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ANALYSIS OF ROCKET, RAM-JET, AND TURBOJET ENGINES FOR SUPERSONIC PROPULSION OF LONG-RANGE MISSILES

II - ROCKET MISSILE PERFORMANCE

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RESEARCH MEMORANDUM

ANALYSIS OF ROCKET, RAM-JET, AND TURBOJET ENGINES FOR SUPERSONIC PROPULSION OF LONG-RANGE MISSILES

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SUMMARY

The theoretical performance of a two-stage ballistic rocket missile having a centerbody and two parallel boosters was investigated for JP4-oxygen and ammonia-fluorine propellants. Both power-plant and missile parameters were optimized to give minimum cost on the basis of the analysis for a range of 5500 nautical miles. After optimum values were found, each parameter was varied independently to determine its effect on performance of the missile.

The missile using the ammonia-fluorine propellant weighs about one-half as much as a missile using JP4-oxygen. Based on an expected unit cost of fluorine in quantity production, the ammonia-fluorine missile has a substantially lower relative cost than a JP4-oxygen missile. Optimum chamber pressures for both propellant systems and for both the centerbody and boosters were between 450 and 600 pounds per square inch. High design altitudes for the exhaust nozzle are desirable for both the centerbody and boosters. For the centerbody, the design altitude should be between 45,000 and 60,000 feet, with the value for ammonia-fluorine lower than that for JP4-oxygen. For the boosters, the design altitude should be 20,000 to 30,000 feet, with the value for the ammonia-fluorine missile higher.

INTRODUCTION

The Complete Study

The role of guided missiles in the nation's weapons system has received much attention in recent years. Latest advances in research and development of engines, aerodynamics, and guidance make practicable the utilization of these missiles for delivery of a warhead at supersonic speeds to a target thousands of miles distant. As an aid to the solution of development and design problems, it is the purpose of this series of reports to study the potentialities of various engines suitable for supersonic propulsion of long-range missiles and to determine those characteristics which result in the best over-all performance of the engine and missile combination. Because of close interrelation of the performance of the engine and airframe, the variations of airframe performance were also investigated at the same time that engine parameters were studied. In order to keep primary emphasis on the characteristics of the engine proper, the material on engine performance is separated from that which considers the over-all missile system. The rocket-engine performance is presented in reference 1, the rocket missile performance in this report, ~~and the ram-jet engine performance in reference 2~~.

Continuing research indicates many improvements that are possible in some of the engine components. The performance of components selected for the engines in this analysis has either been demonstrated in the laboratory or appears, from available data, to be certain of attainment within a reasonable time. Similarly, advanced features of airframe design that are believed possible to develop in a comparable time were also selected.

The principal mission to which attention has been directed is that of a long-range stratigic bombardment missile carrying a 7000-pound pay load a distance of 5500 nautical miles. The configurations studied are all limited to simple two-stage designs consisting of a rocket booster used only during the initial phase of flight and a second stage that flies the remaining distance under its own power. The rocket-propelled missiles are considered to travel along a ballistic trajectory, even though there are serious problems of re-entry into the atmosphere. Although glide and even skip rockets have frequently been proposed for this application, the many problems and uncertainties associated with the aerodynamics of these air-borne types preclude them from the present study.

Evaluating the engine performance, of course, necessitates the use of a realistic yardstick or figure of merit for measuring the performance of the engine-missile combination. The frequently used criterion, gross weight, loses its validity when there are large variations in relative weights of components of different value, such as fuel, engine, structure, and so forth. Accordingly, a criterion of relative cost is used. The relative cost is obtained by assigning a value per unit weight to each of the various components and summing the total component values so obtained. No claim is made for the accuracy of this relative cost, but such a criterion should more nearly provide a figure of merit than does gross weight for long-range missiles that can complete only one mission. However, the resulting gross weight based on minimum cost is also presented. This weight is higher than would be obtained by optimizing the missile for minimum weight.

Rocket Missile Performance

Design studies have been carried out for rocket missiles by several organizations. An analysis of a long-range bomber by the Rand Corporation is reported in reference 3, and more recently considerable work has been done by Consolidated Vultee Aircraft Corporation (ref. 4). Both of these analyses conclude that a long-range rocket-powered ballistic missile can be developed within a reasonable time.

The velocities and altitudes attained near the launching site are sufficient to carry the long-range missiles considered herein along ballistic trajectories to the target without further power. Guidance is assumed to be accomplished from ground-based equipment before the missiles travel more than a few hundred miles; the remainder of the flight is not guided. Most of the flight occurs above the atmosphere. Special design is necessary to permit the warhead to re-enter the atmosphere without destructive decelerating forces or heating.

A parallel three-body configuration was arbitrarily selected for this analysis. The centerbody comprises the second stage of the missile, which is raised to a high altitude and velocity with the aid of the outside bodies, or boosters. During the first-stage flight, the motors in all three bodies draw propellants from the booster tanks. Thus, the second stage begins its flight with full propellant tanks. Drag is included for the part of the boost where it is significant, and cooling is provided for the re-entry of the warhead cone into the atmosphere. Some improvement in range could probably be obtained by adding wings to the boosters and applying lift during the first-stage flight to reduce the gravity losses; furthermore, the addition of wings to the boosters might also permit their recovery and reuse, with a resultant saving in cost per missile. However, the problem of optimizing such a flight plan is beyond the scope of this report.

The purpose of the present analysis is to determine what research is desirable on the power-plant components and to determine the effect of possible improvements on the missile performance. The calculations are presented for two propellant systems, ammonia-fluorine and JP4-oxygen, to afford a comparison of the relative cost of high-energy high-cost propellants with a propellant combination now under extensive development.

Seven independent parameters for the power plant and missile were optimized to give least cost on the basis of the analysis for the specified range. After the optimum values of these parameters were determined, each parameter was varied independently to determine its effect on over-all performance. The seven parameters selected were the chamber pressures and design altitudes of the centerbody and booster engines, the ratios of propellant flow rate to gross weight for the centerbody and boosters, and the ratio of booster diameter to centerbody diameter. The design altitude is that at which the exit pressure of the rocket nozzle equals ambient pressure. The ratio of booster diameter to centerbody diameter was used to determine the weight ratio of the boosters to the centerbody.

The fuel-oxidant ratios for both propellant systems were arbitrarily set at approximately the values giving maximum specific impulse. The cost of the missiles could undoubtedly be reduced somewhat by optimization of this parameter, especially for the ammonia-fluorine missile.

ANALYSIS

Assumptions

The basic assumptions required for the design and flight of the rocket missile are given in this section. Symbols are defined in appendix A, detailed assumptions and weight equations for the missile design are given in appendix B, and equations for calculating the flight trajectory and range are given in appendix C.

Configuration. - Each of the missiles, as sketched in figure 1, consists of three bodies: the centerbody, which carries the cone in which the pay load is housed, and the two boosters symmetrically mounted on either side. Because drag is not a major consideration in a ballistic missile, the aerodynamic characteristics of this arrangement are not considered critical.

Each body has a single rocket engine and pumping plant. During the first-stage flight, the centerbody engine uses propellants from the booster tanks, so that when the boosters are released, the second stage starts with full propellant tanks. The length-diameter ratio of the cylindrical part of the bodies is 5 and that of the nose cones is 4, except for the warhead cone, which is described later. The ratio of booster to centerbody diameter is one of the variables of the analysis.

The propellants and tanks, power plant, and exhaust nozzle were assumed to fill 81 percent of the volume of the missiles, exclusive of that part of the booster nose cones less than 24 inches in diameter. The remaining space is available for structure, piping, and miscellaneous equipment. This filled-volume relation corresponds approximately to that in the V-2 configuration.

Trajectory. - The missiles were assumed to fly a no-lift ballistic trajectory. To eliminate dependence of the analysis on the direction of the flight and latitude of the launching site, the rotation of the earth was neglected. A typical trajectory is shown in figure 2. The thrust of the engines was assumed to be in the direction of flight velocity at all times. To obtain the best possible trajectory, subject to these conditions, the launching angle was varied to give maximum range for each design.

Power plant. - The power plant consists of three turbopump-fed, liquid-propellant rockets, one in each body. They were assumed to operate with constant propellant flow rate during flight in each engine. The centerbody engine was assumed to be flexibly mounted to provide directional control of the missile during the powered flight, but the booster motors were considered rigidly mounted.

The estimated effective specific impulse and power-plant weights are given in reference 1. Separation of the flow in the nozzle was assumed to occur at a ratio of exit-nozzle separation pressure to ambient pressure p_s/p_a of 0.5. A density of 25 pounds per cubic foot was assumed for the power plant, exclusive of the exhaust nozzle.

Pay load and warhead cone. - The pay load is housed in the separable warhead cone that forms the nose of the centerbody. The warhead cone is designed to have minimum weight, consistent with the limitations that it should be aerodynamically stable upon re-entry into the atmosphere, and that its maximum deceleration at this time should not exceed 30 times the acceleration due to gravity. Water is included to provide cooling upon re-entry. The length-base diameter ratio of this cone is 7.8, which appears roughly correct for minimum drag. The equipment associated with guidance of the missile was assumed to be carried in the warhead cone. With these assumptions, the weight of the cone was estimated to be 6000 pounds plus the 7000-pound pay load. A more recent discussion of the re-entry problem is given in reference ~~5~~4. The warhead-cone design is principally a function of re-entry velocity, which does not vary greatly for the range of missile designs considered in this report; therefore, the same warhead-cone weight and cost were used for all rocket missiles.

Guidance. - The missile was assumed to be guided during powered flight by a specialized ground-based radio guidance system as described in reference 4. Only sufficient intelligence need be built into the guidance equipment in the missile to permit accurate execution of the commands of the ground-based guidance system by the control system in the missile. When approximately the correct velocity is attained to reach the target, the main propulsion system is shut off. Final velocity adjustments are then made with very small vernier rockets. This guidance system is appreciably different from the guidance system for an air-borne missile. In any case, the development of a complete system to give the required accuracy and reliability is one of the more difficult problems requiring solution before any type of missile can become effective over 5500 nautical miles. The prospect for development of the guidance system for the ballistic missile appears to be reasonably good (ref. 4).

Structure. - In estimating the structural weight of the ballistic missile, consistency with missiles powered by other engine types is desired. For these types, an effective skin thickness of 0.030 inch is sufficient and also the practical minimum; therefore, this assumption was also adopted for the ballistic missile and is considered conservative. Reference 4 indicates that, for a single-body configuration, pressurized balloon-type propellant tanks have sufficient rigidity without additional structure. However, when balloon-type construction is applied to the configuration considered in the present report, strengthening of the bodies is necessary at the stations where they are joined. A weight equal to 1 percent of the centerbody gross weight was added to each booster in order to provide for the weight of the actual coupling device, the propellant-feed connections from the booster tanks to the second-stage power plant, and associated miscellaneous items.

Component costs. - Unit costs were assigned to the various missile components so that total relative cost of the missile could be evaluated. These unit costs are as follows:

Component	Unit cost per lb
Guidance	72.00
Power plant and controls	38.00
Structure and fins	25.00
Propellant tanks	15.00
Propellants:	
Fluorine	2.00
Ammonia	.04
Liquid oxygen	.05
JP4	.03

For the most part, the unit costs are given in reference ~~4~~ 3, where they are expressed in dollars per pound. Inasmuch as the unit cost per pound is used in the present report only as a weighting factor and the resulting cost estimates might not be suitable for other purposes, the dollar as a unit was discarded and the resulting cost of the missile based on these unit costs was then arbitrarily divided by 750,000 to give relative cost. The relative cost of the warhead cone exclusive of the pay load was estimated to be 0.2072. Because the cost of fluorine in large quantities is uncertain, the effect of variation of fluorine cost was calculated.

Procedure

The seven parameters selected for optimization, together with the assumptions relative to geometry, weights, and relative cost, are sufficient to define the missile design. All calculations were carried out by means of an IBM card-programmed calculator to eight significant figures. The numerical integration process used in the flight calculations is accurate on an absolute basis to about 1 percent on velocity. However, because the errors of integration are systematic, it is permissible to compare ranges differing by considerably less than the absolute accuracy of the range would indicate.

Optimization of parameters. - Optimization of the parameters was carried out for each of the two propellant systems, JP4-oxygen and ammonia-fluorine, as follows. First, a set of values was picked for the seven design parameters, and a missile cost was specified. Then a missile was designed having the assigned cost, and its maximum range was computed. Next, two additional missiles were designed and flown with different values of one of the parameters but with the same values of all other parameters as the first missile. The value of this parameter giving the maximum range was found by interpolation and used in the next set of calculations, which treated still another parameter in the same manner. As the computation proceeded, the range increased. When the range exceeded 6000 nautical miles, cost was reduced to bring range to slightly less than 5500 nautical miles. Each of the seven parameters was again changed in turn in this way and the process repeated until a near-optimum missile was obtained for the specified cost. Keeping the set of optimum values so found for the parameters, the cost was slightly adjusted to obtain a 5500-nautical-mile range. The final adjustment in cost was less than 2 percent.

The process used will lead to an optimum missile if sufficient iterations are carried out. For such an optimum missile, any change or combination of changes in the parameters being optimized would result in a decrease in the range of the missile. A missile having maximum range for a fixed cost is identical to a missile having minimum cost for a fixed range.

Variation of parameters. - With the optimum 5500-nautical-mile missile design for each of the two propellant combinations as a point of departure, changes in single parameters were made to show the effect of these parameters on the range of the missile. Also, the effects of variation of the specific impulse, power-plant weight, structural weight, and tank weight on the range of the optimum missiles were determined. One design was also carried out to show the effect of non-optimum engine parameters.

RESULTS AND DISCUSSION

Optimum Missile

Over-all comparison. - Optimum values of the seven design parameters for a range of 5500 nautical miles are presented in table I. The results of the optimization are summarized in the following table:

	JP4-Oxygen	Ammonia-fluorine
Relative cost	1.2107	1.0594
Total weight, lb	434,220	208,572
Over-all propellant- to gross-weight ratio	0.9088	0.8766
Max. effective specific impulse, lb-sec/lb (av. for all motors at zero ambient pressure)	308.0	367.6
Net initial acceleration, g	0.2057	0.5095

The JP4-oxygen missile is more than twice as heavy as the ammonia-fluorine missile and costs about 15 percent more. The maximum effective specific impulse for zero ambient pressure is shown for the booster and second-stage motors taken together. To offset its lower specific impulse, the JP4-oxygen missile requires a higher propellant- to gross-weight ratio; and, inasmuch as the pay load of the two missiles is the same, a larger gross weight results. As will be shown by subsequent discussion, however, the use of the relative-cost parameter as a basis for optimization leads to gross weights that are somewhat larger than would be obtained if the missiles were designed for minimum gross weight.

Flight plan of optimum missiles. - Some understanding of the characteristics of the two optimum missiles may be gained from study of their trajectories. Figure 3(a) shows the flight plan during powered flight of the two optimum missiles. The ammonia-fluorine missile has a slightly greater velocity at any altitude than the JP4-oxygen missile. The burn-out point and final velocity are very nearly identical for both missiles, so that the ballistic flight shown in figure 2 applies to either.

The velocity and altitude during powered flight are shown as functions of time in figures 3(b) and (c), respectively. The lower initial acceleration of the JP4-oxygen missile is evident, but its effect is offset by longer burning times and by higher acceleration when the tanks are almost empty, so that the final velocity and altitude are about the same.

Engine performance. - The specific impulse of the booster and centerbody motors is shown in figure 4 for the two propellant systems. While the assumption that separation of the flow in the nozzle occurs at a ratio of exit-nozzle separation pressure to ambient pressure p_s/p_a of 0.5 (ref. 1) may be optimistic, it has practically no effect on the impulse of the boosters, increases the centerbody impulse at low altitude only, and thus contributes a very small percentage increase in total impulse. The discontinuity in slope of these curves is caused by the change from separated to unseparated condition in the exhaust nozzle. The greater difference between the effective specific impulse of the first and second stage of the JP4-oxygen missile as compared with the ammonia-fluorine missile is due to the greater difference in chamber pressure and design altitude between the two stages of the JP4-oxygen missile.

The reduction in specific impulse due to drag is plotted against flight velocity in figure 5 for the flight plan of the optimum missiles. The quantity shown is the drag force divided by the propellant flow rate and is a convenient parameter, because it can be subtracted directly from the effective specific impulse. The ammonia-fluorine missile has more drag loss than the JP4-oxygen missile, because at any given velocity it is at a lower altitude, as shown in figure 3(a). Calculations show that the range is approximately 100 miles greater without drag than with drag, which is considered minor; however, if missiles were designed with higher initial accelerations, aerodynamic drag would become more important.

Effect of optimization at constant cost. - Since the estimated unit cost of propellants is low compared with the unit cost of hardware for either propellant combination, it is possible, in the optimization process at fixed cost, to remove a pound of expensive power plant and substitute many pounds of cheap fuel. This process obviously gives higher gross weights and propellant- to gross-weight ratios for the fixed-cost missile than would occur for a minimum-gross-weight missile. However, the process has a limit. For example, a decrease in power-plant weight can result from a reduction of propellant flow rate, chamber pressure, or design altitude. A reduction of chamber pressure or design altitude, however, reduces the effective specific impulse and cancels part of the gain from increased propellant- to gross-weight ratio. A reduction in either flow rate or specific impulse will reduce the acceleration of the missile and increase the time-gravity losses, thus cancelling another part of the gain from increased propellant- to gross-weight ratio. As power-plant weight is reduced, there is, of course, an optimum point beyond which such losses will cause a net reduction in range. Because of the much lower unit cost of JP4-oxygen propellant, the optimum will occur at lower initial acceleration for it than for ammonia-fluorine. This is illustrated in the preceding table, which shows that the net acceleration of the JP4-oxygen missile is less than one-half that of the ammonia-fluorine missile.

By the same reasoning, the low-cost propellant should have lower chamber pressure and design altitude than the more expensive one. Reference to the optimum values given in table I shows that, although the chamber pressure and design altitude of the JP4-oxygen boosters are lower than those for ammonia-fluorine, the chamber pressure and design altitude of the centerbody are higher than those for ammonia-fluorine. To explain this, the centerbody must be considered separately. The centerbody is, in effect, the pay load of the boosters. The greater this load, the larger the boosters must be; therefore, the centerbody for the minimum-cost missile would be near minimum weight. Because the weight of the centerbody is reduced by increasing specific impulse, the optimization resulted in high expansion ratios for the centerbody engines of both missiles. The greater propellant- to gross-weight ratio of the centerbody of the JP4-oxygen missile shown in table II resulted in a greater sensitivity to effective specific impulse than for the ammonia-fluorine centerbody. Because of this greater sensitivity, the optimum expansion ratio of the JP4-oxygen centerbody is higher than that of the ammonia-fluorine centerbody. This greater expansion ratio can be obtained by increasing either design altitude or chamber pressure. The optimum chamber pressure increases somewhat (about 3 percent, but the optimization is not carried out that closely) relative to that of the ammonia-fluorine; but the higher optimum design altitude increases the expansion ratio by 60 percent.

Weight and cost breakdown. - The weight and cost breakdowns for the two optimum missiles are given in tables II and III, respectively. It should be noted that the centerbody-to-missile gross weight ratio is smaller for the JP4-oxygen missile than for the ammonia-fluorine missile, and that the cost of propellants is a large part of the total missile cost for the ammonia-fluorine missile (35.7 percent) but practically negligible for the JP4-oxygen missile (1.9 percent). The relative cost of ammonia-fluorine in the centerbody, however, is less than 5 percent of the total missile cost. It is, therefore, probable that a minimum-cost missile could be designed to use high-performance propellants in the centerbody and low-cost propellants in the boosters. Time did not permit carrying out calculations for this configuration.

Effect of Variation of Design Parameters

Optimized parameters. - The variation of range with each of the design parameters at fixed relative cost is given in figure 6. For each curve, all parameters other than the one being varied are held constant at the values for the optimum missile (table I). The point corresponding to the optimum missile is circled on each curve. At any other point the range could be increased somewhat if the remaining six parameters were reoptimized. Because a large amount of work would be required to calculate the data in this manner, it was not attempted.

Booster-to-centerbody diameter ratio: The effect of the ratio of the weight of the centerbody to the missile gross weight is shown in terms of the ratio of the booster-to-centerbody diameter ratio in figure 6(a). The diameter ratio is related to the weight ratio by the equations given in appendix B. Increasing the diameter ratio shifts fuel and hardware from the centerbody to the boosters and thus changes the staging of the missile. Deviation of this parameter from the optimum value leads to a large loss in range. The large value of the diameter ratio of the optimum JP4-oxygen missile is the result of its low propellant cost.

Design altitude: Another parameter of considerable importance is the centerbody design altitude, shown in figure 6(b). The effect of deviation from the optimum value of this parameter is about the same for both propellant systems, although the optimum design altitude is higher for the JP4-oxygen propellant. Selection of a ratio of exit-nozzle separation pressure to ambient pressure other than 0.5 would alter the optimum value somewhat. The effect of the design altitude of the boosters on range also is shown in figure 6(b). Again, the variation of range with deviation from the optimum values of design altitude is similar for the two propellants.

Ratio of propellant flow rate to gross weight: Figure 6(c) shows the effect of changes in the ratio of propellant flow rate to gross weight on the missile range for the centerbody and the boosters. Increasing the ratio of propellant flow rate to initial gross weight will increase the initial acceleration. For a constant missile cost it will also increase the power-plant weight and reduce the quantity of propellant. There is considerable difference in the shape of the two curves for the two propellant combinations because of the difference in optimum initial acceleration. The range of the JP4-oxygen missile, which has considerably lower initial acceleration than the ammonia-fluorine missile, is much more sensitive to the initial acceleration.

Chamber pressure: The effect on range at constant cost of varying the chamber pressure of the centerbody and booster power plants is shown in figure 6(d). As chamber pressure is increased, the weight of the pumping plant and the specific impulse of the propellant increase. The effect of these changes is very similar for both propellants; and, with the pumping-plant weights described in reference 1, the best chamber pressure lies between 450 and 600 pounds per square inch absolute, with only small differences in range. Reduction of the pumping-plant weights below those assumed will shift the optimum chamber pressures to higher values. Deviation from the optimum chamber pressure causes greater loss for the boosters than for the centerbody power plant.

Simplified missile power plant. - The engine development would be minimized if only one engine size were required and if this engine were designed for low altitude and low chamber pressure. In order to compare the performance of a missile with such a power plant with that of a missile having optimum values of the design variables, a JP4-oxygen missile was designed with identical engines in both boosters and centerbody and having sea-level design altitude and chamber pressure of 300 pounds per square inch absolute. The actual size of the engines and the booster-to-centerbody diameter ratio were optimized. For the same 5500-nautical-mile range, the relative cost of this missile is 1.877 and its gross weight is 800,000 pounds, an increase of 50 percent in cost and 100 percent in weight over the optimum JP4-oxygen missile.

Range. - The approximate relation between missile cost and range (fig. 7) was found by designing several missiles to cost different amounts and computing their range. The optimum values of the parameters for the 5500-nautical-mile range were used for each missile. Therefore, these designs are optimum only for the 5500-nautical-mile range. For other ranges, reoptimization of the design parameters could reduce the cost below that given.

Component weights. - In the actual design of a tactical missile, the weight of the various components will doubtless differ from the values given by the weight equations used herein. Therefore, computations were made with changes in the weight of each of several component groups in both the booster and the centerbody. The effect on range at constant cost of changes in the power-plant, structure, and tank weights is shown in figure 8. These changes were made simultaneously in both the boosters and the centerbody. Changes in component weight change the propellant- to gross-weight ratio. The effect of changes in this ratio are greatest for missiles having the highest ratios. The JP4-oxygen missile is therefore more sensitive to changes in component weights.

The effect of component weight on missile cost at a fixed range is shown in figure 9 for the same components. The effect of simultaneous changes in both booster and centerbody is shown in figure 9 (a). A reduction of 1 percent in the weight of all three component groups will result in 1.3- and 0.8-percent reductions in relative cost for the JP4-oxygen and ammonia-fluorine missiles, respectively.

Figures 9(b) and (c) show the separate effects of changes in the boosters and in the centerbodies as computed with the approximation that, for a constant effective specific impulse, initial acceleration, and range,

$$1 - \lambda^* = (1 - \lambda_1)(1 - \lambda_2) = \text{constant}$$

where λ_1 and λ_2 are the propellant- to gross-weight ratios for the first and second stages, respectively. The value of λ^* was plotted against missile relative cost for several missile designs having the various component weights. Values of relative cost were read from this curve for the values of λ^* corresponding to the standard missile with 5500-nautical-mile range. This approximation should be fairly accurate in the cases where it has been applied but does not include any benefit that could be gained by reoptimizing the other parameters.

The effect of a certain percentage improvement of component weights is greatest for JP4-oxygen, and bigger changes in cost result from changes in the boosters than in the centerbody (figs. 9(b) and (c)). The boosters are, however, larger than the centerbody, and equal changes of weight cause nearly equal changes in cost for both centerbody and booster.

Weight of warhead cone. - The effect of changes in weight of the warhead cone, which might result from changes in weight of the warhead, the structure, or the cooling equipment, is shown in figure 10. The relative cost is assumed to be directly proportional to the weight of the warhead cone. An improvement in the design of the warhead cone, for example, that reduces the weight required for structure and cooling by 1000 pounds with no change in warhead weight will reduce relative cost to approximately 1.12 and 0.98 for JP4-oxygen and ammonia-fluorine, respectively, a reduction of about 8 percent.

Specific impulse. - The effect on range of varying the specific impulse of the optimum missiles is shown in figure 11. For example, if the effective specific impulse of the propellant throughout the flight is 1 percent lower than was estimated, the range of the missile will be decreased about 400 miles (7 percent). The decrease is slightly greater for JP4-oxygen than for ammonia-fluorine. Reference to figure 7 shows that, with the 1-percent decrease in effective specific impulse, the range may be restored to 5500 nautical miles by increasing the relative cost of the JP4-oxygen missile from 1.21 to 1.26 and of the ammonia-fluorine missile from 1.06 to 1.09.

Percent of propellant consumed. - If the missile fails to use all of its propellants for some reason (by failure to hold the proper fuel-oxidant ratio, e.g.), its effective propellant- to gross-weight ratio is reduced, and the range decreases as shown in figure 12. As was previously stated, the standard assumption is that 1 percent of the propellant is left in the tank. If an additional 1 percent were left, the range would be reduced by nearly 750 miles (13 percent). Thus, the effect of small percentage changes of propellant- to gross-weight ratio is nearly 1.9 times as great as similar percentage changes in specific impulse. However, both effects are important. This emphasizes the need for measurement of the amount of propellant left in the tanks, so that both fuel and oxidant are exhausted at the same time.

Unit cost of fluorine. - The much lower gross weight and appreciably lower cost of the ammonia-fluorine missile may be sufficiently important to outweigh the logistic and handling problems associated with this propellant combination. Because the cost of fluorine is not well established and its cost for the ammonia-fluorine missile is a fairly large part of the total cost, the effect of variation in the unit cost per pound of fluorine on the over-all cost of the missile was calculated, and the results are presented in figure 13. The unit cost per pound of 2.0 was selected for fluorine as the basis for optimization. If the missile were reoptimized for each fluorine cost, the relative cost would be reduced somewhat.

For a fluorine unit cost per pound of 2.8, the ammonia-fluorine missile would cost as much as the JP4-oxygen missile for a range of 5500 nautical miles. However, the estimated unit cost per pound of 2.0 is thought to be conservative for the cost of fluorine in large quantities; thus, it appears that some cost difference would exist in favor of the ammonia-fluorine missile. Reference 8 gives an estimate of fluorine cost of 77 cents per pound for a plant capacity of 10 tons per day, which corresponds to a unit cost per pound of 0.77. As can be seen from figure 13, this unit cost results in a relative cost of the missile of 0.83. In addition, the much lower gross weight of the ammonia-fluorine missile remains an advantage.

CONCLUSIONS

Analysis of the missile configuration considered in this report leads to the conclusion that rocket-powered ballistic missiles using either ammonia-fluorine or JP4-oxygen propellants are feasible for ranges of 5500 nautical miles or more. It appears that substantially all the components required for the design of the power plant for a JP4-oxygen missile of this type are in an advanced state of development. The use of ammonia-fluorine as the propellant instead of gasoline-oxygen would reduce the gross weight by half and the cost by 10 to 30 percent, but would, of course, increase development effort required for the power plant.

Fairly broad ranges of the design parameters for the power-plant and missile configuration give near-maximum missile performance. Attempts to design a missile with several parameters too far from optimum can have large effects, as was shown when a simplified power-plant design resulted in doubling the gross weight of a JP4-oxygen missile. The best values found for the ratio of the weight of the centerbody (final stage) to the entire missile are 0.17 to 0.22 for ammonia-fluorine and 0.11 to 0.15 for JP4-oxygen missiles. The best chamber pressures are between 450 and 600 pounds per square inch absolute for all engines and for both propellants. Reduction of pumping-plant weights by further development will tend to shift the best chamber pressures to higher values. High design altitudes are desirable for both the centerbody and the boosters. The best design altitudes for the centerbody are from 45,000 to 60,000 feet, with that for ammonia-fluorine lower than for JP4-oxygen. The best values for the booster are between 20,000 and 30,000 feet, with the ammonia-fluorine value higher. The best net initial acceleration is near 0.2 g for the JP4-oxygen missile and about 0.5 g for the ammonia-fluorine missile.

The design of the missile is not especially sensitive to the requirements of the mission. For example, an increase of 1 percent in range can be obtained with an increase of 0.5 percent in cost. Pay load and cost are roughly proportional. The weight of components, while important, is not critical; for example, a 1-percent reduction in structure, tank, and power-plant weight reduces cost about 1.3 percent for JP4-oxygen and about 0.8 percent for ammonia-fluorine.

Once the missile is constructed, however, it will be sensitive to variation of certain parameters. For example, failure to utilize 1 percent of the propellant in the tanks will cause about 13-percent loss in range; and a 1-percent decrease in specific impulse throughout the powered flight will decrease the range about 7 percent.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, October 8, 1954

APPENDIX A

SYMBOLS

The following symbols are used in the calculations:

A	area, sq in.
A_F	missile frontal area, sq ft
a	acceleration, ft/sec^2
b	constant
C	constant of integration
C_D	drag coefficient
D	drag, lb
d	diameter, in.
F	thrust, lb
f	function
g	instantaneous acceleration due to gravity, ft/sec^2
g_c	gravitational constant, 32.174 $\frac{\text{ft-lb(mass)}}{\text{sec}^2\text{lb(force)}}$
g_v	$g - \frac{v^2}{r}$
g_0	acceleration due to gravity at radius of earth, ft/sec^2
h	altitude, ft
I	specific impulse, lb(force)-sec/lb(mass)
K	constant
l	length, in.
n	ratio of axial acceleration to standard acceleration due to gravity

p_c	chamber pressure, lb/sq in. abs
q	dynamic pressure, lb/sq ft
R	radius of earth, ft
r	distance of missile from center of earth, ft
r_v	distance of missile from instantaneous center of curvature, ft
S	surface area, sq in.
t	time, sec
V	volume, cu in.
v	velocity, ft/sec
W	weight, lb
w	propellant flow rate, lb/sec
X	range along circumference of earth, ft
x	cosine of angle at center of earth between missile and apogee
α, β, γ	constants defining ballistic trajectory
θ	angle between direction of motion and horizontal, radians
ρ	density, lb/cu in.
ρ_a	air density, slugs/cu ft
τ	thickness, in.
ψ	angle at center of earth between missile and launching site, radians

Subscripts:

aux	auxiliaries
B	booster
b	at burn-out
C	centerbody

con	control
cyl	cylindrical
d	design
e	nozzle exit
eff	effective
f	fuel
G	gross
i	at impact
m	engine
n	nozzle
o	oxidant
p	propellant
pp	power plant
s	structure
T	tank
t	nozzle throat
0	initial conditions at launching
1, 2, 3	stations during intervals

APPENDIX B

MISSILE DESIGN PROCEDURE

The equations that specify the weight and volume of each missile component and the procedure used to find solutions to these equations are presented herein.

Assigned Parameters

As mentioned in the body of the report, values were assigned to the following parameters:

(1) Booster design altitude, $h_{d,B}$

(2) Centerbody design altitude, $h_{d,C}$

(3) Booster chamber pressure, $p_{c,B}$

(4) Centerbody chamber pressure, $p_{c,C}$

(5) Ratio of propellant flow rate to gross weight for booster, $(w/W)_B$

(6) Ratio of propellant flow rate to gross weight for centerbody, $(w/W)_C$

(7) Ratio of booster diameter to centerbody diameter, d_B/d_C

(8) Relative cost of complete missile (two boosters and centerbody)

These values, together with the equations relating weight, volume, and cost and the assumptions concerning the configuration, are sufficient to determine the missile design.

Solution of Design Equations

Because the equations are not readily solved in explicit form, the solution is obtained by an iterative technique considering one body at a time. The diameter and gross weight of a booster are estimated. The weight of the booster components is then computed, totaled, and compared with the estimated weight. A new estimate of gross weight is computed with the aid of the Newton-Raphsom method for solving nonlinear algebraic equations (ref. 7).

The diameter of the centerbody is then computed from d_B/d_C, and the weight of the centerbody is determined by the same technique as for the booster. Next, the missile cost is computed and compared with the assigned cost. A new booster diameter is estimated to correspond more closely to the assigned cost. Each iteration with a new diameter includes a new determination of the weight of each stage followed by a new computation of cost. Three iterations are sufficient to get 6- or 7-figure accuracy on the cost parameter. High numerical accuracy (self-consistency) is required to permit the optimization of the parameters by the method described.

Design Equations

From the estimated value of the gross weight of either the centerbody W_C or boosters W_B, the flow rate w for the respective body may be computed from the assigned value of w/W by the following equation:

$$w = W\left(\frac{w}{W}\right) \tag{B1}$$

Since

$$W_{m,aux} = f(h_d,p_c,w)$$

the engine and auxiliary weight for the respective missile bodies may be found from figures 10 and 11 of reference 1 or computed from equations (13) and (14) of that report. The engine of the centerbody is assumed to be gimbal-mounted to stabilize the missile during take-off and to control its flight direction during the power-on phase. Very small vernier rocket engines are assumed to make final adjustment of the velocity after the large engines are shut down. The weight of these control items W_{con} is to be added to the power-plant weight of the centerbody only and is estimated by

$$W_{con} = 0.00162\left(\frac{l_C}{12}\right)^{1/2} W_C \tag{B2}$$

The power-plant weight for the centerbody is then

$$W_{pp;C} = W_{m,aux,C} + W_{con} \tag{B3}$$

while the control weight for the boosters is taken as zero. The volume of the respective missile bodies consisting of a cylinder of a length-diameter ratio of 5 and a cone of length-diameter ratio of 4 is

$$V = 4.97418d^3 \tag{B4}$$

The volume of the respective missile body is assumed to be distributed as follows. Nineteen percent of the missile body volume is assumed to be occupied by structure, piping, instruments, and other miscellaneous items. In addition, the space in the nose cone of a booster ahead of the 24-inch diameter is assumed to be available for instruments and miscellaneous items. The nose of the centerbody is cut off at the 66-inch diameter to accommodate the warhead cone.

The rear section of the missile bodies is assumed to contain the power plant. The volume of the power plant V_{pp} is given by

$$V_{pp} = \frac{W_{pp}}{0.01447} + 0.6362d^2 l_n \tag{B5}$$

where W_{pp} is the power-plant weight and l_n is the length of the divergent section of the nozzle, which is obtained from

$$l_n = 2.105\sqrt{A_t}\left(\sqrt{\frac{A_e}{A_t}} - 1\right) \tag{B6}$$

where

$$A_t = f(p_c, w)$$

$$A_e/A_t = f(p_c, h_d)$$

which may be obtained from figures 8 and 9 of reference 1. The first term on the right side of equation (B5) allows a volume for the power plant based on an average density of 25 pounds per cubic foot, and the second term adds an additional volume proportional to the length of the divergent section of the exhaust nozzle.

The rest of the missile body volume is assumed to be available for propellant tanks, and the tanks are 99 percent filled. The weight of the propellant based on the foregoing assumptions is given by

$$W_p = \rho_p(0.81V - V_{pp} - K_1)0.99 \tag{B7}$$

where K_1 is 14,476 cubic inches for a booster and 301,063 cubic inches for the centerbody.

The fuel weight W_f may be obtained from the propellant weight W_p and the weight-percent fuel in the propellant W_f/W_p:

$$W_f = W_p \frac{W_f}{W_p} \tag{B8}$$

The thickness of the propellant tanks is governed by a hoop stress of 80,000 pounds per square inch resulting from pressurization to 45 pounds per square inch absolute plus the maximum hydrastatic pressure due to the acceleration of the missile; minimum thickness is 0.022 inch.

The fuel tank is assumed to be aft of the oxidant tank and is always cylindrical. The oxidant tank is partly cylindrical and partly conical. From the geometry of the missile, the cylindrical length $l_{o,cyl}$ of the oxidant tank is equal to the length of the cylindrical missile body minus the length of the fuel tank and power-plant compartment:

$$l_{o,cyl} = 5d - \frac{1.5878}{\rho_f} \frac{W_f}{d^2} - 1.5719 \frac{V_{pp}}{d^2} \tag{B9}$$

Based on these assumptions, the thickness of the fuel tanks is given by

$$\tau_f = \frac{n_O W_f}{113,097d} + 0.00253125d \tag{B10}$$

and of the oxidant tanks, by

$$\tau_o = \frac{d}{177,778}\left[0.99\ \rho_o(l_{o,cyl} + 4d - K_2)n + 45\right] \tag{B11}$$

where the acceleration n is given by

$$n = \frac{2F_B + F_C}{2W_B + W_C} \tag{B12}$$

The total acceleration is evaluated at sea level for the booster and just before booster burn-out for the centerbody, inasmuch as these conditions impose the most severe stress on the tanks. The constant K_2 in equation (B11), which accounts for the part of the nose cone not filled by fuel, is equal to 96 inches for the booster and 264 inches for the centerbody.

The tank surfaces are given by

$$S_f = \frac{4.489347}{\rho_f} \frac{W_f}{d} + 1.27235d^2 \tag{B13}$$

$$S_o = 6.3267d^2 + 2.82743d l_{o,cyl} - 2,911 \tag{B14}$$

The total tank weight W_T in each body is given by

$$W_T = 0.284(\tau_f S_f + \tau_o S_o) \tag{B15}$$

The structural weight W_s is taken as proportional to the surface of the missile body, with a term added to account for engine mounts:

$$W_s = 6.2594d^2\tau_s + 0.0034d^{1/2}\ W \tag{B16}$$

where the effective thickness τ_s is taken as 0.03 inch for all missiles.

Two fins are used on each booster for aerodynamic stability. Their weight W_{fin}, estimated in a manner similar to that of reference 3, is

$$W_{fin} = \left(0.00048 + \frac{0.14}{d_B}\right)W_B \tag{B17}$$

The calculated gross weight of the booster W_B is given by

$$W_B = (W_{pp} + W_p + W_T + W_s + W_{fin})_B \tag{B18}$$

and the gross weight of the centerbody W_C, by

$$W_C = (W_{pp} + W_p + W_T + W_s + 13{,}000)_C \tag{B19}$$

The weight of 13,000 pounds is the warhead cone including the pay load.

The relative cost of the complete missile, consisting of two boosters and a centerbody, is computed by summing the products of the weight of each component multiplied by its unit cost, adding the cost of the warhead cone exclusive of the pay load proper, and dividing the sum so obtained by 750,000. In general, the computed gross weight and relative cost do not agree exactly with the estimated gross weight and assigned cost. Adjustments are made and the computation repeated until the computed cost agrees with the assigned cost within 0.005 percent.

APPENDIX C

MISSILE FLIGHT CALCULATIONS

The missile is assumed to fly a no-lift trajectory about a nonrotating earth. Aerodynamic drag is considered for the part of the powered flight where it is significant.

Powered Flight

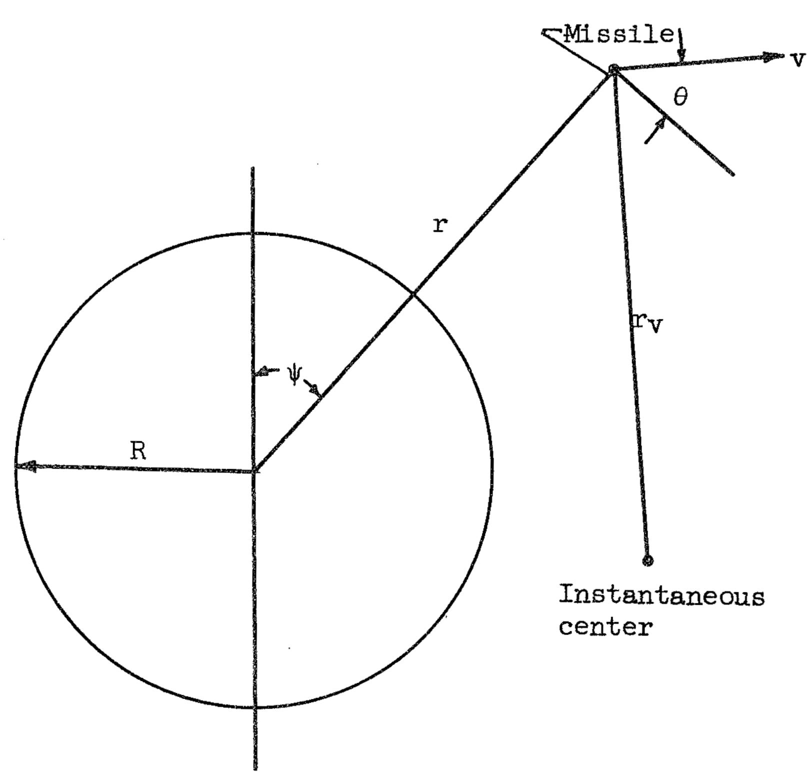
The thrust is assumed to be in the direction of flight. Summing the forces parallel to the direction of flight,

$$\frac{W}{g_c}\frac{dv}{dt} = -\frac{W}{g_c} g \sin \theta - D + F \tag{C1}$$

Summing the forces perpendicular to the direction of flight,

$$\frac{W}{g_c}\frac{v^2}{r_v} = \frac{W}{g_c} g \cos \theta \tag{C2}$$

From the following sketch



$$r_v = \frac{v}{\left(\frac{d\psi}{dt} - \frac{d\theta}{dt}\right)} \tag{C3}$$

With the aid of the identities

$$\frac{d\psi}{dt} = \frac{v \cos \theta}{r} \tag{C4}$$

$$g = g_0\left(\frac{R}{r}\right)^2 \tag{C5}$$

and

$$g_v = g - \frac{v^2}{r} \tag{C6}$$

equations (C1) and (C2) may be written

$$\frac{dv}{dt} = - g_0\left(\frac{R}{r}\right)^2 \sin \theta + g_c \frac{(F - D)}{W} \tag{C7}$$

and

$$\frac{d\theta}{dt} = \frac{-g_v \cos \theta}{v} \tag{C8}$$

The net specific impulse

$$I_{net} = \frac{F - D}{-\frac{dW}{dt}} \tag{C9}$$

where the propellant flow rate $-dW/dt$ is assumed constant for a given stage. Substituting equations (C9) and (C5) into equation (C7) gives

$$dv = - g_0\left(\frac{R}{r}\right)^2 \sin \theta \, dt - I_{net} \, g_c \frac{dW}{W} \tag{C10}$$

Integrating equation (C10) yields

$$\Delta v = g_0 R^2 \overline{\left(\frac{\sin \theta}{r^2}\right)} \Delta t - \bar{I}_{net} \, g_c \, \Delta \ln W \tag{C11}$$

where the bar indicates those terms that actually vary but which, as an approximation, are assumed to be constant at an average value over a short interval of time for stepwise integration.

The missile weight at any time is given by

$$W = W_O + \frac{dW}{dt} t \tag{C12}$$

Equation (C8) may be written

$$\frac{\cos \theta \, d\theta}{1 - \sin^2\theta} = \frac{-g_v dt}{v} \tag{C13}$$

Integrating equation (C13) yields

$$\tanh^{-1}(\sin \theta) = -\int \frac{g_v}{v} dt + C \tag{C14}$$

Let v by approximated by

$$v = b + \bar{a} t \tag{C15}$$

where b is a constant to be determined so that $v = v_1$ when $t = t_1$, and $\bar{a}$ is the average acceleration for a particular interval of time. Then

$$\tanh^{-1}(\sin \theta)\Bigg]_{\theta_1}^{\theta_2} = \frac{-\bar{g}_v}{\bar{a}} \ln\left(1 + \frac{\bar{a}\Delta t}{v_1}\right) \tag{C16}$$

The procedure for integration of the powered flight is as follows: The acceleration at the beginning a_1 and at the end a_2 of the previous interval is extrapolated to give $\bar{a}$ for the center of the present interval with the aid of the following equation:

$$\bar{a} = \frac{3a_2 - a_1}{2} \tag{C17}$$

The average velocity $\bar{v}$ at the center of the interval is computed from

$$\bar{v} = v_2 + \frac{\bar{a}\Delta t}{2} \tag{C18}$$

where v_2 is the velocity at the end of the previous interval and Δt is the integration interval. The value of $\overline{v}$ is used to compute the altitude $\overline{h}$ at the center of the interval from

$$\overline{h} = h_2 + \overline{v} \sin \theta_2 \frac{\Delta t}{2} \tag{C19}$$

where h_2 is the altitude at the beginning of the interval, and the initial value θ_2 is used to approximate the average value of θ for the first half of the interval. The value of $\overline{g}_v$ is then computed from equation (C6) with average values of velocity and radius, where

$$\overline{r} = \overline{h} + R \tag{C20}$$

The $\sin \theta$ at the end of the interval is computed from equation (C16).

The estimated effective specific impulse $\overline{I}_{eff}$ at the altitude $\overline{h}$ is computed from fitted equations.

The aerodynamic drag is computed from

$$D = A_F q C_D \tag{C21}$$

where $q = \frac{1}{2} \rho_a v^2$, and C_D is the drag coefficient computed by linearized theory for bodies of revolution, neglecting interference among the bodies of the configuration.

The net specific impulse I_{net} is computed from

$$I_{net} = I_{eff} - \frac{D}{-\frac{dW}{dt}} \tag{C22}$$

and the value of the velocity at the end of the interval is then computed from equation (C11).

The range at the end of the interval is

$$X_3 = X_2 + \overline{v}\, \overline{\cos \theta}\, \Delta t \tag{C23}$$

where X_2 is the range at the beginning of the interval.

The altitude at the end of the interval is

$$h_3 = h_2 + \overline{v}\, \overline{\sin \theta}\, \Delta t \tag{C24}$$

The specific impulse corresponding to this altitude is then evaluated and corrected for drag by equation (C22)(where drag is evaluated for the end of the interval), and the acceleration of the missile at the end of the interval is computed from equations (C7), (C9), and (C12). The process is then repeated for the next interval. To start the integration, the acceleration at the center of the first interval is approximated for each stage.

Ballistic Flight

The warhead cone is assumed to follow a ballistic path about a nonrotating earth. It is necessary in this problem to consider the difference in height of the missile at burn-out and at impact and to consider the range covered during burning. The solution to the differential equations may be written in closed form (ref. 8). In numerical calculations the values of three parameters α, β, and γ may be computed from the conditions at burn-out:

$$\alpha = -r_b^2 v_b^2 \cos^2\theta_b \tag{C25}$$

$$\beta = 2g_0R^2 \tag{C26}$$

$$\gamma = v_b^2 - \frac{\beta}{r_b} \tag{C27}$$

These parameters are then used to compute the values of the central angle cosines x_b and x_i corresponding to burn-out and to impact, respectively:

$$x_b = \frac{\beta r_b - 2\alpha}{r_b\sqrt{\beta^2 - 4\alpha\gamma}} \tag{C28}$$

$$x_i = \frac{\beta R + 2\alpha}{R\sqrt{\beta^2 - 4\alpha\gamma}} \tag{C29}$$

The total range of the missile is given by

$$\text{Range} = R\left[\cos^{-1}x_b + \cos^{-1}x_i\right] + X_b \tag{C30}$$

where X_b is the range attained during the burning period. When θ is positive at burn-out, $\cos^{-1}x_b$ and $\cos^{-1}x_i$ are between 0 and π if the trajectory intersects the earth.

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TABLE I. - OPTIMUM VALUES OF DESIGN PARAMETERS

Parameter	JP4-Oxygen		Ammonia-fluorine	
	Centerbody	Booster	Centerbody	Booster
Chamber pressure, lb/sq in. abs	538	500	523	556
Design altitude, ft	56,900	21,800	47,250	25,950
$\frac{\text{Propellant flow rate}}{\text{Gross weight}}, \frac{\text{lb}}{(\text{sec})(\text{lb})}$	0.00520	0.00476	0.00512	0.00506
Booster-to-centerbody diameter ratio	1.49		1.30	

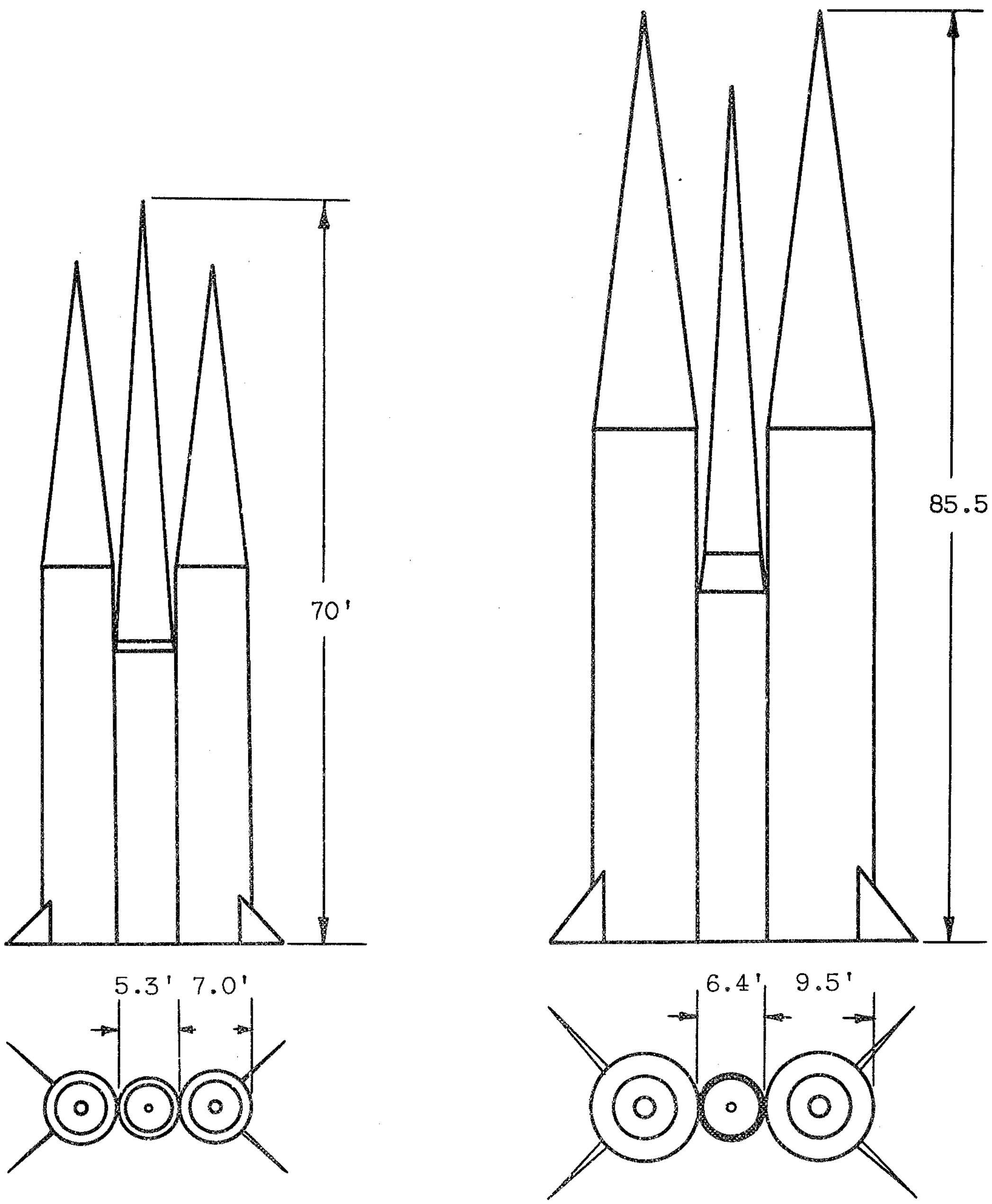
TABLE II. - WEIGHT BREAKDOWN FOR OPTIMUM MISSILE

[Weights in pounds.]

	JP4-Oxygen			Ammonia-fluorine		
	Centerbody	Booster	Complete missile	Centerbody	Booster	Complete missile
Propellant	36,581	179,016	394,613	22,809	80,017	182,843
Tanks	909	2,988	6,885	440	1,176	2,792
Structure	1,032	4,005	9,042	634	2,124	4,882
Motor	690	1,552	3,794	368	741	1,850
Auxiliaries	865	2,681	6,227	502	1,138	2,778
Controls	659	-------	659	427	------	427
Warhead cone	13,000	-------	13,000	13,000	------	13,000
Total	53,736	190,242	434,220	38,180	85,196	208,572
Propellant- to gross-weight ratio, boost stage	0.8245			0.7673		
Propellant- to gross-weight ratio, final stage	.6808			.5974		
Centerbody-to-missile gross-weight ratio	.1238			.1831		

TABLE III. - RELATIVE-COST BREAKDOWN FOR OPTIMUM MISSILE

	JP4-Oxygen			Ammonia-fluorine		
	Centerbody	Booster	Complete missile	Centerbody	Booster	Complete missile
Propellant	0.0022	0.0106	0.0234	0.0471	0.1653	0.3777
Tanks	.0182	.0598	.1378	.0088	.0235	.0558
Structure	.0344	.1335	.3014	.0211	.0708	.1627
Motor	.0350	.0786	.1922	.0186	.0375	.0936
Auxiliaries	.0438	.1358	.3154	.0254	.0577	.1408
Controls	.0333	------	.0333	.0216	------	.0216
Warhead cone	.2072	------	.2072	.2072	------	.2072
Total	0.3741	0.4183	1.2107	0.3498	0.3548	1.0594



(a) Ammonia-fluorine optimum missile. Weight, 209,000 pounds; relative cost, 1.06.

(b) JP4-Oxygen optimum missile. Weight, 434,000 pounds; relative cost, 1.21.

Figure 1. - Optimum missiles.

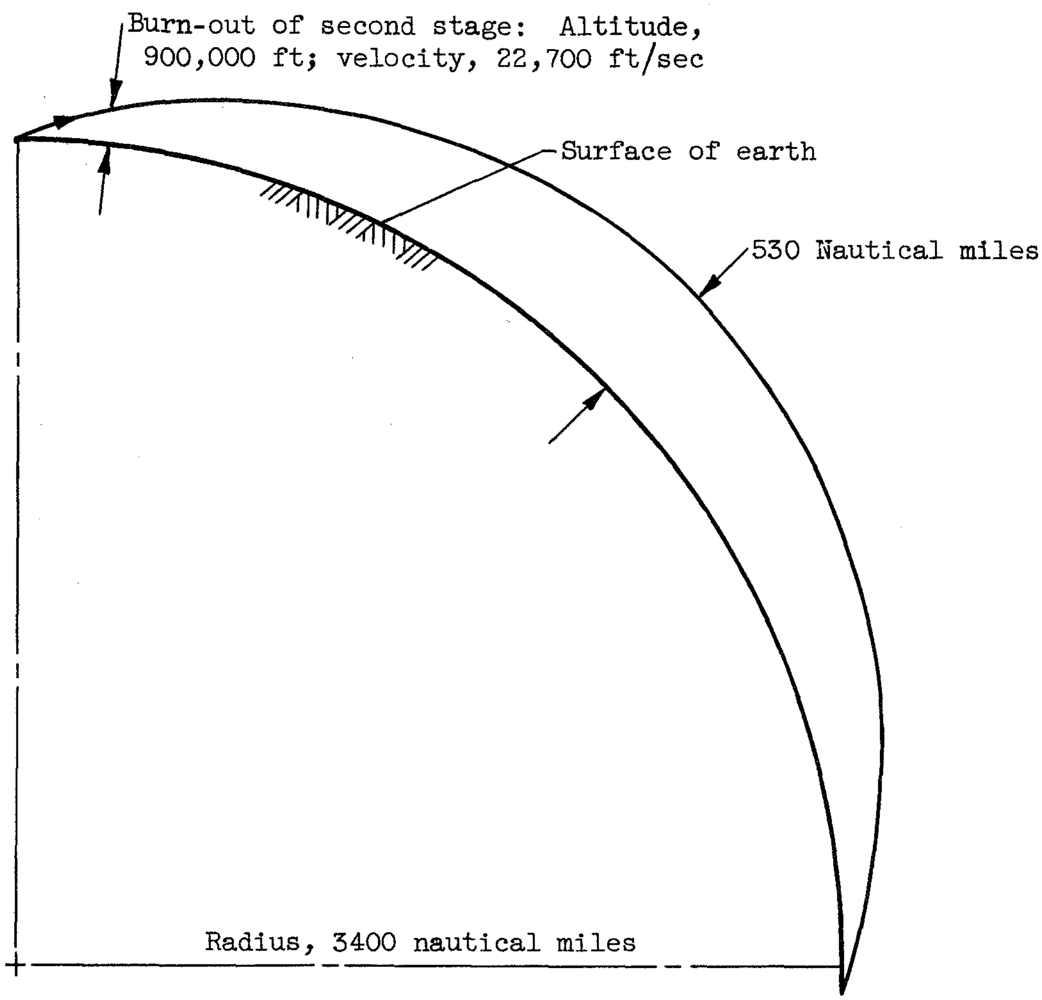
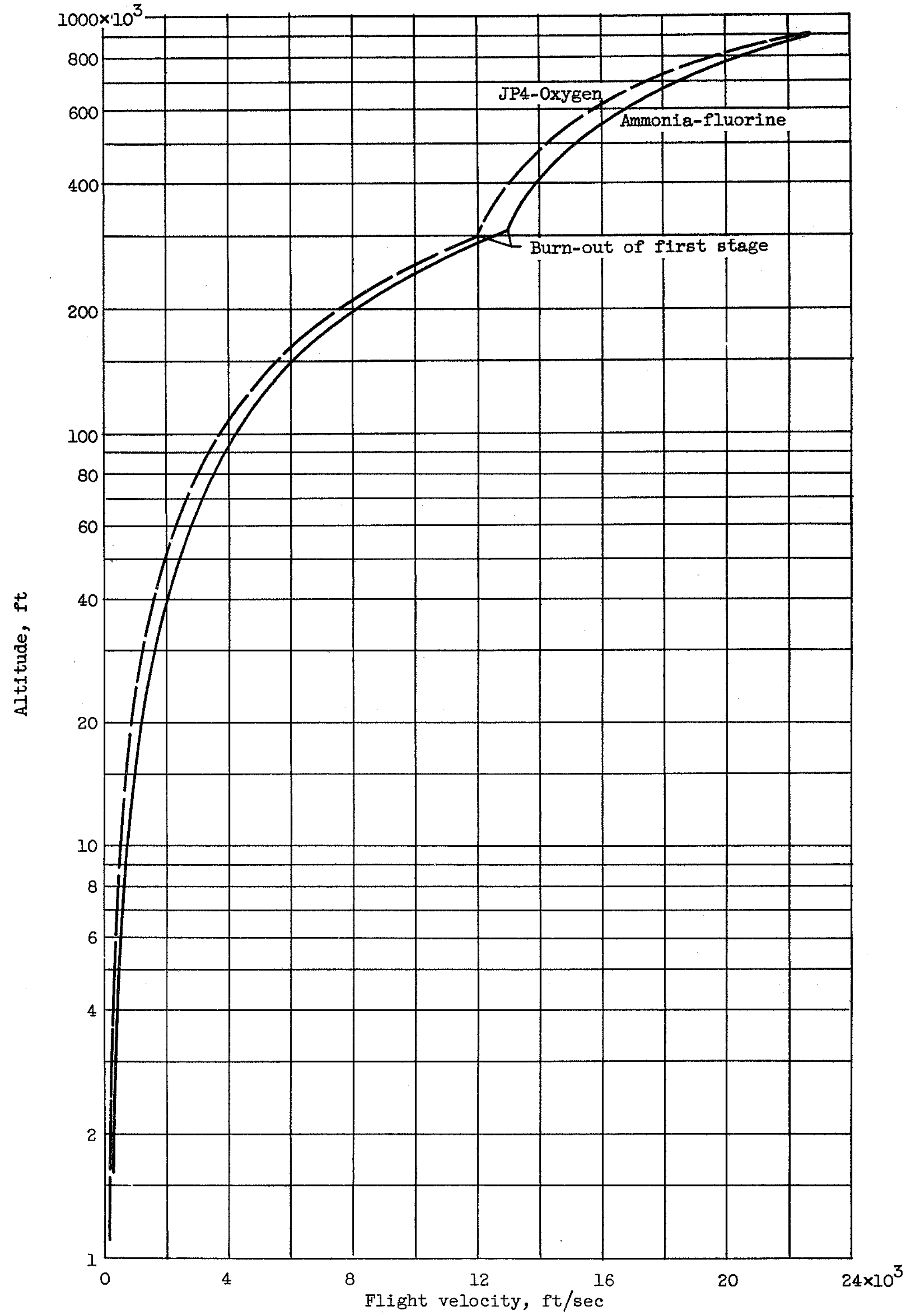


Figure 2. - Complete trajectory for 5500-nautical-mile ballistic missile.



(a) Variation of altitude with velocity.

Figure 3. - Powered-flight characteristics for optimum missiles.

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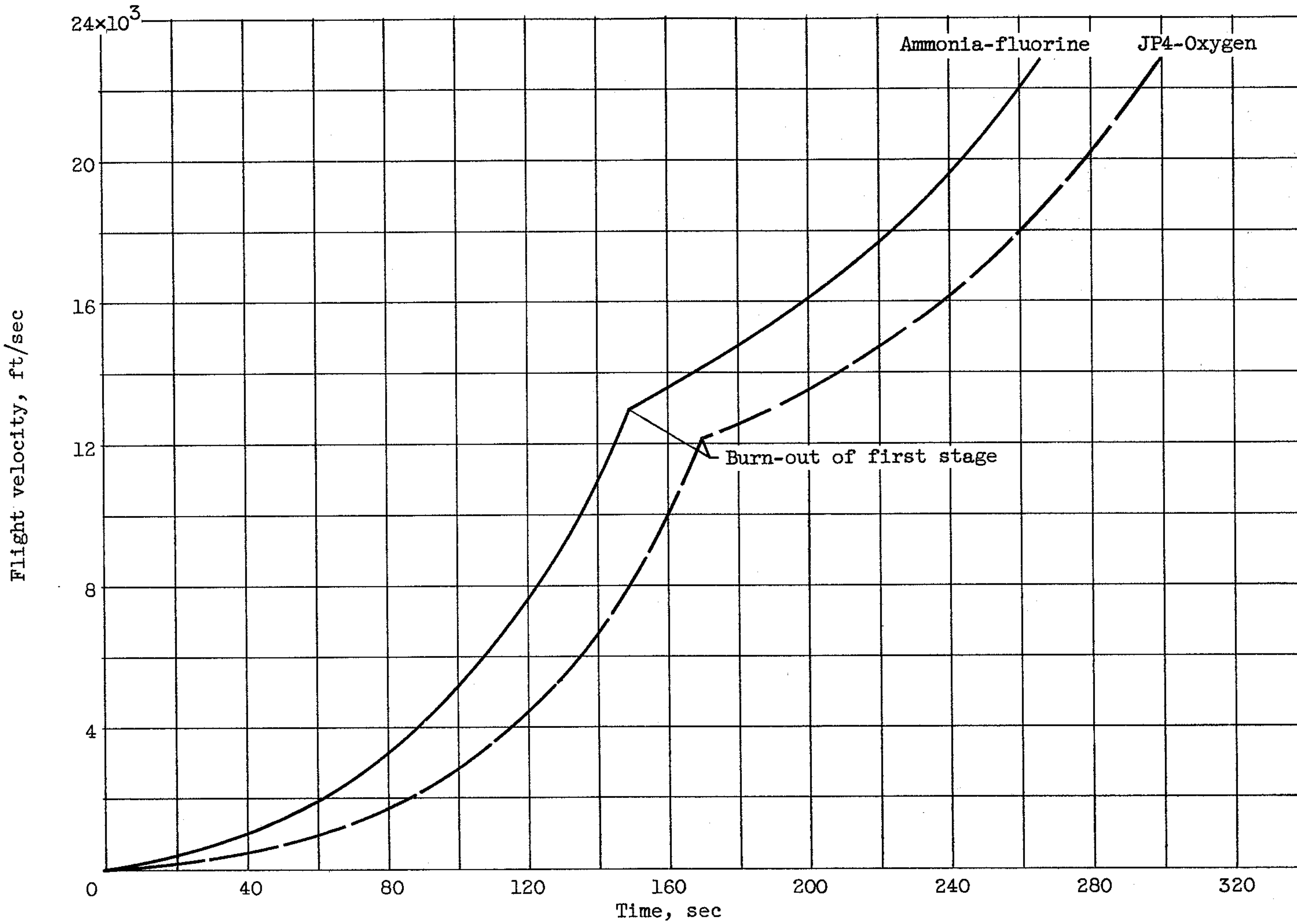
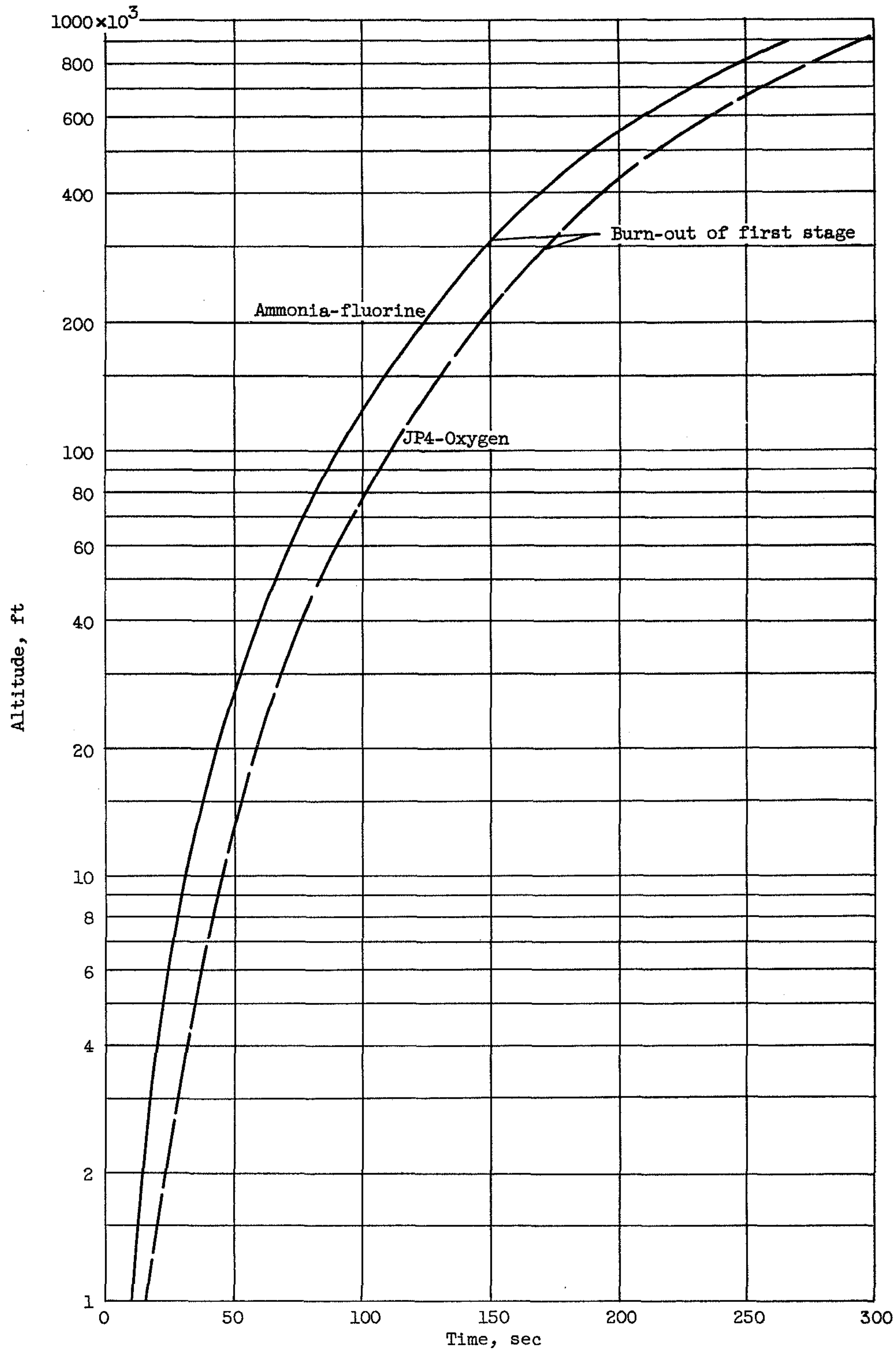


(b) Variation of velocity with time.

Figure 3. - Continued. Powered-flight characteristics for optimum missiles.



(c) Variation of altitude with time.

Figure 3. - Concluded. Powered-flight characteristics for optimum missiles.

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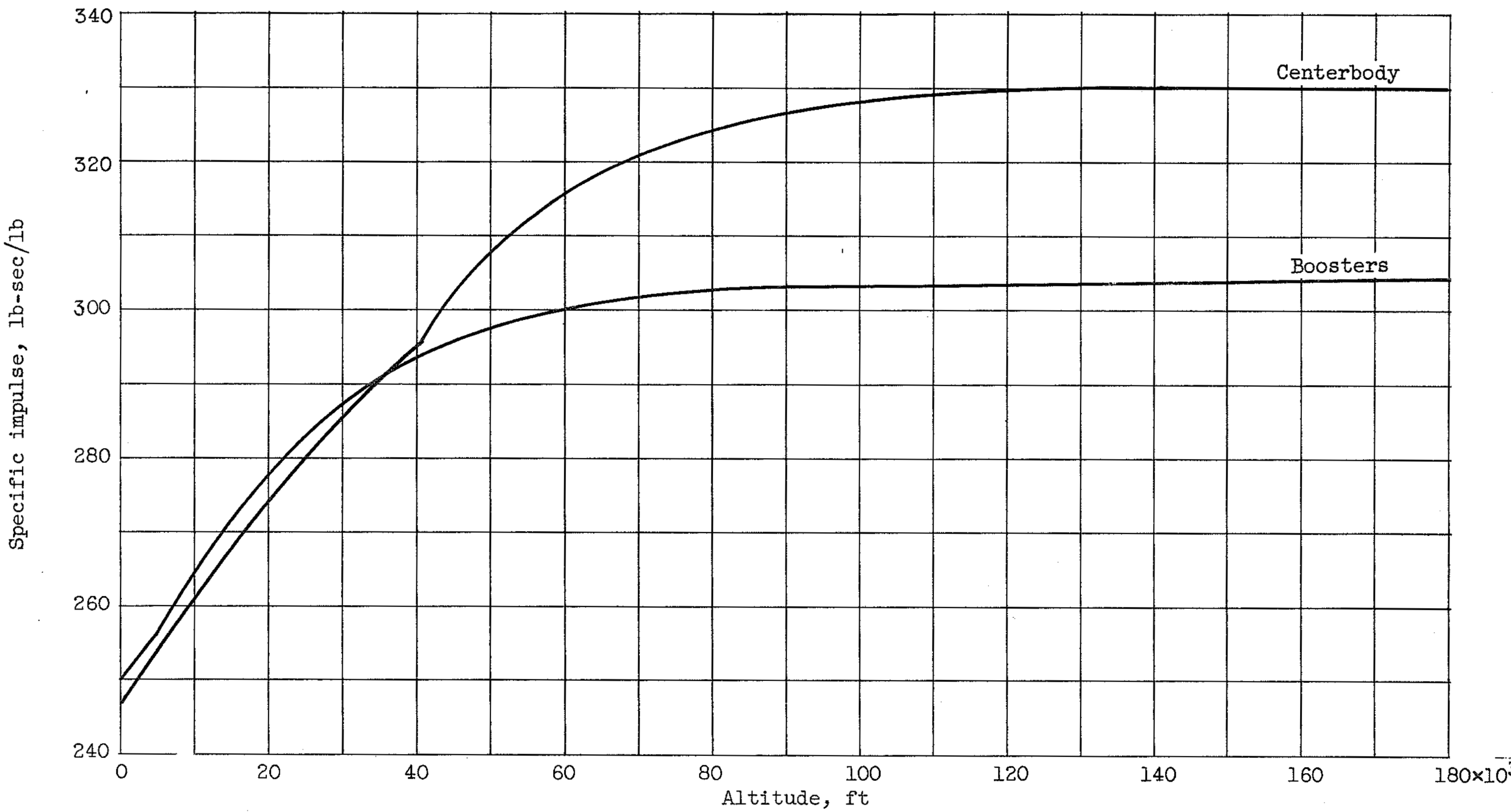
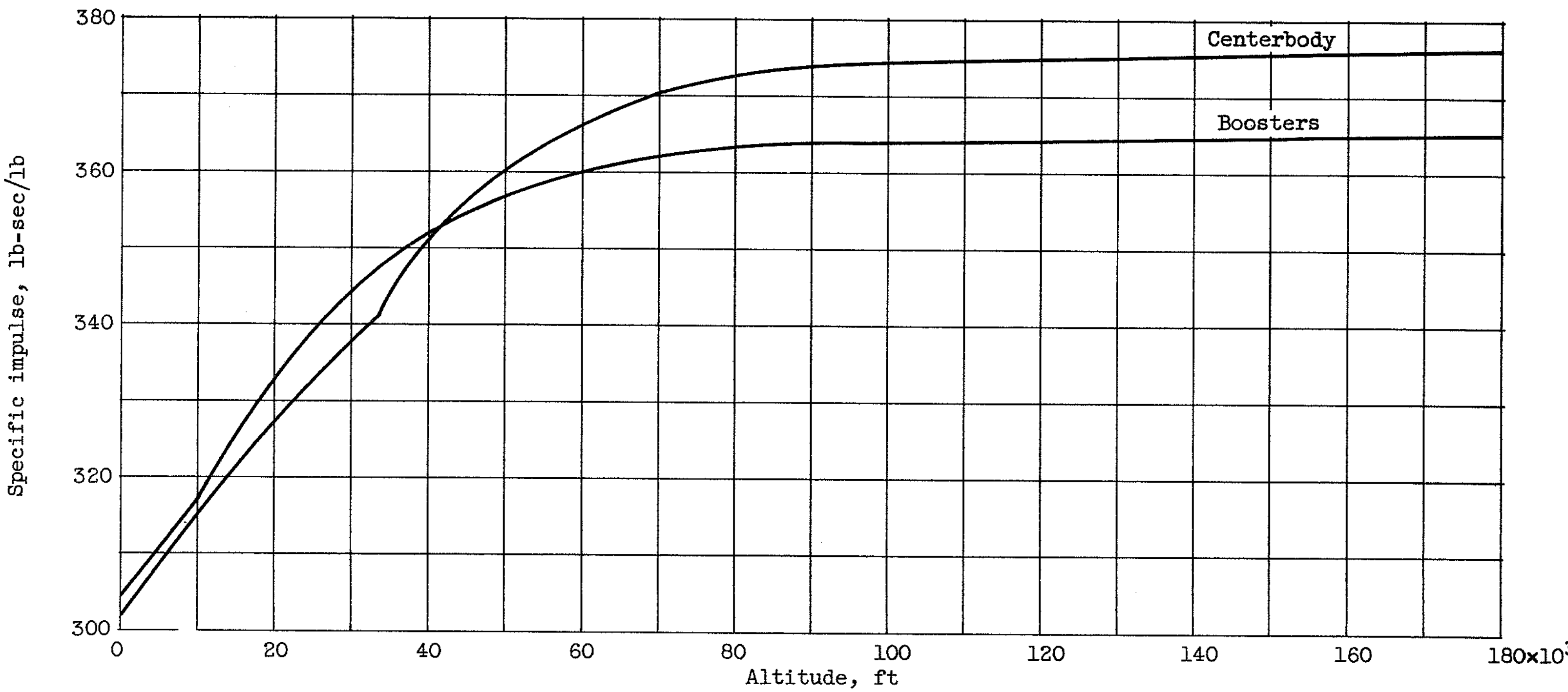


(a) JP4-Oxygen.

Figure 4. - Variation of specific impulse with altitude for optimum missile.



(b) Ammonia-fluorine.

Figure 4. - Concluded. Variation of specific impulse with altitude for optimum missile.

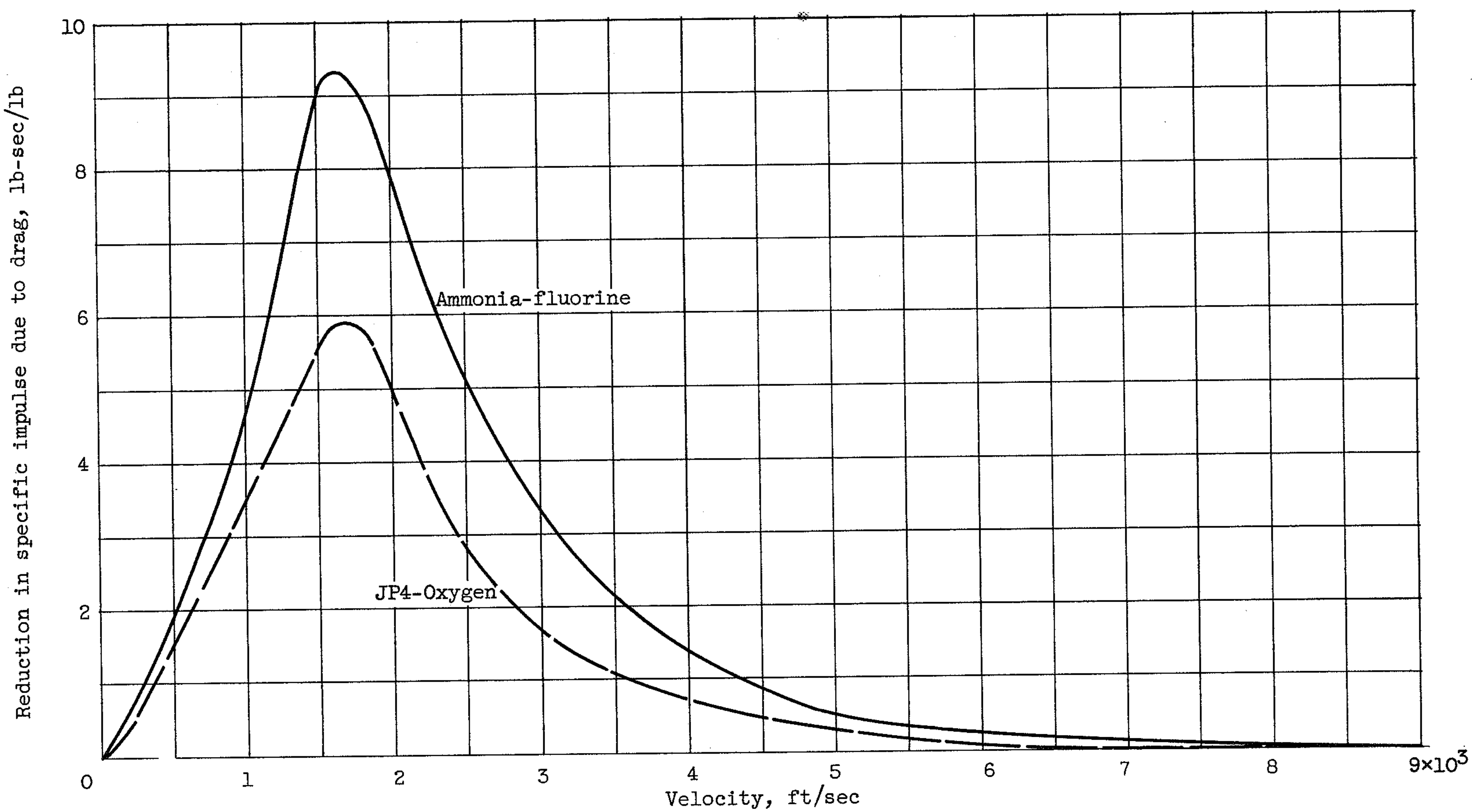
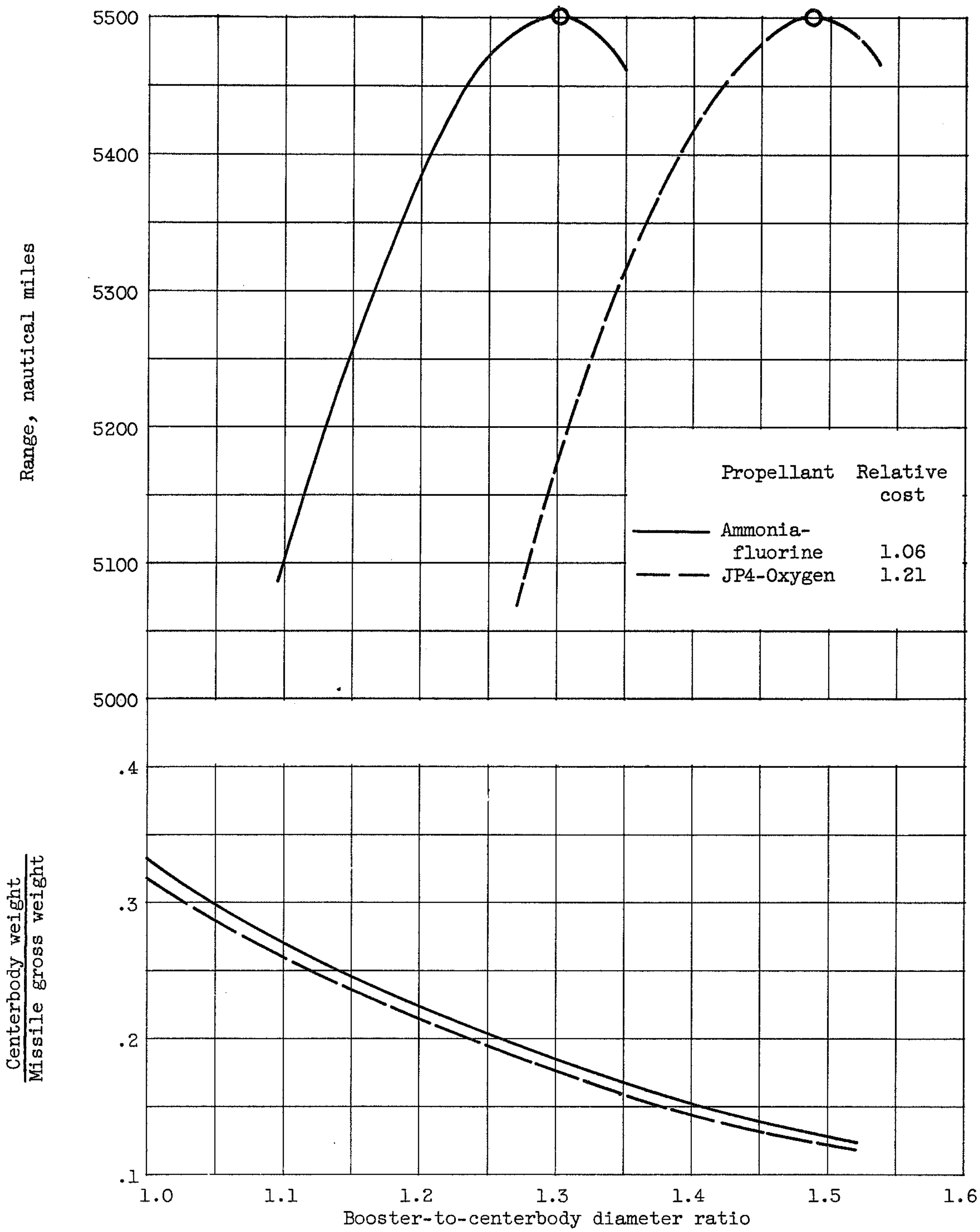
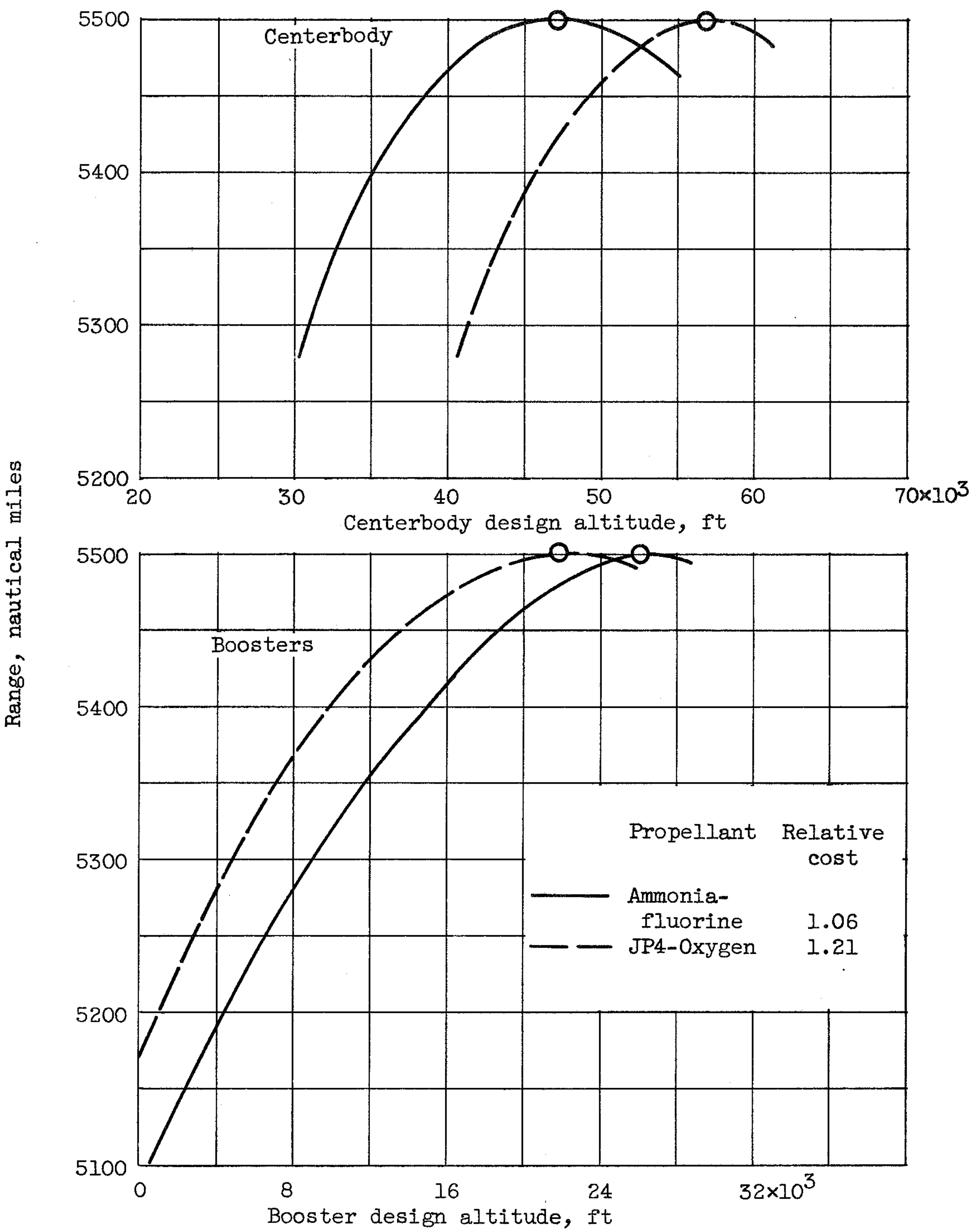


Figure 5. - Reduction in specific impulse due to drag during powered flight for optimum missiles.



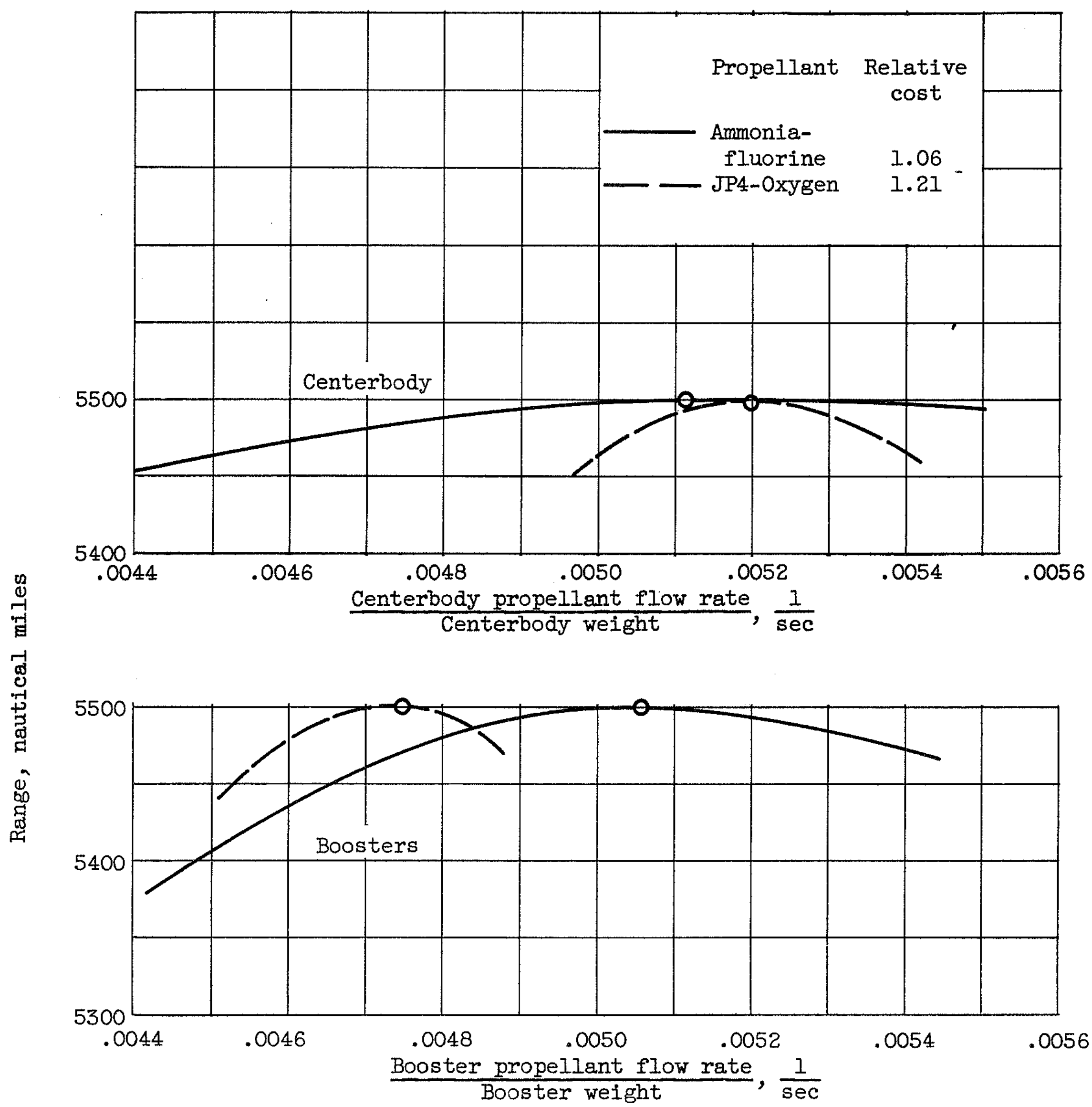
(a) Booster-to-centerbody diameter ratio.

Figure 6. - Variation of range with design parameters at constant relative cost.



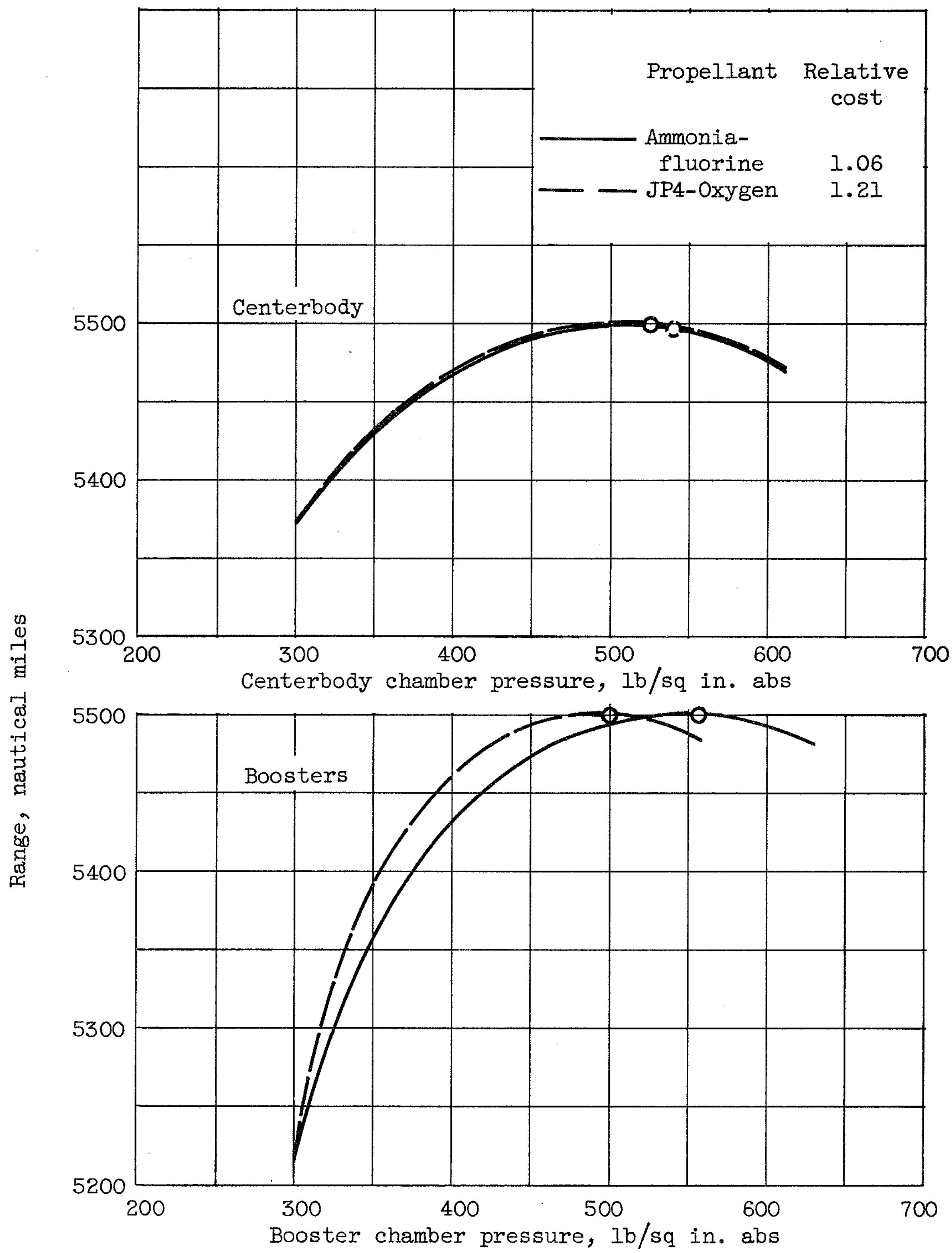
(b) Design altitude.

Figure 6. - Continued. Variation of range with design parameters at constant relative cost.



(c) Specific propellant flow rate.

Figure 6. - Continued. Variation of range with design parameters at constant relative cost.



(d) Chamber pressure.

Figure 6. - Concluded. Variation of range with design parameters at constant relative cost.

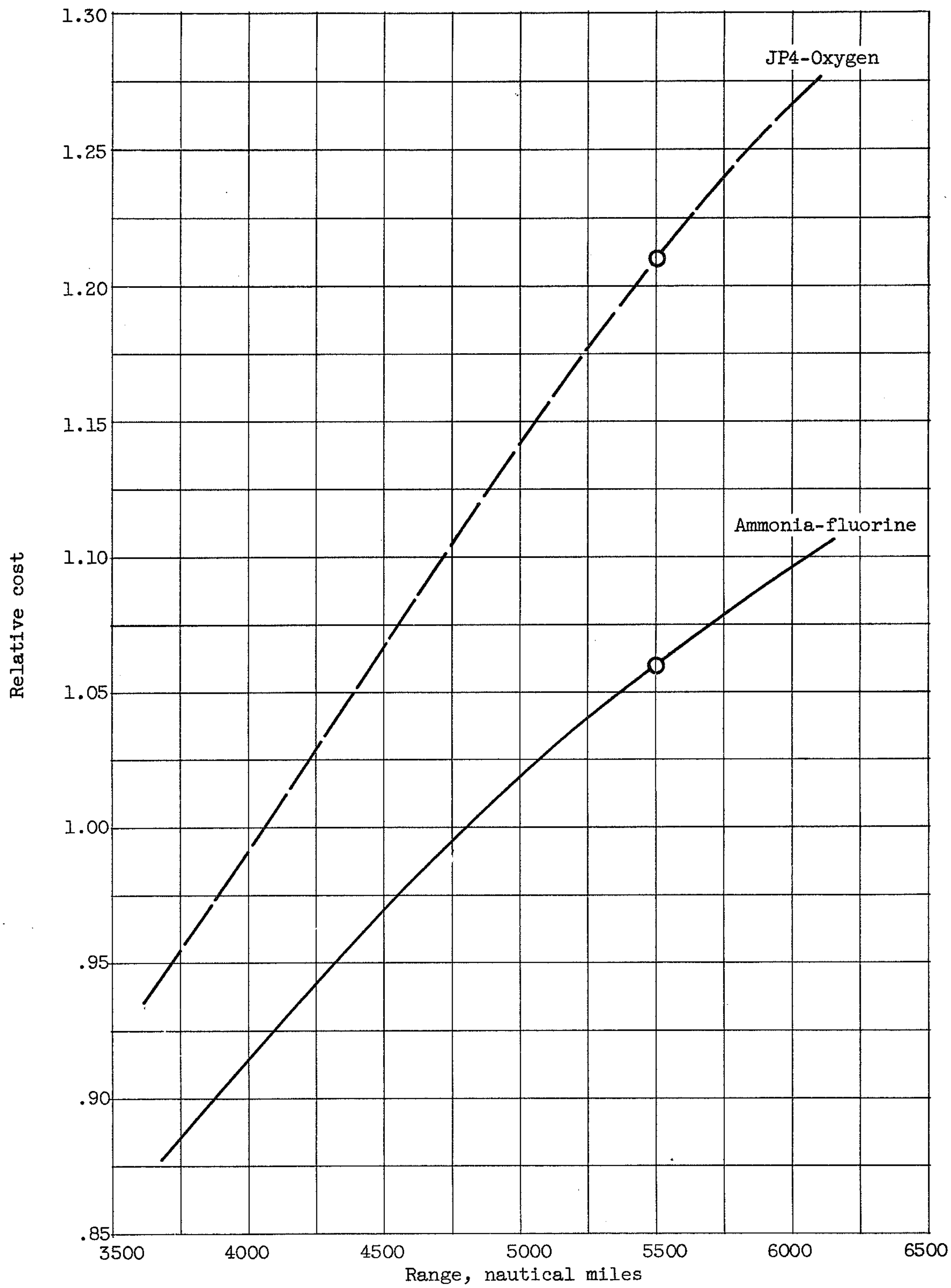
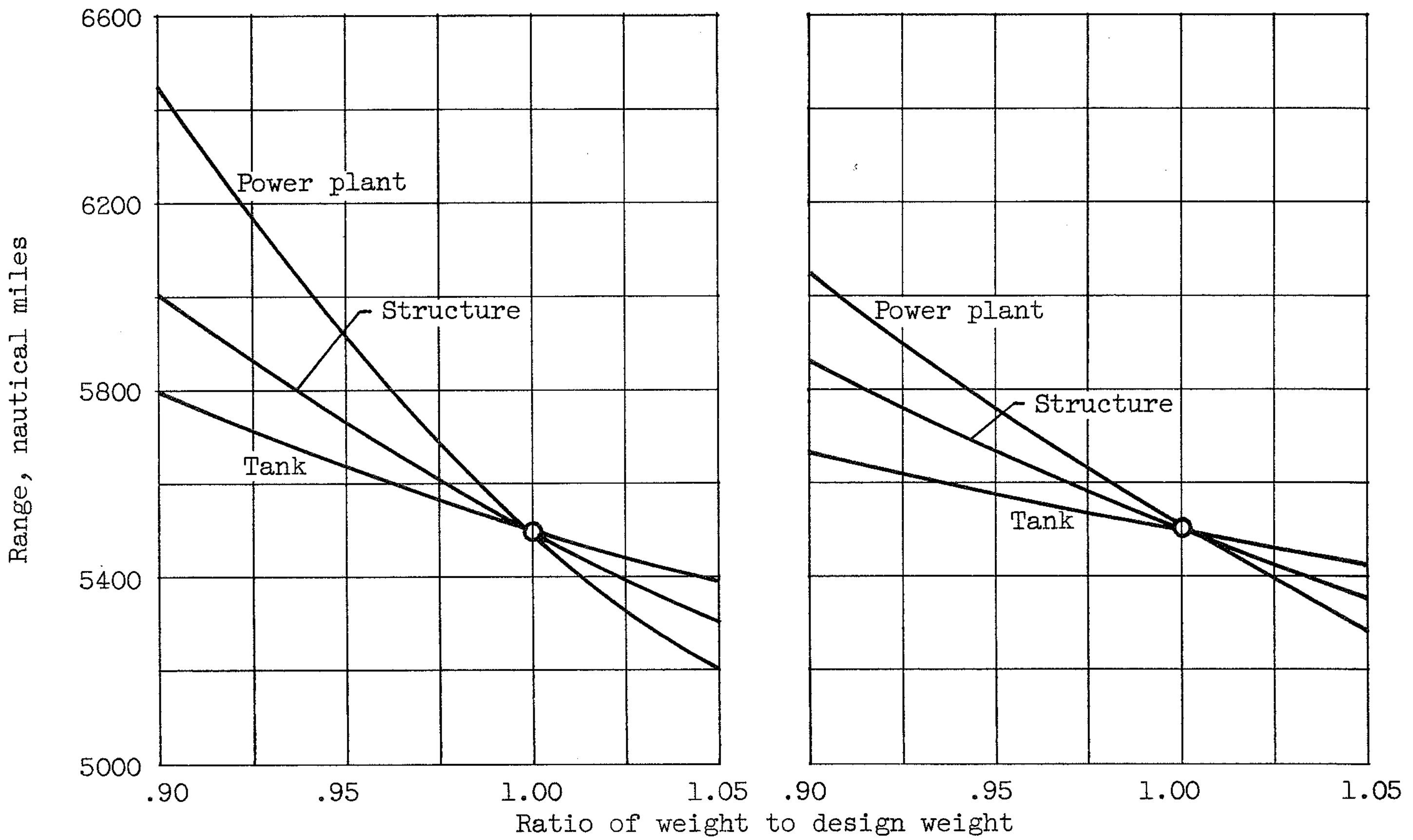


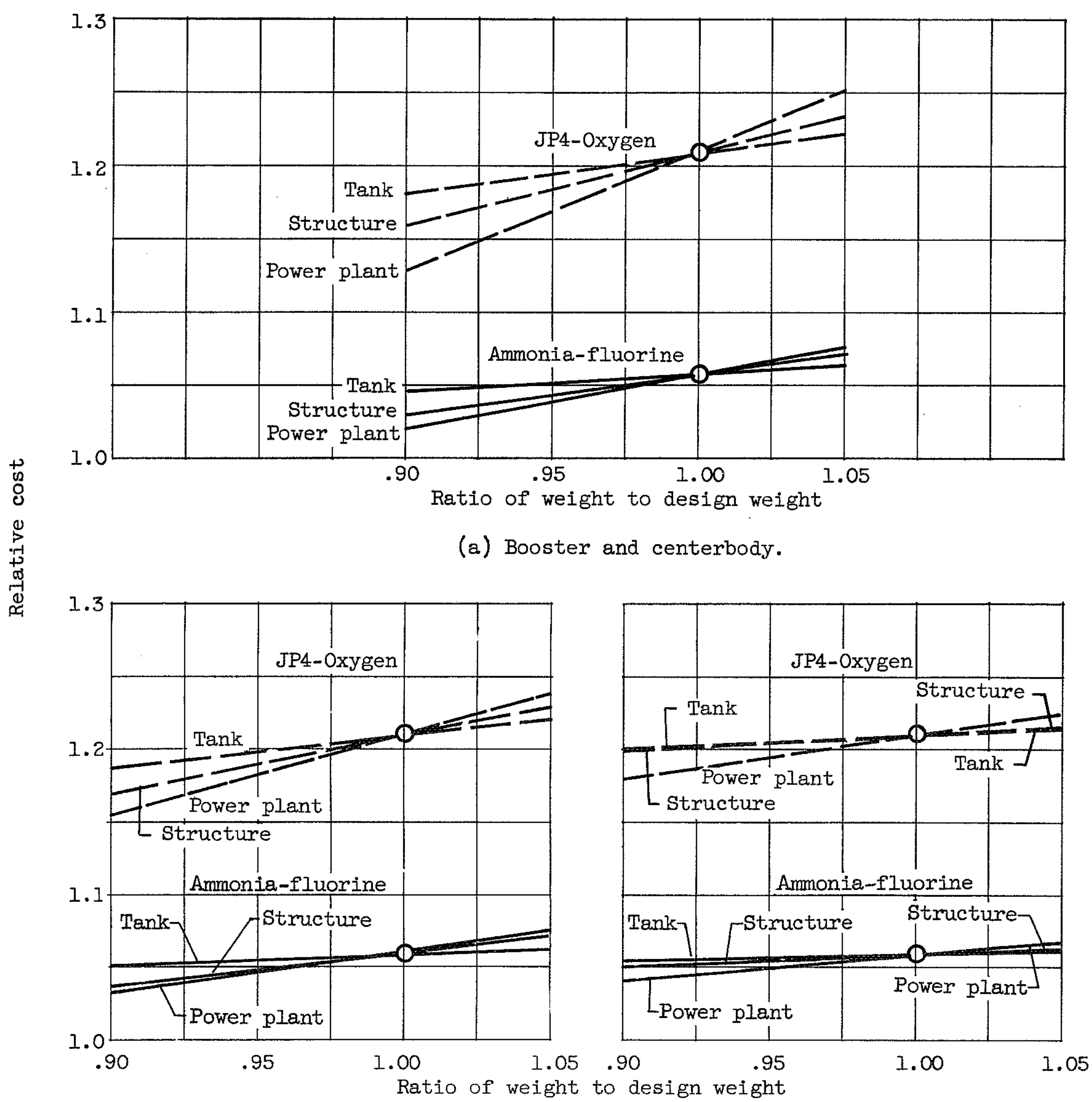
Figure 7. - Effect of range on relative cost of missile.

[redacted]



(a) JP4-Oxygen. Relative cost, 1.21. (b) Ammonia-fluorine. Relative cost, 1.06.

Figure 8. - Effect on range of variation of component weights for boosters and centerbody at constant relative cost.



(a) Booster and centerbody.

(b) Booster.

(c) Centerbody.

Figure 9. - Effect on relative cost of variation of component weights for range of 5500 nautical miles.

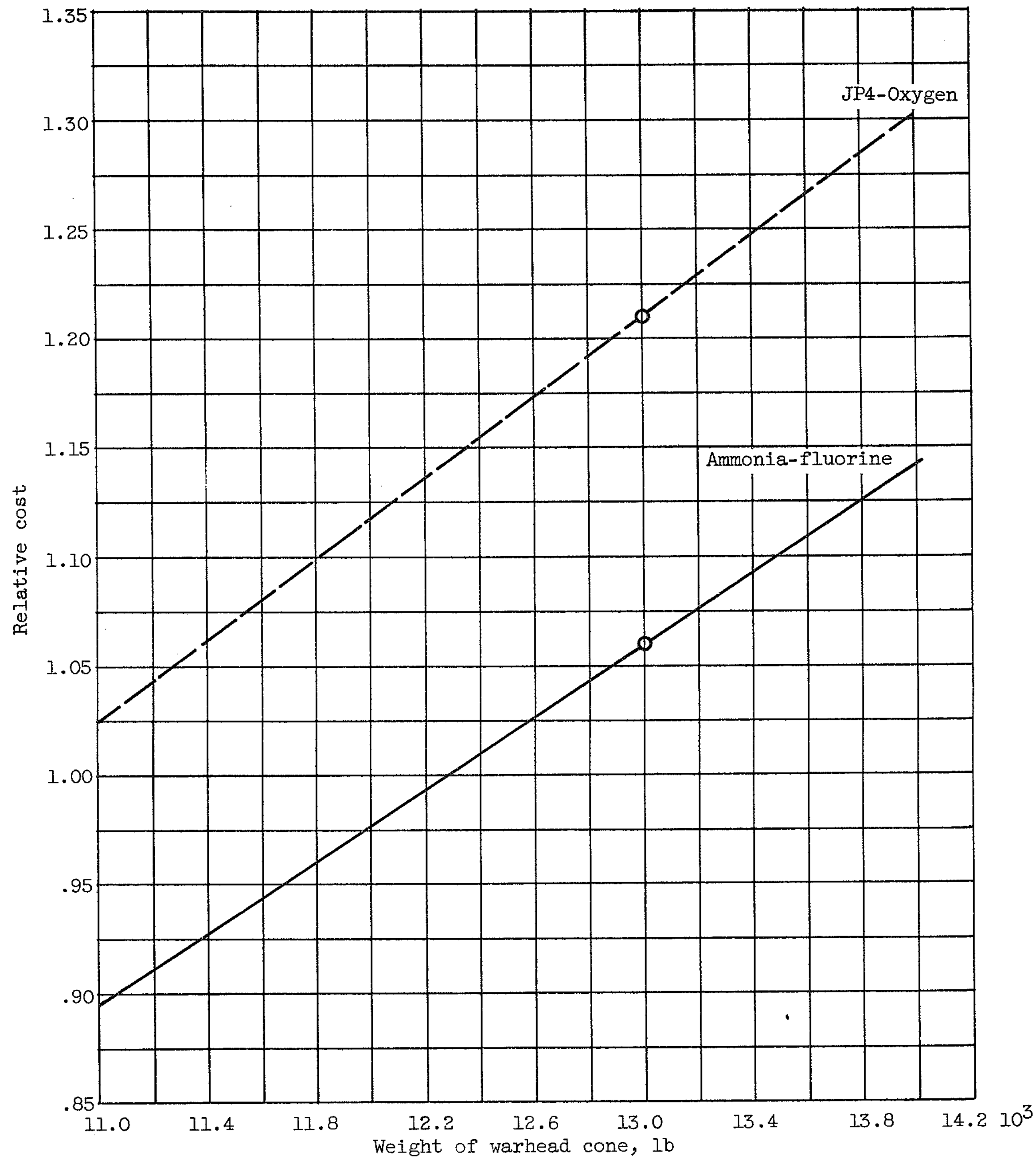


Figure 10. - Effect on relative cost of changes in weight of warhead cone (including warhead, 7000 lb).

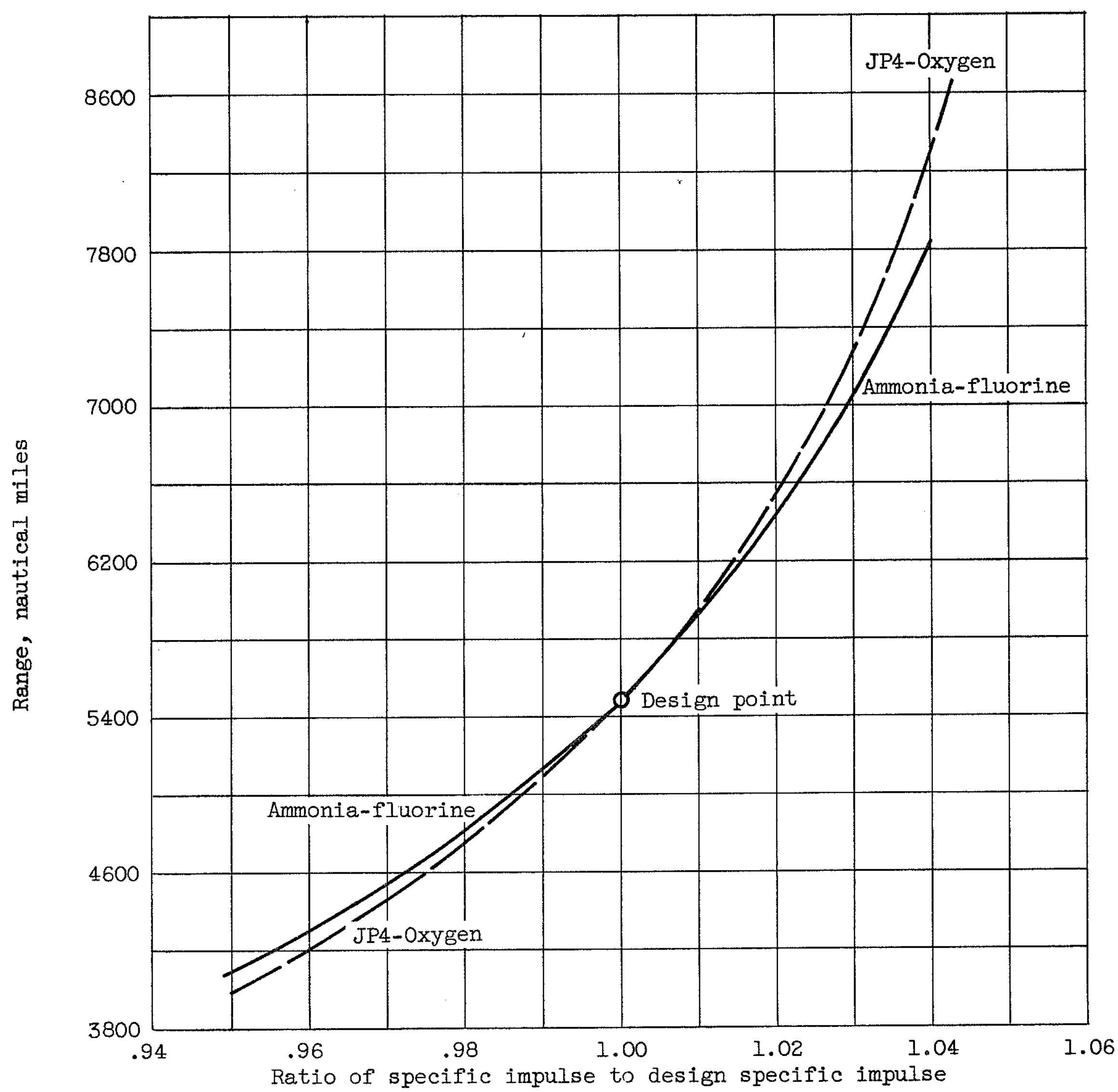


Figure 11. - Effect on range of variation of specific impulse.

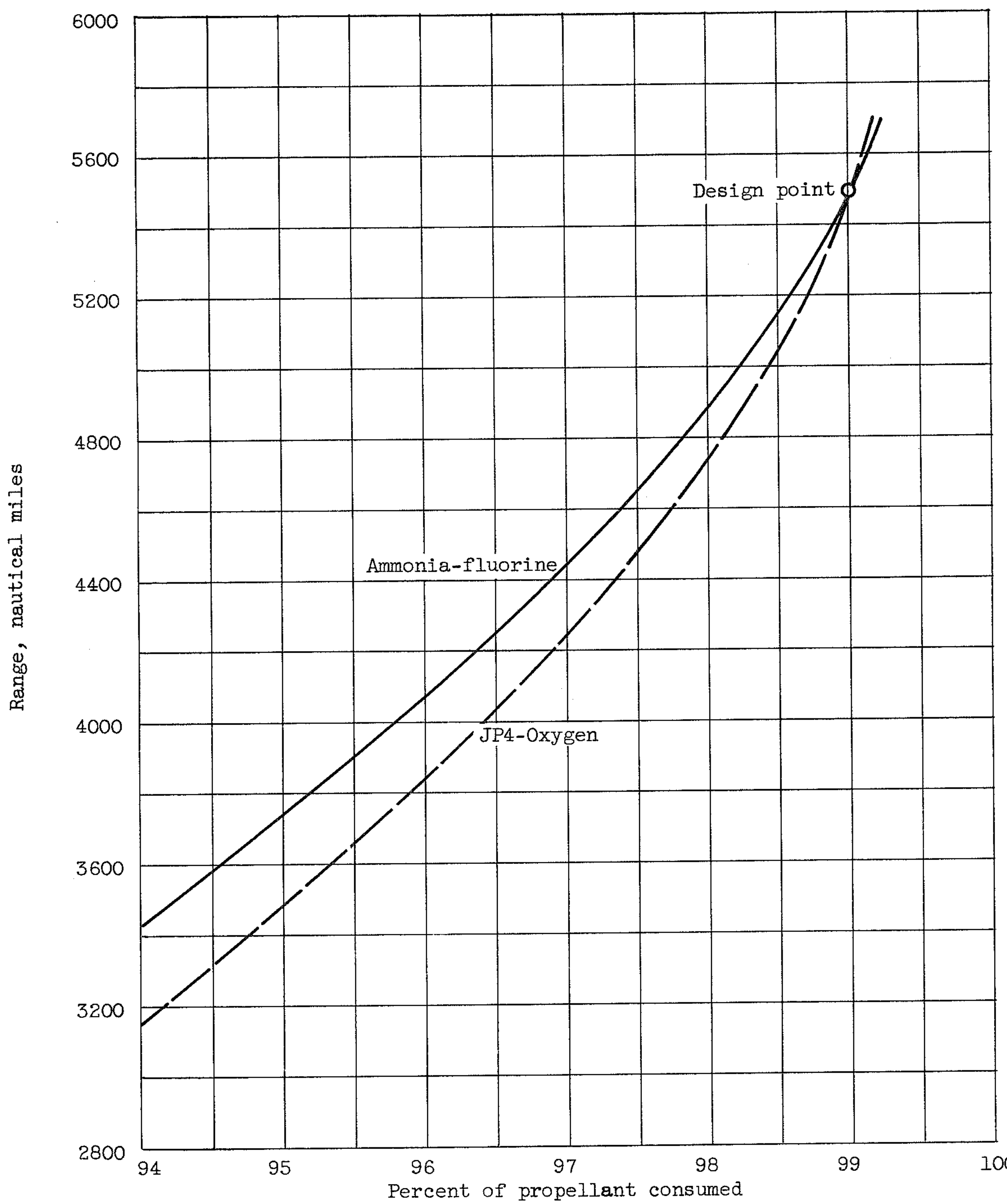


Figure 12. - Effect of failure to utilize all of propellant carried in missile.

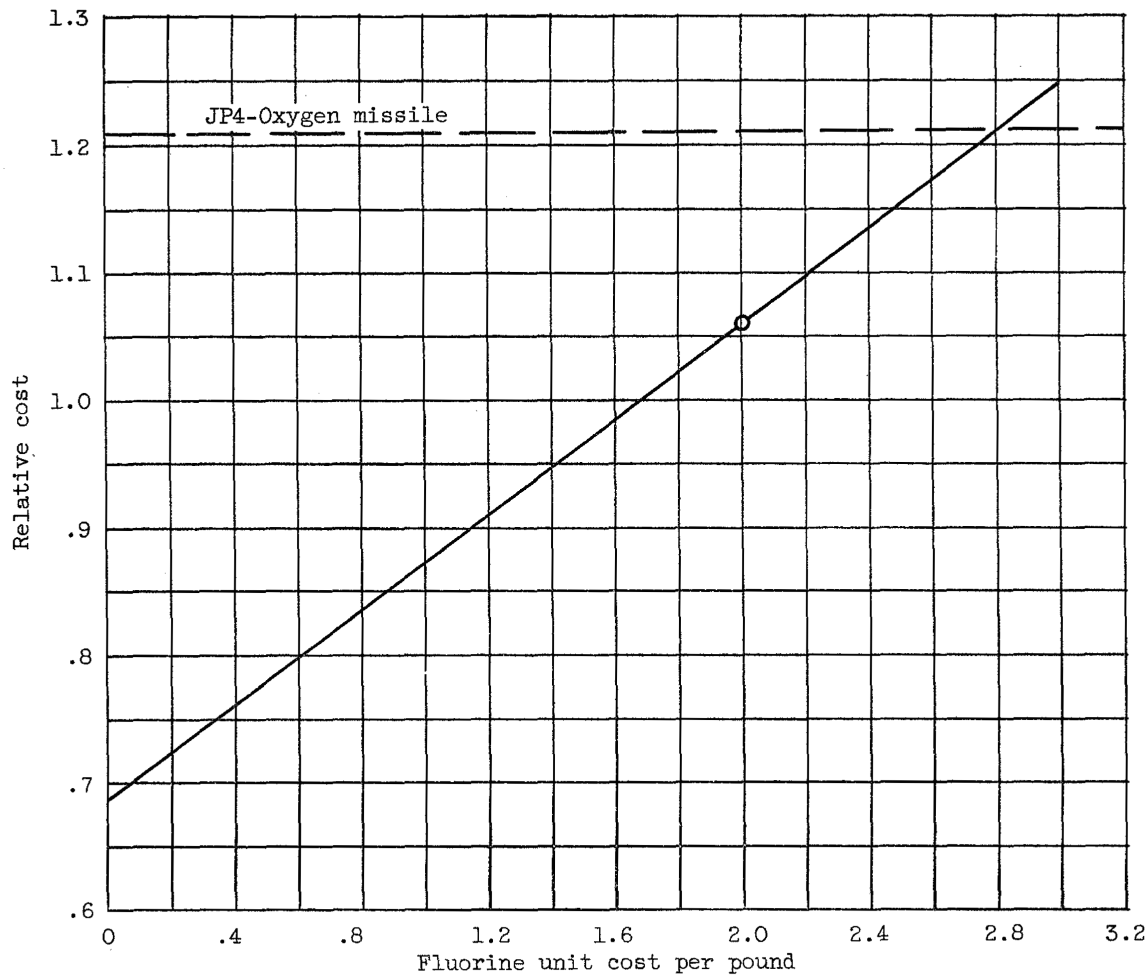


Figure 13. - Effect of unit cost of fluorine on relative cost of ammonia-fluorine missile.

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Unclassified when detached from rest of report

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