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A SINGLE CONJUNCTION RISK ASSESSMENT METRIC: THE F-VALUE

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The Conjunction Assessment Team at NASA Goddard Space Flight Center provides conjunction risk assessment for many NASA robotic missions. These risk assessments are based on several figures of merit, such as miss distance, probability of collision, and orbit determination solution quality. However, these individual metrics do not singly capture the overall risk associated with a conjunction, making it difficult for someone without this complete understanding to take action, such as an avoidance maneuver. The goal of this analysis is to introduce a single risk index metric that can easily convey the level of risk without all of the technical details. The proposed index is called the conjunction "F-value." This paper presents the concept of the F-value and the tuning of the metric for use in routine Conjunction Assessment operations.

INTRODUCTION

THE Goddard Space Flight Center (GSFC) has been performing routine conjunction risk assessment for various NASA missions since January 2005. The Conjunction Assessment (CA) Team at GSFC receives close approach predictions from the Joint Space Operations Center (JSpOC) at Vandenberg Air Force Base. The JSpOC provides predictions in the form of state and covariance information at the time of closest approach (TCA). The GSFC CA Team uses a suite of in-house tools to evaluate each conjunction and assess its potential risk. These additional analyses and assessments are used to assist mission flight operations teams to make mitigation decisions based on the quantified risks.

One shortcoming of this presentation of risk information to mission stakeholders in the past has been the lack of an effective method for conveying a succinct risk level. Individual metrics, such as the miss distance and probability of collision (Pc), do not singly capture the overall risk associated with a conjunction. For example, the Pc could be indicating a risk, but a high Pc may be due to unrealistic covariance sizes on the secondary object due to insufficient tracking. Therefore, the conjunction threat may change as more tracking data are acquired and the problem achieves improved accuracy and precision. There are many quantifiable risk assessment parameters for conjunctions used in risk assessment in addition to the miss distance and Pc. These additional parameters could include, but are not limited to, figures of merit[1] such as conjunction and covariance geometry, or orbit determination solution quality. Our analysis has shown that the overall risk of the conjunction is not based on a single parameter (or figure of merit), but rather an aggregation of these parameters from a global perspective only gained with a detailed knowledge of all the parameters. While conjunction assessment is admittedly a complex problem, being able to convey the risk level in a simple manner is something that has been expressed as very desirable by mission stakeholders. The goal of this analysis is to provide a metric that can serve as this risk level index. The proposed index discussed in detail in this paper is the "F-value", a single value that quantifies the overall risk associated with a conjunction.

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The F-value is a means for quickly conveying the risk of a given conjunction in a high-level view. It is an aggregation of several CA figures of merit to quickly quantify and summarize the inherent risk of a conjunction. While the F-value is very useful in this regard, it is important to note that details used in the generation of the F-value are essential to understanding the conjunction completely. It is not meant to replace the interpretation or disposition of the analyst, but to serve as a notification device, an initial quantification of risk, and to provide a consistent metric of the comparison of conjunction risks. The F-value is analogous to the quarterback (QB) rating used to assess a quarterback's performance in professional football. A QB rating is a complex analytic expression that considers many different performance rating aspects, but rolls all of these details up in a concise metric which is easily interpreted by the casual observer. While the actual details used in these analogies or the F-value are important to understand, the one metric quickly conveys the desired information.

This document defines the abstract concept of the F-value, describes methods for tuning the F-value, and presents an example of how the F-value can be used in conjunction risk assessment operations.

DEFINITION OF F-VALUE

The F-value for a particular conjunction combines any desired number of conjunction risk and quality assessment parameters into one single value. A risk assessment parameter, f_i, is any quantifiable metric that is used in dispositioning the risk associated with a given conjunction, such as miss distance and probability of collision. A quality assessment parameter, f_j, is any quantifiable metric that is used in assessing the quality of the orbit determination solution off of which the close approach predictions are based.

The concept of the F-value is an extension of Fuzzy Set Theory and Fuzzy Logic. In a Fuzzy Set, an element has a degree of membership of that set, i.e. the membership of an element in the set does not have a clear definition but rather a gradual assessment. In classical set theory, an element has a binary inclusion condition – the element is either in the set or is not (1 or 0). In Fuzzy Sets, an element can have a degree of membership, e.g. the element is "somewhat" in the set. A membership function has values that range continuously from 0 to 1. Fuzzy Sets become useful in Fuzzy Logic when several sets can be conditionally satisfied and the logical statement can have a degree of satisfaction.

The basic definition of the F-value is provided in Equation 1, where n is total number of risk assessment parameters, m is the total number of quality assessment parameters, $f_{i/j}$ is the specific F-value of each parameter and $a_{i/j}$ is the weighting of each parameter. The two-term separation delineated in Equation 1 refers to the separate "averaging" of risk assessment parameters and quality assessment parameters.

$$F_{value} = \left(\frac{1}{m} \sum_{i=1}^{m} a_i f_i \right) \cdot \left(\frac{1}{n} \sum_{j=1}^{n} a_j f_j \right) \tag{1}$$

The overall F-value for a conjunction is an average of the f-value of all the risk assessment parameters multiplied by the average of the f-values of all the quality assessment parameters. A specific f-value is a reduction of the actual value of the parameter to a common scale. In other words, an f-value maps the range of all possible values for a particular parameter to a common range of values using a membership function. Once the values have been re-scaled, the aggregation becomes possible. An F-value only considering the risk assessment parameters, and not the quality assessment parameters, is also referred to as the "unscaled" F-value. Conversely, an F-value including both sets of parameters is the "scaled" F-value or, simply, the F-value. These details will be discussed later when the application of the F-value to conjunction assessment operations is performed.

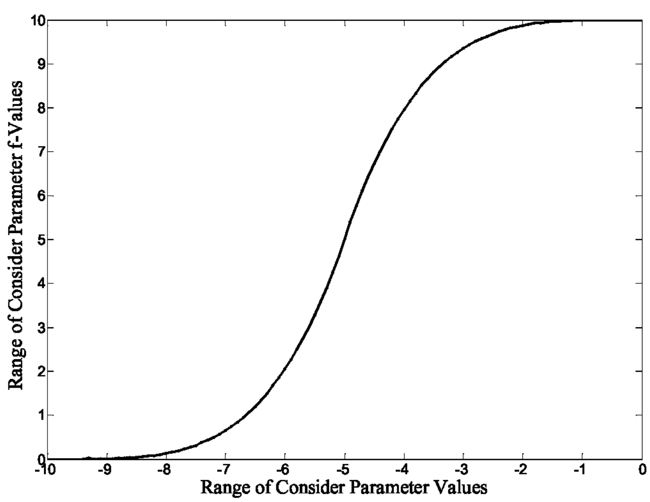
TUNING THE F-VALUE

There are several ways to tune the F-value to meet any specific conjunction assessment requirements or desired preferences. Specifically, there are three tuning mechanisms:

1. The risk or quality assessment parameters used
2. Relationship between an assessment parameter value and associated f-value (i.e. membership function)
3. Weighting of each assessment parameter

In the conjunction F-value model described later, eight risk assessment parameters were used. The F-value model can use any parameter that is quantifiable, i.e. real-valued. For conjunction assessment, any or all of the Figures of Merit[1] could be used; e.g. the relative velocity between the two objects or the conjunction clock angle.

After the number of parameters desired is chosen, the second tuning mechanism is the membership function between the range of possible values and the associated f-value. These relationships can be any continuous function. Typically, membership functions are monotonic in nature as well. Figure 1 provides two simple examples of membership functions. The first example is a traditional "usually" hedge used in Fuzzy Set Theory, and the second is a linear relationship between the range of possible real values and the f-value.



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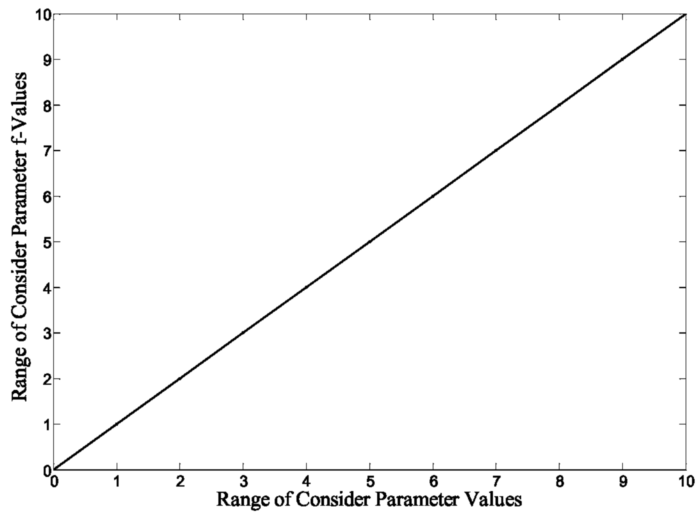

Figure 1: Examples of Parameter Membership Functions

The last tuning mechanism is the final weighting, *a*, of each parameter f-value into the overall, aggregate F-value. The weighting can often be captured in the membership function, but is also an additional tuning mechanism to allow the placement of more trust, or weight, in a particular assessment parameter.

DEFINITION OF CONJUNCTION F-VALUE

An application of the F-value in conjunction risk assessment will now be presented. This F-value has been tuned to accurately quantify the risk for conjunctions with low earth orbiting (LEO) assets based on the Operations Concept in place at NASA GSFC.[2] This F-value will use 5 risk assessment parameters and 3 quality assessment parameters as listed below.

Risk Assessment Parameters:

1. Probability of Collision (Pc)
2. Miss Distance RSS (Root Sum Square)
3. Radial Miss Distance
4. Along-Track Miss Distance
5. Cross-Track Miss Distance

Quality Assessment Parameters:

6. Days to TCA
7. Determinant of the Combined Positional Covariance
8. Days since Last Observation of Object

The first five parameters make-up the "risk factor" of the conjunction being examined; the last three comprise the "quality factor" of the conjunction. Figure 2 - Figure 6 depict the membership functions that map the range of possible values to the f-values of the conjunction risk parameters. These mappings were generated using the operational insight into the relationship between the range of possible values of each specific parameter and its level of risk gained from performing routine CA for NASA/GSFC. For example, the Pc membership function, as shown in Figure 2, increases slowly at first, with Pc values around e-10, e-9, and e-8, then increases quicker, and levels off again for Pc values around e-3, e-2, and e-1. For the miss distance RSS, radial separation, along-track separation, and cross-track separation f-values, a linear relationship is used for the membership function. It is important to note that the f-value for the Pc is mapped to log(Pc) and the f-value for the Covariance is mapped to the log of the determinant of the combined Positional Covariance, or log(Det[COV]). For this analysis, a scale of 0 to 10 is arbitrarily chosen for the risk assessment parameters. An F-value of 0 represents a conjunction that poses minimal, or no, risk. In contrast, an F-value of 10 represents a conjunction that poses significant risk to the safety of that spacecraft. The f-values for the risk assessment parameters, likewise, also range from 0 to 10 to match the previously described scale.

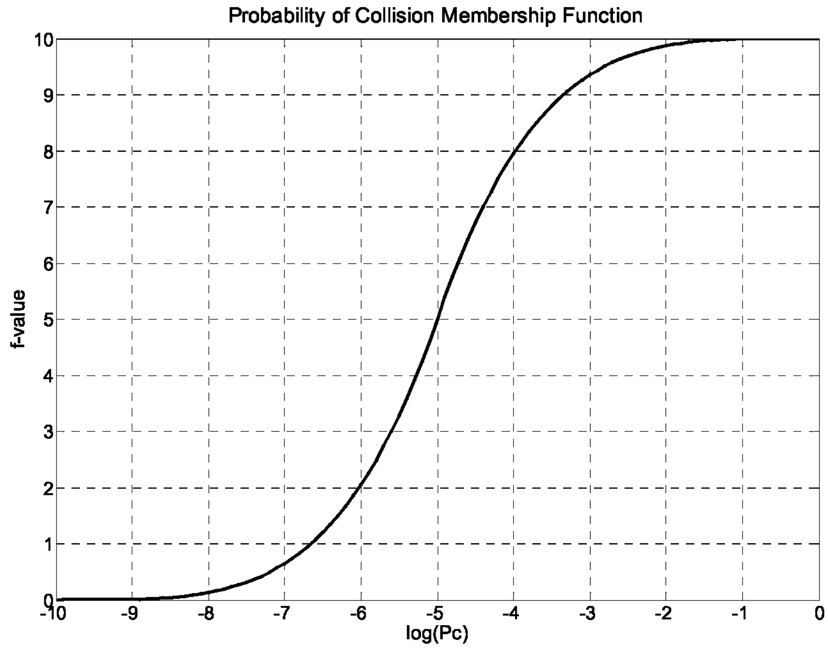

Figure 2: Probability of Collision f-value

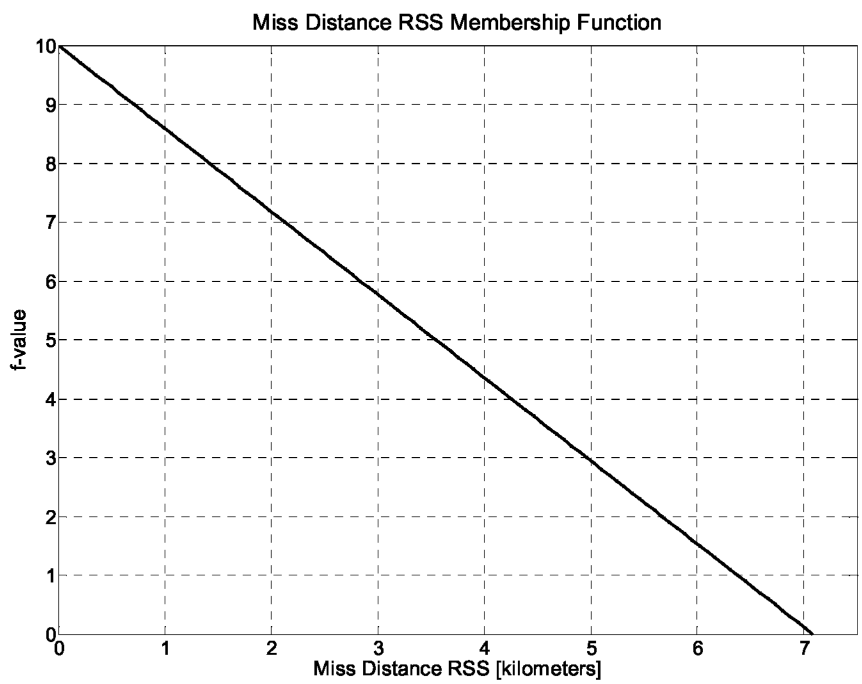

Figure 3: Miss Distance RSS f-value

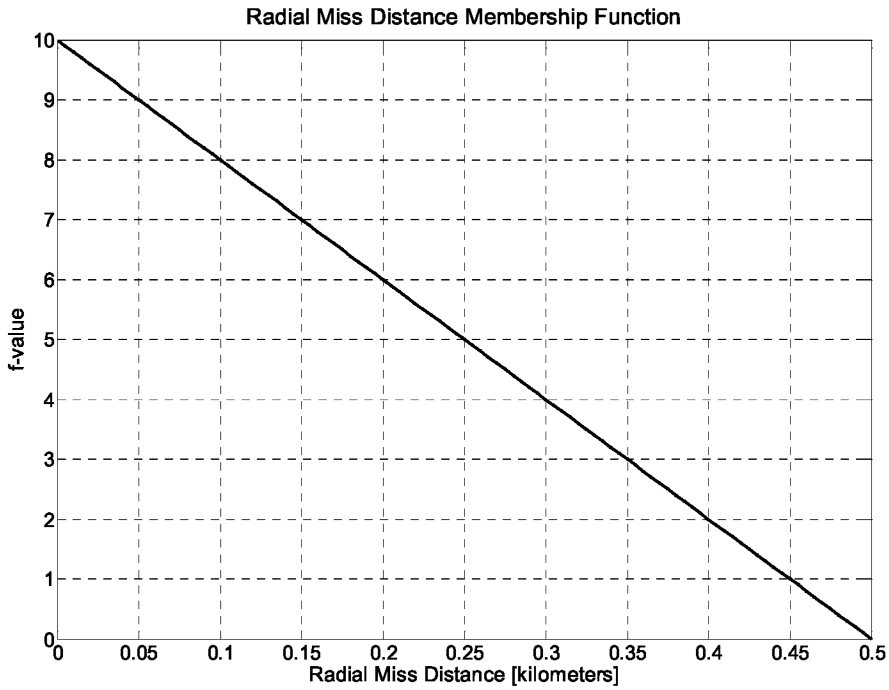

Figure 4: Radial Miss Distance f-value

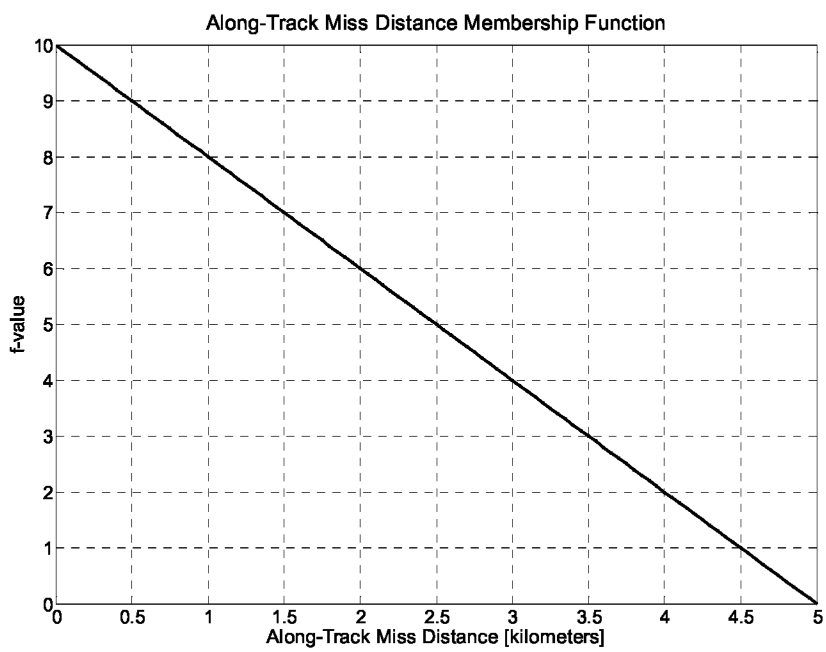

Figure 5: Along-Track Miss Distance f-value

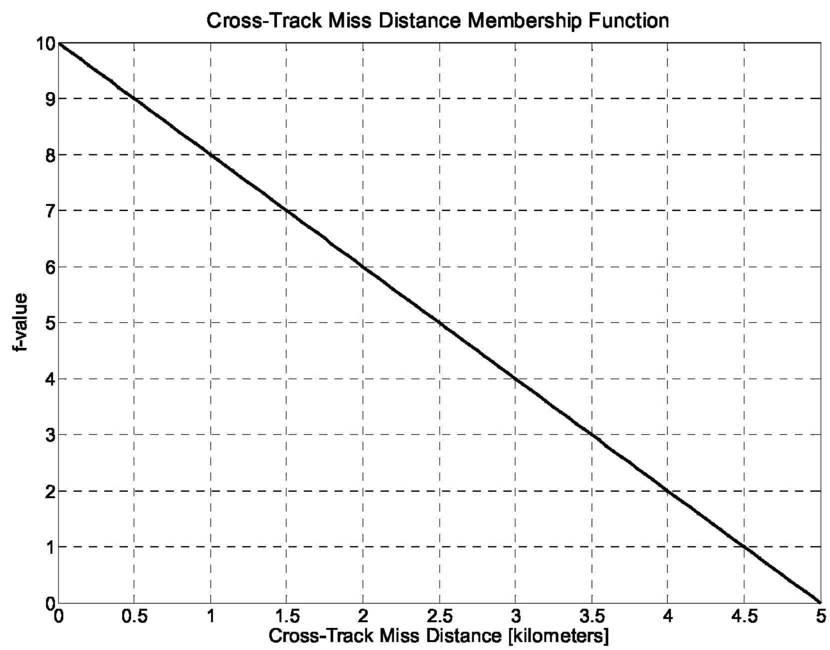

Figure 6: Cross-Track Miss Distance f-value

Figure 7- Figure 9 depict the membership functions that map the range of possible values to the f-values of the conjunction quality assessment parameters. Since the quality assessment parameters are used as a "scaling" mechanism for the conjunction F-value in Equation 1, the range of the f-values for these parameters is 0 to 1. A quality assessment parameter f-value average (or "quality factor") of 0 represents a conjunction with poor quality (ill-defined, unreliable, or untrustworthy) inputs. A quality factor of 1, in contrast, represents a conjunction with well-defined, high quality, or very reliable inputs. The same membership function behavior used for Pc in Figure 2 is applied to the Days to TCA and Combined Covariance membership functions. The Days Since Last Observation uses a linear membership function.

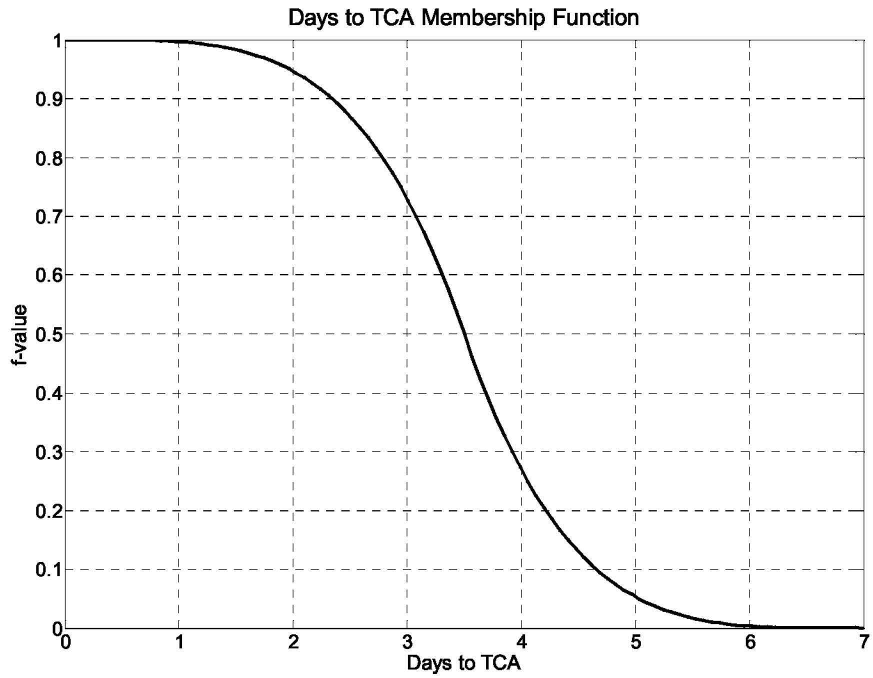

Figure 7: Days to TCA f-value

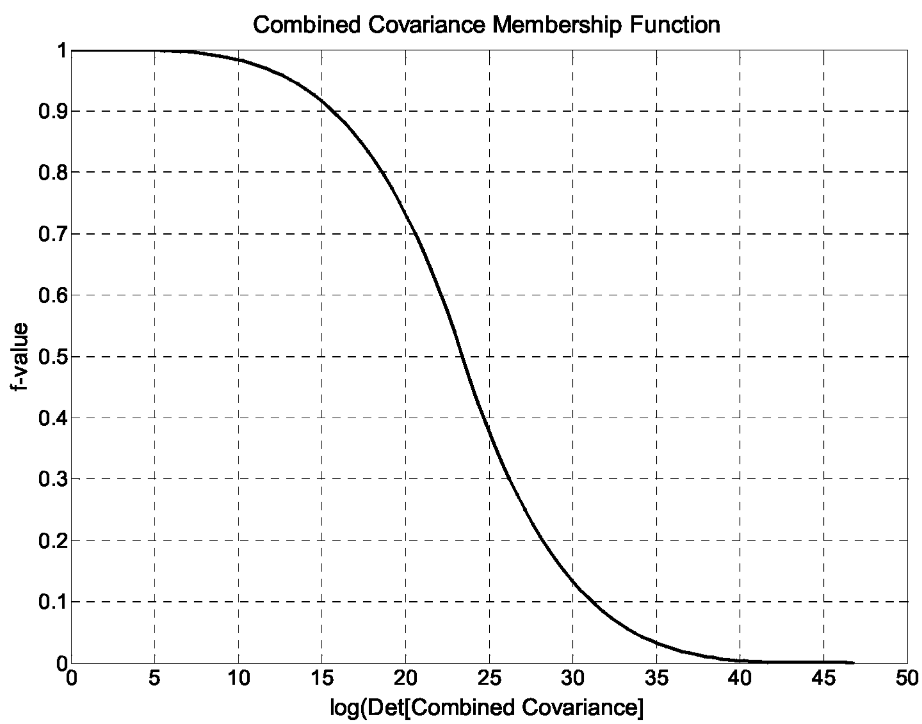

Figure 8: Covariance f-value

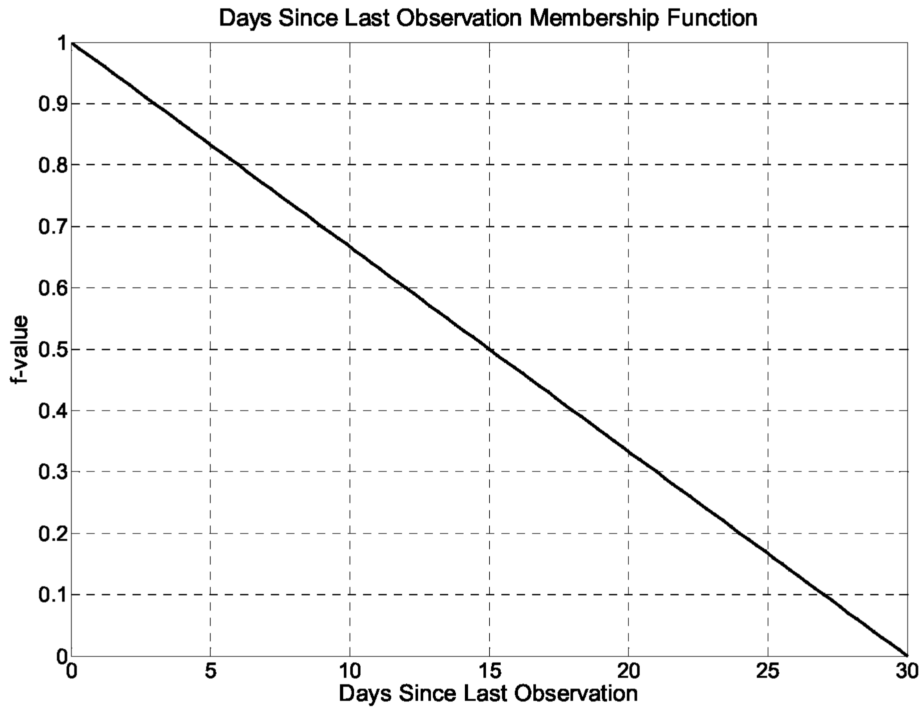

Figure 9: Days Since Last Observation F-value

The use of the quality assessment parameters makes the F-value more meaningful. A particular conjunction could have a high risk due to a small miss distance and high Pc, but the OD solutions for the objects, off of which the close approach predictions are based, could be poor, providing little confidence in those predictions. The quality assessment parameters capture this effect and improve the fidelity in the F-value to predict conjunction risk. In other words, a high risk factor (~10) could be blunted by a low quality factor (~0). The net result of which would be a low F-value and, therefore, a low conjunction risk.

In addition to previously described F-value membership functions, it will also be assumed that all of the parameters will have equal weightings for simplicity of presentation. Weighting risk assessment parameters allows one to put more confidence, or weight, in that parameter measure of risk or quality. Weighting can also be accomplished using the membership function itself as previously described, which is what is done in this example.

CONJUNCTION F-VALUE RESULTS USING OPERATIONAL CA DATA

To illustrate the effectiveness of the F-value described above to quickly quantify the risk magnitude of a conjunction, the F-value concept will be applied to the over 4 years of operational data collected by the GSFC CA Team. Moreover, several examples will be explicitly illustrated and some general statistics will be presented from conjunction assessment operations at NASA GSFC to validate the potential use of the F-value in operations.

The first area of interest is the overall statistics of the F-value and its ability to represent the entire distribution of operational conjunctions observed. Figure 10 is a histogram, with 0.5 F-value bins, illustrating the distribution of conjunction F-values for all operational conjunctions observed.

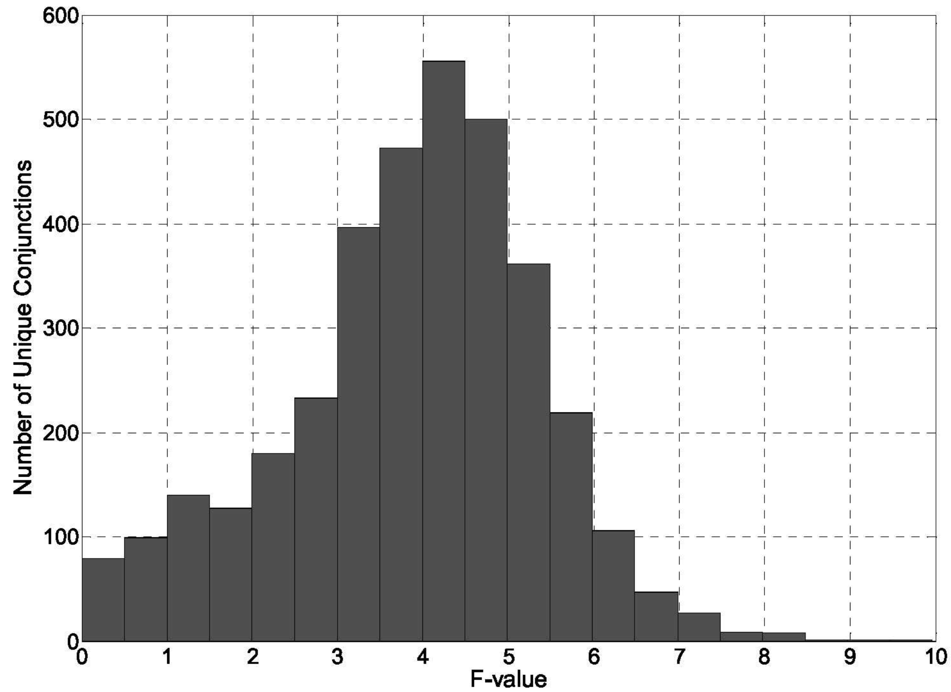

Figure 10: F-value Distribution of Operational Conjunctions Observed

It is observed from Figure 10 that the distribution of F-values observed operationally is quasi-Gaussian distributed, with a mean around 4.5. This distribution agrees with the notional concept that although the flux of conjunctions can be well-determined, the risk posed to an asset spacecraft for conjunction is randomly distributed. The large quantity of events that the GSFC CA Team has seen operationally in concert with the Central Limit Theorem suggests that this distribution should approximate a Gaussian distribution.

Having shown the appropriateness of the F-value in being able to capture the general statistics, it is also important to examine specific conjunction examples. The specific criteria that the F-value should meet are that conjunctions deemed to be high risk, by a careful examination of all the figures of merit by an experienced CA Analyst, should correspond with large F-values and, conversely, that conjunctions with large F-values should correspond to high-risk conjunctions that would be identified by an appropriate examination of all the figures of merit. The former criterion guarantees that high-risk conjunctions are not missed by the F-value while the later guarantees that there are no false alarms. As seen in Tables 1 and 2, the F-value meets both of these criteria. The first cases to be examined are the conjunctions for which the risk was considered high enough to warrant a maneuver to mitigate. Since the development and implementation of the Robotic CA Operations Concept at NASA GSFC in early 2005, there have been 9 risk mitigation maneuvers (RMM) executed by LEO assets. All 9 conjunctions were identified by the CA Team as high interest events. After additional analysis was performed, these 9 conjunctions were deemed to pose a significant enough risk to the asset spacecraft to warrant a RMM recommendation. Table 1 provides information regarding all 9 of these events, including the minimum predicted miss distance and maximum Pc observed throughout the evolution of each conjunction. It should be noted that the maximum Pc computed may correspond to the minimum predicted miss distance listed or may correspond to a different update from the same conjunction. During the prediction of a particular conjunction, many updates are observed. The F-value can be calculated with any update. Hence, the F-value calculation can "evolve" just as the conjunction itself. As the F-value is demonstrated using operational data in the following sections, the maximum F-value refers to the highest F-value achieved over a prediction span and the average F-value refers to the mean of all the F-values observed throughout the prediction span.

Table 1: F-values for Conjunctions where Risk Mitigation Maneuvers Supported by GSFC CA Team

Time of Closest Approach (Z)	Asset	Object	Minimum Predicted Miss Distance [meters][†]	Maximum Pc Computed	Average F-value	Maximum F-value
23 Oct 2005 20:53	Terra	Scout G-1	37	8.15e-2	8.78	9.97
17 Jan 2007 08:43	PARASOL	AnalystSat	43	1.83e-2	7.68	9.48
18 Feb 2007 18:28	SAC-C	SL-8 debris	57	1.04e-3	6.76	7.41
23 Jun 2007 21:44	Terra	Fengyun 1-C debris	18	1.58e-1	7.84	9.67
06 Jul 2007 06:51	CloudSat	Sinah 1	38	4.69e-2	7.98	9.15
27 Jun 2008 15:34	Aura	Triad 1 debris	11	4.80e-1	8.58	9.79
21 Jul 2008 04:38	CloudSat	Delta 1 debris	90	2.89e-3	5.84	8.78
20 Oct 2008 10:59	PARASOL	Fengyun 1-C debris	82	2.11e-2	6.96	7.66
24 Apr 2009 13:29	CloudSat	Cosmos 2251 debris	52	4.75e-2	7.44	9.77

From Table 1, it is observed that the maximum conjunction F-value for all events that required a RMM to be performed is greater than 7. For most RMM events, the maximum conjunction F-value observed is greater than 9. To reiterate, the conjunction F-value ranges from 0 to 10, where an F-value of 0 represents little to no risk posed to the asset spacecraft due to the conjunction and an F-value of 10 represents very significant risk due to the conjunction. The decision to mitigate a given conjunction depends on that mission risk tolerance or threshold as well as their conjunction assessment operations concept. Though the F-value provides a quick and accurate estimation of the overall risk due to a conjunction, setting an acceptable risk level is the prerogative of the project or mission.

Another analysis was performed to correlate the 10 highest F-values observed from the history of operational CA data. Table 2 presents similar data to Table 1 for the highest 10 F-values observed and also includes any actions taken by the mission as a result from the conjunction.

[†] Miss distance computed from close approach predictions provided by JSpOC based on their OD solution, which does not model any planned nominal maneuvers

Table 2: Highest 10 F-values Observed for Conjunctions Supported by GSFC CA Team

Maximum F-value	Average F-value	Asset	Object	Time of Closest Approach (Z)	Minimum Predicted Miss Distance [meters]	Maximum Pc Computed	Comments/ Actions/ Results
9.97	8.78	Terra	Scout G-1	23 Oct 2005 20:53	37	8.15e-2	RMM Performed
9.79	8.58	Aura	Triad 1 debris	27 Jun 2008 15:34	11	4.80e-1	RMM Performed
9.77	7.44	CloudSat	Cosmos 2251 debris	24 Apr 2009 13:29	52	4.75e-2	RMM Performed
9.67	7.84	Terra	Fengyun 1-C debris	23 Jun 2007 21:44	18	1.58e-1	RMM Performed
9.60	7.42	SAC-C	AnalystSat	18 Oct 2008 04:32	142	9.33e-3	No Action Taken
9.48	7.68	PARASOL	AnalystSat	17 Jan 2007 08:43	43	1.83e-2	RMM Performed
9.36	8.47	Aqua	Fengyun 1-C debris	17 Aug 2007 18:06	95	9.62e-3	Maneuver needed to mitigate conjunction risk violated orbit constraints; conjunction risk accepted
9.31	7.76	CALIPSO	81375	09 Oct 2006 15:26	49	1.62-3	Maneuver needed to mitigate conjunction risk violated orbit constraints; conjunction risk accepted
9.30	7.06	EO-1	Moz.5/ Safir/ Rubin-5/Cosmos-3	04 Oct 2008 08:11	80	3.99e-3	RMM Performed‡
9.15	7.98	CloudSat	Sinah 1	06 Jul 2007 06:51	38	4.69e-2	RMM Performed

Of the ten highest F-value conjunctions, 7 resulted in RMM. For the other three conjunctions, the CA Team recommended the risk be mitigated using a maneuver, but the missions opted not to perform the maneuver due to external constraints; i.e. each mission considered the risk of other operational constraints to outweigh that posed by the conjunction.

After observing the comparison of the F-value with the associated action(s) taken by the missions, an in depth analysis of the evolution of the F-value was warranted. These cases highlight the necessity to examine the orbit determination solution being used in the close approach predictions and the ability for the F-value to capture the quality assessment as well as the risk assessment.

The first example chosen is the Aura vs. Triad 1 debris event. The TCA for this conjunction was 27 June 2008 at 15:34 Z. The risk posed to Aura due to the conjunction was deemed high enough to warrant

‡ EO-1 had two orbit maintenance maneuvers planned between first identification of the conjunction and the conjunction event TCA. These maneuvers were predicted to mitigate the risk associated with the no-burn predictions and the CA Team recommended the maneuvers be executed as planned.

a RMM on 26 June (~TCA-24 hours). Table 3 provides the details for the evolution of the conjunction throughout the prediction span.

Table 3: Conjunction Evolution Details for Aura vs. Triad 1 debris

Update:		**6/21/2008 20:09**	**6/23/2008 16:15**	**6/23/2008 21:04**	**6/24/2008 3:52**	**6/24/2008 9:50**
Probability of Collision		7.886E-02	7.298E-04	6.300E-03	1.600E-03	3.950E-03
Miss Distance RSS	km	0.030	0.181	0.137	0.143	0.105
Radial Separation	km	0.007	0.031	0.003	0.038	0.048
Along-Track Separation	km	0.008	0.004	0.004	0.039	0.002
Cross-Track Separation	km	0.028	0.173	0.013	0.132	0.091
Days to TCA	days	5.809	3.972	3.771	3.488	3.239
Days Since Object Last Obs	days	6.157	4.022	4.022	3.601	3.601
Det[Covariance]	m^2	5.173E+11	1.782E+10	1.723E+10	5.610E+09	5.683E+09
Conjunction F-value (Unscaled)		**9.95**	**9.63**	**9.77**	**9.67**	**9.71**
Conjunction F-value (Scaled)		**5.87**	**6.83**	**7.19**	**7.65**	**8.08**
Update:		**6/24/2008 17:06**	**6/24/2008 20:10**	**6/25/2008 11:01**	**6/25/2008 15:43**	**6/25/2008 18:48**
Probability of Collision		5.484E-02	2.324E-02	4.773E-02	3.720E-01	4.076E-01
Miss Distance RSS	km	0.056	0.076	0.061	0.017	0.014
Radial Separation	km	0.028	0.031	0.029	0.001	0.002
Along-Track Separation	km	0.012	0.018	0.018	0.007	0.004
Cross-Track Separation	km	0.047	0.067	0.050	0.016	0.013
Days to TCA	days	2.936	2.809	2.190	1.994	1.865
Days Since Object Last Obs	days	3.198	3.198	2.700	2.172	2.172
Det[Covariance]	m^2	3.542E+09	2.778E+09	1.516E+09	4.789E+08	3.901E+08
Conjunction F-value (Unscaled)		**9.86**	**9.83**	**9.85**	**9.99**	**9.99**
Conjunction F-value (Scaled)		**8.65**	**8.76**	**9.26**	**9.54**	**9.58**
Update:		**6/25/2008 21:10**	**6/26/2008 2:31**	**6/26/2008 10:30**	**6/26/2008 21:57**	
Probability of Collision		4.190E-01	1.786E-02	1.542E-02	1.261E-02	
Miss Distance RSS	km	0.012	0.054	0.055	0.064	
Radial Separation	km	0.001	0.005	0.001	0.011	
Along-Track Separation	km	0.041	0.015	0.014	0.017	
Cross-Track Separation	km	0.011	0.052	0.053	0.061	
Days to TCA	days	1.767	1.544	1.211	0.734	
Days Since Object Last Obs	days	2.172	1.669	1.273	0.776	
Det[Covariance]	m^2	3.942E+08	1.257E+08	8.993E+07	1.050E+08	
Conjunction F-value (Unscaled)		**9.99**	**9.94**	**9.95**	**9.90**	
Conjunction F-value (Scaled)		**9.61**	**9.67**	**9.79**	**9.79**	

Figure 11 illustrates the conjunction F-value trends for the Aura vs. Triad 1 debris event. It can be observed that conjunction was identified with an F-value of 5.87 but reached an F-value greater than 8 by the fourth update and a maximum of 9.79 near the end of the prediction span. The Aura vs. Triad 1 debris presented the second highest F-value observed in the data and, at the time of the event, was certainly one of the most concerning conjunctions observed in CA Operations.

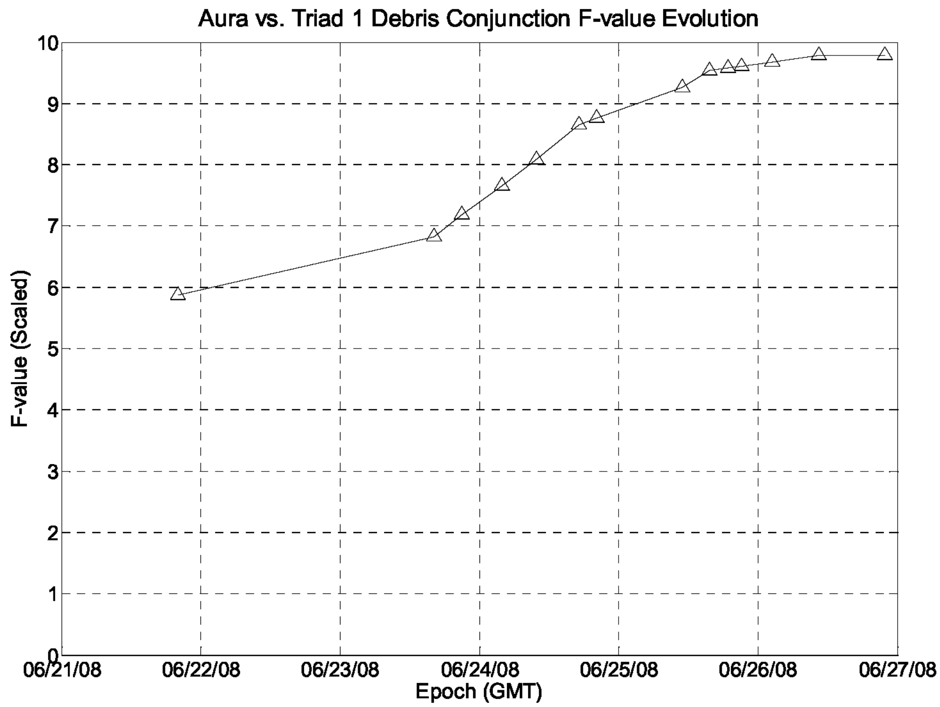

Figure 11: Aura vs. Triad 1 Debris Conjunction F-value Evolution

Comparing and Figure 11, the high risk of this conjunction is much easier observable from the F-value (as shown in Figure 11) rather than the exhaustive details (provided in Table 3). Moreover, the decision point to execute the maneuver was after the tenth update. The information in Table 3 relays the consistent small miss distance, high Pc, and realistic covariance predictions lead to the decision to maneuver. The F-value, and its associated risk scale, more easily conveys that message.

The second case study of PARASOL versus AnalystSat is an example of the F-value correctly describing the true risk posed by a conjunction, prior to increased tasking taking effect. Increased tasking is a term to describe the increase in priority Space Surveillance Network assets to obtain tracking for objects of interest. The initial identification of the conjunction between PARASOL and an AnalystSat showed about an 800 meter miss distance with a probability of conjunction of ~2.6e-3. The unscaled, or risk parameters-only, F-value corresponding to these predictions was 9.44, a fairly high risk indicator. However, the scaled F-value, which includes quality assessment parameters, was a more mild 5.21. Table 4 provides the conjunction details history of this event.

Table 4: Conjunction Evolution Details for PARASOL vs. an AnalystSat

	Update:	10/20/2008 22:41	10/21/2008 19:52	10/22/2008 19:57	10/23/2008 15:55	10/23/2008 19:58
Probability of Collision		2.590E-03	1.432E-03	1.380E-03	0.000E+00	0.000E+00
Miss Distance RSS	km	0.794	1.212	1.412	2.678	2.597
Radial Separation	km	0.006	0.031	0.011	0.144	0.150
Along-Track Separation	km	0.318	0.483	0.560	1.062	1.034
Cross-Track Separation	km	0.728	5.482	1.296	2.454	2.378
Days to TCA	days	6.273	5.391	4.387	3.555	3.387
Days Since Object Last Obs	days	6.813	6.813	6.813	4.189	4.189
Det[Covariance]	m^2	2.180E+16	4.994E+15	6.913E+14	6.748E+12	6.746E+12
Conjunction F-value (Unscaled)		**9.44**	**9.09**	**9.05**	**5.91**	**5.93**
Conjunction F-value (Scaled)		**5.21**	**5.13**	**5.57**	**4.50**	**4.70**

	Update:	10/24/2008 15:55	10/24/2008 21:13	10/25/20098 11:29	10/26/2008 21:42
Probability of Collision		0.000E+00	0.000E+00	0.000E+00	0.000E+00
Miss Distance RSS	km	2.748	2.715	2.576	2.597
Radial Separation	km	0.128	0.129	0.144	0.138
Along-Track Separation	km	1.095	1.078	1.026	1.032
Cross-Track Separation	km	2.518	2.489	2.359	2.379
Days to TCA	days	2.556	2.335	1.241	0.315
Days Since Object Last Obs	days	2.759	2.759	2.197	0.766
Det[Covariance]	m^2	4.125E+11	3.049E+11	1.585E+10	2.156E+08
Conjunction F-value (Unscaled)		**5.93**	**5.95**	**5.97**	**5.98**
Conjunction F-value (Scaled)		**5.41**	**5.51**	**5.77**	**5.91**

This secondary object was identified as an object for which increased tasking was warranted and may potentially help. The object was previously sparsely tracked with several days between observations. After 3 days of updates to this event, this increased tasking began taking effect and yielded a more regular tracking pattern of 1-2 tracks per day. At this point, it is observed from Figure 12 that the unscaled F-value, which considers only risk parameters and not the quality factors, converges with the F-value. This is an example of the F-value accurately portraying the conjunction risk throughout the evolution of the event.

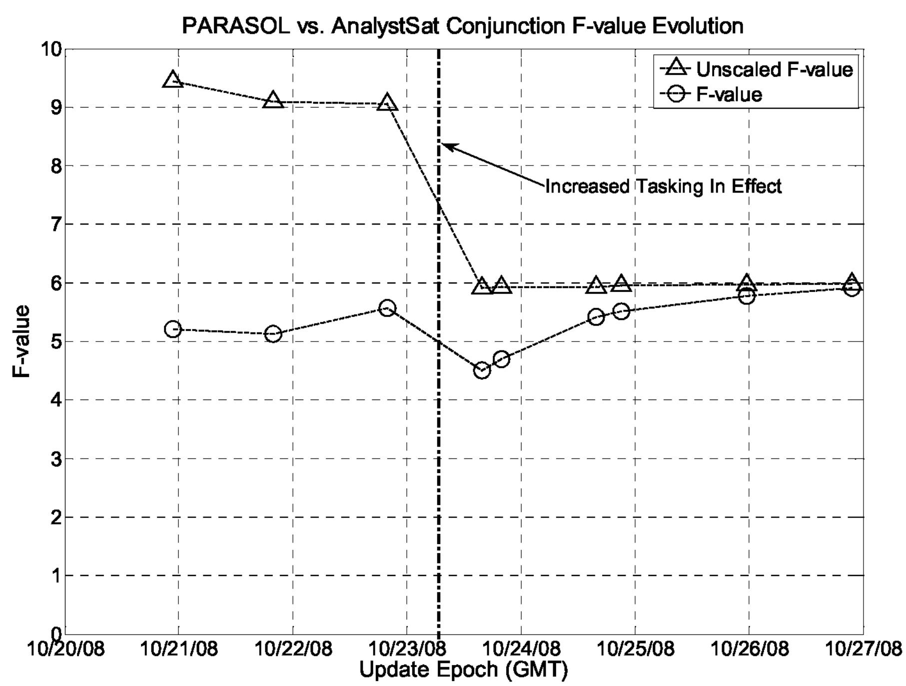

Figure 12: PARASOL vs. an AnalystSat Conjunction F-value Evolution

The example between PARASOL and an AnalystSat demonstrates that a person without a thorough understanding of performing routine conjunction assessment would have dispositioned this event differently if provided the information in Table 4 versus the information in Figure 12. Only having knowledge and understanding of the OD quality assessment and tasking levels would the risk assessments have been the same from the detailed information versus the F-value.

The conjunction between Terra and an AnalystSat provides another example between the unscaled and scaled F-values and the risk associated with a particular conjunction. Based on the risk assessment parameters alone, the unscaled conjunction F-value was high, but due to the quality assessment parameters the actual conjunction F-value was lower. Again an "unscaled" conjunction F-value refers to only the risk factor portion of the F-value computation and not the quality factor. Table 5 provides the close approach details through the evolution of the conjunction.

Table 5: Conjunction Evolution Details for Terra vs. an AnalystSat

	Update:	**8/27/2007 22:29**	**8/29/2007 21:41**	**8/30/2007 22:14**	**9/1/2007 2:45**
Probability of Collision		8.597E-06	8.538E-06	8.527E-06	8.536E-06
Miss Distance RSS	km	1.311	1.113	0.744	0.994
Radial Separation	km	0.002	0.016	0.011	0.006
Along-Track Separation	km	0.732	0.315	0.212	0.280
Cross-Track Separation	km	1.257	1.067	0.713	0.954
Days to TCA	days	5.324	3.357	2.334	1.146
Days Since Object Last Obs	days	19.970	19.970	19.970	19.970
Det[Covariance]	m^2	9.300E+19	3.920E+19	2.878E+19	2.431E+19
Conjunction F-value (Unscaled)		**8.17**	**8.29**	**8.50**	**8.39**
Conjunction F-value (Scaled)		**2.98**	**4.60**	**5.66**	**5.85**

From Table 5, it is observed that the Pc remained around the e-6 level with miss distance predictions around 1 kilometer RSS and < 20 meters of radial separation. Without taking the orbit determination solution quality of the secondary object into consideration, this conjunction indicates a moderate threat to the asset spacecraft. This threat level is apparent in the unscaled conjunction F-value, which remained around 8.2 – 8.5 through the prediction span. However, once the OD is examined, the secondary object was sparsely tracked such that a new track was never observed during the conjunction evolution. In fact the last observation of the secondary object was over 19 days prior to the event TCA. This poor quality OD is reflected in the (scaled) conjunction F-value, which starts out ~3 for the initial prediction and elevates to a high 5 near the end of the prediction span. This F-value is much more aligned with the disposition made by the CA Team at the time of this event, and a more accurate depiction of the threat of this conjunction posed on the asset spacecraft. The unscaled and scaled conjunction F-value trends can be observed in Figure 13.

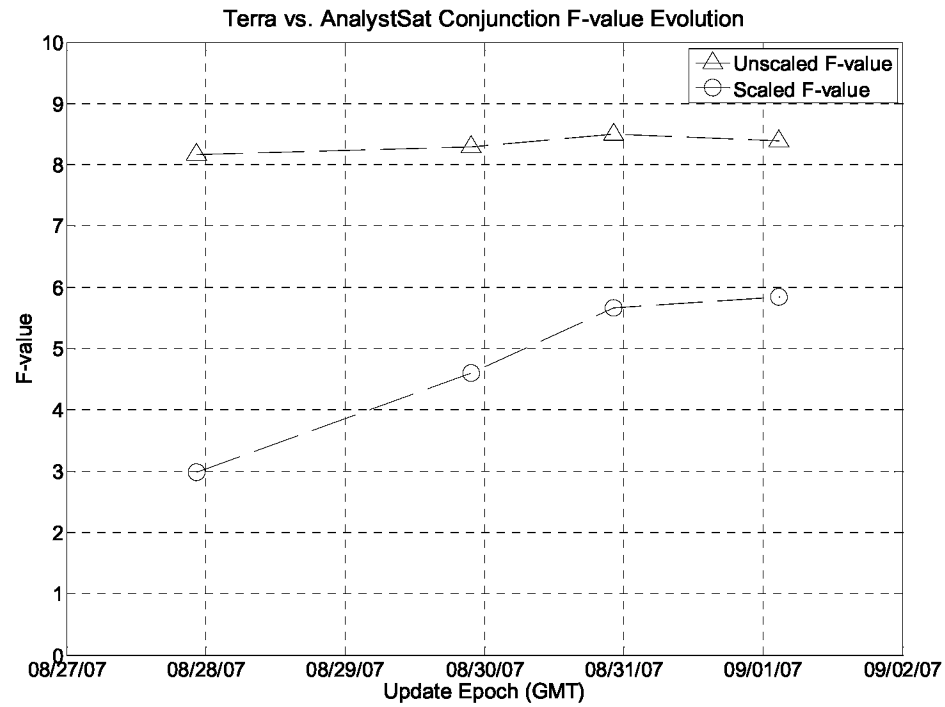

Figure 13: Terra vs. an AnalystSat Conjunction F-value Evolution

CONCLUSION

This analysis introduced the concept of the F-value and its application in conjunction risk assessment. The F-value is a method for quantifying the risk associated with a particular conjunction event into a single value. Its usefulness is observed in the quickness and ease of portraying the risk of a conjunction. While the analysis involved in conjunction assessment is still a necessity, the F-value aggregates all of that information into a portable and understandable value which can be used by mission stakeholders who don't have a detailed understanding of the conjunction evaluation specifics.

This analysis has provided a simple F-value model applied to conjunction risk assessment. Three examples were presented that clearly demonstrate the ability of the F-value to mimic the disposition of conjunction risk as performed by the CA Team. The simple model also demonstrates the ability to hide the underlying analysis in order to quickly portray conjunction risk assessment in the form of a single, quantifiable value. This analysis suggests that the F-value can be used as a single risk index metric that can easily convey the level of risk without all of the technical details.

Based on the encouraging results observed from this analysis, future plans include calculating the F-value operationally. Additionally, these plans include introducing the concept to mission owner/operators and soliciting feedback on its capability and effectiveness. Lastly, the author would like to explore use of the F-value in other orbital regimes.

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