

Complexity Science Applications to Dynamic Trajectory Management: Research Strategies

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The promise of the Next Generation Air Transportation System (NextGen) is strongly tied to the concept of trajectory-based operations in the national airspace system. Existing efforts to develop trajectory management concepts are largely focused on individual trajectories, optimized independently, then de-conflicted among each other, and individually re-optimized, as possible. The benefits in capacity, fuel, and time are valuable, though perhaps could be greater through alternative strategies. The concept of agent-based trajectories offers a strategy for automation of simultaneous multiple trajectory management. The anticipated result of the strategy would be dynamic management of multiple trajectories with interacting and interdependent outcomes that satisfy multiple, conflicting constraints. These constraints would include the business case for operators, the capacity case for the Air Navigation Service Provider (ANSP), and the environmental case for noise and emissions. The benefits in capacity, fuel, and time might be improved over those possible under individual trajectory management approaches. The proposed approach relies on computational agent-based modeling (ABM), combinatorial mathematics, as well as application of “traffic physics” concepts to the challenge, and modeling and simulation capabilities. The proposed strategy could support transforming air traffic control from managing individual aircraft behaviors to managing systemic behavior of air traffic in the NAS. A system built on the approach could provide the ability to know when regions of airspace approach being “full,” that is, having non-viable local solution space for optimizing trajectories in advance.

I. Introduction

This paper outlines a strategy for expanding applications from the modern developments in complexity science, such as agent-based modeling (ABM), into the airspace management domain. The advancements in complexity science and their applications have their roots in the research spawned at the Santa Fe Institute over the past two decades. Applications in many fields outside of aerospace include complex chemical, biological, communications, information, and social systems. Applications in aerospace represent a timely and logical field of endeavor. Future applications are practical in other aerospace, aeronautical, and air transportation system domains.

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trajectory-based operations in the national airspace system (NAS). Existing efforts to develop trajectory management concepts are largely focused on individual trajectories, optimized independently, then de-conflicted with respect to other trajectories, and re-optimized, individually, as possible. The benefits in capacity, fuel, and time are valuable, but perhaps could be greater through alternative strategies. The concept of agent-based trajectories introduced in this paper offers an approach to innovation for automation of simultaneous multiple trajectory management. The anticipated result of the innovation would be dynamic management of multiple trajectories with interacting and interdependent outcomes that satisfy multiple, conflicting constraints. These constraints would include the business case for operators, the capacity case for the Air Navigation Service Provider (ANSP), and the environmental case for noise and emissions. The benefits in capacity, fuel, and time might be improved over those possible under individual trajectory management approaches. The elements of the proposed strategy include computational agent-based modeling (ABM), combinatorial mathematics, application of “traffic physics” concepts to the challenge, and modeling and simulation capabilities. The approach would transform air traffic control from managing individual aircraft behaviors to managing systemic behavior of air traffic in the NAS. A system built on the approach would provide the ability to know when regions of airspace approach being “full,” that is, having non-viable local solution space for optimizing trajectories in advance.

Dynamic trajectory management is a candidate for ABM applications: the ABM methodology provides a means for simultaneously satisfying the competing and often conflicting constraints of business case, airspace capacity case, and environmental case in a heterogeneous environment (varied aircraft types and performance-based communication, navigation, surveillance, and flight performance capabilities). In addition to four-dimensional trajectory modeling in the presence of uncertainty, the ABM methodology also has potential applications to support research related to NASA’s Airspace Program¹.

This paper outlines a new framework for dynamic trajectory management in a NextGen NAS. The framework tools rely on agent-based technologies and computationally efficient combinatorial mathematics and will ultimately allow for achieving system-wide benefits via optimization of domain-specific functions, such as airspace capacity and business-case metrics. The paper introduces the basic concepts and describes an approach to evaluating a critical aspect of controlling complex adaptive systems—human-automation interdependencies.

II. Complexity Science and Agent-Based Technologies

The U.S. airspace is approaching a crisis point since mere operational inefficiency is edging closer to compromising safety, increasing costs to users and the FAA, and producing excess greenhouse gases. Current traffic is straining the existing system of air traffic control (ATC) to the breaking point, and future traffic promises to introduce a whole new set of challenges for which the present system has insufficient capabilities. The current system of traffic management has protocols dating back to World War II, a time when all of the “smarts” were on the ground (radar and radio beacons) and aircraft had a radio, an altimeter, a compass, and (to borrow from novelist J. Levine) a wing and a prayer.

In the intervening years, technologies have evolved on many relevant fronts. The synergy of these technological advances is now at the point where the operation of the U.S. airspace can be fundamentally re-thought. The goal of the FAA and JPDO’s NextGen program follows: “*NextGen moves away from legacy ground-based technologies to a new and more dynamic satellite-based technology....These new capabilities and the highly interdependent technologies that support them will change the way the system operates, reduce congestion, and improve the passenger experience*”ⁱⁱ. Key technologies include ADS-B (Automatic Dependent Surveillance – Broadcast), SWIM (System Wide Information Management), performance-based air traffic management, and NextGen data communications. While early implementation of these technologies is being pursued largely in the interest of their individual benefits, research is needed to pursue the integrated implementation of these technologies for dynamic trajectory management to determine the benefits of systems thinking.

In addition to technological advances, systems thinking about the U.S. air transportation system offers an approach for managing the interdependencies of all the system players. In the context of the science of complex systems, systems thinking has been around for decades, from the cybernetics of the 1950s to the general systems theory of the 60s and 70s^{iii, iv, v}. In the field of systems thinking, just as in the case of the national airspace, practical advance is tied to technological innovation. Many of the ideas systems science put forth in preceding decades did not achieve fruition or application until the 1990s and beyond because comprehensive understanding of those systems required ubiquitous access to powerful computing resources.

This requirement was met in the emergence of the desktop computer with associated languages and operating systems capable of building, simulating, and interacting with complex systems. Quantitative advances (in faster, smaller, cheaper, and more flexible computation) eventually resulted in qualitative advancements, namely in powerful and flexible simulation. Thus, simulation became a third category of scientific inquiry, complementing (even

completing) the traditional roles of theory and experimentation. For instance, aircraft testing that used to happen in wind tunnels or structures laboratories now more frequently occurs in the form of experiments by computer simulation. This maturation means that airspace architecture and procedure development can now be evaluated and implemented by applying the advances from systems thinking to the challenges of dynamic air traffic management.

III. Complex Adaptive Systems

The latest incarnation of systems science is often called CAS (Complex Adaptive Systems) and is generally associated with the Santa Fe Institute (SFI)^{vi, vii, viii}. Scientists at the SFI and elsewhere have studied systems as diverse as ecological communities, immune system response, stock markets, automobile traffic, and crime waves. CAS takes its lead from biological systems and develops computational methodologies for simulating, analyzing, and controlling man-made systems. Agency, Open Systems, and Adaptation. These three critical characteristics are common to natural and man-made systems.

Agency: CAS are characterized by many “computational agents”— entities capable of decision making, utilizing some degree of autonomy. In general, these agents do not have access to the entire state of the system, but rather are constrained to “live by rules,” using data that are local and relevant. These agents may be stock traders, flocking birds^{ix}, ants, flight path trajectories, or even aircraft with or without pilots. Figure 1 depicts simple but well-designed rules that produce an amazingly realistic and fluid motion.

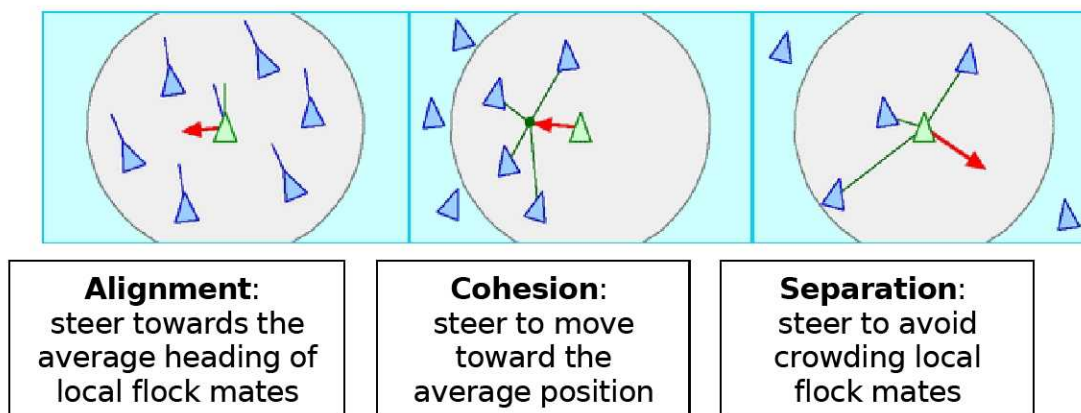


Figure 1. – Example of Agent Rules (C. Reynolds, 1989 ^x)

Open Systems: CAS may gain or lose agents and even types of agents because of their internal dynamics, such as individuals in an ecosystem being born or dying, generating the emergence or extinction of entire species. This openness prevents many real-world complex systems from being treated by traditional analytical techniques borrowed from statistical physics and economics that were built using the more tractable mathematics of closed systems. These traditional methods produced such triumphs as the ideal gas law and general equilibrium theory in economics. However, many modern systems are open and cannot be considered in isolation from their surroundings.

Adaptation: Agents are able to “learn” and modify their behavior over time in response to their histories so as to optimize their behavior. This ability generally means a higher probability of survival in natural systems and more cost-effective behavior in engineered systems. Their ability to adapt to disturbances and disruptions in the system is also advantageous since most natural systems are perpetually “noisy” and error-prone.

IV. What Can CAS Do for the U.S. Air Transportation System?

A. Coping with Complexity, Past

The U.S. airspace is facing a “complexity catastrophe,” of sorts, engendered by too many moving parts, too many constraints, and too many required decisions to be managed by a completely human-centered system. Even though it is technically possible to know the state of the entire airspace at a given time, it is computationally overwhelming to use this quantity of information in a practical way. For example, the complexity of computing the next course of action for each aircraft increases exponentially with the number of aircraft and rapidly becomes intractable. Therefore, any solution is going to require reducing the effective complexity of the system.

The current ATC system does exactly that. It reduces complexity by introducing constraints to the system such as the division of airspace into sectors, fixed airways between ground navigation stations, vertical and horizontal separation protocols, and procedural separation. This constrained system has worked well for decades, although at the cost of reduced efficiency and capacity. However, this system is now reaching the end of its effectiveness, largely because components that were previously treated as separable (for instance cruise trajectory management and tower-controlled air traffic management near large airports) are now becoming coupled due to increasing traffic density. As a result, the current methods of reducing complexity will no longer work efficiently. In other words, the model of component-independence approximates the real system to a lower level of fidelity than is required to ensure both he efficiency and the safety of the future NAS.

B. Learning from the Internet

The U.S. airspace may be able to benefit from the Internet business and operating models in terms of managing complexity and improving scalability while introducing fewer efficiency-killing constraints. Instead of behaving like a huge information packet-switching network, the NAS is comprised of a person- and cargo-switching network, and the “packets” are aircraft^{xi}. The Internet was originally designed to be robust and fail-safe, particularly in the event of a nuclear war. It has achieved this goal by incorporating redundancy and distributed control. It works reasonably efficiently; it has local rules (routing protocols) that when implemented by many routers in parallel produce an aggregate systemic result that is fairly optimal in terms of resource utilization. Since these rules were well designed, the Internet continues to operate effectively even though it has far outgrown its original defense system ambitions in terms of size, complexity, and diversity of applications. The Internet has thus proven to be an adaptive and scalable system.

The U.S. airspace, in contrast, is much more “high-touch” than the Internet. On the one hand, it requires massive human intervention for safe operations. On the other hand, human operators are not infallible, especially in the presence of high workloads. To keep the workload for air traffic controllers manageable, this human intervention component will need to be reduced, as it has been with the Internet. Granted, aircraft are more important than the average bit of information in data packets, and unlike bits of information cannot be split up, copied at will, sent via various routes, and reassembled at their destination (at least currently outside the realm of science fiction). Nonetheless, several key concepts, in particular Agency, Redundancy, and Adaptation, are transferable:

Agency: Internet packets contain meta-information, including, for example, destination. The information packets then interact with local rules (routers) to get where they are going without having to refer to a central authority. This approach dramatically reduces computational complexity and dependency. In a NextGen NAS, the aircraft can contain much more meta-information that has been the practice in the past (including trajectory information). This information would be the source of enhanced interactions between centralized and distributed interactions, in the interest of reduced workloads and improved robustness and performance (aircraft fuel and time and airspace capacity).

Redundancy: The Internet works even when some of its parts do not because it has multiple pathways for an information packet. The interaction of local routing rules with multiple pathways generates a large number of optional future paths at each decision point, making the system as a whole robust. In a NextGen NAS, dynamic trajectories would include multiple pathways as well, built to accommodate business case preferences.

Adaptation: The local rules for routing information allow the system to adapt to disturbance, such as routers or transmission lines being down. Adaptation is essential to robustness, along with redundancy and agency. In a NextGen NAS, dynamic adaptation of trajectories for continual re-planning in the face of disruption is essential for improving system performance.

It seems reasonable to expect that the scalability and reduced complexity of operating the NAS could benefit from the applications of such Internet lessons.

C. Coping with Complexity, Future

At the highest level, maintaining a manageable workload for air traffic controllers will require shifting some of their work in an intelligent fashion. A logical transition involves transferring this work to the aircraft in the airspace by providing them the information and the tools to act as agents on their own behalf. If this is accomplished carefully, the capacity, safety margin, and efficiency of use of the airspace can all be significantly improved while increasing the robustness and adaptability of the ATC system.

Implementing this transition properly requires the appropriate hardware and software for aircraft to identify the current position and anticipated trajectory, or *intent*, of each aircraft. In this context, *intent* means not only knowing where an aircraft is going, but also knowing its negotiation protocols and preference structure for interacting with other aircraft that may perturb its path. To achieve this capability electronically will require a formal computational language by which aircraft can negotiate with each other on the use of the airspace as well as default and fail-safe backups. As a naïve example of an agent-based protocol, the following communication between two aircraft could occur: “If projected conflict is positive, we both perturb our trajectories by 10 degrees left and recompute possible conflict until possible conflict is negative.” A complete communication protocol would include sophisticated conflict resolution (and other) protocols in a dynamic trajectory optimization system.

V. Coping with Complexity in NextGen

Designing such protocols will require accounting for the business case parameters, as well as airspace capacity and environmental considerations in NextGen trajectory management. The approach would start with theoretical considerations of system stability and mechanism design, evolve through computational testing and simulation in increasingly sophisticated and realistic model environments, and eventually produce a set of protocols for testing in a simulation environment with humans and automation interacting. In-flight evaluation of such an architecture would logically follow. Figure 2 illustrates the relationships among air traffic operations (controller), and the flight operations (dispatcher/trajectory manager), and the aircraft fleet (pilots) that would be served by automation of trajectory management. Where current air traffic operations primarily involve the controller and the pilot, NextGen operations would leverage stronger interactions of the flight operations service for trajectory management. The research questions posed by these interactions would include the interdependencies of humans and automation involved in the tactical and strategic management of trajectories.

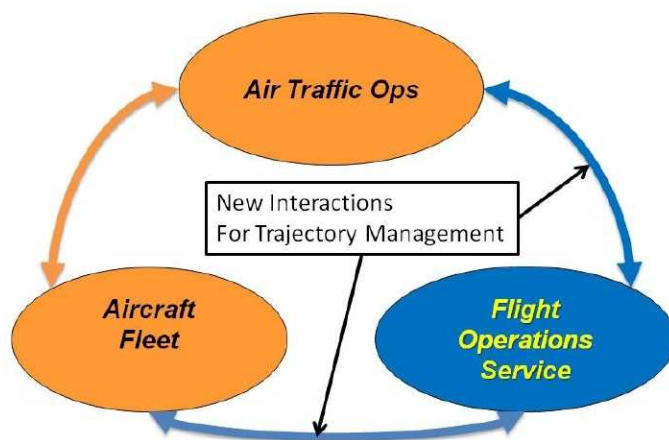


Figure 2. – Integrated NextGen Dynamic Trajectory Management System

Some current thinking regarding trajectory optimization takes the approach that operators would independently compute multiple acceptable flight plans, queued in priority according to their business case considerations. Then when airspace disruptions occur, the air traffic managers and pilots would institute the next most preferable flight plan in the queue. This approach might be unnecessarily labor intensive. Automating trajectory optimization could bring various benefits beyond workload reduction. Those benefits could include optimal performance of the airspace with simultaneous consideration of multiple operators’ business plans, fuel efficiency considered from the standpoint of multiple trajectories rather than trajectories considered independently, and related emissions and noise considerations. This concept is the foundation of the agent-based trajectory thinking proposed in this paper.

VI. Relevant Components of CAS to NextGen Dynamic Trajectory Management

A. Game theory^{xii}

Originally developed in the 1940s to model military conflict and economic choice, game theory has modern relevance in the field of mechanism design^{xiii, xiv, xv}: How does one design local rules of interaction (game rules) so that a larger system composed of subsets of agents interacting with each other by local rules of interaction produces an efficient and robust result for the entire system? Nature is rife with examples. For example, ants in an ant colony are not aware of the state of the entire colony, but their local decisions in foraging and defense behaviors serve to further the colony and optimize its chances to survive and prosper. Another example is the food system in New York City, which at any one time has only a three-day supply. However, a complex system of commerce and transport markets ensures access to food is uninterrupted all the way from farm to table without micromanagement by a central authority. These examples serve as metaphors for managing air traffic.

This methodology of aligning local rules and incentives with consequent system behavior would be useful in designing protocols for collision avoidance, separation, and trajectory optimization between pairs or small numbers of aircraft in a local or regional space. The resulting method would simultaneously ensure safe operation, overall system effectiveness, and a level economic/business landscape for all players in the airspace. This is the logical next step beyond creating local rules by which aircraft avoid each other, as is currently the subject of exploratory research^{xvi}.

B. Traffic physics

The science of traffic physics is a new field emerging at the boundary of the study of agent behavior and statistical physics. To date, the science has largely been applied to roadway vehicle dynamics because of the significant societal and financial import. In addition, traffic systems are perceived to be highly suboptimal and offer ready access to large amounts of data^{xvii, xviii}. This research has applicability to other many-agent systems in addition to roadways^{xix}. The utility of the science is the ability to define systemic measures that are independent of the particular behaviors of each agent in a traffic system, much as the pressure exerted by a gas on its container is independent of the details of motion of each individual molecule. Analyzing traffic systems is also computationally extensive, another reason why the field is so new. As the Nobel physicist Murray Gell-Mann once said, “Imagine how hard physics would be if particles could think.” Systems of cars or aircraft are essentially thinking particles, serving as evidence of Gell-Mann’s insight.

Physical systems consisting of many particles are often characterized in terms of *phase*, such as liquid, solid, or gaseous. The phase is a property of an entire system, rather than of any of its particular components (molecules, cars, or aircraft). The dependence of systemic properties on a small number of critical parameters can be shown on a phase diagram. As an example, the three well-known phases for water are dependent on temperature and pressure. Adjusting these parameters can control the phase of water.

Systems of interacting agents in freeway traffic also have phases that correspond to free-flowing (“liquid”) or jammed (“solid”) traffic. Traffic also has phases that do not have analogues in physical systems, such as backwards-flowing waves of stalled traffic mixed with moving traffic. Phase diagrams for vehicle traffic look somewhat different than the water phase diagram above because there is one critical parameter (vehicle population density) to adjust instead of two (temperature and pressure). Just as the water molecules obey certain laws (conservation of energy and momentum, equipartition of energy), the traffic “molecules” obey simple laws. They attempt to get where they are going as quickly as possible (with an upper limit) and interact with other vehicles, avoiding collisions and following at a safe distance. In vehicle traffic, throughput (or capacity) of a roadway increases with density to a certain point after which a marked decrease is observed; hence, the emergence of a traffic jam.

In Figure 3, the black x's are experimental data, the grey boxes are simulation data, and the lines are theoretical upper and lower bounds illustrating the bounds of maximum flow and congested flow.

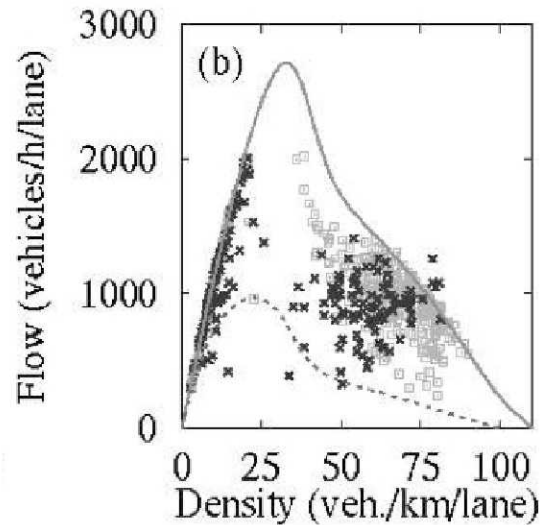


Figure 3. – Traffic Phase Diagram (D. Helbing, 2002^{xx})

These phase analysis techniques have yet to be applied to the three-dimensional motion of aircraft, but there is no fundamental reason why this cannot be done. Formulating a traffic physics paradigm for aircraft is important in that it allows one to formulate a general answer to a ubiquitous and important question: “When is the airspace ‘full’?” without having to specify the details of a particular air traffic scenario. Furthermore, it can be used as a guide to engineering different local rules such as: “Can the airspace capacity be increased by a different choice of “particle interaction rules” (*i.e.*, aircraft navigation protocols)? Characterizing a system by its phase is different from using measures that are associated with individual agents or with small departures from equilibrium, such as Lyapunov exponents^{xxi}. Framing the system as a collection of agents lays the groundwork for characterizing the system’s overall behavior or state.

C. Optimization and Logical Phase Transitions

A key part of creating a safe, robust, flexible, and efficient air traffic system is defining metrics that are measurable and can be optimized. Certain aspects of trajectory optimization are already well understood and implemented in some cases^{xxii}. Phase transitions were discussed in the context of physical systems of particles and traffic in the section above, but phase transitions also exist in *logical* systems such as schedules or other optimization problems. In a general optimization problem, the number of possible solutions will decrease (unless it is already zero) as the number of constraints increases. This decrease is often not gradual but rather sudden, and it looks like a typical physical phase transition such as that between water and ice with a sharp and well-defined boundary.

Almost all optimization problems can be reduced to a construct known as SAT, short for “satisfiability^{xxiii}.” The critical parameter for this phase transition is not temperature or pressure or vehicle density, but rather the density of constraints—the x-axis in Figure 4.

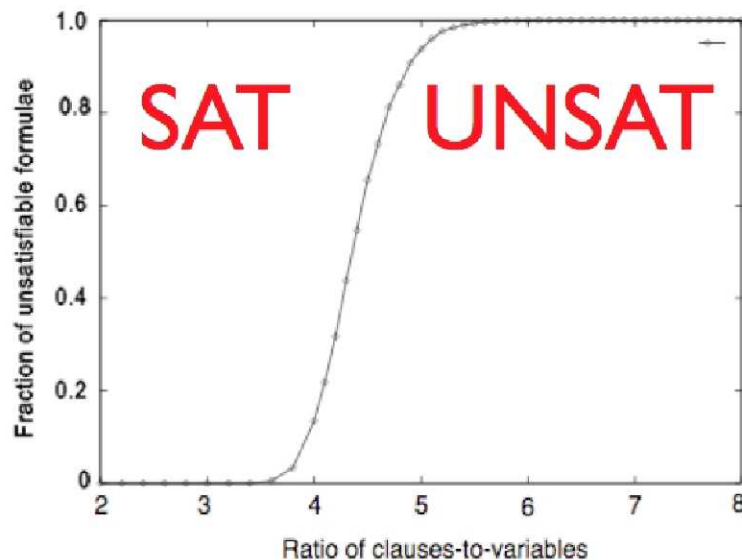


Figure 4. – Satisfiability Phase Transition (Kirkpatrick and Selman, 1994^{xxiv})

By taking the agent’s point of view, the logical and physical definitions for systems can be intuitively connected. For example, if the logical agent (molecule or car or aircraft) has, on average, no options as to where to go next, then the physical system freezes up or the traffic jams, or the system is “full.” Kirkpatrick and Selman^{xxv} and others established the mathematical connection between logical and physical phase transitions in the last ten years. This connection between the theory of optimization and the phase transition between viable and non-viable solution regions can provide insight into the definition of metrics for robustness and flexibility, both of which are related to the presence and “on the fly” accessibility of alternate solutions for an aircraft’s trajectory.

Robustness can be interpreted as the presence of many solutions to a problem, and flexibility could be interpreted as the ease with which one can transition between these solutions^{xxvi}. The solution of general SAT systems provides compelling insight^{xxvii} into these abstract concepts as Figure 5 shows.

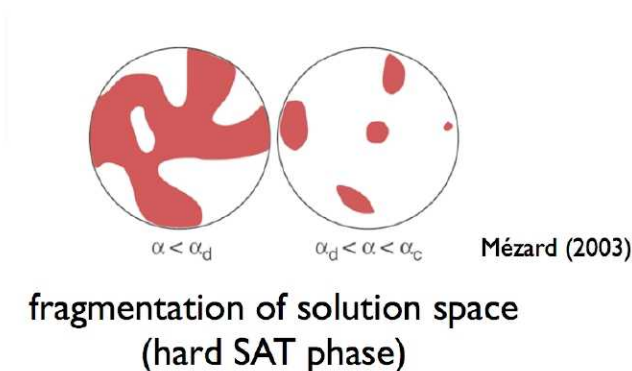


Figure 5. – Solution Space of SAT Problems – Near Phase Transition^{xxviii}

The solution space transforms from connected (on the left) to disconnected (right) as the phase transition boundary is approached, meaning that flexibility disappears before robustness does. This near-phase-transition phenomenon is also appealing because it means that there is an advance warning of the onset of a phase transition, something extremely useful in systems where humans might intervene to avoid undesirable dynamics. This phenomenon hints at the

possibility of an emerging role for NextGen air traffic management systems: managing systemic behavior rather than micromanaging individual aircraft behavior.

VII. Conclusion

Three related thoughts on research strategies for dynamic trajectory management are offered in this paper: (1) agent-based trajectories may offer a means and form of optimizing *en masse* among the business case constraints, airspace capacity constraints, and environmental constraints for air traffic management; (2) agent-based trajectory management may offer an approach to exploring and configuring the interdependencies of the human and automation involved in managing aircraft flight paths as a set rather than independent and individual, one-at-a time movements; and (3) agent-based approaches to airspace management may provide a tool for applying the concepts of traffic physics to the challenges of airspace capacity management.

Taken together, these thoughts represent a promising strategy that should complement airspace management research and applications, with potential benefits to the topics of interest in the current NASA Airspace Program, including the following^{xxix}:

- Collaborative decision making techniques involving multiple agents
- Optimal allocation of separation assurance functions across humans and automation and air and ground systems
- Optimization techniques to address demand/capacity imbalances
- Traffic complexity monitoring and prediction
- Weather assimilated into ATM decision-making
- Environmental metrics and assessments of new concepts and technologies
- The effect of traffic congestion on integration of UAVs into the NAS
- The requirements for, and the development of, a simulation environment to test UAV integration in the NAS

This paper outlines a new framework for dynamic trajectory management in a NextGen NAS. The framework tools rely on agent-based technologies and computationally efficient combinatorial mathematics and should ultimately allow for achieving system-wide benefits via optimization of domain-specific functions such as airspace capacity and business-case metrics. The paper introduces the basic concepts and describes an approach to evaluating a critical aspect of controlling complex adaptive systems – human-automation interdependencies.

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