frequency (Hz)	3B Loss Factor $\eta$	3B Loss Factor Increase $\Delta\eta$
3B-O-S	683.0	0.0018	---	684.4	0.0047	---
3B-S-S	680.6	0.0019	0.0001	682.0	0.0044	-0.0003
3B-R-S	682.3	0.0047	0.0029	683.7	0.0072	0.0025
3B-RL-S	680.1	0.0184	0.0174	681.2	0.0206	0.0159

A shaker test was then performed with inductor sizes 0.38 H, 0.69 H, and 1.27 H, and various resistances. Figure 11 shows the results of that test. Figure 12 illustrates the resonant peak reduction with various shunt circuits. With the 0.69 H inductor and a resistance of 2 k $\Omega$  or less, the desired loss factor can be achieved in a non-spinning test. A future spin test will be required to demonstrate the RL circuit effectiveness under centrifugal load.

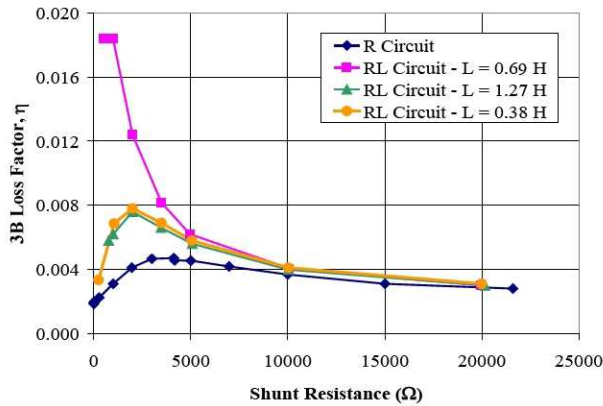


Figure 11. Shaker Test Results – 3B-R-S and 3B-RL-S

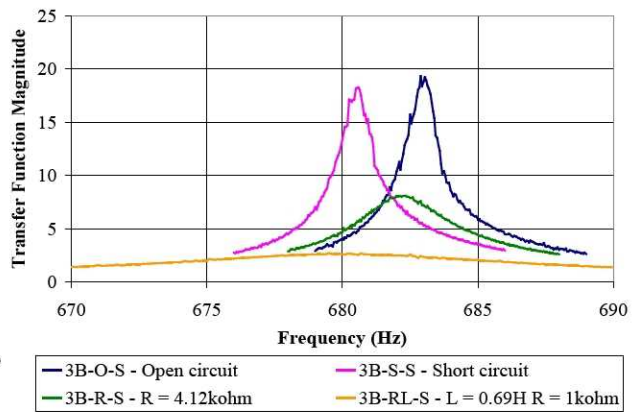


Figure 12. Transfer Functions – Plate Tip Velocity to Clamp Acceleration (in/s/G)

### B. Spin Test Results

Before performing spinning vibration tests, the plates were run to a maximum speed of 4000 rpm for one minute in the Dynamic Spin Facility. They were then removed from the rig and inspected to check the integrity of the plates, piezoelectric material, and wiring. In addition, the piezoelectric capacitance was checked before and after vibration spin testing, with minimal difference.

Next, the plates were vibration tested in the spin rig in rotor speed sweeps up and down from 2000 to 3800 rpm. Figure 13 shows both the predicted and experimental resonance frequencies as a function of rotor speed for the 3B-O-C open circuit configuration. Each point shown represents a resonance peak occurring when the engine order excitation frequency matched the third bending mode resonance frequency.

The loss factor of the open circuit (3B-O-C) and R-circuit (3B-R-C) cases is shown in Fig. 14 as a function of rotor speed. Note that the baseline loss factor (3B-O-C) is similar in the spin rig as for the shaker test based on tip measurements (about 0.004). The increase in loss factor  $\Delta\eta$  is also similar. In the shaker test  $\Delta\eta = 0.0025$  for the tip velocity calculation, and for the spin test  $\Delta\eta$  varies from 0.001 to 0.003. This comparison is not ideal, since the shaker and spin rig dynamics are different, and the shaker test is performed in air while the spin test is performed in vacuum. However, it does give a sense of the relative effectiveness of the R circuit versus the open circuit.

As in the non-spinning test, the R circuit does increase the plate damping; however, it does not give the desired loss factor of 0.01. A test is planned in the near future to include the 0.69 H inductor in the spin rig.

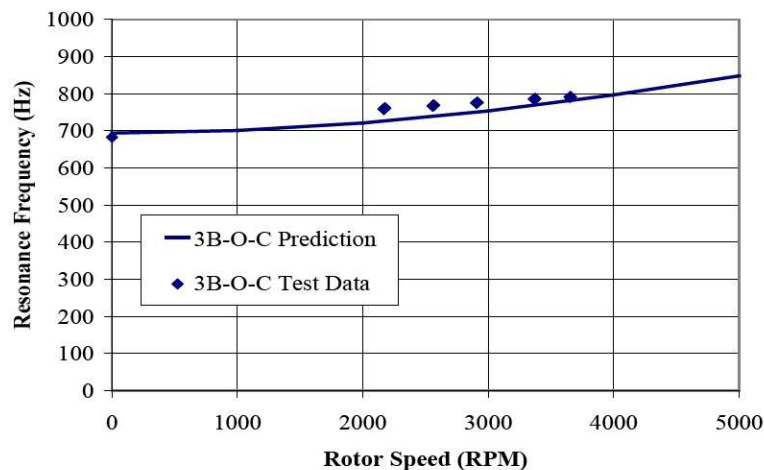


Figure 13. Resonance Frequencies – 3B-O-C



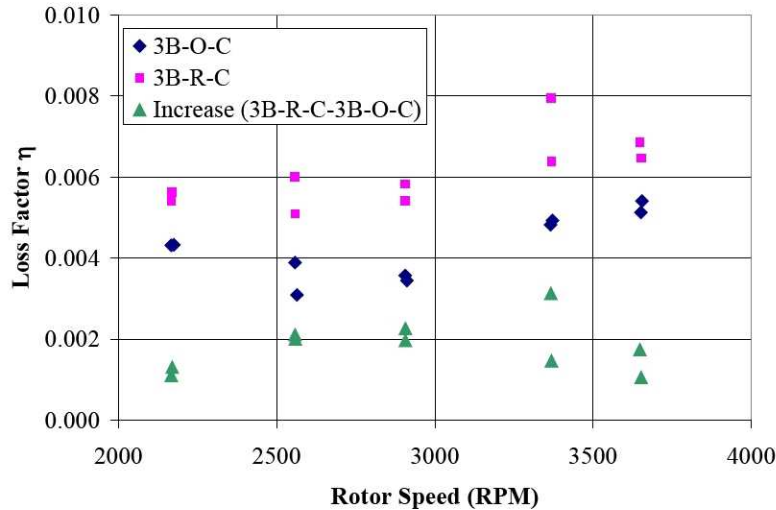


Figure 14. Loss Factor – 3B-O-C and 3B-R-C

## VI. Summary

Spin and shaker testing were conducted on passively-shunted piezoelectric-damped rectangular plates to study the feasibility of damping turbomachinery blades. The plate was designed so that the first three bending mode frequencies were below 1000 Hz due to spin testing excitation bandwidth. Under centrifugal loading, the mean strain in the patch was maintained below material limits by placing the patch toward the blade tip, in an area of high resonant stress for the targeted third bending mode. The surface-mounted patch maintained integrity with no observable changes in electrical properties after cyclic centrifugal testing. An increase in loss factor of 0.001-0.003 based on tip velocity was achieved in the third bending mode location with a resistive shunt circuit implementation and no external power source. With a simple resistor, the circuit size was small, but the damping was not sufficient. The tuned shunt circuit with a wound inductor yielded very high damping in shaker tests, and will be tested in the spin rig in the near future. The inductor, sized for the 700-800 Hz range due to a spin test bandwidth limit of 1000 Hz, is quite large. A higher-frequency mode in a turbomachinery blade will reduce the inductor to a reasonable size – approximately an order of magnitude smaller at 2 kHz. The shunt circuit maintained integrity under centrifugal loading. With the resistive shunt circuit, there was minimal effect of the resonant frequency shift from 0 rpm to maximum operating speed. The inductor, tuned to a 3000 rpm resonance frequency, effectively damped the 0 rpm resonance frequency in shaker testing. However, spin testing must still be done to show this inductor functions well over the entire rotor speed range.

In conclusion, testing shows that shunted piezoelectric materials have the ability to reduce plate vibrations under centrifugal loading at room temperature and under vacuum. However, a resistive shunt alone will probably not be sufficient to achieve the desired damping goal. The addition of an inductor to form a tuned shunt circuit should increase damping considerably. Future work will include spin testing with a tuned circuit that includes an air-core inductor, and a tapered test article that targets a dominant high cycle fatigue bending mode. Researchers at GRC are also developing, with planned testing, newer high-temperature piezoelectric materials and high electro-mechanical coupled piezoelectric materials. Planned studies also include realistic implementation and testing of the piezoelectric material within a blade.

## Acknowledgments

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