Elsevier Editorial System(tm) for Journal of Hydrology
Manuscript Draft

Manuscript Number:

Title: Decomposition of the Mean Squared Error & NSE Performance Criteria: Implications for Improving Hydrological Modelling

Article Type: Research Paper

Keywords: mean squared error; Nash-Sutcliffe efficiency; model performance evaluation; calibration; multiple criteria; hydrologic modelling; criteria decomposition; diagnostic analysis

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April 7, 2009

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Dear Mr. Georgakakos,

Please find enclosed our manuscript entitled "Decomposition of the Mean Squared Error & NSE Performance Criteria: Implications for Improving Hydrological Modelling" submitted for publication in the Journal of Hydrology, authored by Hoshin V. Gupta, Harald Kling, Koray K. Yilmaz, Guillermo F. Martinez-Baquero.

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2 Decomposition of the Mean Squared Error & NSE Performance Criteria: Implications for
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Abstract:

The mean squared error (MSE) and the related normalization, the Nash-Sutcliffe efficiency (NSE), are the two criteria most widely used for calibration and evaluation of hydrological models with observed data. Here, we present a diagnostically interesting decomposition of NSE (and hence MSE), which facilitates analysis of the relative importance of its different components in the context of hydrological modelling, and show how model calibration problems can arise due to interactions among these components. The analysis is illustrated by calibrating a simple conceptual precipitation-runoff model to daily data for a number of Austrian basins having a broad range of hydro-meteorological characteristics. Evaluation of the results clearly demonstrates the problems that can be associated with any calibration based on the NSE (or MSE) criterion. While we propose and test an alternative criterion that can help to reduce model calibration problems, the primary purpose of this study is not to present an improved measure of model performance. Instead, we seek to show that there are systematic problems inherent with any optimization based on formulations related to the MSE. The analysis and results have implications to the manner in which we calibrate and evaluate environmental models; we discuss these and suggest possible ways forward that may move us towards an improved and diagnostically meaningful approach to model performance evaluation and identification.

Keywords:

mean squared error; Nash-Sutcliffe efficiency; model performance evaluation; calibration; multiple criteria; hydrologic modelling; criteria decomposition; diagnostic analysis

1 Introduction

The mean squared error (MSE) criterion and its related normalization, the Nash-Sutcliffe efficiency (NSE, defined by Nash and Sutcliffe 1970) are the two criteria most widely used for calibration and evaluation of hydrological models with observed data. The value of MSE depends on the units of the predicted variable and varies on the interval [0.0 to inf], whereas NSE is dimensionless, being scaled onto the interval [-inf to 1.0]. As a consequence, the NSE value - obtained by dividing MSE by the variance of the observations and subtracting that ratio from 1.0 (Eq. 1 and Eq. 2) - is commonly the measure of choice for reporting (and comparing) model performance. Further, NSE can be interpreted as a classic skill score (Murphy 1988), where 'skill' is interpreted as the comparative ability of a model with regards to a baseline 'model', which in the case of NSE is taken to be the 'mean of the observations' (i.e., if NSE ≤ 0, the model is no better than using the observed mean as a predictor). The equations are:

$$MSE = \frac{1}{n} \cdot \sum_{t=1}^{n} \left(x_{s,t} - x_{o,t} \right)^2 \qquad \text{Eq. 1}$$

$$NSE = 1 - \frac{\sum_{t=1}^{n} \left(x_{s,t} - x_{o,t} \right)^2}{\sum_{t=1}^{n} \left(x_{o,t} - \mu_o \right)^2} = 1 - \frac{MSE}{\sigma_o^2} \qquad \text{Eq. 2}$$

where n is the total number of time-steps, $x_{s,t}$ is the simulated value at time-step t, $x_{o,t}$ is the observed value at time-step t, and μ_o and σ_o are the mean and standard deviation of the observed values. In optimization MSE is subject to minimization and NSE is subject to maximization.

While the NSE criterion may be a convenient and popular (albeit gross) indicator of model skill, there has been a long and vivid discussion about the suitability of NSE (McCuen and Snyder 1975, Martinec and Rango 1989, Legates and McCabe 1999, Krause et al. 2005, McCuen et al. 2006, Schaefli and Gupta 2007) and several authors have proposed modifications - e.g. Mathevet et al. (2006) proposed a bounded version of NSE and Criss and Winston (2008) proposed a volumetric efficiency to be used instead of NSE. One of the main concerns about NSE is its use of the observed mean as baseline, which can lead to overestimation of model skill for highly seasonal variables such as runoff in snowmelt dominated basins. A comparison of NSE across basins with different seasonality (as is often reported in the literature) should therefore be interpreted with caution. For such situations, various authors have recommended the use of the seasonal or climatological mean as a baseline model (Garrick et al. 1978, Murphy 1988, Martinec and Rango 1989, Legates and McCabe 1999, Schaefli and Gupta 2007).

It is now generally accepted that the calibration of hydrological models should be approached as a multi-objective problem (Gupta et al. 1998). Within a multiple-criteria framework, the MSE and NSE criteria continue to be commonly used, because they can be computed separately for (1) different types of observations (e.g. runoff and snow observations; Bergström et al. 2002), (2) different locations (e.g. runoff at multiple gauges; Madsen 2003), or (3) different subsets of the same observation (e.g. rising and falling limb of the hydrograph; Boyle et al. 2000). More generally, however, different types of model performance criteria - such as NSE, coefficient of correlation, bias, etc. - can be computed from multiple variables and/or at multiple sites (see Anderton et al. 2002, Beldring 2002, Rojanschi et al. 2005, Cao et al. 2006, and others).

When handled in this manner, the model calibration problem can be treated as a full multiple-criteria optimization problem resulting in a 'Pareto set' of non-dominated solutions (Gupta et al 1998), or reduced to a related single-criterion optimization problem by combining the different

(weighted) criteria into one overall objective function. Numerous examples of the latter approach exist in the literature where NSE or MSE appear in an overall objective function (e.g. Lindström 1997, Bergström et al. 2002, Madsen 2003, van Griensven and Bauwens 2003, Parajka et al. 2005, Young 2006, Rode et al. 2007, Marce et al. 2008, Wang et al. 2009), because it conveniently enables the application of efficient single-criterion automated search algorithms, such as SCE (Shuffled Complex Evolution, Duan et al. 1992) or DDS (Dynamically Dimensioned Search, Tolson and Shoemaker 2007).

When using multiple criteria in evaluation, it has to be considered that some of these criteria are mathematically related, which is not always recognized (Weglarczyk 1998). For example, it is possible to decompose the NSE criterion into separate components, as shown by Murphy (1988) and Weglarczyk (1998), which facilitates a better understanding of how different criteria are interrelated and thereby enable more insight into what is causing a particular model performance to be 'good' or 'bad'. Equally important, the decomposition can provide insight into possible trade-offs between the different components.

In this paper we present a diagnostically interesting decomposition of NSE (and hence MSE), which facilitates analysis of the relative importance of different components in the context of hydrological modelling, and show how model calibration problems can arise due to interactions among these components. Based on this analysis, we propose and test alternative criteria that can help to avoid these problems. The analysis is illustrated by calibrating a simple precipitation-runoff model to daily data for a number of Austrian basins having a broad range of hydro-meteorological characteristics, and evaluating the results on both the calibration and an independent 'evaluation' period. The results clearly demonstrate the problems that can be associated with any calibration based on the NSE (or MSE) criterion. The analysis and results have interesting implications to the

manner in which we calibrate and evaluate environmental models; we discuss these and some possible ways forward in the discussion and conclusions sections.

2 Decomposition of model performance criteria

2.1 Decomposition of NSE

A decomposition of criteria based on mean squared errors reveals that there are three distinctive components, represented by the correlation, the conditional bias, and the unconditional bias, as evident in Eq. 3 which shows a decomposition of NSE (Murphy 1988, Weglarczyk 1998).

$$NSE = A - B - C \qquad \text{Eq. 3}$$

with: $A = r^2$

$$B = \left[r - \left(\sigma_s / \sigma_o\right)\right]^2$$

$$C = \left[\left(\mu_s - \mu_o\right) / \sigma_o\right]^2$$

where r is the linear correlation coefficient between x_s and x_o, and (μ_s, σ_s) and (μ_o, σ_o) represent the first two statistical moments (means and standard deviations) of x_s and x_o respectively. The quantity A measures the strength of the linear relationship between the simulated and observed values, B measures the conditional bias, and C measures the unconditional bias (Murphy 1988).

However, an alternative way in which to reformulate Eq. 3 is given below as Eq. 4.

$$NSE = 2 \cdot \alpha \cdot r - \alpha^2 - \beta_n^2 \qquad \text{Eq. 4}$$

with: $\alpha = \sigma_s / \sigma_o$

120 $$\beta_n = (\mu_s - \mu_o)/\sigma_o$$

121 where the quantity α is a measure of relative variability in the simulated and observed values, and
122 β_n is the bias normalized by the standard deviation in the observed values (note that β_n = sqrt(C)).

123 Eq. 4 shows that NSE is composed of three components, two of which relate to the ability of the
124 model to reproduce the *first and second moments of the distribution of the observations* (i.e. mean
125 and standard deviation), while the third relates to the *ability of the model to reproduce timing and*
126 *shape* as measured by the correlation coefficient. The ideal values for the three components are
127 $r = 1$, $\alpha = 1$, and $\beta_n = 0$. From a hydrological perspective, 'good' values for each of these three
128 components are highly desirable, since in general we aim at matching the overall volume of flow,
129 the spread of flows (e.g. flow duration curve), and the timing and shape of (for example) the
130 hydrograph (Yilmaz et al. 2008). It is clear, therefore, that optimizing NSE is essentially a search
131 for a balanced solution among the three components, which is similar to the multiple-criteria
132 approach of computing an overall (weighted) objective function from several different criteria as
133 discussed in the introduction.

134 However, in using NSE we must be concerned with two facts. First, the bias (μ_s - μ_o) component
135 appears in a normalized form, scaled by the standard deviation in the observed flows. This means
136 that in basins with high runoff variability the bias component will tend to have a smaller
137 contribution (and therefore impact) in the computation and optimization of NSE, possibly leading to
138 model simulations having large volume balance errors. In a multiple-criteria sense, this is
139 equivalent to using a weighted objective function with a low weight applied to the bias component.

140 Second, and equally serious, the quantity α appears twice in Eq. 4, exhibiting an interesting (and
141 problematic) interplay with the linear correlation coefficient r. It is easy to show, by taking the first
142 derivative of NSE (in Eq. 4) with respect to α, that the maximum value of NSE is obtained when

143 $\alpha = r$. And, since r will always be smaller than unity, this means that in maximizing NSE we will
144 tend to select a value for α that *underestimates the variability in the flows* (more precisely, we will
145 favour models/parameter sets that generate simulated flows that underestimate the variability).

146 Taking these two facts together, we note that when $\beta_n = 0$ and $\alpha = r$, then the NSE is equivalent to
147 r^2, which is the well-known coefficient of determination. Therefore, r^2 can be interpreted as a
148 maximum (potential) value for NSE if the other two components are able to achieve their 'optimal'
149 values.

150 Fig. 1 illustrates the relationship of NSE with r and α, while assuming that β_n is zero (β_n is only an
151 additive term, anyway). For a given r the 'optimal' α for maximizing NSE lies on the 1:1 line,
152 although the ideal value of α is on a horizontal line at 1.0. This theoretical relationship is illustrated
153 in Fig. 1a. Of course, not all combinations of r and α may be possible with a hydrological model
154 due to restrictions imposed by the model structure, feasible parameter values and input-output data.
155 However, Fig. 1b shows a real example in which random sampling of the parameter space actually
156 seems to cover a large portion of the theoretical criteria space. Since the model used here (HyMod
157 model, Boyle 2000) is a simple, but representative, example of watershed models in common use,
158 the problematic interplay between α and r is likely to be of importance for any type of hydrological
159 model that is optimized with NSE.

160 **Fig. 1 near here**

161 Further, the same exact problems will arise when using MSE as a model calibration criterion. We
162 can substitute Eq. 4 into Eq. 2, and thereby obtain Eq. 5 which shows the related decomposition of
163 the MSE criterion, consisting (again) of three error terms, but here all three of them are additive.

$$MSE = 2 \cdot \sigma_s \cdot \sigma_o \cdot (1 - r) + (\sigma_s - \sigma_o)^2 + (\mu_s - \mu_o)^2 \qquad \text{Eq. 5}$$

From Eq. 3, Eq. 4 and Eq. 5 it should be immediately obvious that many different combinations of the three components can result in the same overall value for MSE or NSE, respectively, potentially leading to considerable ambiguity in the comparative evaluation of alternative model hypotheses. The relative contribution of each of these components to the overall MSE can be computed as:

$$f_i = F_i / \sum_{j=1}^{3} F_j \qquad \text{Eq. 6}$$

with: $F_1 = 2 \cdot \sigma_s \cdot \sigma_o \cdot (1 - r)$

$F_2 = (\sigma_s - \sigma_o)^2$

$F_3 = (\mu_s - \mu_o)^2$

2.2 Alternative model performance criteria

As discussed above, a peculiar feature of the NSE criterion is the problematic interplay between α and r, which is likely to result in an underestimation of the variability in the flows. One way to overcome this is by inflating the observed variability as indicated by Eq. 7, while at the same time preserving the mean of the observations and their linear correlation with the simulations. Using Eq. 7 with Eq. 4 results in Eq. 8, which represents a 'corrected' version of NSE:

$$x^*_{o,t} = c \cdot (x_{o,t} - \mu_o) + \mu_o \qquad \text{Eq. 7}$$

$$NSE_{cor} = \frac{1}{c} \cdot 2 \cdot \alpha \cdot r - \frac{1}{c^2} \cdot \alpha^2 - \frac{1}{c^2} \cdot \beta_n^2 \qquad \text{Eq. 8}$$

where c is correction factor to inflate the variability in the observed flows. It can be easily shown that if c is set equal to $1/r$, it will assure that a value of $\alpha = 1$ will now maximize NSE_{cor} (as opposed to $\alpha = r$ maximizing NSE).

Alternatively, instead of trying to come up with a 'corrected' NSE criterion, since MSE and NSE can be decomposed into three components, the whole calibration problem can instead be viewed from the multi-objective perspective, by focusing on the correlation, variability error and bias error as separate criteria to be optimized. In doing this, it makes sense to enable a better hydrological interpretation of the bias component by using the ratio of the means of the simulated and observed flows (β) for this further analysis - as opposed to using β_n. With this formulation, using β instead of β_n, all three of the components now have their ideal value at unity.

Fig. 2 shows an example for the trade-off between the three components for a simple hydrological model using random parameter sampling. The plot shows a distinctive Pareto front in the three-dimensional criteria space. If it is desired to select a compromise solution from the Pareto front, one possible approach is to compute for all points the Euclidian distance from the ideal point and then to subsequently select the point having the shortest distance (Eq. 9). Since all three of the components are dimensionless numbers, we are able to obtain a reasonable solution for the Euclidian distance in the un-transformed criteria space. Alternatively, a re-scaling of the axes in the criteria space is easily obtained via Eq. 10. In this paper, we will only explore the use of the KGE criterion (Eq. 9), which is equivalent to setting all three scaling factors of Eq. 10 to unity.

Fig. 2 near here

$$KGE = 1 - ED \qquad \text{Eq. 9}$$

$$KGE_s = 1 - ED_s \qquad \text{Eq. 10}$$

197 with: $ED = \sqrt{(r-1)^2 + (\alpha-1)^2 + (\beta-1)^2}$

198 $ED_s = \sqrt{[s_r \cdot (r-1)]^2 + [s_\alpha \cdot (\alpha-1)]^2 + [s_\beta \cdot (\beta-1)]^2}$

199 $\beta = \mu_s / \mu_o$

200 where ED is the Euclidian distance from the ideal point, ED_s is the Euclidian distance from the ideal
201 point in the scaled space, β is the ratio between the mean simulated and mean observed flows, i.e. β
202 represents the bias; s_r, s_α and s_β are scaling factors that can be used to re-scale the criteria space
203 before computing the Euclidian distance from the ideal point, i.e. s_r, s_α and s_β can be used for
204 adjusting the emphasis on different components.

205 Analogous to Eq. 6 we can compute the relative contribution of the three components with Eq. 11.

$$g_i = G_i / \sum_{j=1}^{3} G_j \qquad \text{Eq. 11}$$

206 with: $G_1 = (r-1)^2$

207 $G_2 = (\alpha-1)^2$

208 $G_3 = (\beta-1)^2$

209 2.3 Notes on regression lines

210 As is well known, the slope of the regression lines and the coefficient of correlation are related (Eq.
211 12 to Eq. 14). Since different 'optimal' values for α are obtained by the NSE and KGE criteria, this
212 also leads to implications for the regression lines.

$$r^2 = k_s \cdot k_o \qquad \text{Eq. 12}$$

$$k_s = \frac{Cov_{so}}{\sigma_s^2} = \frac{r}{\alpha} \qquad \text{Eq. 13}$$

$$k_o = \frac{Cov_{so}}{\sigma_o^2} = r \cdot \alpha \qquad \text{Eq. 14}$$

with: $r = \dfrac{Cov_{so}}{\sigma_s \cdot \sigma_o}$

where Cov_{so} is the covariance between the simulated and observed values, k_s is the slope of the regression line when regressing the observed against the simulated values, and k_o is the slope of the regression line when regressing the simulated against the observed values.

Murphy (1988) has already noted that for NSE the conditional bias term B in Eq. 3 will vanish only if the slope of the regression line k_s is equal to unity (i.e. regressing the observed against the simulated values), which is desirable in the context of the 'verification' of forecasts. This means that for a given forecast (simulated value), the expected value of the observed value lies on the 1:1 line (assuming a Gaussian distribution). As discussed before, the optimal value of α that maximizes NSE is given by a model simulation for which α is equal to r. As evident in Eq. 13 this results in $k_s = 1$, but at the same time this also implies that $k_o = r^2$ (Eq. 14). Because r^2 will always be smaller than unity, this means that we will, in general, tend to underestimate the slope of the regression line when regressing the simulated against the observed values. The tendency will be for high values (peak flows) to be underestimated and for low values (recessions) to be overestimated in the simulation.

In brief, for maximizing NSE the optimal values for k_s and k_o are unity and r^2, respectively. In the case of KGE, the optimal value for α is at unity, which means that for maximizing KGE the optimal values for both k_s and k_o are equal to r. Again, since r is smaller than unity we will tend to underestimate high values and overestimate low values.

In considering this, it should be noted that both approaches for computing the regression lines (regressing observed against simulated values, or vice versa) are valid, but have different interpretations. In the context of runoff simulations, when using k_s we are basing the evaluation on the expected error in simulation of the observed runoff being zero for a given simulated runoff, which is a sensible approach when making runoff forecasts under 'normal' conditions. However, if we are interested in the 'unusual' runoff conditions - such as runoff peaks - then a more sensible approach would be to use k_o, where we are interested in the question, "If a flood occurs, can we forecast (simulate) it?", whereas in the case of k_s such a runoff peak is 'averaged out'. Fig. 3 illustrates this with typical scatter plots for runoff simulation. In this example, k_s is close to unity, suggesting unbiased forecasts (Fig. 3a), and at the highest simulated flows of around 10 m^3/s the small number of observed flows (runoff peaks) that are well above the regression line are 'averaged out' by the larger number of observed flows that occur slightly below the regression line. However, it is clear that whenever a runoff peak above 10 m^3/s occurs, there is a clear tendency for underestimation in the simulation (Fig. 3b).

These problems arise because the distribution of runoff is usually highly skewed. If k_o is of higher interest, then the use of NSE may cause problems, since the simulated runoff will tend to underestimate the peak flows. In the case of the KGE criterion, we will also have a tendency towards underestimation, but not as severe as with the NSE. Note that for extreme low-flows, similar considerations as for the runoff peaks apply (but here we will tend to overestimate the low-flow).

252 **Fig. 3 near here**

253 # 3 Case Study

254 To examine and illustrate the implications of the theoretical considerations presented above we
255 applied a simple conceptual precipitation-runoff model to several basins. Using NSE (Eq. 2) and
256 KGE (Eq. 9) as model performance criteria, two different sets of parameters were obtained for each
257 basin by calibration against observed runoff data. For each parameter set we compare the overall
258 model performance as evaluated by the NSE and KGE criteria and, in addition, conduct a detailed
259 analysis of the criterion components. Further, we also examine the model performance on an
260 independent 'evaluation' period.

261 ## 3.1 Study area

262 For this study we used the forty-nine mesoscale Austrian basins (Fig. 4) used in the regionalization
263 study reported by Kling and Gupta (2009). All are pre-alpine or lowland basins where snowmelt
264 does not dominate runoff generation. They vary in size from 112.9 km^2 to 689.4 km^2, with a median
265 size of 287.3 km^2, and a mean elevation range from 232 m to 952 m above sea level. The basins
266 represent a wide range of physiographic and meteorological properties, with the most important
267 land-use types being forest, grassland and agriculture. According to the Hydrological Atlas of
268 Austria (BMLFUW 2007), the long-term mean annual precipitation in the basins ranges from 507 to
269 1929 mm, and the corresponding runoff ranges from 44 to 1387 mm, resulting in a large range of
270 runoff coefficients (from 9 to 72 percent). Thus, both wet and dry basins are included. Fig. 5 shows
271 a diagnostic plot where normalized actual evapotranspiration is plotted against normalized
272 precipitation (both variables are scaled by potential evapotranspiration); it indicates that most of the
273 basins are energy limited and only a few of the basins are water limited.

Fig. 4 near here

Fig. 5 near here

3.2 Data basis

We used observed daily data for the period September 1990 to August 2000; the first two years were used as a warm-up period, the next five years for calibration, and the final three years for independent evaluation. Observed catchment outlet runoff data were used for parameter calibration in each of the basins. Precipitation inputs were based on daily data from 222 stations, regionalized using the method of Thiessen-Polygons. Air temperature inputs were based on data from 98 stations, regionalized via linear regression with elevation. Potential evapotranspiration inputs were based on monthly fields of potential evapotranspiration (Kling et al. 2007) with a spatial resolution of 1x1 km. The monthly potential evapotranspiration data were disaggregated to daily time-steps by using daily data from 21 indicator stations, where the daily potential evapotranspiration was computed using the Thornthwaite-method (Thornthwaite and Mather 1957).

3.3 Hydrological model

A simple, conceptual, spatially distributed daily precipitation-runoff model similar to the HBV model (Bergström 1995) was used; the model was previously applied to these same basins by Kling and Gupta (2009). The model uses a 1x1 km^2 raster grid for spatial discretization of the basins. However, for simplicity, the current study assumes uniform parameter fields. Inputs to the model are precipitation, air temperature, and potential evapotranspiration. The model consists of a snow module, soil moisture accounting, runoff separation into different components, and a routing module. Snowfall is determined from precipitation data using a threshold temperature, and snowmelt is computed with the temperature-index method (see e.g. Hock 2003). Rainfall and

296 snowmelt are input to the soil module, where runoff generation is computed via an exponential
297 formulation that accounts for current soil moisture conditions (see e.g. Bergström and Graham
298 1998). Actual evapotranspiration depletes the soil moisture store; the rate of actual
299 evapotranspiration depends on current soil moisture conditions and potential evapotranspiration.
300 Runoff is separated into fast (surface flow) and slow (base flow) components by two linear
301 reservoirs having different recession coefficients. A further linear reservoir is used to simulate
302 channel routing of the runoff. Fig. 6 shows the conceptual structure of the model (the snow module
303 is not shown). The model equations are presented in Kling and Gupta (2009). Table 1 lists the most
304 important parameters of the model.

305 To reduce dimensionality of the parameter calibration problem, some of the model parameters are
306 set to plausible values and are not further calibrated. This applies to snow parameters, because snow
307 is of limited importance in the basins of this study, and to the channel routing parameters, which are
308 of limited importance at the daily time-step (the values of Kling and Gupta (2009) are used). In
309 addition, the critical soil moisture for reducing actual evapotranspiration is set to a constant value.
310 The six remaining parameters were calibrated using the Shuffled Complex Evolution optimization
311 algorithm (SCE, Duan et al. 1992), using six complexes.

312 **Fig. 6 near here**

313 **Table 1 near here**

314 3.4 Results

315 The optimization runs resulted in two parameter sets for each basin. Optimization using the
316 'optNSE' method results in parameter sets 'θ_{optNSE}' that yield optimal runoff simulations
317 maximizing NSE (Eq. 4), while optimization using the 'optKGE' method results in parameter sets
318 'θ_{optKGE}' that yield optimal runoff simulations maximizing KGE (Eq. 9). A standard method for

reporting model performance in precipitation-runoff modelling studies is to present scatter plots of NSE between calibration and evaluation periods (see e.g. Merz and Blöschl 2004). Fig. 7 displays such a scatter plot; as expected, for many basins the NSE deteriorates when going from the calibration to the evaluation period (Fig. 7a). Similar results are obtained for KGE (Fig. 7b).

Now, there can be different reasons for deterioration of model performance on the evaluation period. These include over-fitting of the parameters to the calibration period, non-stationarity between the calibration and evaluation periods, lack of 'power' in the objective function, etc. Instead of falsification and model rejection, which would be a logical conclusion from such a result, it is common practice to simply report the deterioration in the model performance and then to move on. In our case, we can report that when moving from calibration to evaluation period the median NSE has decreased from 0.76 to 0.59 and the median KGE has decreased from 0.86 to 0.72, but what hydrological meaning do these numbers have? Here, an analysis of the different components that constitute the overall model performance enables us to learn much more about the model behaviour, differences between the calibration and evaluation periods, and also differences between basins.

Before analysing the criterion components it is interesting to note the relationship between NSE and KGE. Fig. 7 shows that when optimizing on KGE (optKGE) there is a strong correlation between the values obtained for the KGE and NSE criteria (Fig. 7d). However, when optimizing on NSE (optNSE), the correlation between the values obtained for NSE and KGE is lower (Fig. 7c). The reasons for this will become much clearer later in this section, but briefly it is useful to keep in mind that optimization on KGE strongly controls the values that the α and β components can achieve, whereas optimization on NSE constrains these components only weakly.

Fig. 7 near here

342 The relative contributions of the criterion components to the overall model performance obtained
343 via optimization are shown in Fig. 8 (see Eq. 6 for optNSE and Eq. 11 for optKGE). The obtained
344 (optimized) model performance is dominated by the component representing r (dark grey), whereas
345 the other components representing the bias (light grey) and the variability (medium grey) of flows
346 have only small relative contributions. This applies for all 49 basins and for both optimization on
347 NSE (Fig. 8a) and KGE (Fig. 8b). However, a low relative contribution of a component to the final
348 value of the (optimized) model performance does not necessarily imply that the model performance
349 criterion is, in general, insensitive to this component. Instead, the relative contribution of a
350 component can be small because of (1) low 'weight' of the component in the equation for
351 calculating the overall model performance, and/or (2) the value of the component is close to its
352 optimal value. As a consequence of (2), the relative contribution of the components representing the
353 bias and the variability of flows can become large for non-optimal parameter sets.

354 To illustrate these considerations, Fig. 8c and Fig. 8d show the relative contribution of the criterion
355 components using random parameter sampling for a selected basin (Glan River). The sampled
356 points are arranged from left to right in order of decreasing performance for the selected criterion.
357 With decreasing overall model performance (either NSE or KGE) there is a general tendency for the
358 relative contribution of r to decrease and for the other two components to become much more
359 important. In some cases only the component representing the bias is dominant, whereas in other
360 cases only the component representing the variability of flows is dominant. This clearly indicates
361 that both NSE and KGE are sensitive to all three of the components. From a multi-objective point of
362 view this is definitely desirable, because it means that by calibrating on the overall model
363 performance we can substantially improve the components representing the bias and the variability
364 of flows. Here of course we should remember that in NSE the bias is normalized by the standard

365 deviation of the observed flows and that the 'optimal' α is equal to r. Hence, with NSE it is not
366 necessarily assured that from a hydrological point of view good values for α and β are obtained.

367 **Fig. 8 near here**

368 The cumulative distribution functions for the NSE, r, α, and β measures as obtained with optNSE
369 and optKGE in the calibration and evaluation periods are shown in Fig. 9. Looking first at the
370 results for the NSE criterion (Fig. 9a), we see that while the NSE obtained by optNSE is larger than
371 with optKGE, the difference is rather small. This indicates that by calibrating on KGE, we have
372 obtained only a slight deterioration in overall performance as measured by NSE. Further, although
373 there is a pronounced reduction in NSE from calibration to evaluation period, the reduction is
374 similar for both optNSE and optKGE.

375 However, the change in NSE tells us little that is diagnostically useful about the causes of this
376 'deterioration' in overall model performance. Of more interest, are the values obtained for the three
377 criterion components. The results for the calibration period are discussed first. Note that the
378 distribution of r is almost identical with either optNSE or optKGE (Fig. 9b, filled symbols),
379 indicating that both of the criteria have achieved similar hydrograph match in terms of shape and
380 timing. However, for the other two components, optKGE has achieved considerably better results.
381 Fig. 9c shows that there is a strong tendency for underestimation of α by optNSE (filled circle
382 symbols), due to which only 18 percent of the basins are within 10 percent of the ideal value at
383 unity, whereas for optKGE (filled triangle symbols) all of the basins are within 10 percent of the
384 ideal value. Similarly optKGE yields good results for β (Fig. 9d), with all of the basins having a
385 bias of less than 10 percent, while for optNSE 16 percent of the basins have a bias of greater than
386 10 percent. In general, optKGE results in a β value that is much closer to the ideal value at unity
387 than with optNSE. Thus, the use of optKGE has resulted in all of the basins having α and β close to

388 their ideal values of unity during calibration. This now explains why we get such a high correlation
389 between NSE and KGE in Fig. 7d; because both α and β are now almost constant across the basins
390 (here close to unity), the equations for KGE and NSE both become approximately linear functions
391 of r, and in fact we tend towards the relationship $NSE(\theta_{optKGE}) = 2*KGE(\theta_{optKGE})-1$.

392 Next we examine what happens for the evaluation period. In general, we see that the statistical
393 distributions of the three components have changed. The cumulative distribution function of r has
394 shifted to lower values in a consistent manner for both optNSE and optKGE (Fig. 9b), so that both
395 methods yield again very similar results for timing and shape. However, the optKGE calibrations
396 have retained a median value of α close to unity (the same as during calibration) while the overall
397 variability in the distribution has increased around the median value (Fig. 9c). This indicates that
398 the statistical tendency to provide good reproduction of flow variability *persists* into the evaluation
399 period, but there is an increase in the noise so that the distribution has become much wider. In
400 contrast, the optNSE results continue to show a systematic tendency to underestimate α (variability
401 of flows) during the evaluation period along with a considerable increase in random noise.
402 Similarly, the cumulative distribution function of β obtained by both methods remains centred close
403 to its calibration value while showing an increase in the variability (Fig. 9d). The small shift in the
404 median value may be caused by the fact that there is approximately 5 % less precipitation during the
405 evaluation period. Clearly, the KGE criterion has provided model calibrations that are statistically
406 more desirable during calibration while providing results that remain statistically more consistent on
407 the independent evaluation period.

408 **Fig. 9 near here**

409 An interesting observation is that in a few basins the paradoxical case occurs where *all three of the*
410 *criterion components improve with optKGE, but the value of NSE decreases* when compared to the
411 NSE obtained with optNSE (Table 2). The reason for this is the interplay between the terms α and r

412 in the NSE equation (illustrated nicely in Fig. 1). It is therefore actually (counter intuitively)
413 possible for both α and r to get closer to unity while NSE gets smaller. This is, of course, because
414 optimization on NSE seeks to make $\alpha = r$, and therefore '*punishes*' solutions for which α is close to
415 the ideal value of unity, while r will always be smaller than unity.

416 **Table 2 near here**

417 As discussed earlier, it is likely that optimization with NSE will yield results where α is close to r.
418 Fig. 10a shows a comparison between r and α obtained by the two optimization cases for all of the
419 basins. In general, when optimizing with NSE, the value of α is indeed very similar to r, which
420 means that the variability of flows is systematically underestimated (as shown above), and α
421 approaches the ideal value of unity in only one of the 49 basins. In contrast, when optimizing with
422 KGE, the value of α is close to the ideal value of unity for most of the basins.

423 Consequently, as expected from the theoretical discussion, systematically different results are
424 obtained by optNSE and optKGE for the slopes of the regression lines (Fig. 10b), where the cases
425 of regressing the simulated against the observed values (k_o, Eq. 14) and regressing the observed
426 against the simulated values (k_s, Eq. 13) are distinguished. In general, when using optNSE the value
427 of k_s is close to the ideal value at unity, but k_o is significantly smaller than one. In the case of
428 optKGE both k_s and k_o are smaller than one, but the underestimation is not as large as for k_o with
429 optNSE. Note (from Eq. 12) that the only way that we can have both k_s and k_o equal to one is for r
430 to be equal to unity, which would only happen if the model and data were perfect.

431 **Fig. 10 near here**

432 Finally, we report briefly on the optimal parameter values obtained using optNSE and optKGE.
433 Interestingly, even though the statistical properties of the streamflow hydrographs (as measured by
434 α and β) did change significantly (Fig. 9), for many basins the *parameter values* did not change by

large amounts (compared to the feasible parameter range) when moving from optNSE to optKGE (Fig. 11). The correlation between the parameter values of optNSE and optKGE is at least 0.80 for all six of the parameters, and for three of the parameters the correlation is larger than 0.90. For the parameter *K1* the values are slightly smaller with optKGE, which has the effect of higher peaks and quicker recession of surface flow. Also the parameter *K3* decreases with optKGE, which has the effect of a less dampened base flow response. Given the function of these two parameters in the model structure, a reduction in the parameter values has the effect of increasing the value of the α measure. In addition, we see an increase in the percolation parameter *K2*, which results in more surface flow and less base flow, with the overall effect of increasing the value of α.

The function of the parameters $S1_{max}$ and *Beta* in the model is mainly to control the partitioning of precipitation into runoff and evapotranspiration (thereby controlling the water balance), and as a consequence these parameters mainly affect the β measure. However, these parameters also affect the α measure and parameter interaction between $S1_{max}$ and *Beta* complicates the analysis. Given the function of these parameters in the model, the β measure should increase with a decrease in either $S1_{max}$ and/or *Beta*, but this is not obvious from Fig. 11, because a decrease in $S1_{max}$ can be compensated by an increase in *Beta*, and vice versa.

For the parameter $S2_{crit}$ no clear tendency of change is visible. Here it should be mentioned that there was no change in the parameter values in sixteen of the basins for which the parameter values were at their lower bounds (4 basins) and upper bounds (12 basins), respectively. Note that these 16 points also contribute to the rather high correlation observed.

Fig. 11 near here

On a visual, albeit subjective, basis a comparison of the parameter sets obtained by optNSE and optKGE reveals that in many of the basins the two parameter sets are almost indistinguishable, but

458 nevertheless the criterion components have changed. As an example, Fig. 12a displays a
459 comparison of the parameters obtained by optNSE and optKGE for the Glan River. Apparently the
460 parameter values are quite similar, although the α measure and to a lesser extent the β measure have
461 both improved when using optKGE (see Table 2). For many basins, the difference in each of the
462 parameters was found to be only a small percentage of the overall feasible range (Fig. 12b); in 14 of
463 the 49 basins, all of the six parameters have changed by less than 10%, and in only a few of the
464 basins did two or more parameters change by a significant amount. For the latter, the changes may
465 also (at least in part) be a consequence of parameter interactions; for example, there is a clear
466 tendency for *K2* and $S2_{crit}$ to increase/decrease simultaneously, and this fact must, of course, also be
467 considered when interpreting the scatter plots in Fig. 11.

468 **Fig. 12 near here**

469 4 Discussion

470 A decomposition of the NSE criterion shows that this measure of overall model performance can be
471 represented in terms of three components, which measure the linear correlation, the bias and the
472 variability of flow. By simple theoretical considerations, we can show that problems can arise in
473 model calibrations that seek to optimize the value of NSE (or its related MSE). First, because the
474 bias is normalized by the standard deviation of the observed flows, the relative importance of the
475 bias term will vary across basins (and also across years), and for cases where the variability in the
476 observed flows is high, the bias will have a low 'weight' in the computation of NSE. Second, there
477 will be a tendency for the variability in the flows to be systematically underestimated, so that the
478 ratio of the simulated and observed standard deviations of flows will tend to be equal to the
479 correlation coefficient. As a consequence, the slope of the regression line (when regressing
480 simulated against observed values) will be smaller than one, so that runoff peaks will tend to be

481 systematically underestimated. This finding may seem to contradict the general notion that
482 optimization on NSE will improve simulation of runoff peaks. In fact NSE is generally found to be
483 highly sensitive to the large runoff values, because of the (typically) larger model and data *errors*
484 involved in the matching of such events, and this fact is separate from the general (theoretical and
485 practical) tendency to underestimate the runoff peaks. Of course, when it is of interest to regress the
486 observed against the simulated values then an optimization on NSE can yield desirable results, since
487 in such a case the optimal slope of the regression line for maximizing NSE is equal to unity.

488 These theoretical considerations were all supported by the results of the modelling experiment. Of
489 course, in such an experiment, not all solutions within the theoretical criteria space are possible
490 because of constraints regarding the model structure, parameter ranges, and available data.
491 However, it was found that the simple model was capable of achieving good solutions for the bias
492 and the variability of flows with only slight decreases in the correlation coefficient. The
493 optimization task therefore becomes one of specifying the objective function in such a way that it is
494 capable of achieving such a solution as an optimal solution (i.e. simultaneously good solutions for
495 bias, flow variability and correlation). Apparently, this was not the case with NSE, and we therefore
496 formulated an alternative criterion (KGE) that is based on an equal weighting of the three
497 components (correlation, bias, and variability measures). Of course the correlation will not, in
498 general, reach its ideal value of unity, but an optimization on KGE resulted in the other two
499 components being indeed close to their ideal values. Thus, the use of KGE instead of NSE for
500 model calibration improved the bias and the variability measure considerably while only slightly
501 decreasing the correlation.

502 The simulation results were also examined for an independent evaluation period. In general, the
503 overall model performance and the individual components deteriorated in a statistical sense. It is at
504 least partially likely that this is due to the rather short lengths of the calibration and evaluation

505 periods used in this study (five and three years, respectively). Further, it should be noted that this
506 study has not accounted for either the uncertainty in the parameter values or the uncertainty in the
507 computed statistics, which would require a more rigorous Bayesian approach. Nevertheless, the
508 results clearly show that optimization using NSE tends to underestimate the variability of flows on
509 the calibration period, and that this behaviour tends to persist into the evaluation period. Further, the
510 bias in the calibration period is well constrained with KGE, but not with NSE, whereas in the
511 evaluation period (with overall poorer bias) the results with NSE are only slightly inferior to KGE.

512 An interesting result is that for many basins the optimal parameter values changed by only small
513 amounts (relative to the feasible range) when using KGE instead of NSE. In the KGE optimization
514 there was a tendency to decrease the recession parameters of surface flow and base flow to simulate
515 a flashier hydrograph, and thereby improve the value of the variability measure. Because of
516 parameter interactions there was no clear tendency of a change in the parameters for the bias
517 measure. In general, this suggests that the values of multiple criteria can be improved by making
518 only small changes in the parameter values. This emphasizes the importance of the relative
519 sensitivity of the criterion components to changes in the parameter values. On the one hand, this is a
520 desirable effect during calibration, because we want to have measures that are actually sensitive to
521 the parameter values, thereby theoretically increasing parameter identifiability. On the other hand,
522 this raises important questions for parameter regionalization, because even a small 'error' in a
523 parameter value could result in poor values of individual measures, thereby causing poor overall
524 model performance.

525 The attempt to explain the relationships between changes in the parameters and values of the
526 criterion components relates to the idea of diagnostic model evaluation, as proposed by Gupta et al.
527 (2008) and tested by Yilmaz et al. (2008) and Herbst et al. (2009). The idea behind diagnostic
528 model evaluation is to move beyond aggregate measures of model performance that are primarily

529 statistical in meaning, towards the use of (multiple) measures and signature plots that are selected
530 for their ability to provide hydrological interpretation. Such an approach should improve our ability
531 to diagnose the causes of a problem and to make corrections at the appropriate level (i.e. model
532 structure or parameters). The theoretical development presented in this paper, shows one simple,
533 statistically founded approach to the development of a strategy for diagnostic evaluation and
534 calibration of a model. Clearly, the measures used in this study have some diagnostic value. The
535 bias and variability measures represent differences in matching of the means and the standard
536 deviations (the first two moments) of the probability distributions of the quantities being compared.
537 Their appearance in NSE and MSE indicates that these performance criteria give importance to
538 matching these two long-term statistics of the data. From a hydrological perspective, these statistics
539 relate to the properties of the flow duration curve, in which issues of timing and shape of the
540 dynamical characteristics of flow are largely ignored. These statistics will therefore be mainly
541 controlled by aspects of model structure and values of the parameters that determine the general
542 partitioning of precipitation into runoff, evapotranspiration and storage (i.e. overall water balance)
543 and, further, the general partitioning of runoff into fast and slow flow components (e.g. see Yilmaz
544 et al 2008). Meanwhile, all other differences between the statistical properties of the observed and
545 simulated flows such as timing of the peaks, and shapes of the rising limbs and the recessions of the
546 hydrograph (i.e. autocorrelation structures), are lumped into the (linear) correlation coefficient as an
547 aggregate measure. A logical next step would be to further decompose the correlation coefficient
548 into diagnostic components that represent different aspects of flow timing and shape (e.g.
549 autocorrelation structure). Further, a distinction between different states (modes) of the hydrological
550 response - such as driven and non-driven (see e.g. Boyle et al. 2000) – may also prove to be
551 sensible. Such considerations are left for future work.

552 Before entering into our concluding remarks, we should point out that the primary purpose of this
553 study was not to design an improved measure of model performance, but to show clearly that there
554 are systematic problems inherent with any optimization that is based on mean squared errors (such
555 as NSE). The alternative criterion KGE was simply used for illustration purposes. An optimization
556 on KGE is equivalent to selecting a point from the three-dimensional Pareto front with the minimal
557 distance from the ideal point. Many different alternative criteria would also be sensible, but
558 ultimately it has to be understood that each single measure of model performance has its own
559 peculiarities and trade-offs between components. In the case of KGE probably the most problematic
560 characteristic is that the slope of the regression lines will tend to be smaller than one, albeit not as
561 strongly as with NSE (when regressing simulated against observed values). Because of the simple
562 design of the KGE criterion it is straightforward to understand the trade-offs between the
563 correlation, the bias and the variability measure. These trade-offs are more complex in the case of
564 NSE.

565 If single measures of model performance are used we deem it to be imperative to clearly know the
566 limitations of the selected criterion. It then will depend upon the type of application whether these
567 limitations are of concern or not. The decomposition presented here highlights the fact that identical
568 values of the NSE criterion are not necessarily indistinguishable - as is commonly (and erroneously)
569 assumed in the literature in arguments relating to equifinality (Beven and Binley 1992, Beven and
570 Freer 2001) - since the criterion components may be quite different. Thus, when evaluating or
571 reporting results based on calibration with NSE, information about the correlation, bias, and
572 variability of flows should also be given (interestingly, this was already proposed by Legates and
573 McCabe (1999), although they did not discuss the interrelation between NSE and its three
574 components). Ultimately the decision to accept or reject a model must be made by an expert
575 hydrologist, where such a decision is best based in a multiple-criteria framework. To this end, an

576 analysis of the components that constitute the overall model performance can significantly enhance
577 our understanding of model behaviour and provide insights helpful for diagnosing differences
578 between models, basins and time periods within a hydrological context.

579 5 Summary and conclusions

580 In this study a decomposition of the widely used Nash-Sutcliffe efficiency (NSE) was applied to
581 analyse the different components that constitute NSE (and hence MSE). We present theoretical
582 considerations that serve to highlight problems associated with the NSE criterion. The results of a
583 case study, where a simple precipitation-runoff model was applied in several basins, support the
584 theoretical findings. For comparison we show how an alternative measure of model performance
585 (KGE) can overcome the problems associated with NSE.

586 In summary, the main conclusions of this study are:

587 • The mean squared error and its related NSE criterion consists of three components,
588 representing the correlation, the bias and a measure of variability. The decomposition
589 shows that in order to maximize NSE the variability has to be underestimated. Further, the
590 bias is scaled by the standard deviation in the observed values, which complicates a
591 comparison between basins.

592 • Given that NSE consists of three components, an alternative model performance criterion
593 KGE is easily formulated by computing the Euclidian distance of the three components
594 from the ideal point, which is equivalent to selecting a point from the three-dimensional
595 Pareto front. Such an alternative criterion avoids the problems associated with NSE (but
596 also introduces new problems).

- The slopes of the regression lines are directly related with the three components. NSE is suitable if the interest is in regressing the observed against the simulated values, but less suitable for regressing the simulated against the observed values. This means that if NSE is used in optimization, then runoff peaks will tend to be underestimated. The same applies for KGE, but the underestimation will not be as severe.

- After optimization, the component representing the linear correlation dominates the model performance criterion for both NSE and KGE. For non-optimal parameters sets any of the three components can be dominant in NSE or KGE.

- Even with a simple precipitation-runoff model it is possible to obtain runoff simulations where the mean and variability of flows are matched well, and the linear correlation is still high. However, this applies only for optimization with KGE, since NSE does not consider such a solution as 'good'.

- The optimal parameter values may, in practice, only change by small amounts when using KGE instead of NSE as the objective function for optimization (as in our example). This emphasizes the importance of considering the sensitivity of the three components to perturbations in the parameter values.

This study reinforces the argument that model calibration is a multi-objective problem (Gupta et al 1998), and shows that a decomposition of the calibration criterion into components can help to greatly enhance our understanding of the overall model performance (and, by extension, the differences in model performance between model structures, basins and time periods). To compute these components is a straightforward task and should be included in any evaluation of model simulations. Ultimately, such an approach may help in the design of diagnostically powerful evaluation strategies that properly support the identification of hydrologically consistent models.

6 Acknowledgements

Funding was provided for Harald Kling as an Erwin Schrödinger Scholarship (grant number J2719-N10) by FWF, Vienna, Austria. Partial support for Hoshin Gupta and Koray Yilmaz was provided by the Hydrology Laboratory of the National Weather Service (Grant NA04NWS4620012) and by SAHRA (Center for Sustainability of semi-Arid Hydrology and Riparian Areas) under the STC program of the National Science Foundation (agreement EAR 9876800). Partial support for Koray Yilmaz was also provided by NASA's Applied Sciences Program (Stephen Ambrose) and PMM (Ramesh Kakar) of NASA Headquarters.

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725

726 Table 1: Parameters of the model. Parameters in brackets were not calibrated.

parameter	units	feasible range	description
$S1_{max}$	mm	50 - 700	soil storage capacity
Beta	/	0.1 - 25	exponent for computing runoff generation
($S1_{crit}$)	/	(0.6)	critical soil moisture for actual evapotranspiration
K1	h	10 - 500	recession coefficient for surface flow
K2	h	10 - 1000	recession coefficient for percolation
$S2_{crit}$	mm	0 - 15	outlet height for surface flow
K3	h	500 - 10000	recession coefficient for base flow
(*K4*)	h	(0 - 10)	recession coefficient for distributed routing

727

728 Table 2: 'Paradoxical' examples for NSE and components in three basins (results for the calibration
729 period). All three components (r, α, β) improve but the overall model performance measured by
730 NSE decreases with the parameter set obtained by optKGE.

basin	method	NSE [/]	KGE [/]	r [/]	α [/]	β [/]
Zaya River	optNSE	0.484	0.685	0.714	0.871	1.019
	optKGE	0.452	0.732	0.733	1.026	1.001
Pitten River	optNSE	0.742	0.828	0.863	0.899	1.028
	optKGE	0.730	0.865	0.866	1.004	1.016
Glan River	optNSE	0.786	0.855	0.888	0.912	1.028
	optKGE	0.776	0.888	0.889	1.002	1.007

731

732

733 Figure captions:

734 Fig. 1: Relationship of NSE with α and r (β_n is assumed to be zero): (a) theoretical relationship, (b)
735 illustrative example obtained by random parameter sampling with a hydrological model (Leaf
736 River, Mississippi, USA, 1924 km^2, 11 years daily data, HyMod model; only those points where
737 $\beta_n^2 < 0.01$ are displayed). Contour lines indicate values for NSE. See colour version of this figure
738 online.

739 Fig. 2: Example for three-dimensional Pareto front of r, α and β. *ED* is the Euclidian distance
740 between the 'optimal' point and the ideal point, where all three measures are 1.0. Glan River,
741 Austria, 432 km^2, 5 years daily data, HBV model variant, random parameter sampling.

742 Fig. 3: Typical scatter plots depicting simulated and observed runoff ($r = 0.86$ and $\alpha = 0.90$) and
743 fitted regression lines: (a) regression against simulated runoff ($k_s = 0.96$) and (b) regression against
744 observed runoff ($k_o = 0.77$). Pitten River, Austria, 277 km^2, 5 years daily data, HBV model variant,
745 parameters optimized on NSE. Note, that in (a) and (b) the identical data points are plotted, but the
746 axes are flipped.

747 Fig. 4: Map showing locations of the 49 Austrian basins used in this study. Also depicted are the 49
748 gauges and 222 precipitation stations.

749 Fig. 5: Relationship between index of evaporation and index of wetness for the 49 Austrian basins.
750 The index of wetness is computed as the ratio between precipitation (P) and potential
751 evapotranspiration (ETp). The index of evaporation is computed as the ratio between actual
752 evapotranspiration (ETa) and ETp. Data represent long-term means from the period 1961 to 1990
753 and are taken from Hydrological Atlas of Austria (BMLFUW 2007).

754 Fig. 6: Conceptual model structure (the snow module is not shown). Parameters in brackets are not
755 calibrated.

Fig. 7: Scatter plots of overall model performance: cal = calibration period, eval = evaluation period. Note that in (a) two points are located outside the plotting range because of negative NSE values in the evaluation period.

Fig. 8: Stacked area plots showing the relative contribution of the components for NSE and KGE in the calibration period: (a) optNSE in 49 basins, (b) optKGE in 49 basins, (c) and (d) random parameter sampling in the Glan River basin.

Fig. 9: Cumulative distribution functions for NSE, r, α, and β as obtained with optNSE and optKGE in the calibration and evaluation periods.

Fig. 10: Relationship between (a) r and α and (b) the slope of the regression lines k_s and k_o.

Fig. 11: Scatter plots of optimal parameters obtained by optNSE and optKGE. Parameter values are normalized by the feasible parameter range (Table 1); the parameters *Beta*, *K1*, *K2* and *K3* are log-transformed before normalization.

Fig. 12: Comparison of the parameter sets obtained by optNSE and optKGE: (a) normalized parameter values of θ_{optNSE} and θ_{optKGE} in the Glan River basin, (b) difference in the normalized parameter values (computed as θ_{optKGE}-θ_{optNSE}), displayed for all basins.

Figure 1

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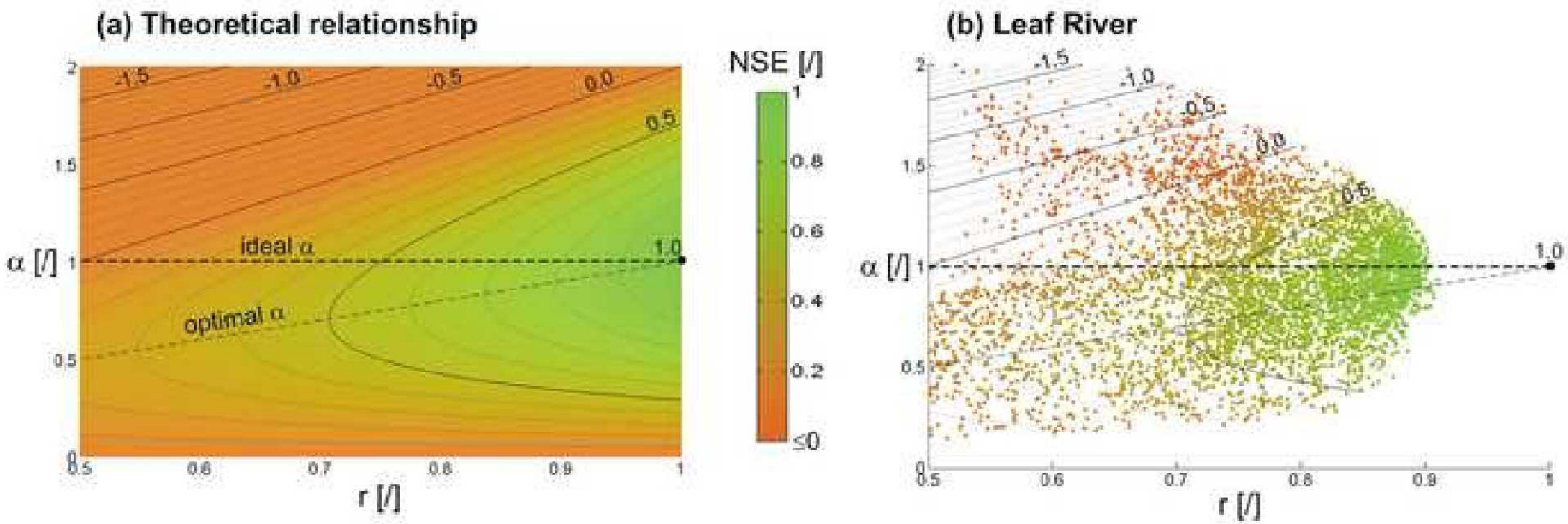

Figure 2

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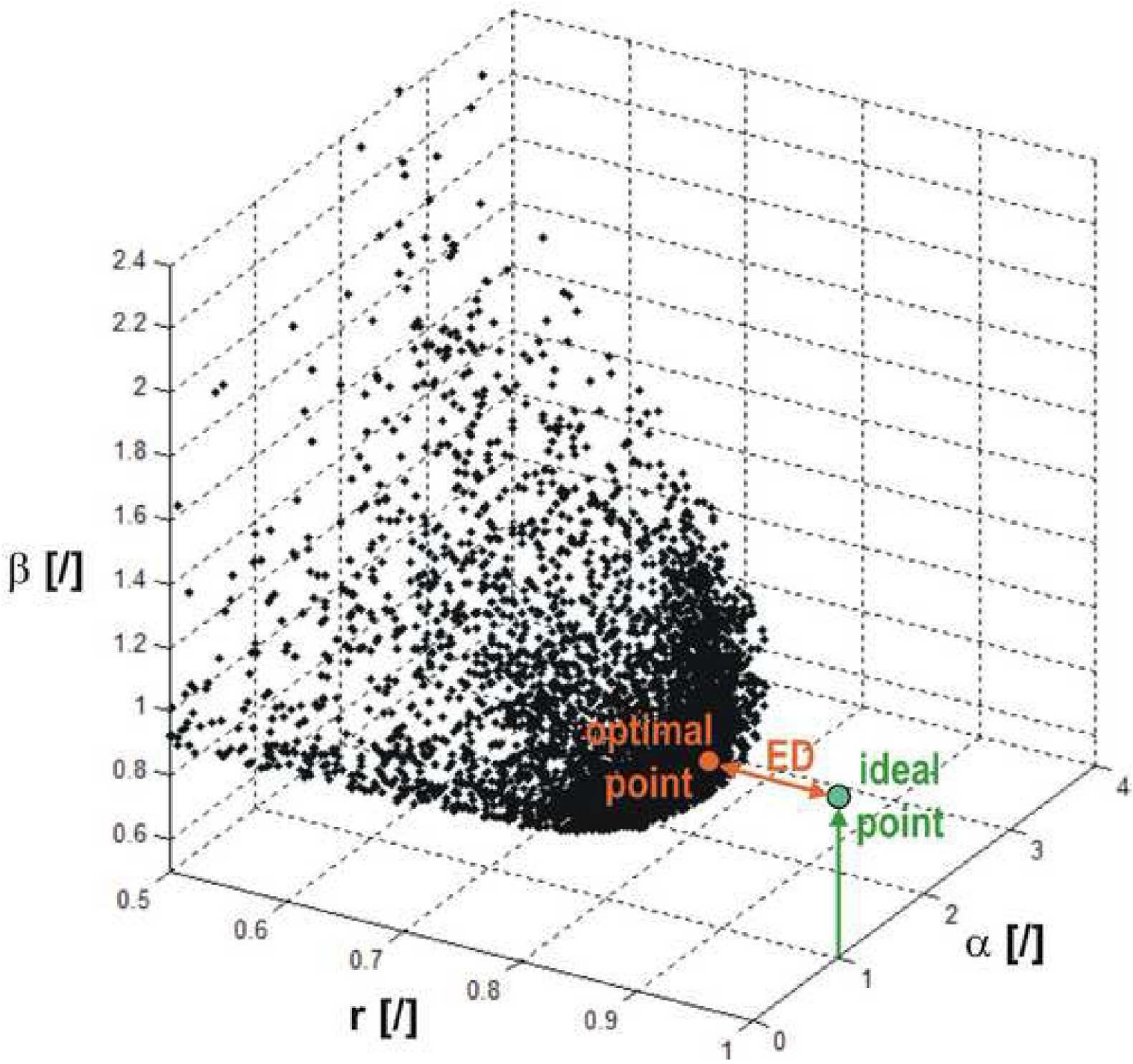

Figure 3

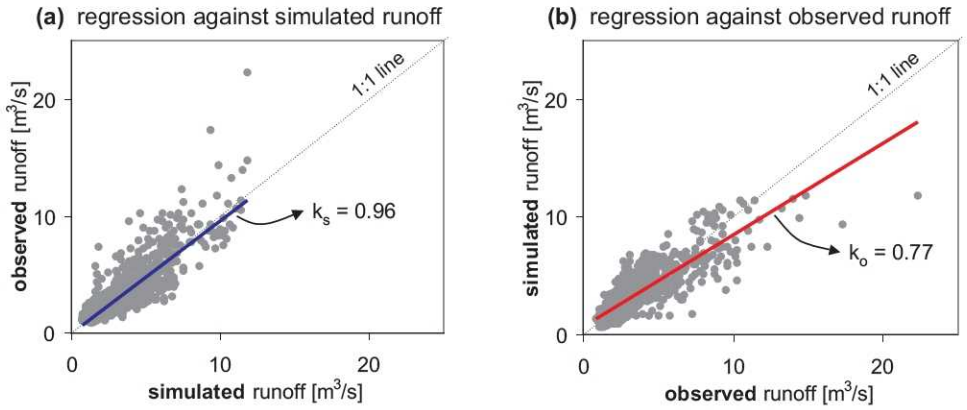
(a) regression against simulated runoff
1:1 line
k_s = 0.96
observed runoff [m³/s]
simulated runoff [m³/s]
0
10
20
(b) regression against observed runoff
1:1 line
k_o = 0.77
simulated runoff [m³/s]
observed runoff [m³/s]
0
10
20

Figure 4

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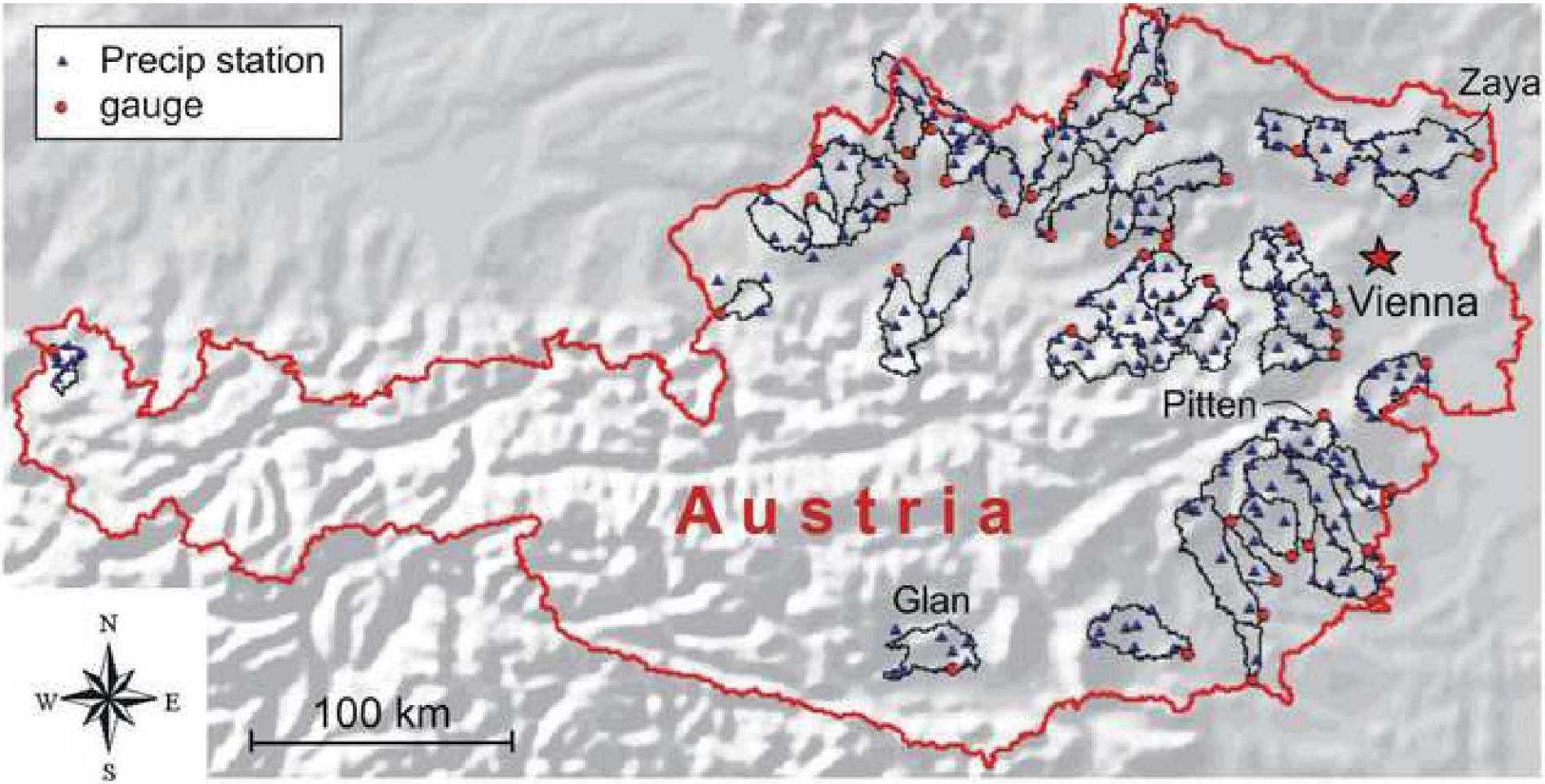


Figure 5

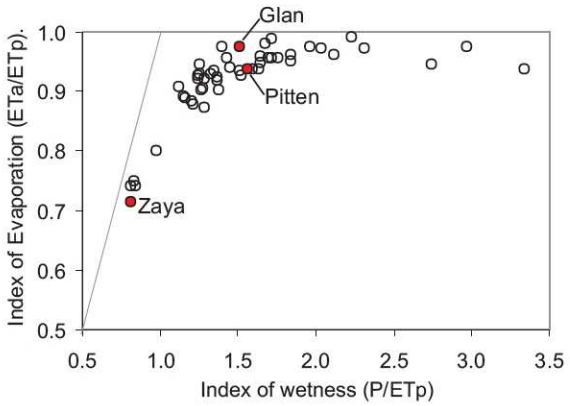

Glan
Pitten
Zaya
Index of Evaporation (ETa/ETp).
Index of wetness (P/ETp)
1.0
0.9
0.8
0.7
0.6
0.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5

Figure 6

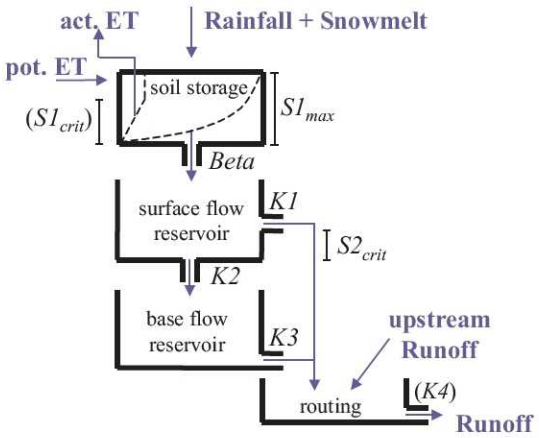

act. ET
Rainfall + Snowmelt
pot. ET
soil storage
$(S1_{crit})$
$S1_{max}$
Beta
surface flow reservoir
K1
$S2_{crit}$
K2
base flow reservoir
K3
upstream Runoff
routing
(K4)
Runoff

Figure 7

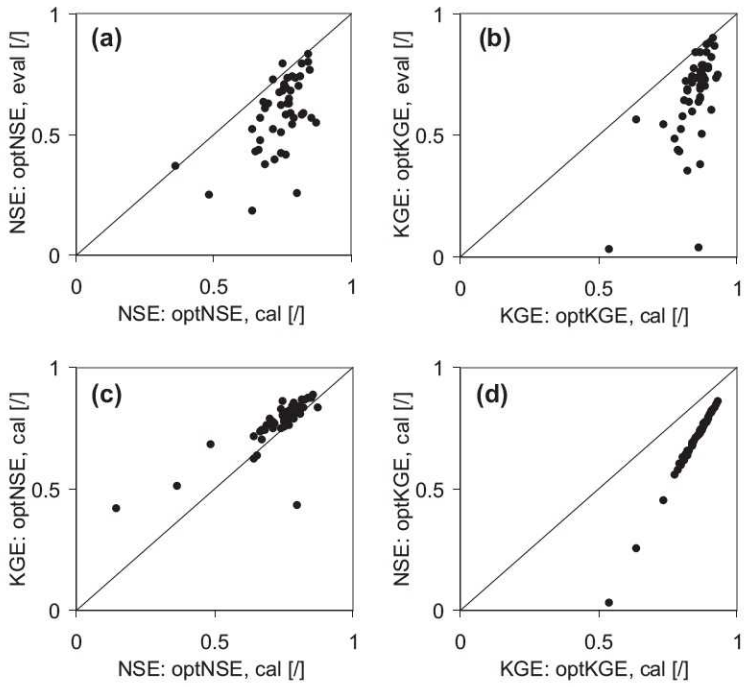

(a)
NSE: optNSE, eval [/]
NSE: optNSE, cal [/]
(b)
KGE: optKGE, eval [/]
KGE: optKGE, cal [/]
(c)
KGE: optNSE, cal [/]
NSE: optNSE, cal [/]
(d)
NSE: optKGE, cal [/]
KGE: optKGE, cal [/]
0
0.5
1

Figure 8

Click here to download high resolution image

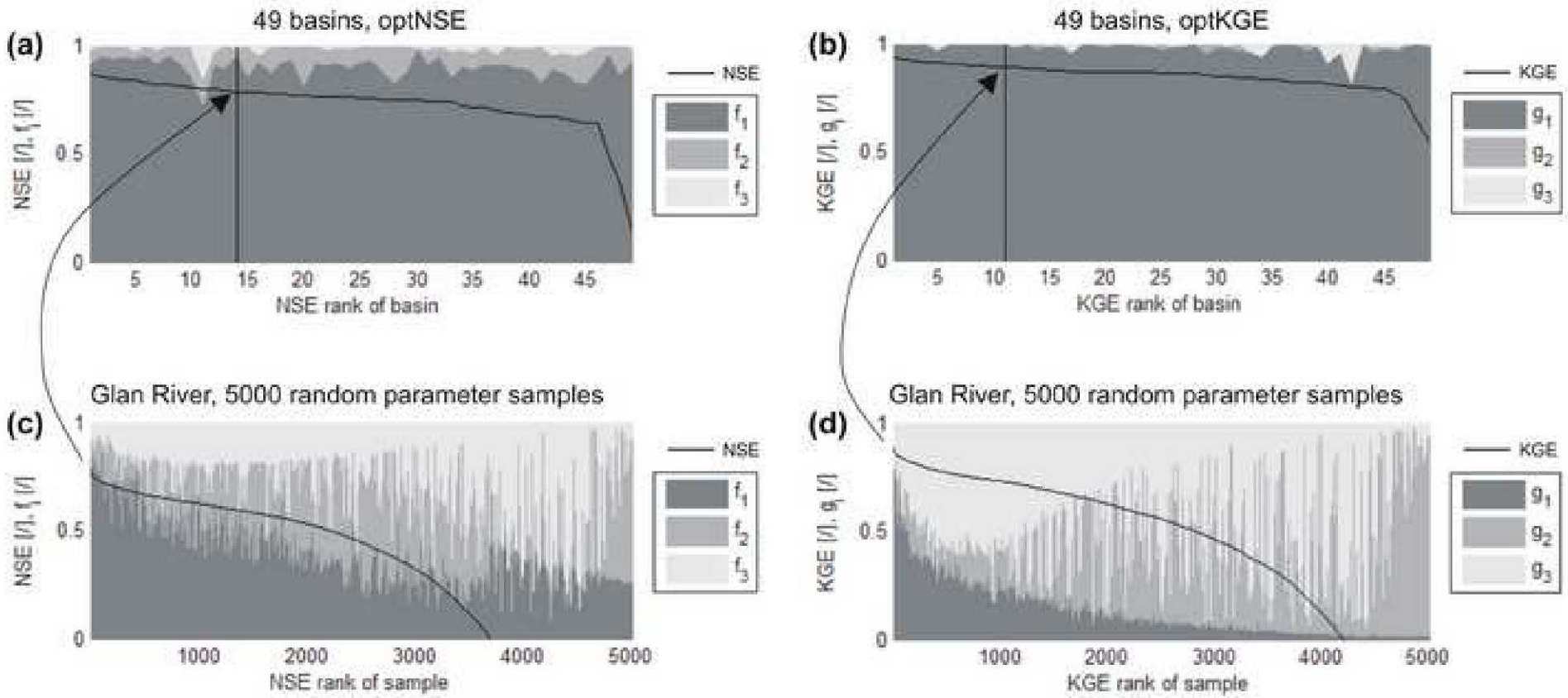


Figure 9

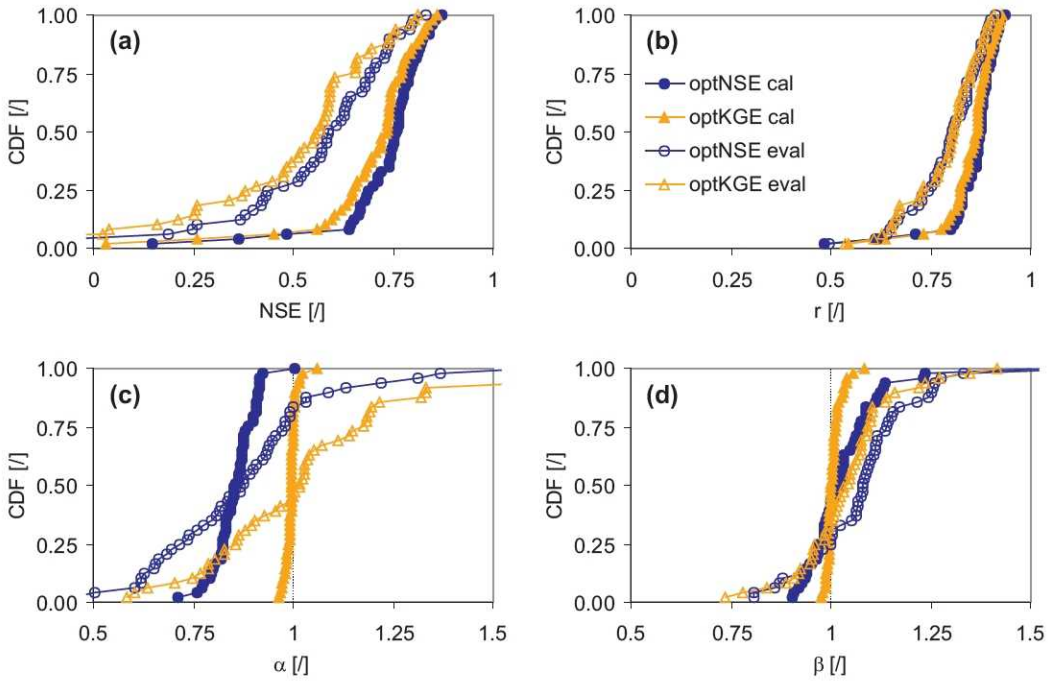

(a)
(b)
(c)
(d)
CDF [/]
NSE [/]
r [/]
α [/]
β [/]
optNSE cal
optKGE cal
optNSE eval
optKGE eval
1.00
0.75
0.50
0.25
0.00
0
0.25
0.5
0.75
1
1.25
1.5

Figure 10

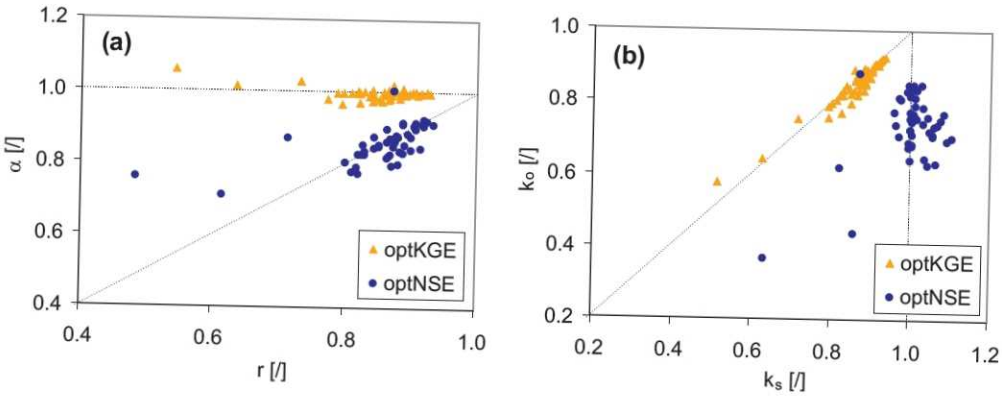


Figure 11

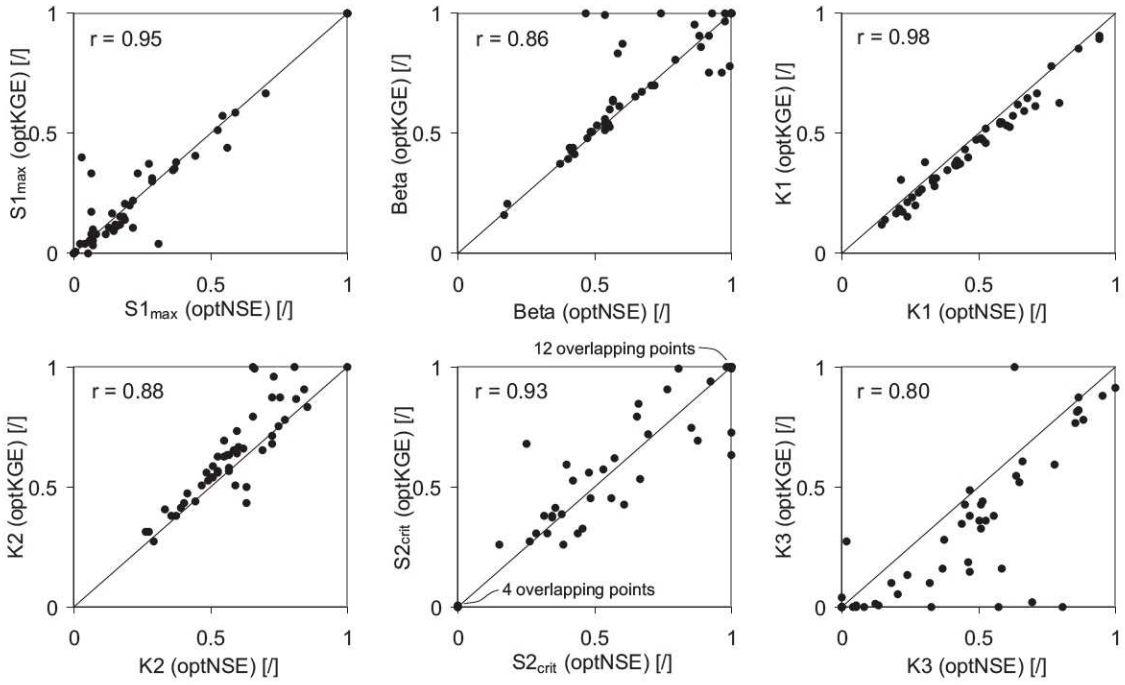
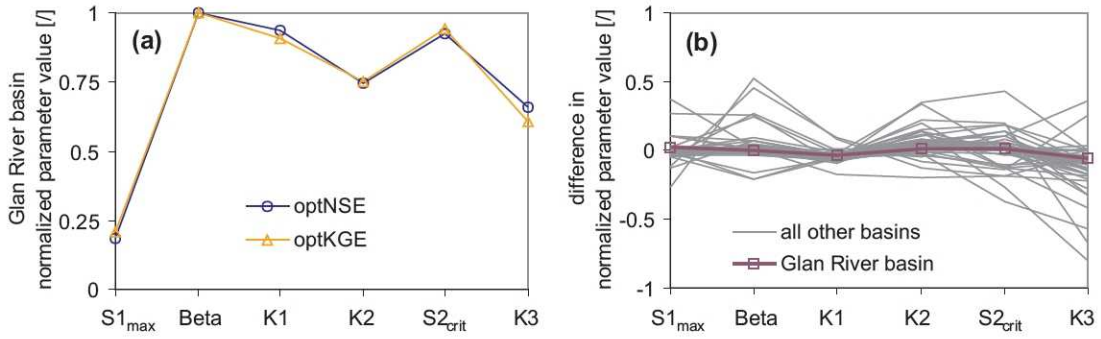

r = 0.95
S1max (optKGE) [/]
S1max (optNSE) [/]
r = 0.86
Beta (optKGE) [/]
Beta (optNSE) [/]
r = 0.98
K1 (optKGE) [/]
K1 (optNSE) [/]
r = 0.88
K2 (optKGE) [/]
K2 (optNSE) [/]
12 overlapping points
r = 0.93
4 overlapping points
S2crit (optKGE) [/]
S2crit (optNSE) [/]
r = 0.80
K3 (optKGE) [/]
K3 (optNSE) [/]
0
0.5
1

Figure 12


(a)
Glan River basin
normalized parameter value [/]
1
0.75
0.5
0.25
0
optNSE
optKGE
S1max
Beta
K1
K2
S2crit
K3
(b)
difference in
normalized parameter value [/]
1
0.5
0
-0.5
-1
all other basins
Glan River basin
S1max
Beta
K1
K2
S2crit
K3

Gupta, H. V. Kling, H., Yilmaz, K. K., Martinez-Baquero, G.F. 2009: Decomposition of the Mean Squared Error & NSE Performance Criteria: Implications for Improving Hydrological Modelling, Journal of Hydrology.

SUMMARY

The mean squared error (MSE) and the related normalization, the Nash-Sutcliffe efficiency (NSE), are the two criteria most widely used for calibration and evaluation of hydrological models with observed data. Here, we present a diagnostically interesting decomposition of NSE (and hence MSE), which facilitates analysis of the relative importance of its different components in the context of hydrological modelling, and show how model calibration problems can arise due to interactions among these components. The analysis is illustrated by calibrating a simple conceptual precipitation-runoff model to daily data for a number of Austrian basins having a broad range of hydro-meteorological characteristics. Evaluation of the results clearly demonstrates the problems that can be associated with any calibration based on the NSE (or MSE) criterion. While we propose and test an alternative criterion that can help to reduce model calibration problems, the primary purpose of this study is not to present an improved measure of model performance. Instead, we seek to show that there are systematic problems inherent with any optimization based on formulations related to the MSE. The analysis and results have implications to the manner in which we calibrate and evaluate environmental models; we discuss these and suggest possible ways forward that may move us towards an improved and diagnostically meaningful approach to model performance evaluation and identification.