WFSC
WAVEFRONT SENSING & CONTROL GROUP

Status on Iterative Transform Phase Retrieval applied to the GBT Data

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Image-Based Wavefront Sensing and Control of the NRAO Green Bank Radio Telescope

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Collaborators: Dr. Richard Prestage / Assistant Director, National Radio Astronomy Observatory (NRAO) Green Bank Operations, Green Bank, WV, Todd Hunter, Frank Ghigo, NRAO Green Bank Operations, Green Bank, WV, Bojan Nikolic, Mullard Radio Astronomy Observatory, Universiy of Cambridge, UK

Business Area: Astrophysics, Communications / Navigation Systems, Exploration Systems.

Overview

Introduction
- Phase Retrieval / NASA Projects
- JWST TRL-6

GBT Data / Notes:
- Data Format and Sampling
- Ray Trace Model & Wavefront
- Symmetry of GBT Data
- Pupil and Fourier Model
- Pupil Amplitude

PR Simulations
- Wavefront derived from GBT Data symmetry
- Wavefront Sensing accuracy and Coherent / Incoherent Assumptions

GBT Results

Applications and Technology Development

- NASA Investments in Image-Based WFSC
  - Developments through JWST Pre Phase-A and Phase-B,
  - WFSC Demonstrated to TRL-6 using the Ball Aerospace TBT,
  - Have investigated a number of performance and implementation details, e.g., optimal diversity defocus, bandpass, phase wrapping, Branch Points,
  - Compact Supercomputing Architecture utilizing DSPs

Date	*Projects*
1990	Hubble Primary Mirror Aberration Determination
1994	Mars Observer Camera In-flight Diagnosis
1996	Cassini ISS Narrow Angle Camera Verification Testing
09/1998	NASA Developmental Comparative Active Telescope Testbed (DCATT)
01/1999	NASA Wavefront Control Testbed (WCT)
01/2000	NASA Wavefront Control Testbed 2 (WCT-2)
01/2002	NASA Wavefront Control Testbed 3 (WCT-3)
08/2000	IRAC Testing (Spitzer Space Telescope)
08/2001	Phase Retrieval Camera
04/2002	RIVMOS Testing
07/2002	NIRSpec Microshutter (MSA) Testbed
09/2002	HUBBLE Simulator Hardware (CASTLE)
04/2003	TPF's High Contrast Imaging Testbed (HCIT)
04/2003	Mercury Laser Altimeter (MLA)
06/2003	NASA Fixed Lens WFS Testing
08/2003	JWST AMSD Mirror Testing with a Phase Retrieval Camera
07/2003	Ball Aerospace RA-6 (Boulder, CO)
10/2003	GSFC EUNIS Testing
07/2004	IRMOS Modeling
09/2004	Keck I (Kamuela, HI)
08/2004	HST Wide-Field Camera III
07/2005	Palomar 200" Telescope Adaptive Optics System (PALAO) Calibration
10/2005	JWST Testbed Telescope (TBT; Ball Aerospace, Boulder, CO)

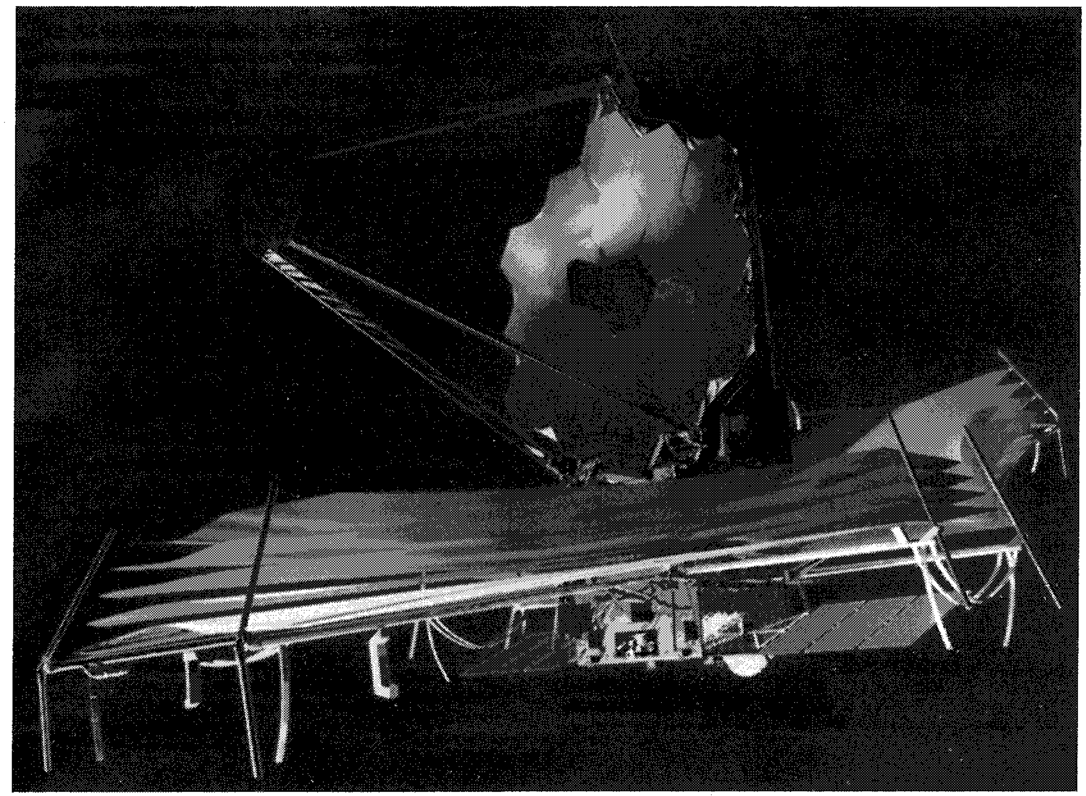
James Webb Space Telescope (JWST)

- Successor to the Hubble Space Telescope
- Current Launch Date is 2013

- 18 Segment PM
- 6.5 meter aperture
- Orbit at L2

NORTHROP GRUMMAN
Space Technology

Ball

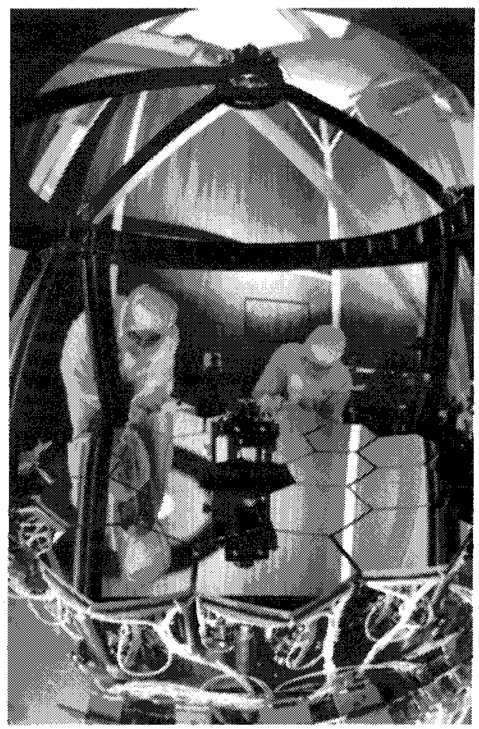


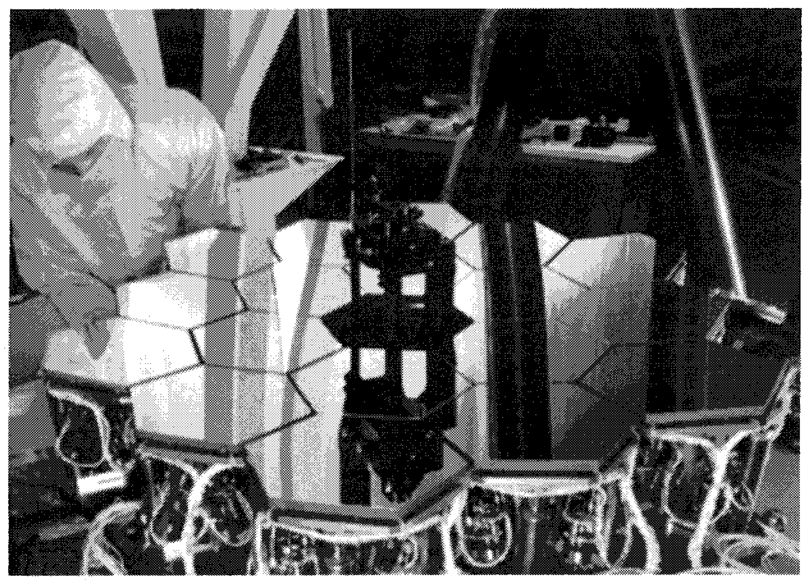
Testbed Telescope (TBT)

Flight traceable, 1/6 scale, 18 segment design

Algorithm Performance requirements dictated by NASA's **TRL-6**

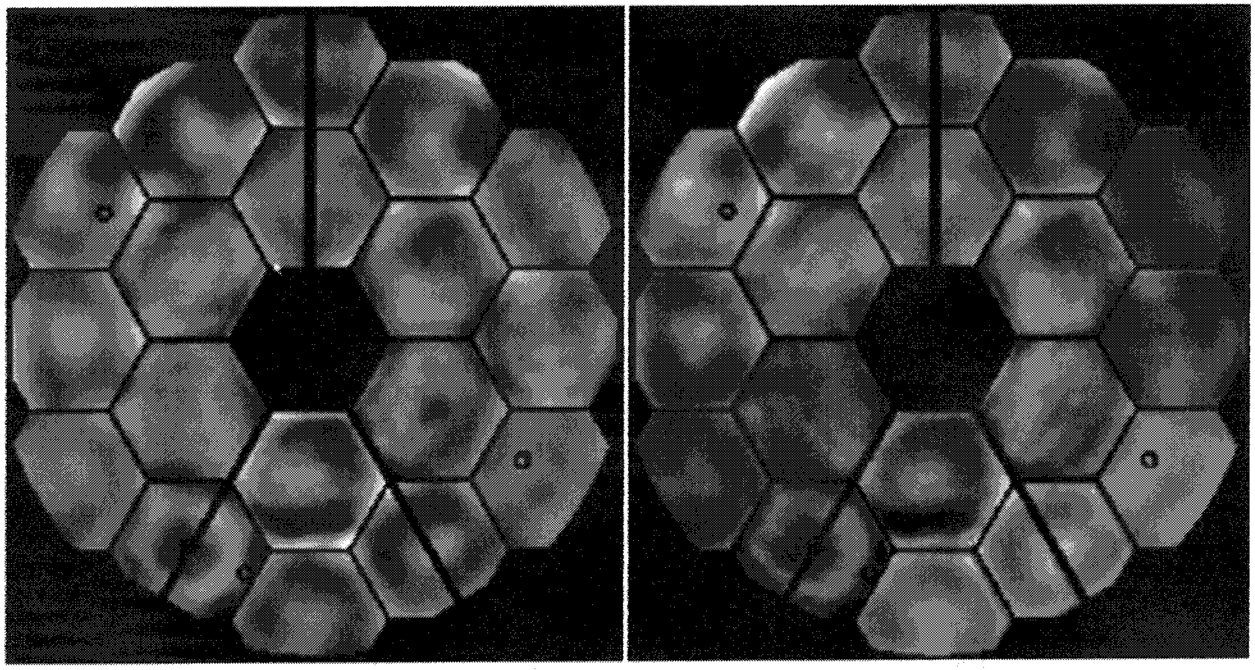
- Testbed provides functionally accurate simulation platform for developing deliverable WFSC algorithms and software,
- Used to perform TRL-6 end to end testing,
- a solution is a fine-phasing algorithm that incorporates feedback,
- an adaptive diversity function, eliminates Branch Points, and Wrapping





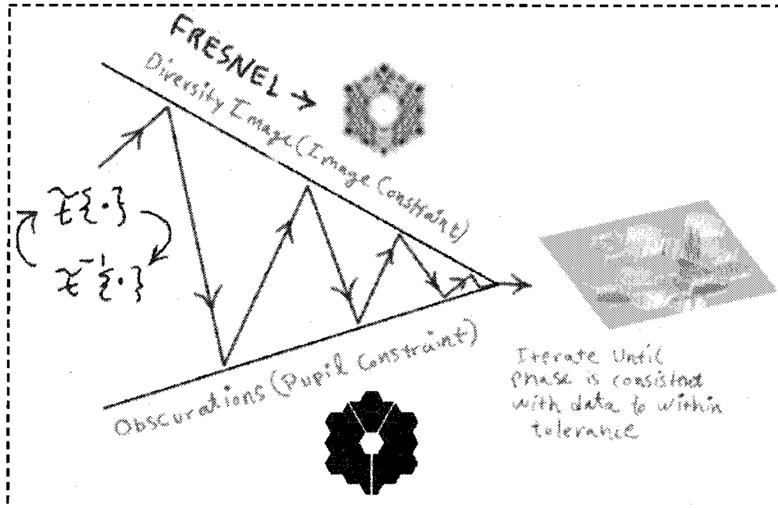
TRL-6 Comparison with Interferometer

Phase Retrieval: Interferometer:



Phase Retrieval Approaches

- Two main approaches commonly used:
  - Iterative Transform (ITA)
    - Gerchberg-Saxton
    - Misell-Gerchberg-Saxton
    - HDA (extends dynamic range)



  - Parametric (non-linear least squares model fitting)
    - Solve for aberration coefficients
    - Solve for point-point phase in the pupil

For JWST - adopted a hybrid approach that incorporates features of both types of algorithms.

min [Objective Function]:

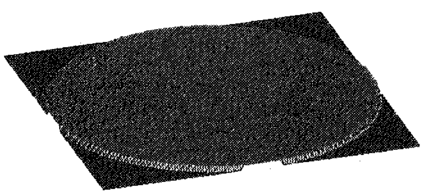
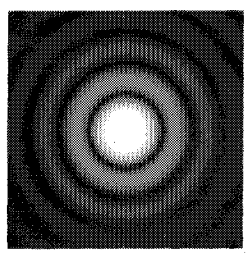
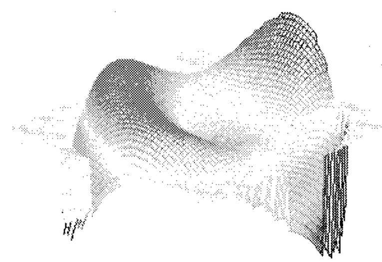
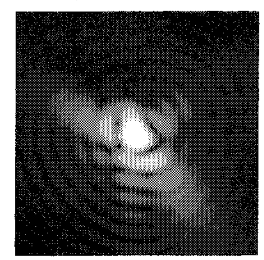
$$\frac{\partial}{\partial a_m}\sum_{j=1:N}\left\| psf_{data,j} - psf_{model,j}(\lambda, f_{\#}, Pixel, \Phi_{Div}, \vec{a})\right\|^2 = 0$$

† e.g., W. H. Southwell, "Wavefront Analyzer using a Maximum Likelihood Algorithm," J. Opt. Soc. Am. A3, 396-399 (1977).

Concept:

- phase from intensity data? (intensity → r, phase → θ)
  $$z = x + i\,y = r\,e^{i\theta}$$
- complex numbers:
  $$\Rightarrow |z|^2 = r^2 e^{i\theta} e^{-i\theta} = r^2 \neq r^2(\theta)$$
- phase part is decoupled from intensity

- phase-recovery fact - optical aperture scatters phase information into the intensity data
- star image –normally like an airy disk for a circular aperture:

- intensity is now a function of the phase: $$r^2 = r^2(\theta)$$
- algorithm: indirectly recover phase from intensity.

Earlier Work using ITA with Radio Antennas

- 1985, D. MORRIS ‘Phase retrieval in the radio holography of reflector antennas and radio telescopes’, IEEE Trans., AP-33, pp.749-755
- 1988, D. Morris, et al., “Radio holography measurement of the 30-m millimeter radio telescope ...,” Astron. Astrophys., vol. 203, p. 399.
- 1991, D. MORRIS, et. al, ‘Experimental assessment of phase retrieval holography of a radiotelescope’, IEE Proc. H, 138, pp. 243-247
- 1994, A. Greve, D. Morris, et. al., "Astigmatism in Reflector Antennas: Measurement and Correction," IEEE Trans ANTENNAS & Prop VOL. 42, NO. 9
- 1996, D. Morris, Simulated Annealing Applied to the Misell algorithm for phase retrieval, IEE Proc - Microw Antennas Prop , Vol 143, No 4, August I996

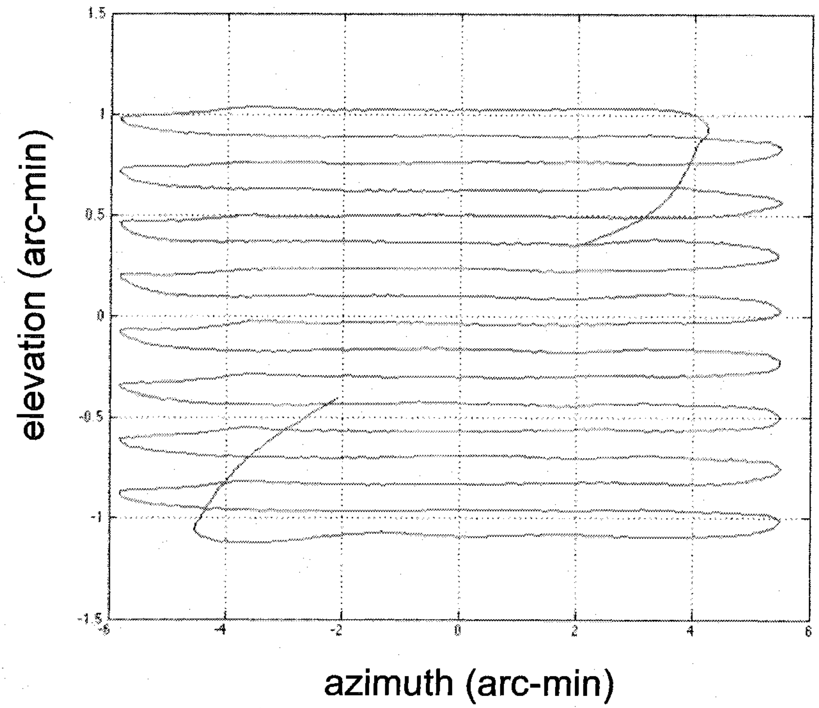
Notes / Understanding of GBT Data

- Consists of two feeds (pixels), two polarizations,
- Separated by 58 arc-seconds,
- Output of receivers is differenced to minimize the effect of sky-brightness variations.
- Effective response of the telescope is modeled as the real beam convolved by two delta functions separated by 58" in the azimuth direction
  - aberrations due to both of the feeds being off (and on opposite sides of) the optical axis are negligible?
  - if this is not negligible, then a "single beam convolved by two delta functions" assumption may not be valid.

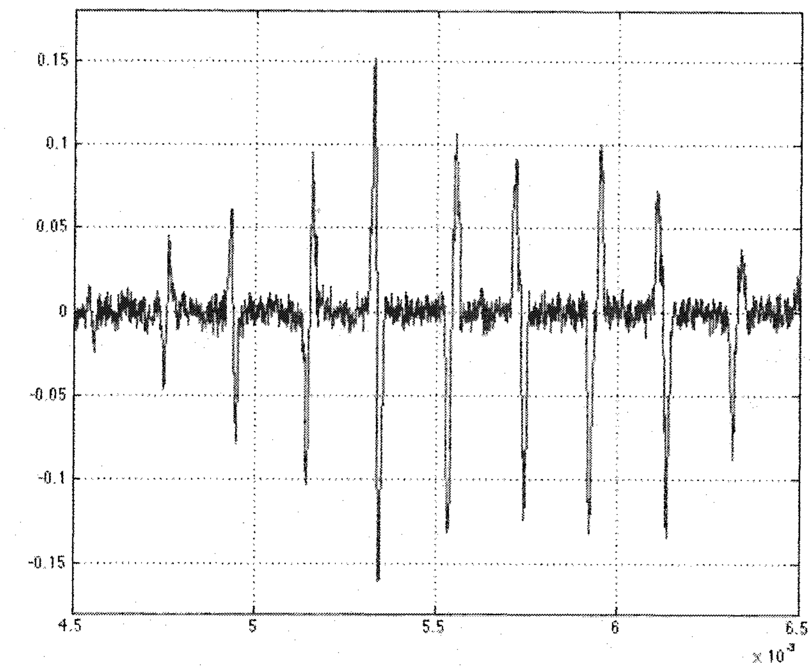
Raw Data Contributed by the NRAO

- Data Filename: s114-l-db.fits, April 2005
- Read: `dx, dy, fnu, ufnu, ttime` ([5806×1 double])

Scan Pattern: (plot dx, dy) :



Signal vs time (plot fnu, ttime):

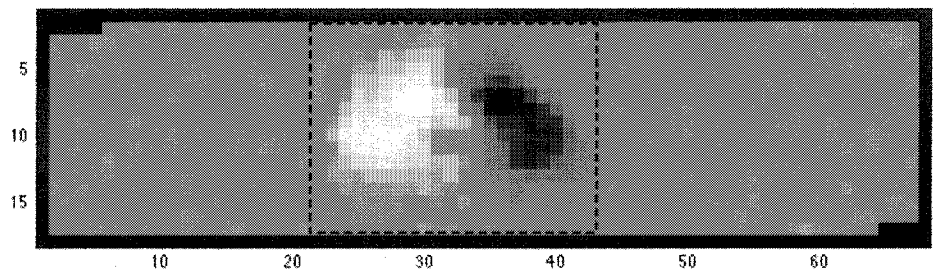


NRAO Data: Non-uniform data samples are interpolated:

- Data values: dx (azm) , dy (elv) are used to form a rectangular coordinate array.

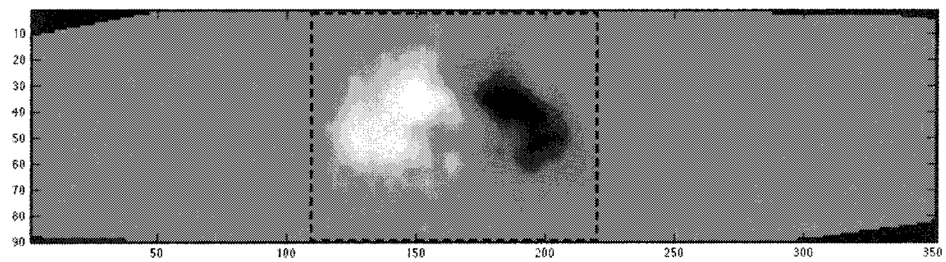
Two Options:

down-sample in x:



17×68

up-sample in y:

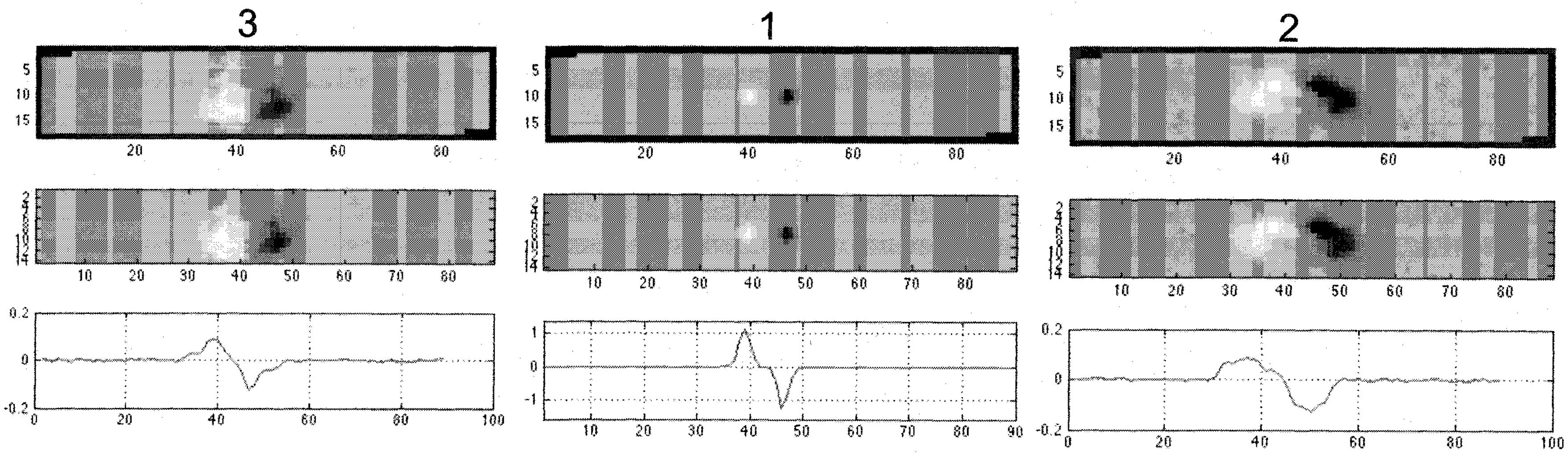


90×351

- First interpolated to a uniform rectangular grid (azm-elev),

- A rectangular coordinate grid of 17 by 68 is formed and then the 5806 fnu data values are interpolated to this grid using cubic interpolation.

NRAO Data & Sampling



ν = 43.1 GHz; λ = 6.96 mm

<u>Azimuth direction</u> (x), approximately 350 samples/per scan line.

Sampling in Azimuth = 3600*(180/pi)*(dx(251)-dx(250)) = **2.42 Arcsec / pixel** = px = 1.1732e-05 radians

Qx = λ /(D*px) = 6.96e-3 / (100*1.1732e-05) = 5.9325

<u>Elevation direction</u> (y), 17 scan lines

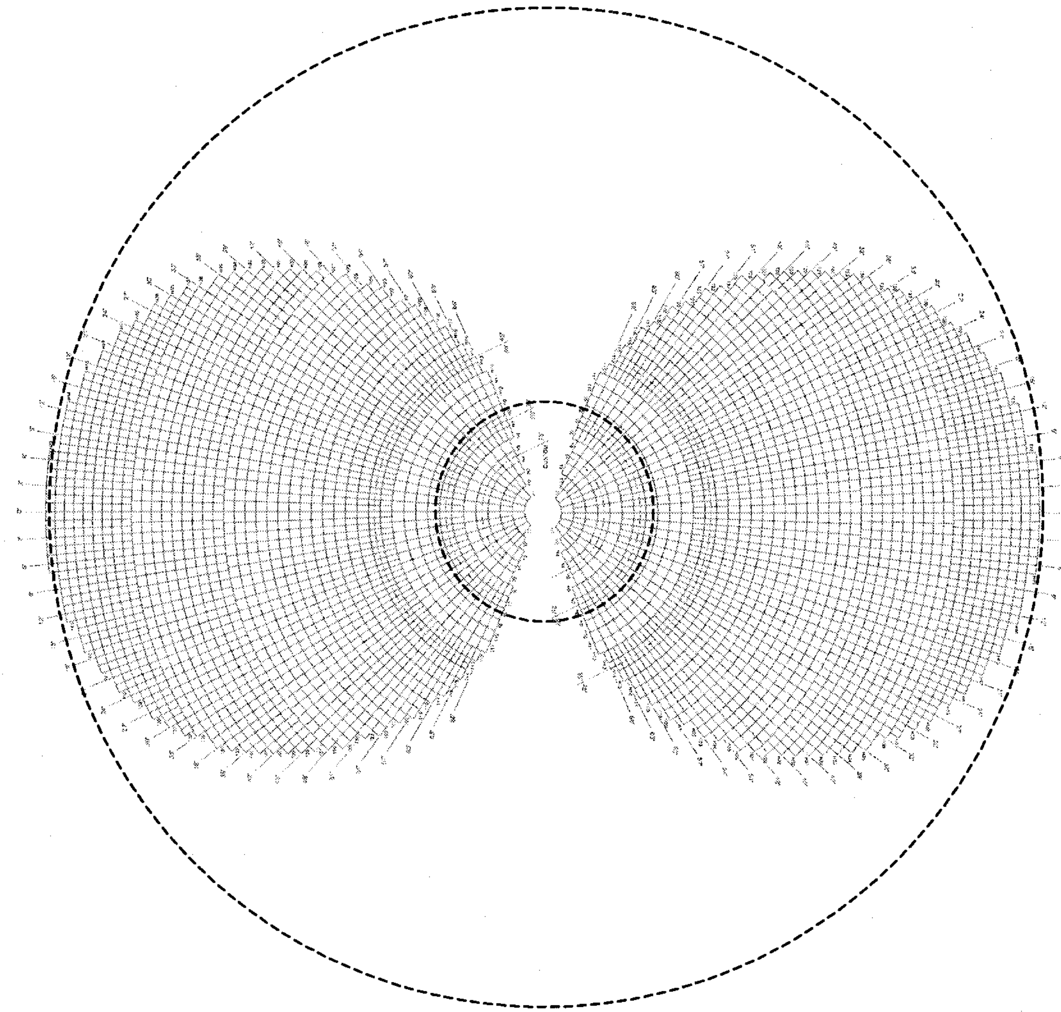
Sampling in Elevation = 3600*(180/pi)*(dy(5600)-dy(250))/17 = **7.5 Arcsec / pixel** = py = 3.6361e-05 radians

Qy = λ /(D*px) = 6.96e-3 / (100*3.6361e-05) = 1.9141

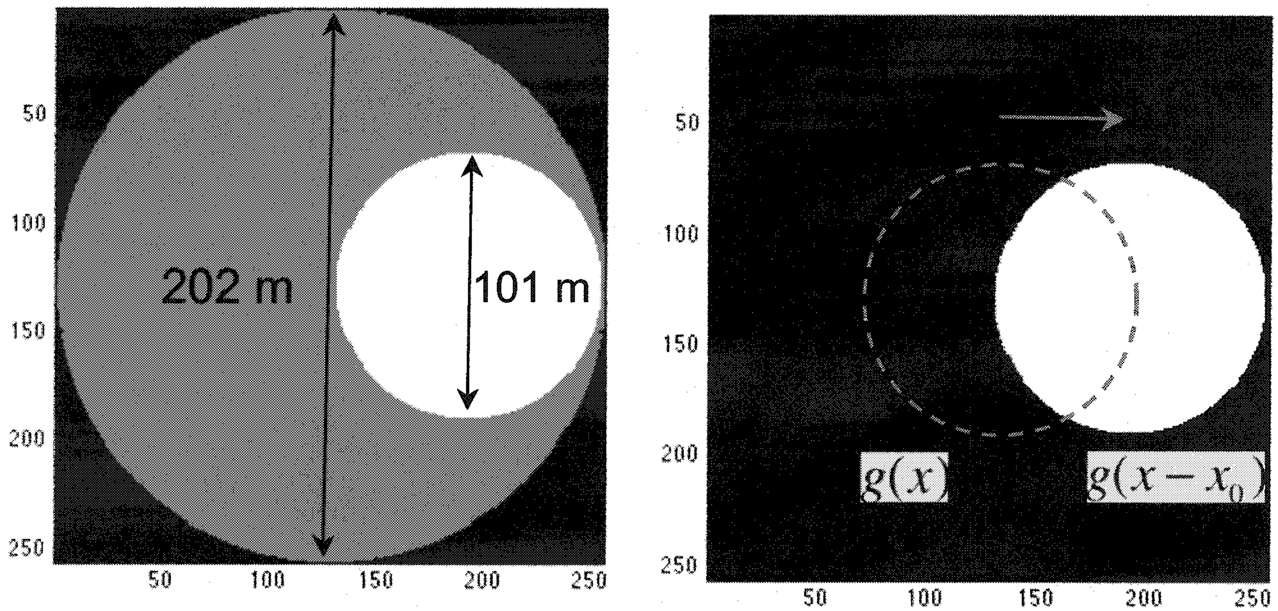
Nyquist sampling is 7.2 arcsec / sample, Q = 2; Under-sampled by 0.96 in <u>Elevation</u>; over-sampled by 2.97 in <u>Azimuth</u>

NRAO GBT Aperture

- Panels are arranged in such a way that rings are concentric with a parent parabola.
- Zemax design: GBT is setup as a single off-axis section of the parent parabola.



GBT Fourier Model



Note that Translation Shift of Fourier Transform produces a phase factor:

$$\Im[g(x-x_0)] = G(\omega)e^{-i2\pi\omega x_0}$$

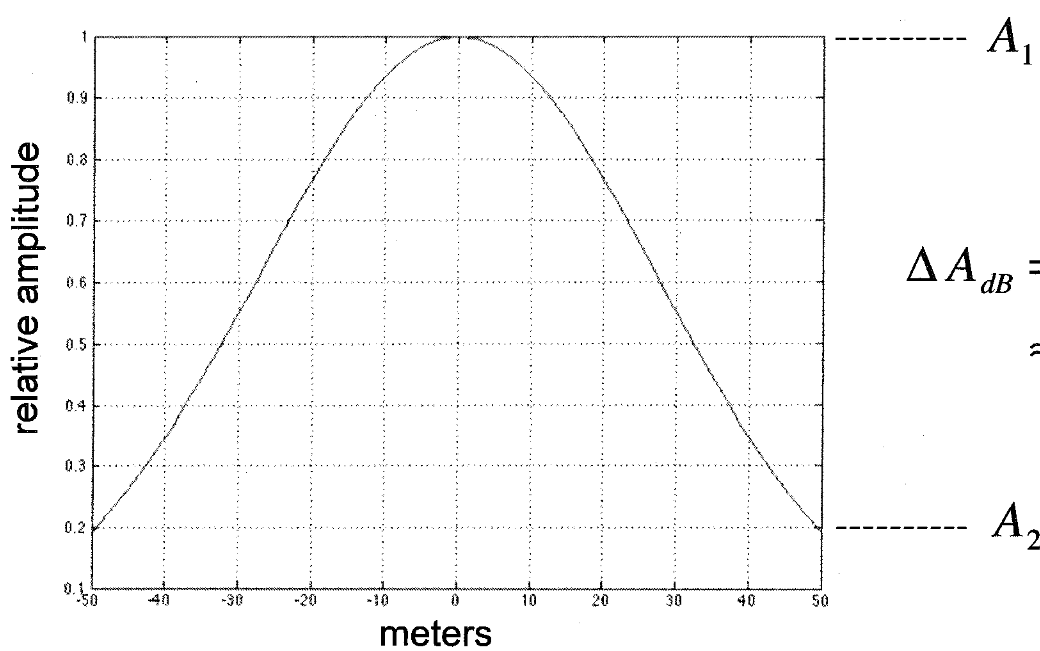
Pupil Illumination - I

Cool Link: edge taper in radio astronomy (Cheng / Mangum):
http://www.alma.nrao.edu/memos/html-memos/alma197/memo197.html

$$A(r) = 1, \qquad (uniform)$$
$$= \exp[-\alpha (r / r_0)^2], \qquad (tapered\ Gaussian)$$
$$\alpha = (T_e / 20)\ln 10, \qquad (edge\ taper\ factor)$$

$$T_e \equiv edge\ taper\ in\ dB$$

Using the formula for edge taper in dB:



amplitude variation:

$$\Delta A_{dB} = 10\log[(A_1 / A_2)^2] = 20\log(1/0.18)$$
$$\approx 15\ dB$$

Pupil Illumination - II

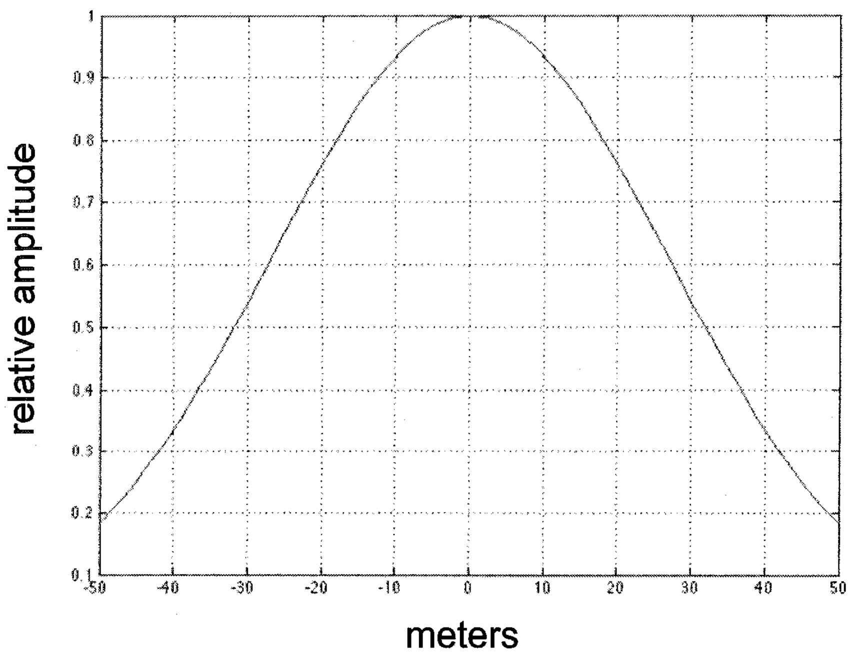
$$A(r) = 1, \qquad (uniform)$$

$$= \exp[-\hat{r}^2 / 2\sigma^2], \qquad (tapered\ Gaussian)$$

$$with\ \ \sigma = 0.3,$$

from PTCSPN47.pdf

… the aperture plane amplitude distribution, that is, the illumination of the primary surface. This was approximated as a well-centered and circular Gaussian with a width (in radius-normalized units) defined by <u>σ = 0.3</u>, which corresponds to 14.5 dB of illumination taper at the edge of the dish …



$$\sigma \approx 0.55$$

amplitude variation:

$$\Delta A_{dB} = 10 \log[(A_1 / A_2)^2] = 20 \log(1/0.18)$$

$$\approx 15\ dB$$

Challenge for ITA Phase Retrieval

- Two incoherently subtracted irradiance values appear in the GBT data.
- Data collection process, $I = I_1 - I_2$
- For the ITA approach to work, these irradiance values should be the result of one FFT.
- So make the approximation that:

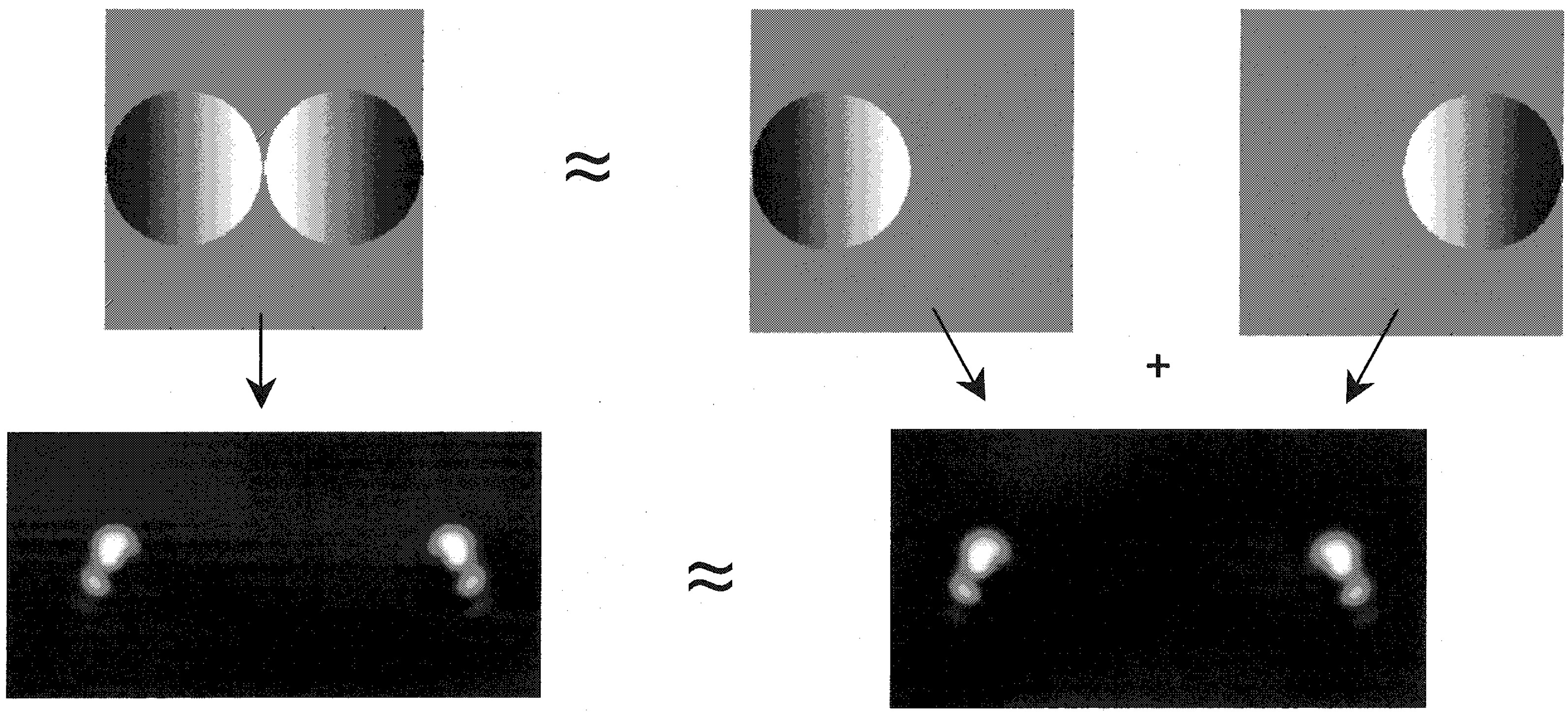
Coherent Approximation for Incoherent Data:

$$\left|\Im\{A_L(-\theta_t) + A_R(+\theta_t)\}\right|^2 \approx \left|\Im\{A_L(-\theta_t)\}\right|^2 + \left|\Im\{A_R(+\theta_t)\}\right|^2$$

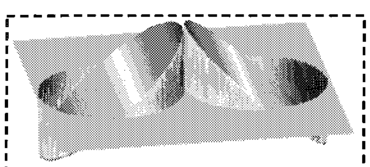
or simply $I \approx I_L + L_R$

Coherent Approximation for Incoherent Data

$$\left|\Im\{A_L(-\theta_t)+A_R(+\theta_t)\}\right|^2 \approx \left|\Im\{A_L(-\theta_t)\}\right|^2 + \left|\Im\{A_R(+\theta_t)\}\right|^2$$



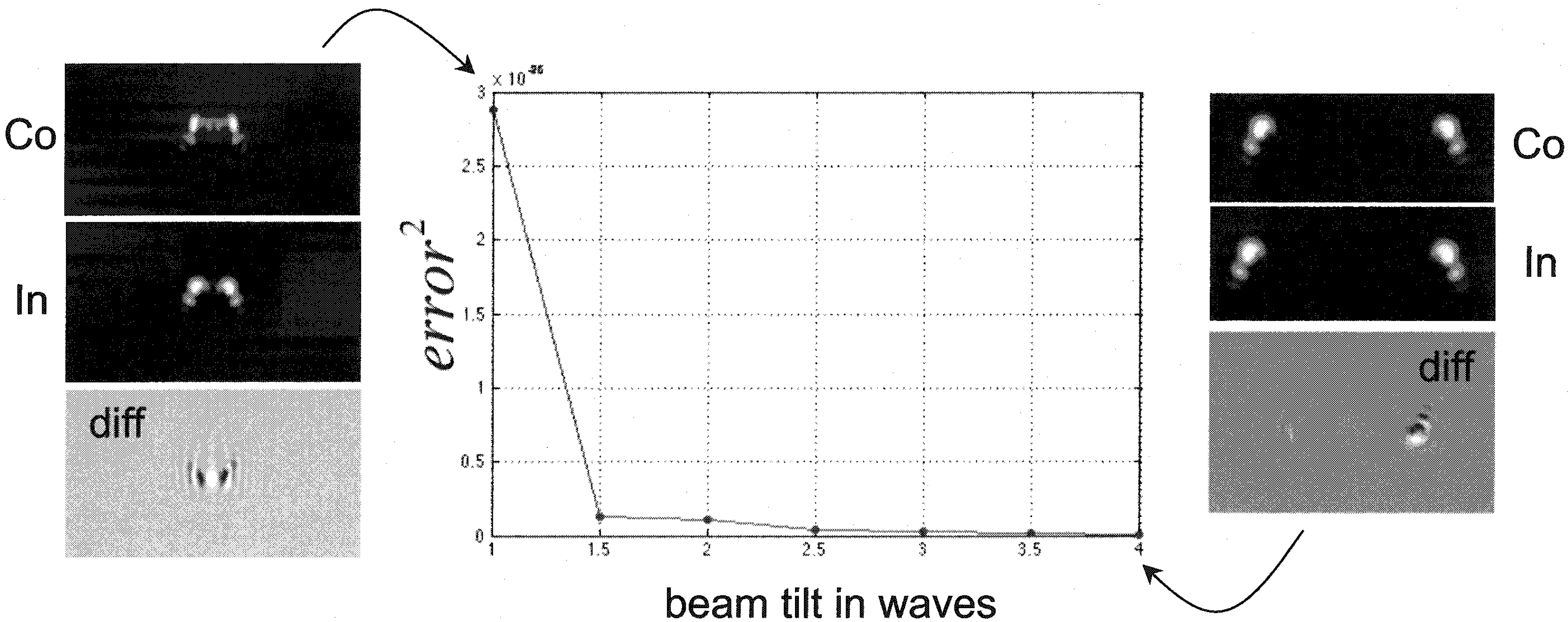
beam tilt:



Validity of Approximation?

- Good approximation for large tilt (i.e., there is little interference)
- Plot of squared error as a function of tilt:

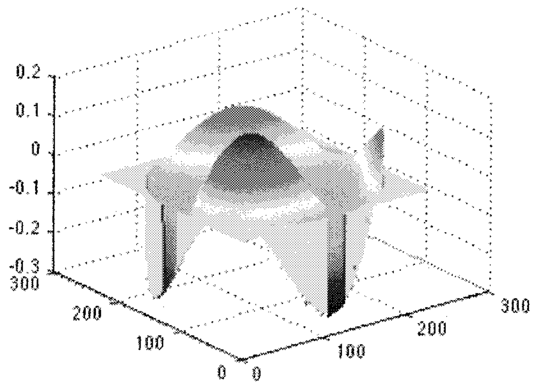
$$error^2 = \sum_k \sum_j \left| I_{coherent}(j,k) - I_{incoherent}(j,k) \right|^2$$

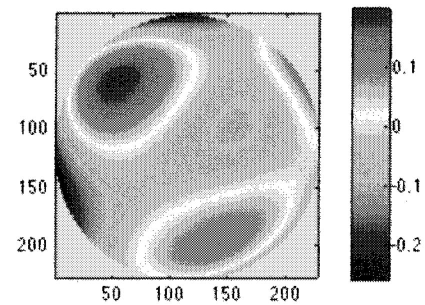


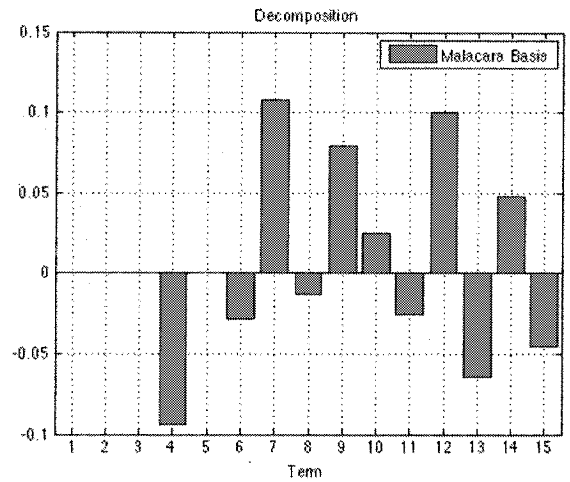
How does error propagate to Phase Retrieval?

Proof of Concept: PR Simulation

Input Wavefront:







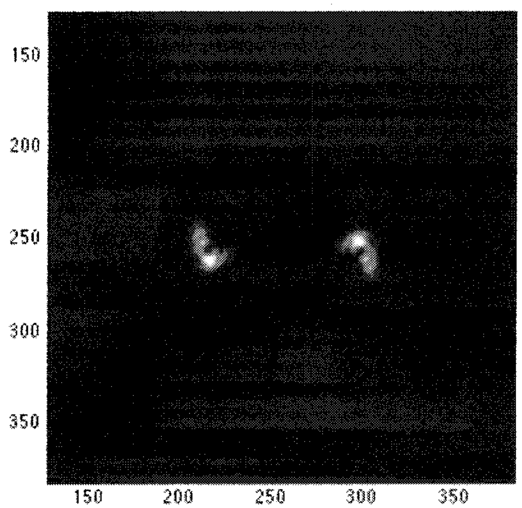
RMS = 557 μ; λ/13; PV = 0.4561 λ

Malacara Basis Set:

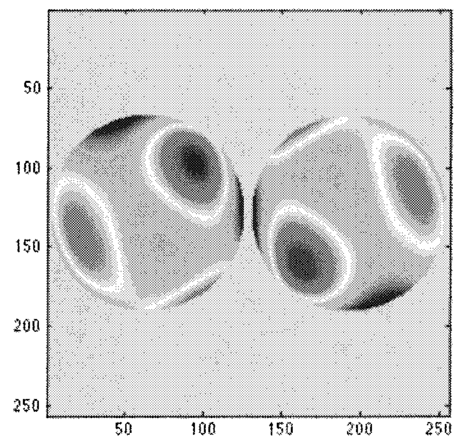
#	radial	azimuth	term	aberration
1	0	0	1	piston
2	1	0	$r \sin \alpha$	y-tilt
3	1	1	$r \cos \alpha$	x-tilt
4	2	0	$r^2 \sin 2\alpha$	45° astig (1st order)
5	2	1	$2r^2 - 1$	defocus
6	2	2	$r^2 \sin 2\alpha$	0° astig (1st order)
7	3	0	$r^3 \sin 3\alpha$	30° trefoil
8	3	1	$r(3r^2 - 2) \sin \alpha$	y-coma
9	3	2	$r(3r^2 - 2) \cos \alpha$	x-coma
10	3	3	$r^3 \cos 3\alpha$	0° trefoil
11	3	3	$r^4 \sin 4\alpha$	22.5° tetrafoil
12	3	3	$(4r^4 - 3r^2) \sin 2\alpha$	45° astig (2nd order)
13	3	3	$6r^4 - 2r^2 - 1$	spherical
14	3	3	$(4r^4 - 3r^2) \cos 2\alpha$	0° astig (2nd order)
15	3	3	$r^4 \cos 4\alpha$	0° tetrafoil

Check: Coherent PR Simulation

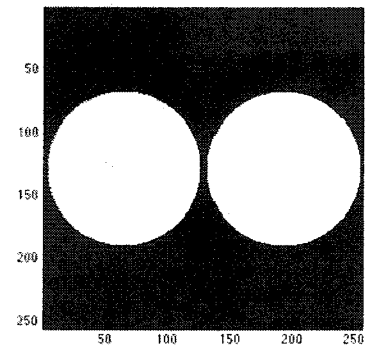
Image on 1-side of focus:



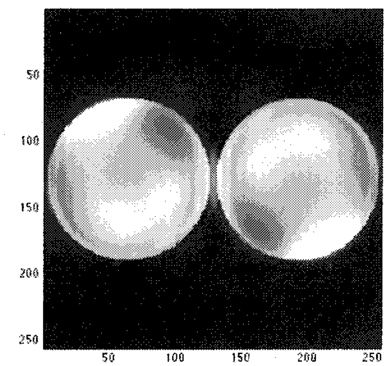
Dual aperture model:



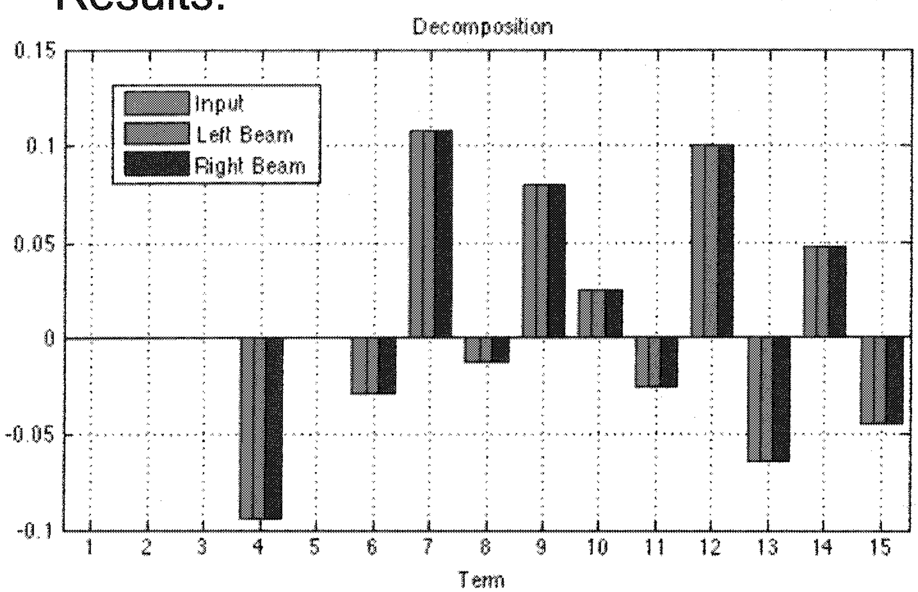
Pupil Amplitude:



Recovered:

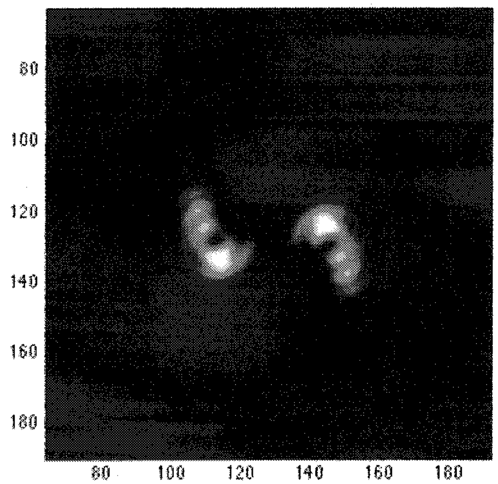


Results:

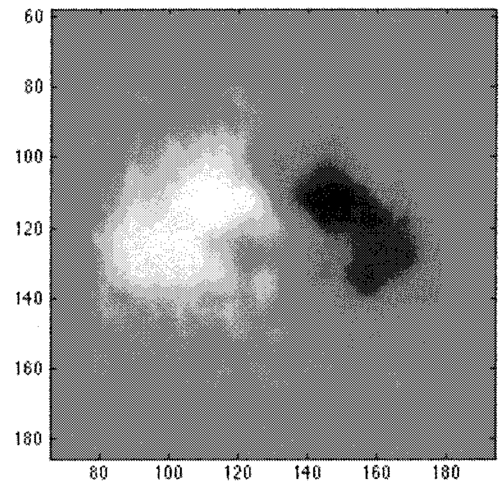


Comment on GBT Beam Symmetry

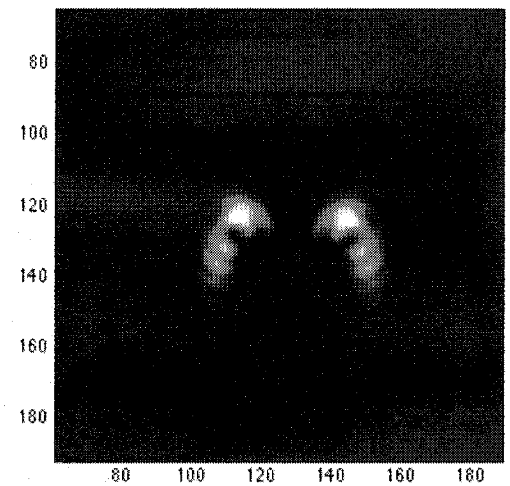
Model

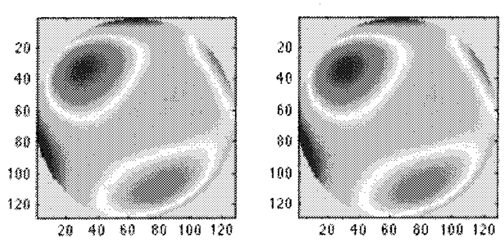


Data

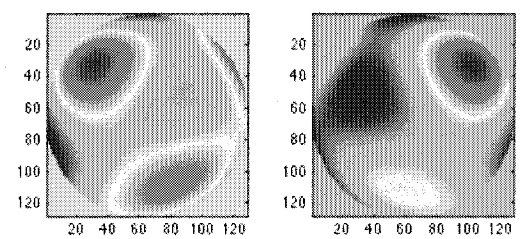


Incoherent Model



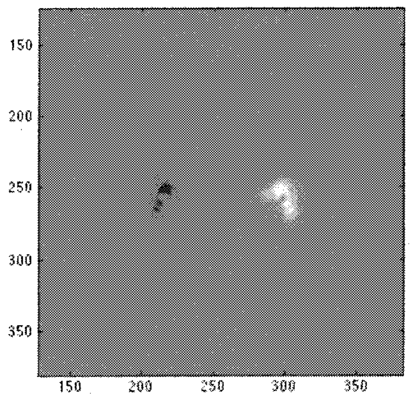


Let PR solve for the wavefront that is consistent with the data:

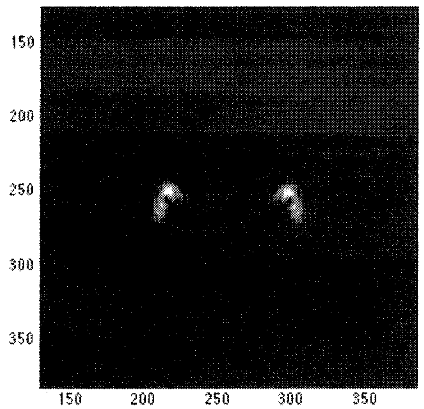


Incoherent PR Results (simulation)

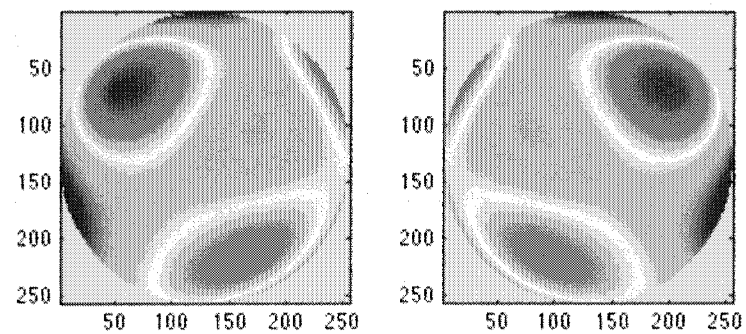
Incoherent Data:



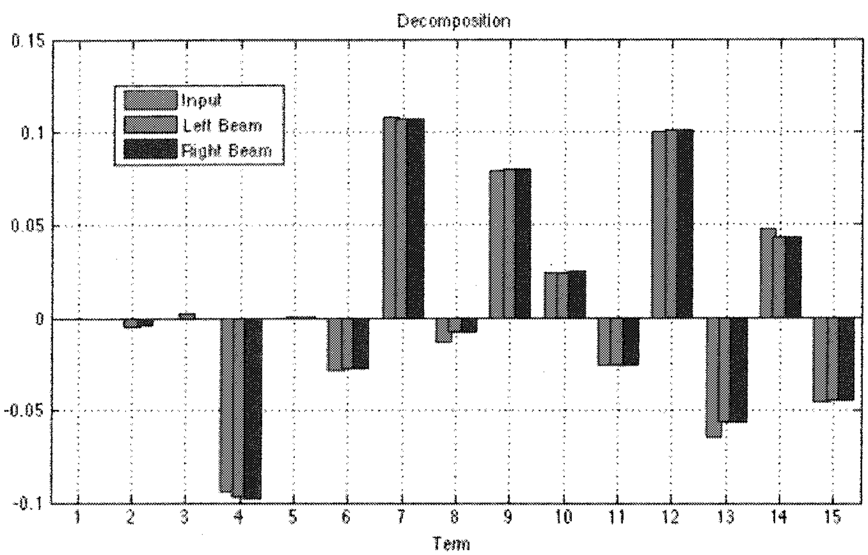
abs()



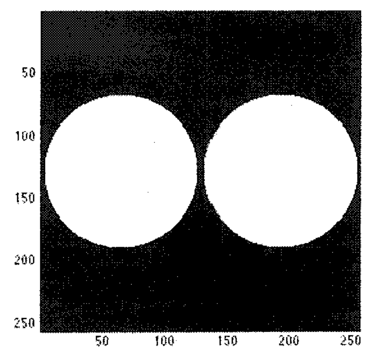
dual wavefront:



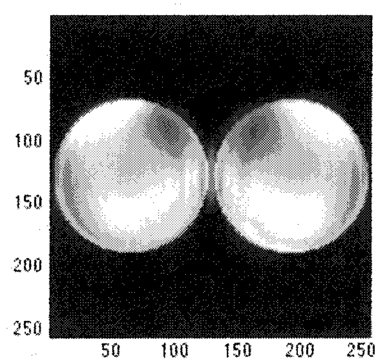
Results:



Pupil Amplitude:



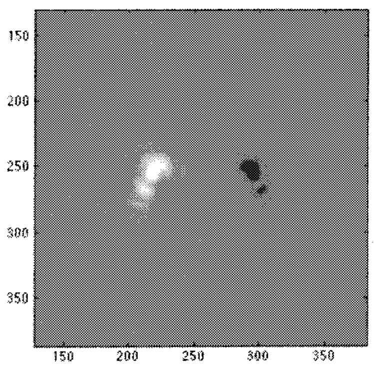
Recovered:



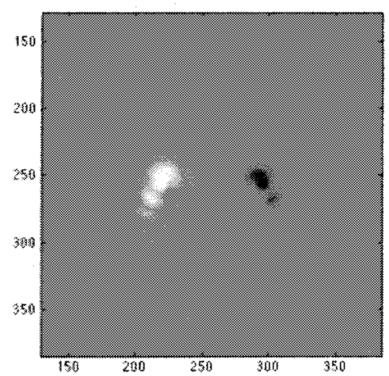
Incoherent PR Results - worst case

-- Simulation: 2 waves beam tilt

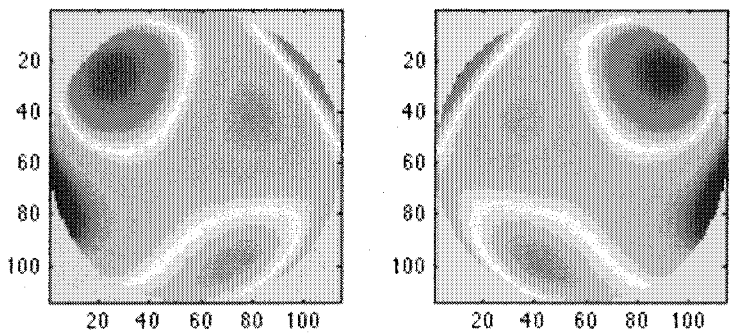
Incoherent Data:



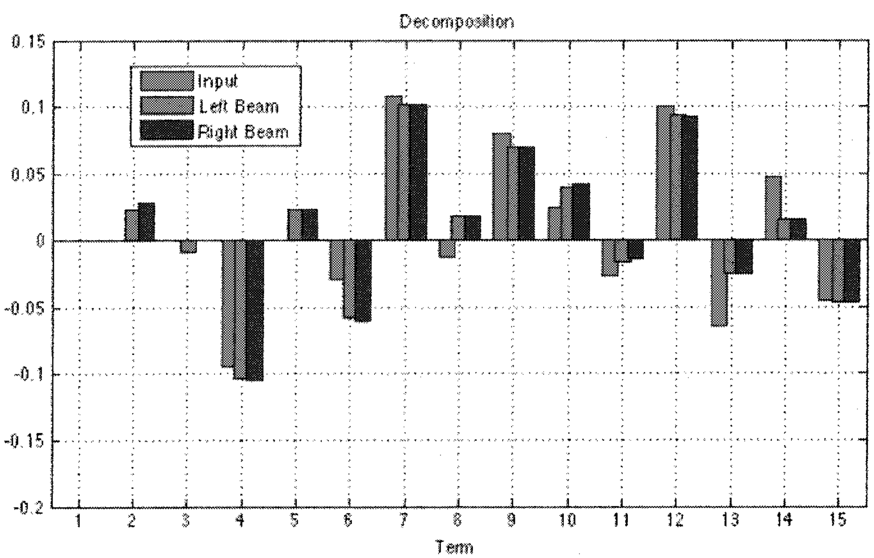
Model:



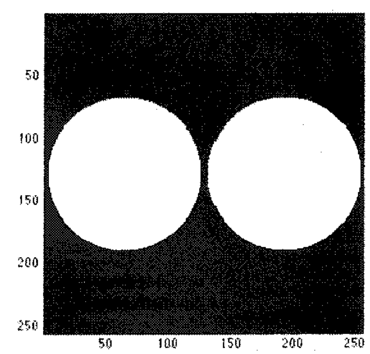
dual wavefront:



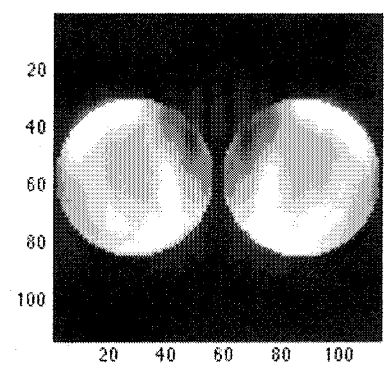
Results:



Pupil Amplitude:

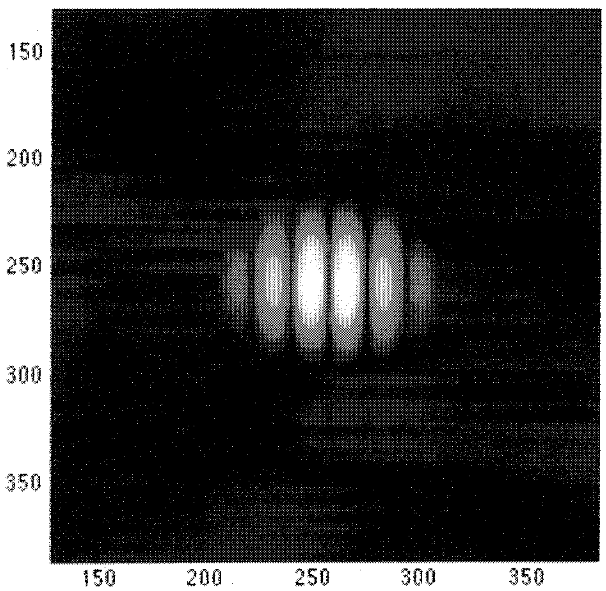


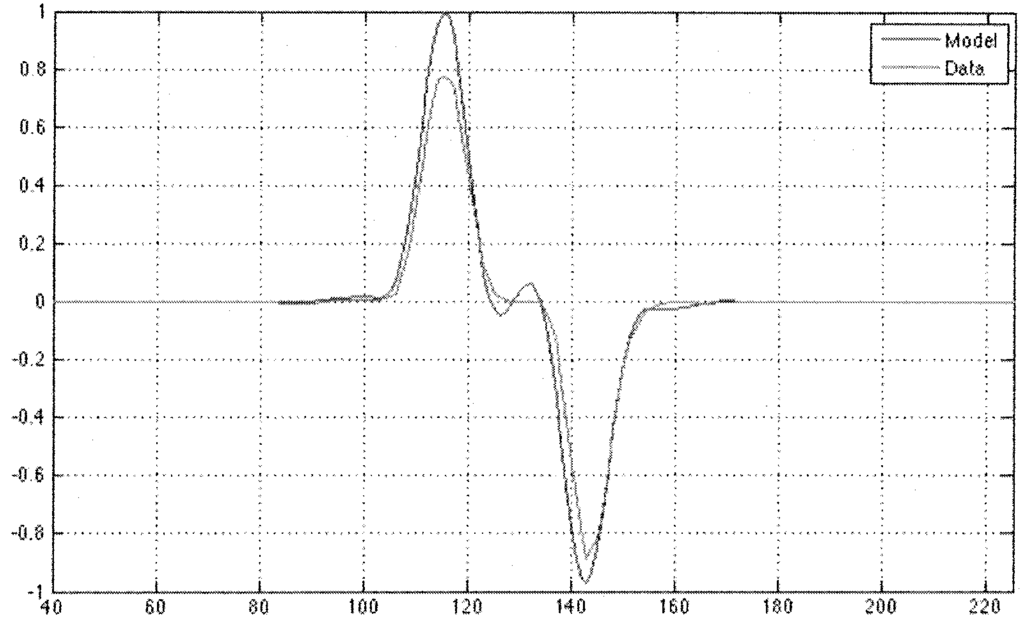
Recovered:



Estimate Initial Sampling Parameters from focused GBT Data

- Q ≈ 4.5
- Beam tilt ≈ 1.5 λ

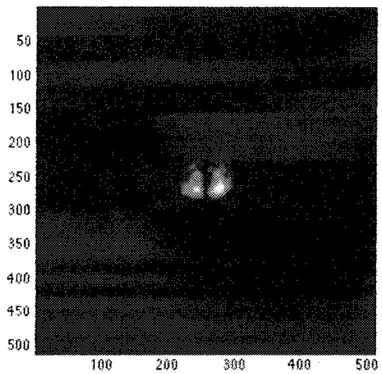




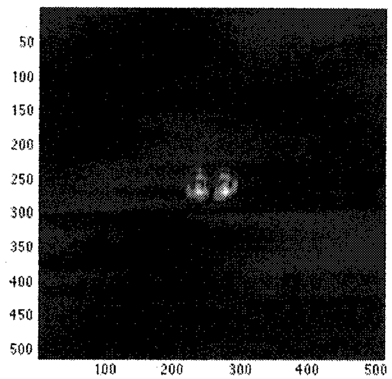
Can also tune parameters by matching the FFT of the data

Wavefront Sensing Results applied to GBT Data - 3

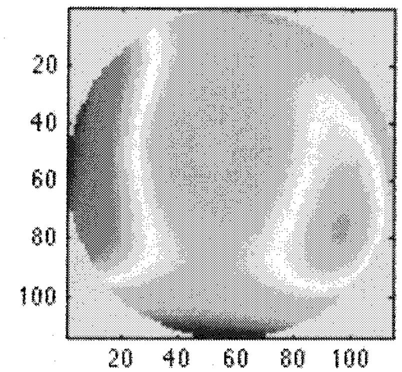
GBT Data:

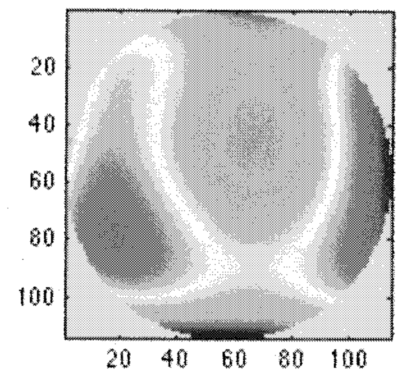


Model:

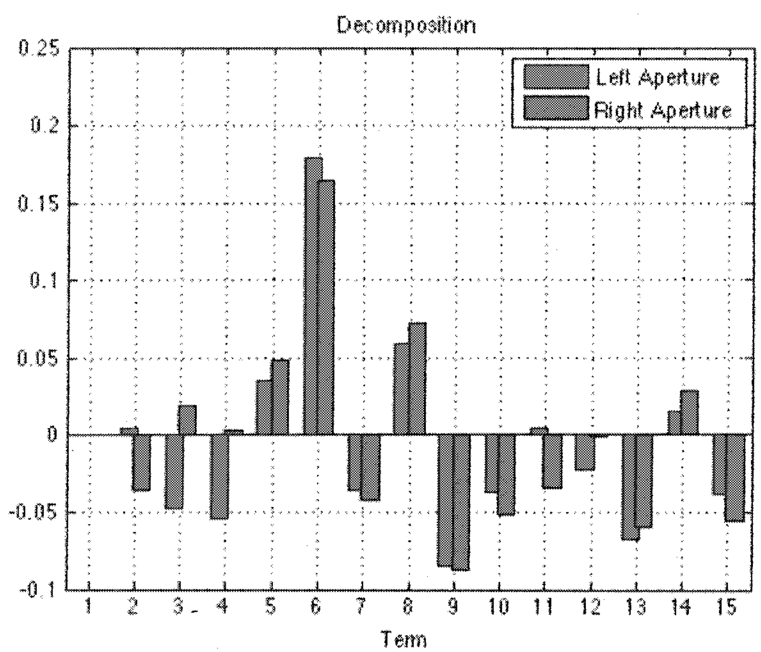


wavefront:

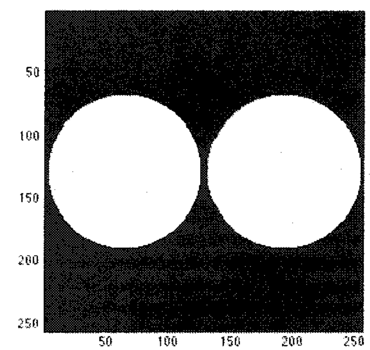




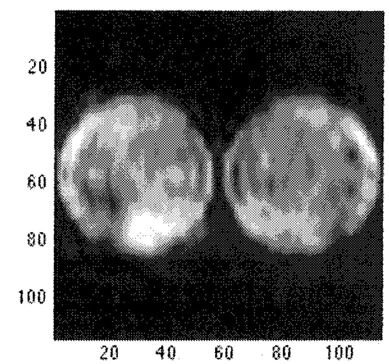
Results:



Pupil Amplitude:

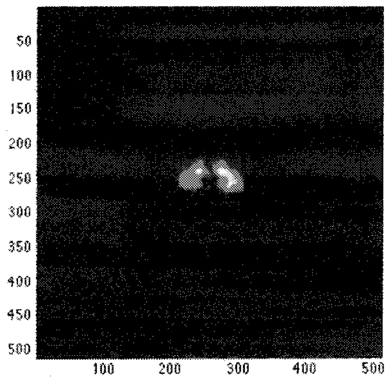


Recovered:

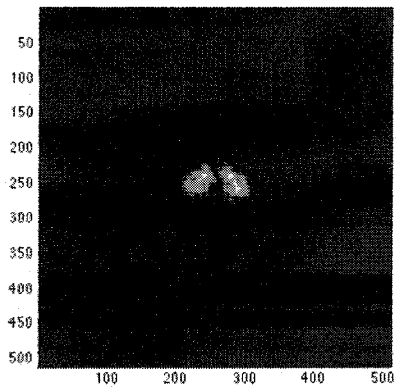


Wavefront Sensing Results applied to GBT Data - 2

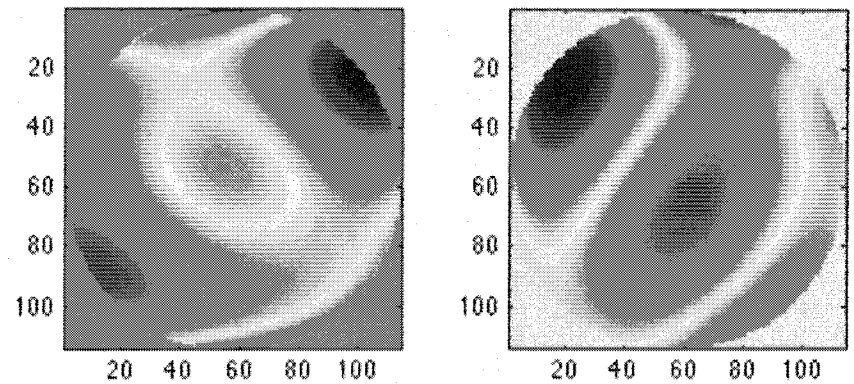
GBT Data:



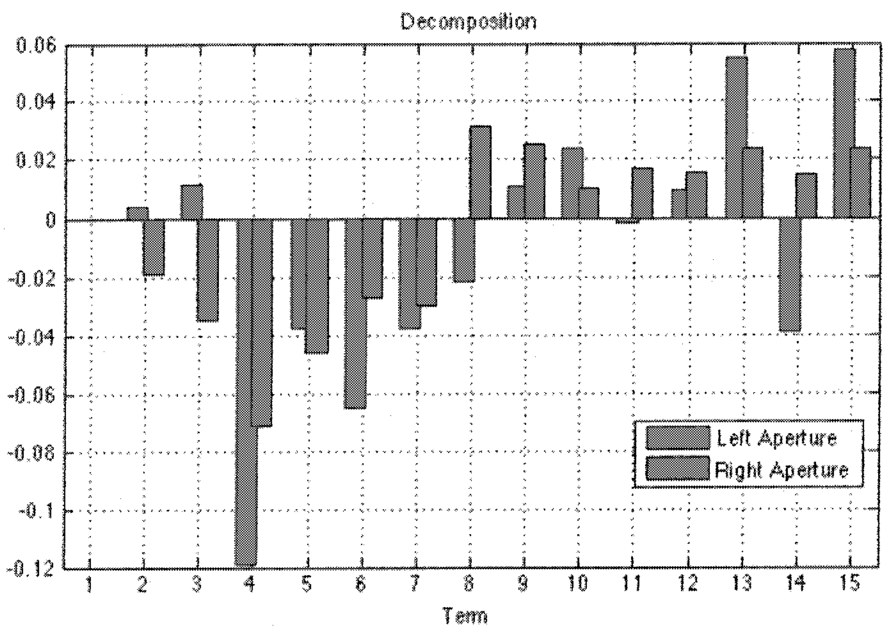
Model:



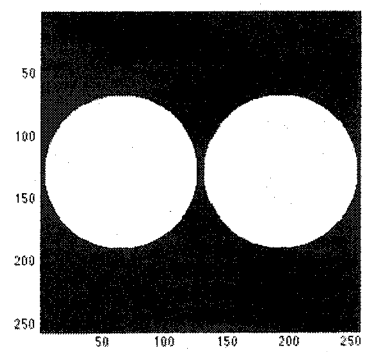
wavefront:



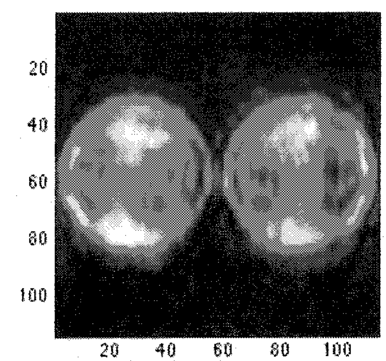
Results:



Pupil Amplitude:



Recovered:



Summary

- In principle, coherent ITA PR may work on incoherent GBT data,
- Errors increases as beam tilt decreases