A MATHEMATICAL MODEL OF A SIMPLE AMPLIFIER USING A FERROELECTRIC TRANSISTOR

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ABSTRACT

This paper presents a mathematical model characterizing the behavior of a simple amplifier using a FeFET. The model is based on empirical data and incorporates several variables that affect the output, including frequency, load resistance, and gate-to-source voltage. Since the amplifier is the basis of many circuit configurations, a mathematical model that describes the behavior of a FeFET-based amplifier will help in the integration of FeFETs into many other circuits.

Keywords: FeFET; FFET; ferroelectric transistor; analog amplifier; model

INTRODUCTION

The use of ferroelectric field-effect transistors (FeFETs) to create simple amplifiers has not been extensively studied. Moreover, the FeFET's unique characteristics of hysteresis and nonlinearity result in an amplifier that displays properties that are different from those of a MOSFET-based amplifier [1]. The FeFET amplifier's behavior has never been modeled. Thus, a mathematical-based model that describes a FeFET-based amplifier was created. The model is based on empirical data and incorporates several variables that affect the output, including frequency, load resistance, and gate-to-source voltage. The amplifier is the basis of many circuit configurations, and creating an easy-to-use model that illustrates the behavior of the FeFET amplifier will greatly aid in the integration of FeFETs into many other circuits.

FeFET AMPLIFIER CIRCUIT CONFIGURATION

A FeFET simple amplifier is built using a FeFET and a resistor. Figure 1 illustrates the FeFET amplifier circuit configuration. As can be seen from the figure, the resistor, R, is connected to the source of the FeFET. The input voltage, V_{in}, is provided from the gate of the transistor to ground. Thus, V_{in} includes both V_{GS} and the voltage across the resistor. V_{DD} is the voltage from the drain of the FeFET to ground, thereby including both V_{DS} and the voltage across the resistor. The output voltage, V_{out}, is measured at the source of the transistor and above the resistor.

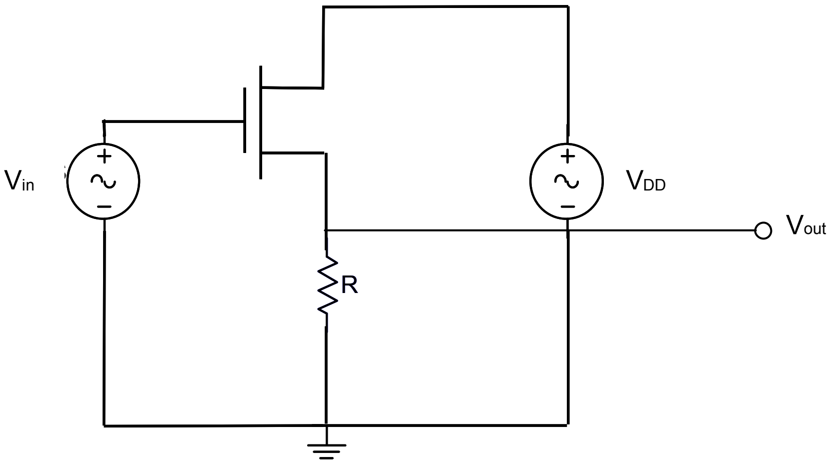

Figure 1 FeFET amplifier circuit configuration

DESCRIPTION OF THE MODEL

The model was created in Microsoft Excel and is based on the equations [2] for the drain current, I_D, that are derived from the Fermi-Dirac equation. When the gate voltage is on,

$$I_D = \frac{I_{DSAT} - B * Log(t_1)}{e^{k(Vp-Vgs)} + 1} \quad . \tag{1}$$

When the gate voltage is off,

$$I_D = \frac{I_{DSAT} - B * Log(t_1)}{e^{k(Vgs-Vp)} + 1} \quad . \tag{2}$$

In the two previous equations, I_D is the drain current, I_{DSAT} is the drain current when the FeFET is in saturation, B is the drain current decay coefficient, V_{gs} is the gate-to-source voltage V_{GS}, V_p is a polarization voltage, which is the gate voltage at which half the saturation current is reached, k is a constant defining the rate of change of the function, and t_1 is the elapsed time in seconds since the last poling. More needs to be said about the polarization voltage, V_p. Each FeFET has two different V_{p}'s, one is for the negatively-poled drain current, while the other is for the positively-poled current. This provides the hysteresis characteristic of the FeFET's I-V relationship.

The model allows the user to enter the desired value for several of the parameters that affect the output. Thus, the user can define values for V_{DD}, load resistance (R_{load}), input frequency, amplitude of V_{in} ($V_{in,amp}$), offset of V_{in} ($V_{in,offset}$), phase shift, and frequency coefficient. $V_{in,offset}$ was included in order for the model to be able to account for offsets in the input voltage. The phase shifting parameter *phase* is not the precise angular phase shift of the signal; rather it is a tunable parameter that is used to figuratively represent the amount of phase shift desired. Therefore, it is not a measurable parameter; it is an adjustable, user-specified parameter. The frequency coefficient is used to regulate the effect of frequency. Other parameters incorporated into the model include positive and negative V_p ($V_{p,pos}$ and $V_{p,neg}$, respectively), k, which is the constant given in Equations (1) and (2), and maximum I_D,. I_{Dmax} was needed for the calculation of I_D. The parameters $V_{p,pos}$, $V_{p,neg}$, k, and I_{Dmax} are determined through experimentation.

EQUATIONS OF VARIABLES

Besides the parameters defined by the user and those obtained by experimentation, the model also includes variables that are determined using simple mathematical equations. These parameters are V_{in}, $V_{in,phased}$, *Polarization1*, *Polarization2*, I_{Dpos}, I_{Dneg}, I_D, $I_{D,FA}$, and V_{out}. V_{in} is the value of the input voltage that takes into account both the amplitude of the input voltage and an offset, if given. The value of the input voltage that includes $V_{in,amp}$, $V_{in,offset}$, and the phase shift is $V_{in,phased}$. *Polarization1* and *Polarization2* are parameters that reflect the polarization property of ferroelectric material. I_{Dpos} is the drain current found using positive $V_{p,}$ $V_{p,pos}$, when the gate voltage is on, while I_{Dneg} is the drain current found using negative V_p, $V_{p,neg}$, when the gate voltage is on. I_D is the final value of the drain current. $I_{D,FA}$ is the frequency adjusted value of I_D. Thus, $I_{D,FA}$ adjusts I_D based on the input frequency and the frequency coefficient. Finally, V_{out} is the output voltage. The equation for each parameter is given in the following set of equations.

$$\mathrm{V_{in} = V_{in,amp}*sin(radian\ value) + V_{in,offset}} \tag{3}$$

$$\mathrm{V_{in,phased} = V_{in,amp}*sin(radian\ value + phase\ shift) + V_{in,offset}} \tag{4}$$

At radian value 0,

$$\mathrm{Polarization1 = 0} \tag{5}$$

At all other test points,

$$\text{Polarization1} = \begin{cases} 1 \text{ if } V_{in} > V_{p,pos} & (6) \\ \text{the value of Polarization1 at the preceding test point if } V_{in} < V_{p,pos} & (7) \end{cases}$$

At radian value 0,

$$\text{Polarization2} = 0 \quad (8)$$

At all other test points,

$$\text{Polarization2} = \begin{cases} 0 \text{ if } V_{in} < V_{p,neg} & (9) \\ \text{the value of Polarization1 at this test point if } V_{in} > V_{p,neg} & (10) \end{cases}$$

$$I_{Dpos} = I_{Dmax} / (e^{k*(Vp,pos - Vin,phased)} + 1) \quad (11)$$

$$I_{Dneg} = I_{Dmax} / (e^{k*(Vp,neg - Vin,phased)} + 1) \quad (12)$$

$$I_D = \begin{cases} I_{Dpos} \text{ if Polarization2} = 1 & (13) \\ I_{Dneg} \text{ if Polarization2} = 0 & (14) \end{cases}$$

$$I_{D,FA} = I_D\text{*frequency coefficient*}\log_{10}\text{frequency} \quad (15)$$

$$V_{out} = I_{D,FA}{*}R_{load} \quad (16)$$

The V_{in} and $V_{in,phased}$ equations are based on the sinusoidal wave equation [3] given by

$$x(t) = A*sin(\omega t + \theta) + D, \quad (17)$$

where A is the amplitude of the signal, ω is the radian frequency in radians per second, t is time in seconds, θ is the phase angle in radians, and D is the DC offset. In the equation for V_{in}, $V_{in,amp}$ is the amplitude, *radian value* is ωt, and $V_{in,offset}$ is the DC offset. There is no phase shift in V_{in}; however, this variable exists in the equation for $V_{in,phased}$. In order to fully explain the term *radian value* in the equations for V_{in} and $V_{in,phased}$, the method of measurement used in the model must first be discussed. The model measures the parameters at specific test points. The value of these test points are the nonnegative numbers. The degree equivalent of each test point was chosen to be (360/30) times the value of the test point. This relationship is given as follows

$$\textit{degree value} = \textit{test point} * (360/30) \tag{18}$$

The radian equivalent of the degree value of each point is utilized throughout all the equations for the variables. Since ωt is equivalent to $2\pi ft$, where f is frequency, and t in this model is a multiple of the period $1/f$ ($t=n/f$, where n is a nonnegative number), then ωt can be reduced to $2\pi n$. However, *degree value* can be rewritten as $360*n/30$, whose radian equivalent is given by $2\pi n/30$. Thus, ωt in Equation (17) is replaced with $2\pi n/30$ in the equations for V_{in} and $V_{in,phased}$, and the term $2\pi n/30$ is denoted by *radian value*.

The equation for each of the other variables is derived from the basic properties of ferroelectric material. The polarization variables, *Polarization1* and *Polarization2*, are initially set to zero, then altered based on the value of V_{in} at the current test point. If V_{in} is greater than $V_{p,pos}$, then *Polarization1* is set to 1, otherwise it is set to its value at the preceding test point. *Polarization2* is set to 0 if V_{in} is less than $V_{p,neg}$. If V_{in} is greater than $V_{p,neg}$, *Polarization2* takes the value of *Polarization1* at the current test point. The equations for I_{Dpos} and I_{Dneg} are derived from the Fermi-Dirac-based equation for I_D when the gate voltage is on, where V_p in Equation (1) is replaced with $V_{p,pos}$ for I_{Dpos} and $\mathrm{V}_{\mathrm{p},neg}$ for I_{Dneg}, I_{DSAT} is replaced with I_{Dmax}, and V_{gs} is replaced with $V_{in,phased}$. The term

$B*log(t_1)$ in Equation (1) is ignored in this model with little loss of accuracy. I_D equals I_{Dpos} if *Polarization2* is 1, but if *Polarization2* is 0, then I_D equals I_{Dneg}. V_{out} is determined simply using Ohm's Law with the load resistance and the frequency-adjusted value of I_D. This set of equations along with the user-defined and experimentation-based parameters provide a very good approximation of the output of a FeFET simple amplifier.

MODELED AND MEASURED DATA ANALYSIS

The accuracy and efficiency of the model created was examined by comparing the output of the model with empirical data for specific test cases. It is important to note that this model is the first mathematical-based computer model to characterize the behavior of a FeFET simple amplifier. Thus, this model is a first attempt and can be considered a preliminary version of more sophisticated models that can be created in the future.

Given the user-defined and experimental-based parameters, the model outputs a plot of V_{in} and V_{out} with respect to time. As was aforementioned, time is given as a multiple of the period. Several representative test cases were examined. For each test case, two sets of parameter values are included: one set leads to a model output where the amplitude of $V_{ou,}$ $V_{out,amp}$, is as close to the oscilloscope value as the model allows, while the other set results in a waveform for V_{out} that is very similar in shape to that of the oscilloscope. For each set of parameter values, a table of the values and a plot of the modeled output are given. The modeled plots are compared to the oscilloscope plot. For all the test cases, the sum of $V_{in,amp}$ and $V_{in,offset}$ never exceeds 8V to ensure that the FeFET is not burned.

Test Case I

For the first test case, V_{DD} was set to 0.709V, the amplitude of V_{in} was 6V, the frequency was 100Hz, and the load resistance was selected to be 35kΩ. No offset was included in the input signal. The following table lists the values of the user-defined parameters that resulted in $V_{out,amp}$ that is greater than that of the oscilloscope, but as small as the model allowed.

Table 1 Initial parameter values for Test Case I

Parameter	**Value**
V_{DD}	0.709V
R_{load}	35kΩ
Frequency	100Hz
$V_{in,amp}$	6V
$V_{in,offset}$	0V
Phase	1.5
I_{Dmax}	6.00E-04A
$V_{p,pos}$	2.5V
$V_{p,neg}$	-0.5V
k	0.75
Frequency Coefficient	0.007

The parameter *phase* was set to 1.5 in order to reproduce the phase shift seen in the oscilloscope output of the empirical measurement, which is shown in Figure 2. It was observed that a positive value for *phase* caused the output signal to be shifted to the left, while a negative value for *phase* led to a rightward shift of the output signal. Thus, in this test case, the output signal leads the input signal. The value of the parameter *k* was selected after experimenting and noting the effect of changing that parameter. For this test case, decreasing *k* below 1 removed the clipping effect at the peaks of the output signal, and further decreasing *k* below 0.8 removed the clipping effect at the troughs of the output signal. A very small value for *frequency coefficient* was chosen to ensure that

$V_{out,amp}$ was as small as possible. Decreasing *frequency coefficient* decreases $I_{D,FA}$, as seen in Equation (15), which leads in turn to a decrease in V_{out}. The values of I_{Dmax}, $V_{p,pos}$, and $V_{p,neg}$ were determined from previous experiments and will remain unchanged throughout all the test cases. Figures 2 and 3 show the oscilloscope output and the modeled output of the signals, respectively.

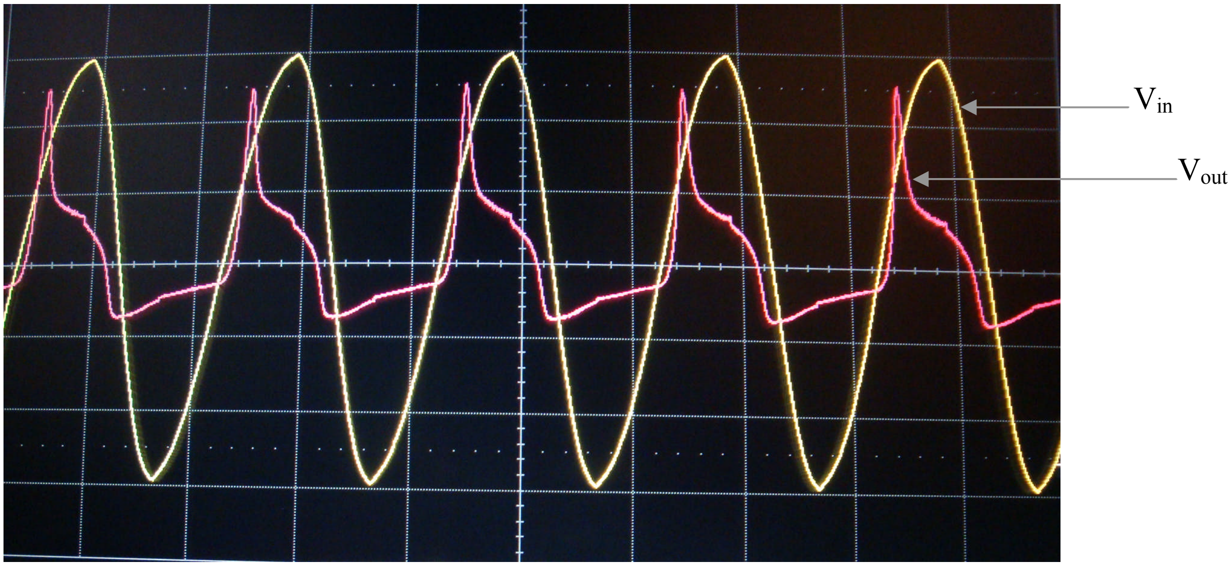

Figure 2 Oscilloscope output for Test Case I

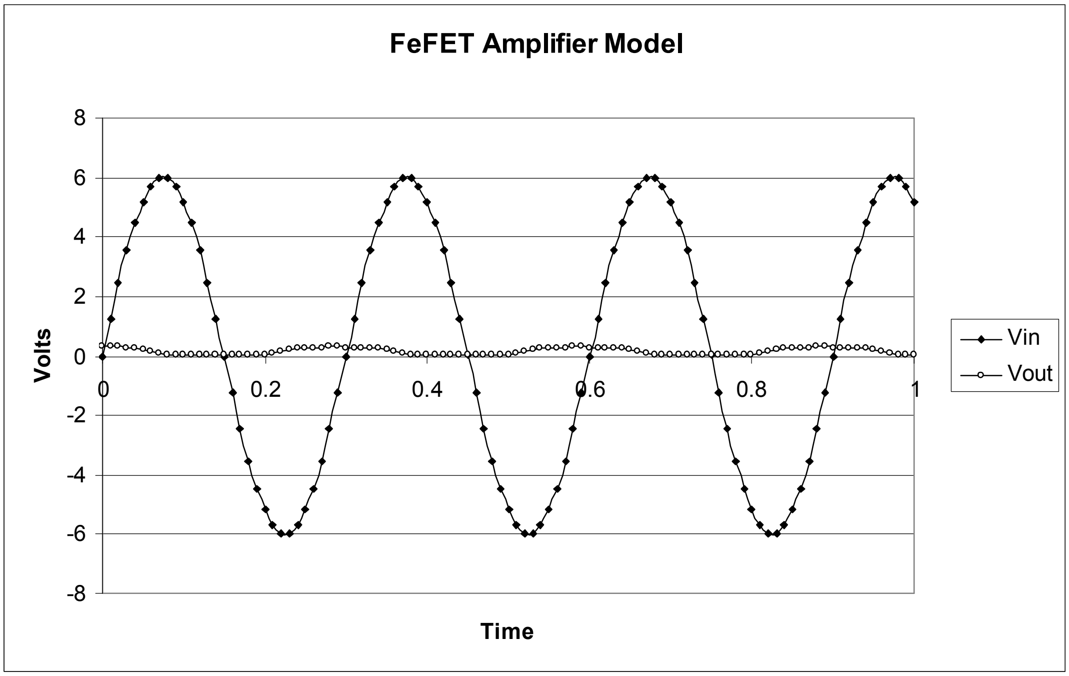

Figure 3 Modeled output for Test Case I based on initial parameter values

In this and all upcoming oscilloscope plots, it must be noted that the input and output waveforms are not plotted on the same coordinate system, despite their sharing the same axes. For example, in Figure 2, the coordinate system of V_{in} is partitioned into divisions that are 2V each, whereas V_{out}'s coordinate system is divided into 5mV per division blocks. Therefore, $V_{in,amp}$ is 6V, while $V_{out,amp}$ is only 11.8mV.

The maximum value of V_{out} as measured by the oscilloscope is 11.8mV, which is significantly larger than the amplitude of the model's V_{out}, as seen in Figure 3. However, this is the smallest value of $V_{out,amp}$ that still displays the shape of the signal. Comparing Figures 2 and 3, one might not believe that the model accurately illustrates the behavior of the FeFET amplifier, but increasing *frequency coefficient* to 0.15 and decreasing k to 0.3 produces the plot shown in Figure 4, which closely resembles the oscilloscope output. The given value of the *frequency coefficient* was found to produce the desired amount of frequency effect. At 0.3, k was small enough to model the jumps in the peaks and troughs visible in the oscilloscope output of this test case. Table 2 lists the new parameter values.

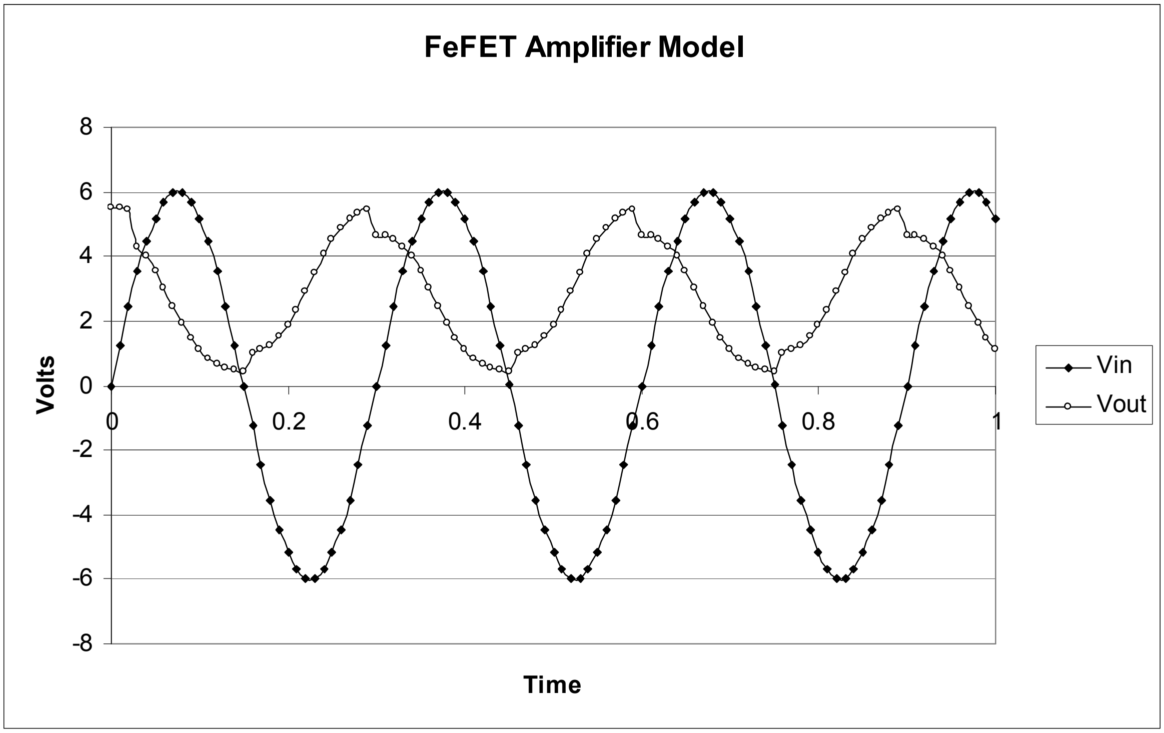

Figure 4 Modeled output for Test Case I based on new parameter values

Table 2 New parameter values for Test Case I

Parameter	Value
V_{DD}	0.709V
R_{load}	35kΩ
Frequency	100Hz
$V_{in,amp}$	6V
$V_{in,offset}$	0V
Phase	1.5
I_{Dmax}	6.00E-04A
$V_{p,pos}$	2.5V
$V_{p,neg}$	-0.5V
k	0.3
Frequency Coefficient	0.15

Comparing the modeled waveforms of Figure 4 with the measured waveforms shows that, in fact, significant similarities exist between the two plots. The modeled output signal displays the same jumps in its peaks and troughs as those of the oscilloscope output signal. Moreover, the input and output waveforms of the model intersect at almost the same locations as the measured waveforms. Despite these parallels, one particular divergence existing between the two output signals is the fact that V_{out} of the model never takes on both positive and negative values i.e., V_{out} is either greater than or equal to zero or less than or equal to zero, depending on the chosen values of the parameters. This fact will be noted in all the test cases. Further improvements in the model may resolve this issue. Even though the amplitude of V_{out} is not correct since *frequency coefficient* has been significantly increased for the purpose of depicting the details of V_{out}, overall, V_{out} of the model has the same shape as the oscilloscope's V_{out} and nearly the same values when *frequency coefficient* is small enough.

Test Case II

For the second test case, V_{DD} was again set to 0.709V, the amplitude of V_{in} was decreased to 2V, $V_{in,offset}$ was set to 2V, the frequency was 1MHz, and the load resistance was selected to be 600kΩ. The following table lists the values of the user-defined parameters.

Table 3 Initial parameter values for Test Case II

Parameter	Value
V_{DD}	0.709V
R_{load}	600kΩ
Frequency	1MHz
$V_{in,amp}$	2V
$V_{in,offset}$	2V
Phase	-0.75
I_{Dmax}	6.00E-04A
$V_{p,pos}$	2.5V
$V_{p,neg}$	-0.5V
k	1
Frequency Coefficient	0.0002

In this test case, a negative value for *phase* was chosen since the output signal lags the input signal. The magnitude of *phase* is much smaller for this test case than for the previous test case since, as can be seen in the oscilloscope output of Figure 5, the input and output signals are out of phase by a small amount. The removal of the clipping effect was again noted when the value of *k* was decreased. However, decreasing *k* below 1 resulted in more rounded peaks and troughs than desired, so *k* was set to 1. The value of 0.0002 was selected for *frequency coefficient* because it led to the smallest $V_{out,amp}$ while maintaining the desired shape of V_{out}. Figure 6 shows the modeled output.

Figure 5 Oscilloscope output for Test Case II

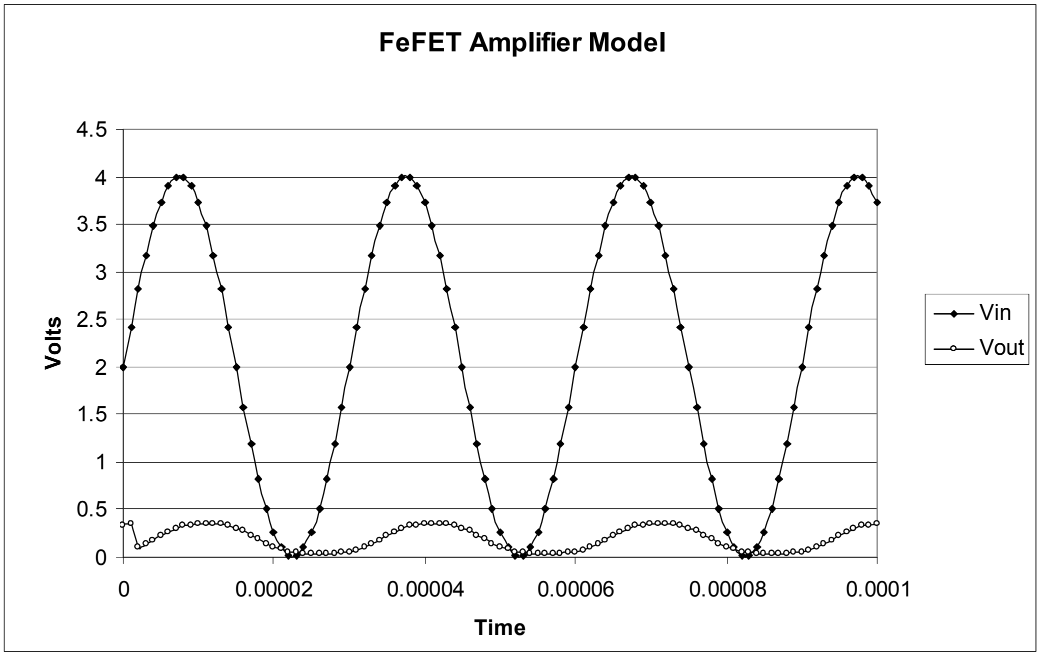

Figure 6 Modeled output for Test Case II based on initial parameter values

The modeled output displays the same phase shift present in the oscilloscope output. Moreover, $V_{out,amp}$ of the model is as close as possible to $V_{out,amp}$ of the oscilloscope output, which is 74mV, while displaying the same trends as the empirical output signal. To further verify the accuracy of the model, plots were obtained for this test case when *frequency coefficient* was increased to 0.002. Figure 7 shows the modeled output, and the table below summarizes the parameter values.

Table 4 New parameter values for Test Case II

Parameter	**Value**
V_{DD}	0.709V
R_{load}	600kΩ
Frequency	1MHz
$V_{in,amp}$	2V
$V_{in,offset}$	2V
Phase	-0.75
I_{Dmax}	6.00E-04A
$V_{p,pos}$	2.5V
$V_{p,neg}$	-0.5V
k	1
Frequency Coefficient	0.002

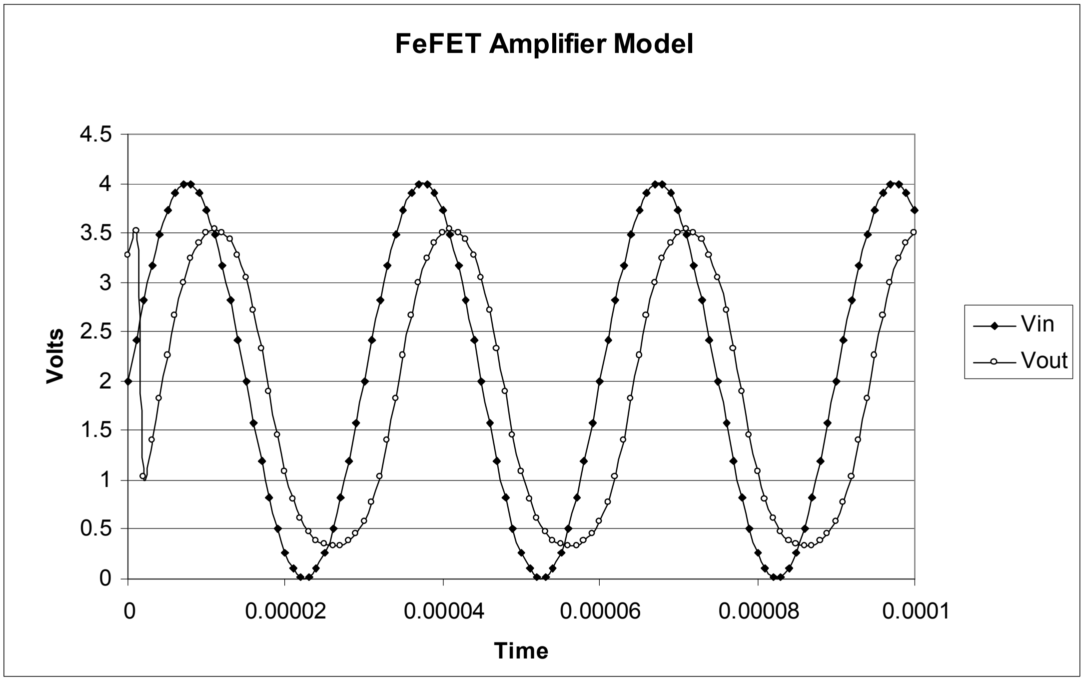

Figure 7 Modeled output for Test Case II based on new parameter values

The plots of Figure 7 look very similar to those generated by the oscilloscope. The modeled plots display the same shape and intersect at almost the same points as the oscilloscope plots. It can be concluded that the plots of Figure 6 have the correct, expected shape but were too small to be effectively compared to the measured results.

Test Case III

For the final test case, V_{DD} remained 0.709V, the amplitude of V_{in} was further decreased to 1V, 3V of offset were applied to V_{in}, the input frequency was 100kHz, and R_{load} was 560kΩ. These values and the other user-defined parameters are summarized in Table 5.

Table 5 Initial parameter values for Test Case III

Parameter	**Value**
V_{DD}	0.709V
R_{load}	560kΩ
Frequency	100kHz
$V_{in,amp}$	1V
$V_{in,offset}$	3V
Phase	1.5
I_{Dmax}	6.00E-04A
$V_{p,pos}$	2.5V
$V_{p,neg}$	-0.5V
k	2
Frequency Coefficient	-0.00015

This list of parameter values is different from those presented previously in several aspects. Here, *frequency coefficient* is a negative value. It was determined that a negative *frequency coefficient* most accurately represents the fact that the maximum value of V_{out} on the oscilloscope is 75mV, and its minimum value is -50mV, as seen in Figure 8. In this figure, the output signal is plotted on a coordinate system with 50mV per division. Another point of interest is the fact that the measured output signal is lagging the input signal, but a positive value of *phase* was selected. This can be attributed to the fact that the negative *frequency coefficient* flipped the plot of V_{out} across the x-axis, thereby making V_{out} lead V_{in}. This accounts for the positive value of *phase*, since a leading signal is given by positive *phase*. The parameter k was set to 2 because larger values for this parameter resulted in the clipping of the output signal. Thus, comparing the modeled plots of Figure 9 with the measured plots of Figure 8, it can be seen that the modeled V_{out} is close in value to the measured V_{out}.

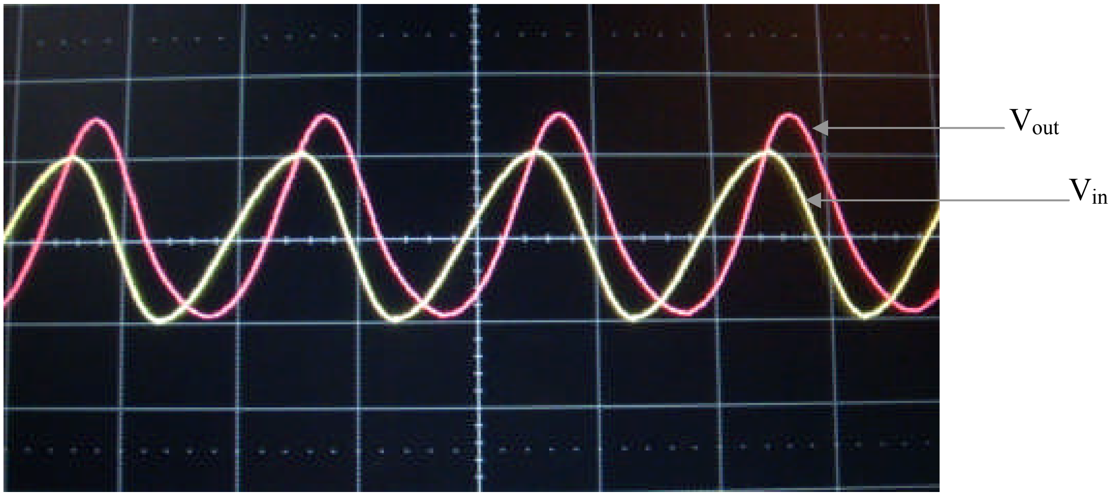

Figure 8 Oscilloscope output for Test Case III

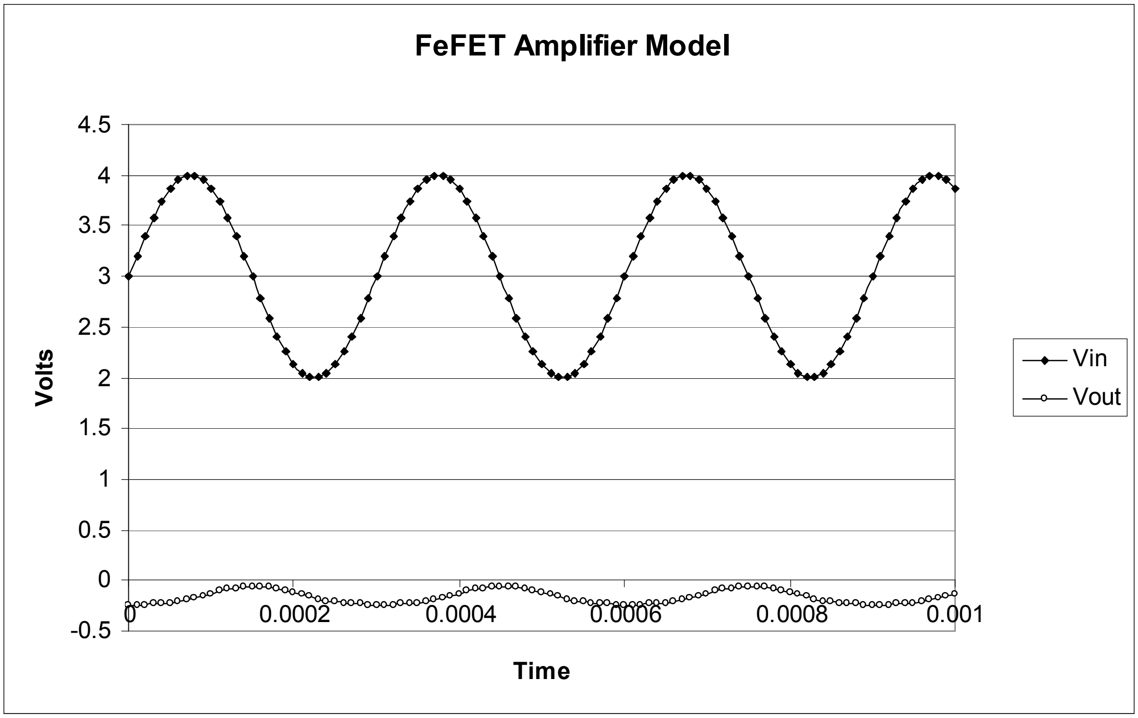

Figure 9 Modeled output for Test Case III based on initial parameter values

To obtain plots of V_{out} and V_{in} that closely resemble the oscilloscope plots, several changes were made to the parameter values. First, *frequency coefficient* was changed to a positive value, and its magnitude was increased to 0.0035. Since *frequency coefficient* was then positive, *phase* was changed to a negative value so that V_{out} lagged V_{in} as shown in the oscilloscope plots. A value of 1.3 was chosen for *k* because higher values resulted in more rounded peaks and troughs for V_{out} than desired. The following table lists the new parameter values. The modeled plots are shown in Figure 10.

Table 6 New parameter values for Test Case III

Parameter	**Value**
V_{DD}	0.709V
R_{load}	560kΩ
Frequency	100kHz
$V_{in,amp}$	1V
$V_{in,offset}$	3V
Phase	-1.5
I_{Dmax}	6.00E-04A
$V_{p,pos}$	2.5V
$V_{p,neg}$	-0.5V
k	1.3
Frequency Coefficient	0.0035

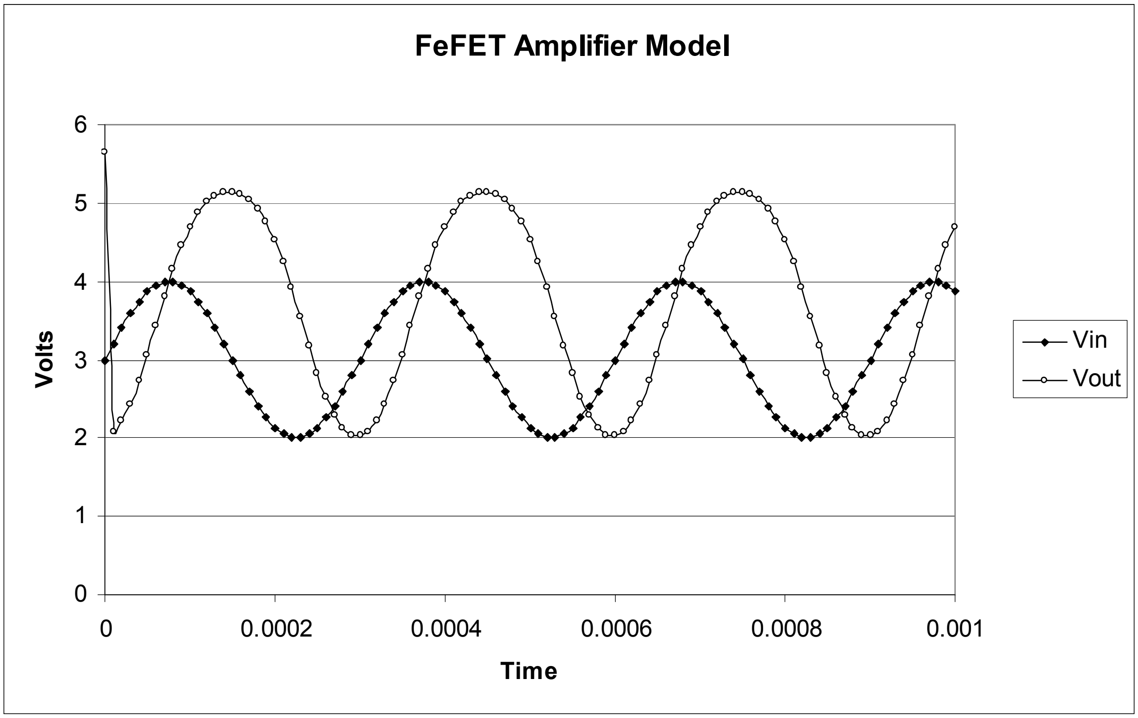

Figure 10 Modeled output for Test Case III based on new parameter values

These plots for V_{in} and V_{out} closely resemble the empirically-derived plots. As with the previous test cases, these plots verify the correctness of the model. It is clear that the desired results are obtainable using this model.

CONCLUSION OF DATA ANALYSIS

From these test cases, it can be concluded that the model's results are not only comparable but similar to the oscilloscope outputs. The model's output plots have the same shape and exhibit the same trends as those produced by the oscilloscope. Examining the mathematical equations behind the model brings to light their simplicity and the model's ease of use. Therefore, this model is accurate enough to be used now and efficient enough to be easily improved for further study.

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