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EXPERIMENTS ON THE RECOVERY OF WASTE HEAT  
IN COOLING DUCTS

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## SUMMARY

Tests have been conducted in the N.A.C.A. full-scale wind tunnel to investigate the partial recovery of the heat energy which is apparently wasted in the cooling of aircraft engines. The results indicate that if the radiator is located in an expanded duct, a part of the energy lost in cooling is recovered; however, the energy recovery is not of practical importance up to airplane speeds of 400 miles per hour. Throttling of the duct flow occurs with heated radiators and must be considered in designing the duct outlets from data obtained with cold radiators in the ducts.

## INTRODUCTION

The possibility of recovering a small part of the heat energy wasted in the cooling air of aircraft engines has been indicated by theoretical considerations (reference 1). Calculations show that if the waste heat is added at a pressure above that of the external stream to the air flowing through a duct, the momentum of the air is increased and a thrust may be derived. Experiments have been made in the N.A.C.A. full-scale wind tunnel to verify this energy recovery in wing cooling ducts as an extension of the previous tests (reference 2) which were made with cold radiators.

The wing and duct arrangements are the same as described in reference 2. The heat was added in the duct by an electrical resistance unit used to simulate the radiator. Tests were made of four of the ducts in which the effects of adding heat were determined by measurements of the drag, quantity of air flow through the duct, air temperature rise across the heater, static pressure at the heater, and the electrical input. The section drag coefficients of the duct were measured by means of the momentum method to avoid the inaccuracy arising from measurements of small changes in the duct drag as differences in the drag on the complete wing.

The experiments were of necessity planned to utilize the available equipment and power resources, and optimum tunnel speeds and power input could not be attained. The tests should therefore be considered as preliminary to indicate the nature of the effects and to develop the testing technique.

### SYMBOLS

- Q, heat content or mass flow.
- $\gamma$ , ratio of specific heats,  $C_P/C_V$ .
- $C_P$ , specific heat at constant pressure.
- T, absolute temperature.
- p, absolute pressure.
- $\eta_Q$ , efficiency of heat cycle.
- $\eta_M$ , efficiency of momentum recovery.
- $\eta_T$ , over-all efficiency of heat exchange.
- $\rho$ , density.
- $\sigma$ , density ratio.
- V, air speed.
- $V_R$ , velocity at the radiator based on the frontal area of the radiator ( $V_R = V_1$  for cold radiator).
- $V_1/V$ , flow ratio.
- q, dynamic pressure.
- c, chord of wing at cooling duct.
- H, total head.
- $c_{d_0}$ , section profile drag coefficient.
- $A_R$ , radiator frontal area.

- K, ratio of pressure drop across radiator to dynamic pressure at face of radiator ( $\Delta P/q_R$ ).
- P, power input.
- $\Delta P$ , pressure drop across radiator.
- $\Delta P_1$ , power recovered from decrease in drag.
- $\Delta P_2$ , power to equalize flow.
- $\Delta P_3$ , power loss due to heating of radiator.
- $\Delta P_Q$ , power regained from heat cycle.

### RÉSUMÉ OF THEORY

#### Efficiency of Heat Recovery

If heat is added to the air flowing through a duct (fig. 1) the increase in kinetic energy of the air equals the heat added in the duct,  $Q$ , minus the heat retained in the exit,  $Q_1$ . The efficiency of the heat cycle,  $\eta_Q$ , is therefore equal to

$$\eta_Q = \frac{Q - Q_1}{Q}$$

Since

$$Q = C_p(T_2 - T_1)$$

and

$$Q_1 = C_p(T_3 - T_0)$$

then

$$\eta_Q = \frac{(T_2 - T_1) - (T_3 - T_0)}{T_2 - T_1}$$

For the adiabatic change in the expanding and contracting sections of the duct,

$$\frac{T_1}{T_0} = \left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_3} = \frac{T_2 - T_1}{T_3 - T_0}$$

therefore

$$\eta_Q = 1 - \frac{T_3 - T_0}{T_2 - T_1} = 1 - \left(\frac{p_0}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad (1)$$

Although this efficiency,  $\eta_Q$ , indicates the proportion of the heat energy available to increase the kinetic energy of the flow, a further efficiency of conversion of the kinetic energy into thrust must be considered. The increase in kinetic energy per second is equal to

$1/2 m (V_4^2 - V_0^2)$ . The force on the duct is the increase in momentum per second equal to  $m (V_4 - V_0)$ . Since the airplane velocity is  $V_0$ , the energy recovery from this thrust equals  $m V_0 (V_4 - V_0)$ . The efficiency of the conversion of the kinetic energy into thrust power equals

$$\eta_M = \frac{m V_0 (V_4 - V_0)}{1/2 m (V_4^2 - V_0^2)} = \frac{2V_0}{V_4 + V_0}$$

*Handwritten notes:*  
 $\eta_M = \frac{m V_0 (V_4 - V_0)}{1/2 m (V_4^2 - V_0^2)} = \frac{2V_0}{V_4 + V_0}$   
 where if  $V_4$  is infinite, with limit  $V_4 \rightarrow \infty$ ,  $\eta_M \rightarrow 0$   
 without  $V_4$   $\eta_M = 1$

The over-all efficiency of the heat exchange,  $\eta_Q \eta_M$ , equals

$$\eta_T = \left[ 1 - \left(\frac{p_0}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \right] \left[ \frac{2V_0}{V_0 + V_4} \right] \quad (2)$$

In the usual case,  $V_0 \cong V_4$ , and  $\eta_M$  is about 1. A curve of  $\eta_Q$  against  $V$  is shown in figure 2. In computing the value of  $\eta_Q$  for this curve it has been assumed that 0.8 of the free stream dynamic pressure is converted into static pressure in the duct.

## Power Loss in Heated Radiator

The net gain realized by adding heat in a duct is decreased by the higher power consumption in the heated radiator. The power absorbed by a radiator is proportional to  $\rho_R V_R^3$ . For equal cooling with a constant temperature difference at the radiator the  $\rho_R V_R$  is essentially independent of the absolute air temperature, and since

$$\rho_R = \rho_1 \frac{T_1}{T_R}$$

then

$$V_R = V_1 \frac{T_R}{T_1}$$

and

$$\rho_R V_R^3 = \left( \frac{T_R}{T_1} \right)^2 \rho_1 V_1^3$$

If the mean radiator temperature,  $T_R$ , is assumed to be  $\frac{T_1 + T_2}{2}$ , then

$$\rho_R V_R^3 = \left( \frac{T_1 + T_2}{2T_1} \right)^2 \rho_1 V_1^3 \quad (3)$$

For a rise in air temperature  $T_2 - T_1$  of  $100^\circ$  F., the radiator loss may be increased as much as 20 percent. This power loss defined as  $\Delta P_3$  is independent of air speed and is, therefore, of greatest importance at the lower air speeds in which case the energy recovery from the heat cycle is only a fraction of the energy lost in

the radiator. The value of the function  $\left( \frac{T_1 + T_2}{2T_1} \right)^2 - 1$

which is the ratio of the increased power absorbed in the hot radiator to the power absorbed in the cold radiator is shown in figure 3 for various values of  $T_2 - T_1$ .

### Throttling of Flow in Heated Ducts

The addition of heat in a radiator duct also serves to throttle the air flow. It may be readily shown that adding heat to a slow-moving air stream does not greatly change the total pressure, so that at the outlet the total pressure is about the same with either the hot or the cold radiator. The static pressure at the outlet is independent of the conditions within the duct, so that the dynamic pressure at the outlet must also be the same for either case. (Subscripts h and c in the following equations refer to hot and cold radiators.)

Since

$$q_3 = 1/2 \rho_3 V_3^2$$

and

$$\rho_{3h} = \rho_{3c} \times \frac{T_{3c}}{T_{3h}}$$

then

$$V_{3h} = V_{3c} \sqrt{\frac{T_{3h}}{T_{3c}}}$$

therefore

$$\rho_{3h} V_{3h} = \rho_{3c} V_{3c} \sqrt{\frac{T_{3c}}{T_{3h}}} \quad (4)$$

and

$$\Delta \rho_3 V_3 = 1 - \sqrt{\frac{T_{3c}}{T_{3h}}}$$

For a temperature rise of the heated air of about 100° F. the mass flow will be decreased about 10 percent. The outlet area of wing cooling ducts should therefore be

slightly larger than indicated by tests with cold radia-

tors. Values of the function  $1 - \sqrt{\frac{T_{3c}}{T_{3h}}}$ , which repre-

sents the decrease in flow due to the heat, are plotted in figure 3 for various values of  $T_2 - T_1$ .

#### EXPERIMENTAL METHOD AND TESTS

The tests were conducted in the N.A.C.A. full-scale wind tunnel which is described in reference 3. The duct and wing arrangement shown in figure 4 are described in reference 2, and a drawing of the wing showing the installation of the heating unit is shown in figure 5.

Four ducts were investigated which were similar in all respects except for the inlet size and position. In the nomenclature of reference 2, these ducts are:

<u>Number</u>	<u>Designation</u>
1	4.6 - 2a - 4 - 70
2	6.0 - 2a - 4 - 70
3	4.9 - 0 - 4 - 70
4	6.0 - 0 - 4 - 70

The ducts will be hereafter referred to as No. 1, 2, etc.

The electrical heater used in the duct consisted of grids of flat bars connected so as to absorb the available power of 100 kw. The resistance units were connected so that more of the power was absorbed at the middle of the duct than at the edges.

The temperatures of the air behind the heater were measured by means of four calibrated thermocouples. The temperature of the air in the wake behind the wing was measured by a vertical rack of thermocouples which enabled the distribution of heat across the wake to be determined.

The quantity of air flowing through the duct and the



pressure in the rear of the radiator were determined from measurements of the total and static pressure over a section of the duct near the outlet. The pressure drop through the heater grid was measured for a range of air speeds.

The section drag coefficients of the ducted wing were computed from the Jones equation (reference 4) from measurements made by means of a wake-survey rack. The wake rack consists of a comb of total head tubes and a separate comb of static tubes each about 2 feet long. A simultaneous photographic record of the pressures in all the tubes was obtained by means of a multiple manometer.

The total and static pressure in the duct, air temperatures behind the radiator, air temperatures in the wake, electrical input to heater, and wake pressures were measured for each of the ducts at a lift coefficient of  $C_L = 0.24$  (corresponding approximately to the high-speed condition) at a tunnel speed of 83 miles per hour. The wake measurements were made for the lateral stations at the duct center and 1 foot from the duct center.

## RESULTS AND DISCUSSION

In the usual case the section-drag coefficients of a wing are obtained by means of the Jones equation (reference 4) as

$$c_{d_o} = \frac{2}{c} \int^W \sqrt{\frac{(H - p)}{(H_o - p_o)}} \left[ 1 - \sqrt{\frac{(H - p_o)}{(H_o - p_o)}} \right] \quad (5)$$

When heat is added in the wing the air density in the wake is changed, and the equation should be modified to

$$c_{d_o} = \frac{2}{c} \int^W \sqrt{\frac{\sigma(H - p)}{(H_o - p_o)}} \left[ 1 - \sqrt{\frac{(H - p_o)}{\sigma(H_o - p_o)}} \right] \quad (6)$$

in which  $\sigma$  equals  $\rho_{4h} / \rho_o$ . In order to check the assump-

tion used in deriving equation (6), measurements of the drag and heat content were made at a number of different distances behind the trailing edge for one of the wing duct arrangements. These results are shown in figure 6, and it may be noted that the drags measured at different distances behind the trailing edge are in reasonable agreement. The low value obtained at the 72-inch station may be due to the lower accuracy of the momentum method at the greater distance behind the trailing edge.

The heat contents at the different stations are in good agreement. The effect of the heat in decreasing the drag is clearly shown in figure 6 by the low momentum loss in the part of the wake in which the heat was concentrated.

A summary of the test results is shown in table I. The reduction in the section-drag coefficient due to adding heat in the duct is about 0.0020 for all the ducts. This drag increment varied from about 10 to 15 percent of the section-drag coefficient of the wing and duct, and was therefore measured with considerable accuracy.

Representative curves are shown in figure 7 of the total and static pressure near the duct outlet, from which data the quantity of air flow through the duct was computed. The flow through the duct was throttled from 10 to 15 percent by the addition of the heat. This is slightly more than would be calculated from the data of figure 3; however, the effect of the increased resistance of the heated radiator on the flow which was omitted from the previous discussion may readily account for the difference. Data showing the effects of varying the resistance in the duct on the air flow are shown in reference 2.

The horsepower input into the duct per foot was obtained by averaging the integrated value of the heat content in the wake at the two measuring stations (fig. 6). The temperature difference across the radiator was determined from the thermocouples.

The power recovered per foot of duct width,  $\Delta P_1$ , was calculated from the decrease in drag and the velocity, as

$$\Delta P_1 = \frac{(\Delta c_d q c) V}{550}$$

10

Since

$$V = 83.0 \text{ miles per hour}$$

and

$$c = 9.67 \text{ feet}$$

then

$$\Delta P_1 = 37.8 \Delta c_d$$

A comparison of the duct resistance, with and without heat, must necessarily be made for equal mass flows. The comparison is made in table I for the flow corresponding to the heated radiator case, and it is therefore necessary to credit the cold duct with the difference in internal power for the two flows. This power difference,  $\Delta P_2$ , was computed on the assumption that the external losses remained constant for the range over which the  $V_R/V$  changes.

The power loss in the radiator is  $Q\Delta P$  in which  $\Delta P$  is  $Kq_R$ . The value of  $K$  was determined from measurements of the pressure drop across the radiator, the values of which are shown in figure 8 for a range of air speeds.

Since

$$Q\Delta P = \frac{K\rho V_1^3 A_R}{2 \times 550}$$

then

$$\Delta P_2 = \frac{KA_R \rho (V_{1c}^3 - V_{1h}^3)}{1100}$$

The increased power absorbed by a hot radiator,  $\Delta P_3$ , has been discussed in the résumé of the theory. Knowing the temperature difference,  $T_2 - T_1$ , the increased power due to heating may be obtained from figure 3 as a fraction of the power absorbed by the cold radiator. This increment tends to decrease the useful power that may be recovered in the duct.

In order to compute the power regained due to the adiabatic expansion of the air after the heat is added, it

is necessary to debit the measured drag differences with the power required to adjust the flows to the same quantity and to credit it with the power lost due to the higher resistance of the hot radiator. That is,

$$\Delta P_Q = \Delta P_1 - \Delta P_2 + \Delta P_3$$

The values of  $\Delta P_Q$  which are given in table I show a considerable scatter which is due largely to the fact that they are relatively small differences of larger quantities. The cycle efficiency is obtained from  $\Delta P_Q/P = \eta_Q$ . The experimental  $\eta_Q$  obtained from the foregoing relation and the theoretical value obtained from equation (1) using the pressure behind the radiator for the term  $p_1$  are shown in table I.

The experimental efficiencies are lower than the theoretical ones and the ratios of the two are somewhat inconsistent. As an average of the four cases the experimental efficiencies are about one-half of the theoretical. It will be noted that the efficiencies are low, which is due to the relatively slow speed at which the tests were made. The efficiency varies as  $V^2$  as shown in figure 2.

Design application.- From analysis of the results it is indicated that if only the heat from the radiator is added in the duct from the standpoint of performance calculations the effect of the heat may usually be neglected; that is, the duct efficiencies obtained with cold radiators may be applied with sufficient accuracy to represent the operating condition. Assuming a high speed of 400 miles per hour and a conversion of 0.8 of the dynamic pressure into static pressure in the duct, a theoretical efficiency,  $\eta_Q$ , of 4 percent is indicated (fig. 2). Further assuming the average value of 0.4 for the ratio of the heat in the cooling air to the brake horsepower, the maximum power return possible is 1.6 percent of the brake horsepower. Since the actual efficiency may be only one-half of the theoretical, the actual maximum power regain may be less than 1 percent of the brake power. From this value must be subtracted the power loss due to the heating of the radiator and several other minor losses (reference 1).

The actual regain in power even for the higher speed airplanes may, therefore, be less than 1 percent of the engine power which is smaller than the possible accuracy of the calculations.

The effect of the heat must be considered, however, in calculating the quantity of air that will flow through the duct. It will be necessary to provide a larger duct outlet and in some cases a larger radiator than indicated from tests made with cold radiators. In the present tests with temperature differences of about 80° to 100° F. across the heater, the air-flow quantities were decreased from 10 to 15 percent below those for the cold radiator tests.

This paper has discussed the possibility of recovering a part of the energy which is apparently lost in the cooling of engines. It is of interest to point out that if the heat of the exhaust is discharged into the cooling duct, a substantial recovery of power may be made. This recovery of power will be no greater, however, than that resulting from discharging the exhaust gases rearward by a suitable exhaust pipe. In effect, the cooling duct becomes an exhaust pipe, and its use for this purpose will be primarily dictated by the arrangement and detail design of the airplane. For some designs the cooling duct will provide a convenient place for expanding the exhaust gases so that they may be discharged more nearly at the free-stream velocity.

Langley Memorial Aeronautical Laboratory,  
National Advisory Committee for Aeronautics,  
Langley Field, Va., April 3, 1939.

## REFERENCES

1. Meredith, F. W.: Note on the Cooling of Aircraft Engines with Special Reference to Ethylene-Glycol Radiators Enclosed in Ducts. R. & M. No. 1683, British A.R.C., 1936.
2. Nickle, F. R., and Freeman, Arthur B.: Full-Scale Wind-Tunnel Investigation of Wing Cooling Ducts. Confidential Memorandum, N.A.C.A., 1938.
3. DeFrance, Smith J.: The N.A.C.A. Full-Scale Wind Tunnel. T.R. No. 459, N.A.C.A., 1933.
4. The Cambridge University Aeronautics Laboratory (B. Melville Jones): The Measurement of Profile Drag by the Pitot- Traverse Method. R. & M. No. 1688, British A.R.C., 1936.

TABLE I

Duct	$c_d$ cold	$c_d$ hot	$\Delta c_d$	$v_1/v$ cold	$v_1/v$ hot	Pressure rise at radiator $P_1 - P_0$ lb./ sq.ft. hot	Power input hp./ft. P	Change in air temp. at rad. $T_2 - T_1$ F deg.	Total power recov- ered hp./ft. $\Delta P_1$	Power required to pro- vide equal flow hp./ft. $\Delta P_2$	Power lost due to heat- ing of radiator hp./ft. $\Delta P_3$	Power regained from ex- pansion hp./ft. $\Delta P_Q$	Experi- mental effi- ciency of heat cycle $\eta_Q, \%$	Theoret- ical ef- ficiency of heat cycle $\eta_Q, \%$
1	.0147	.0127	.0020	.432	.398	11.1	44.3	83	.076	.094	.056	.038	.086	.148
2	.0154	.0133	.0021	.437	.389	10.9	46.0	98	.079	.127	.060	.012	.026	.146
3	.0180	.0159	.0021	.354	.302	7.7	54.0	111	.079	.089	.034	.024	.045	.102
4	.0187	.0164	.0023	.331	.286	7.2	65.0	98	.087	.069	.024	.042	.065	.095

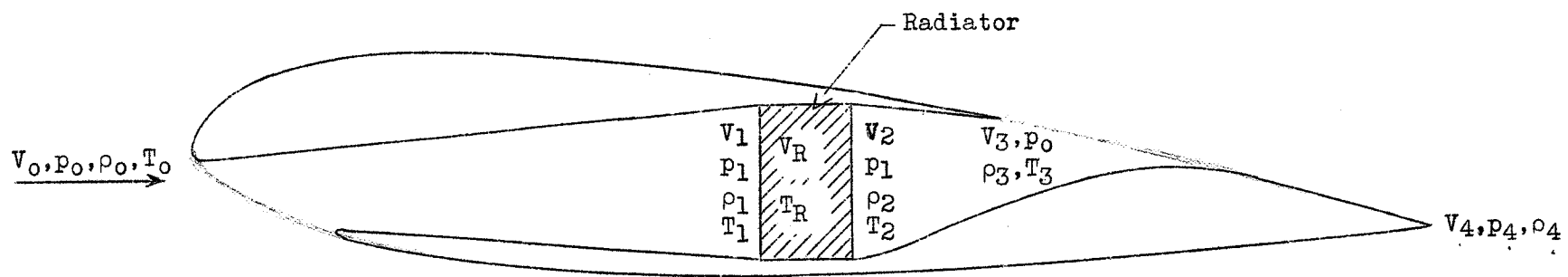


Figure 1.



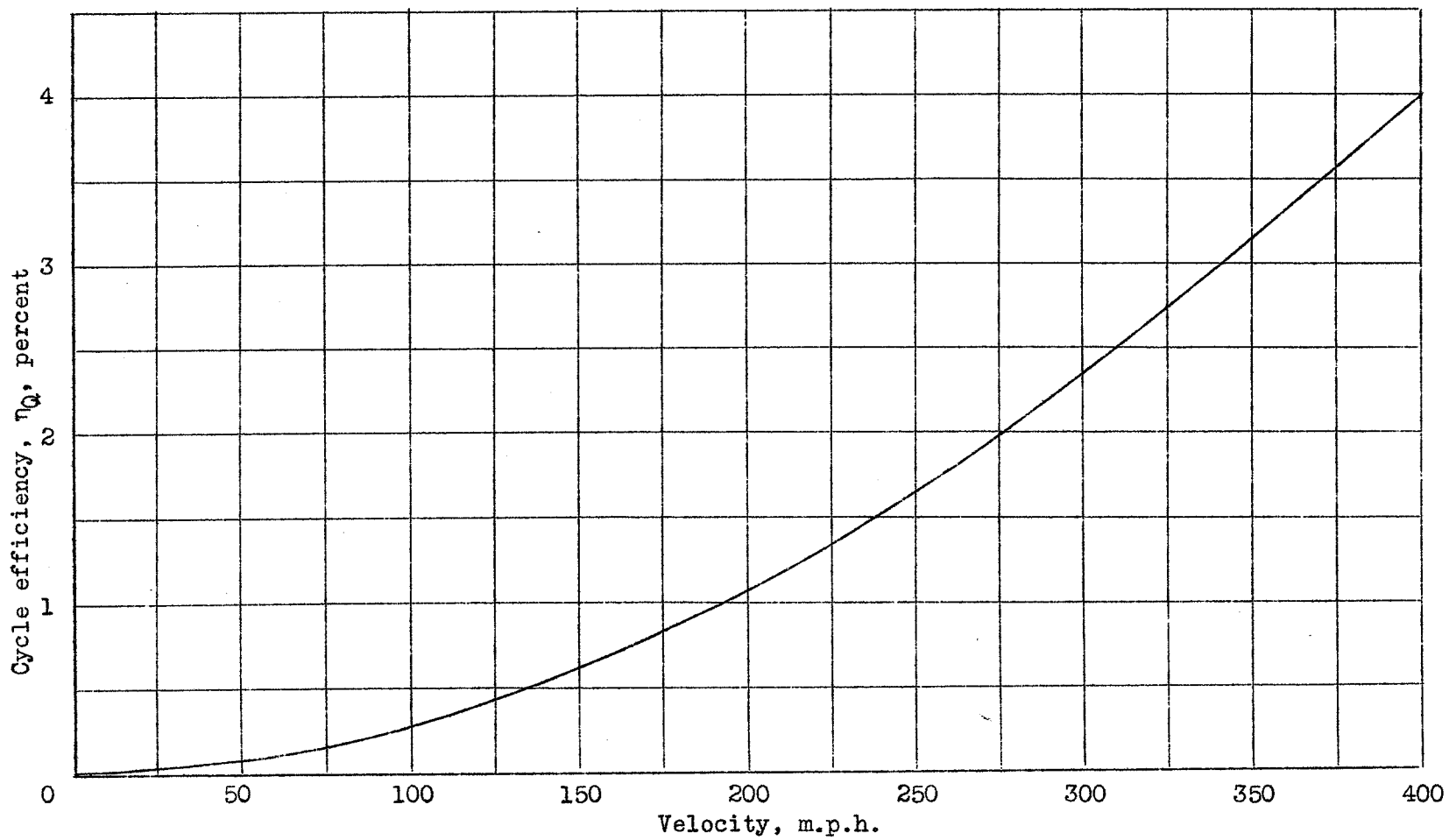


Figure 2.

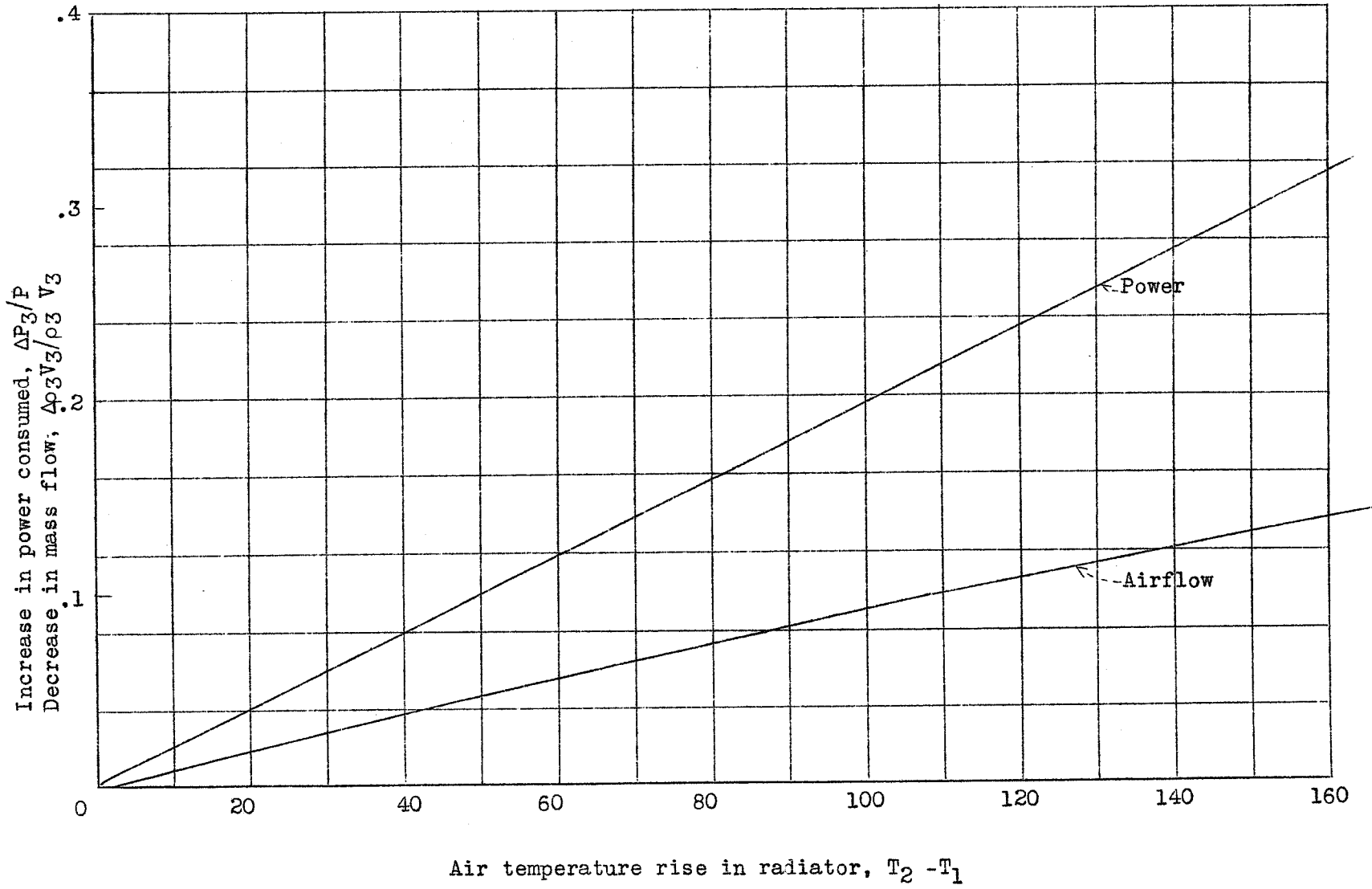


Figure 3.

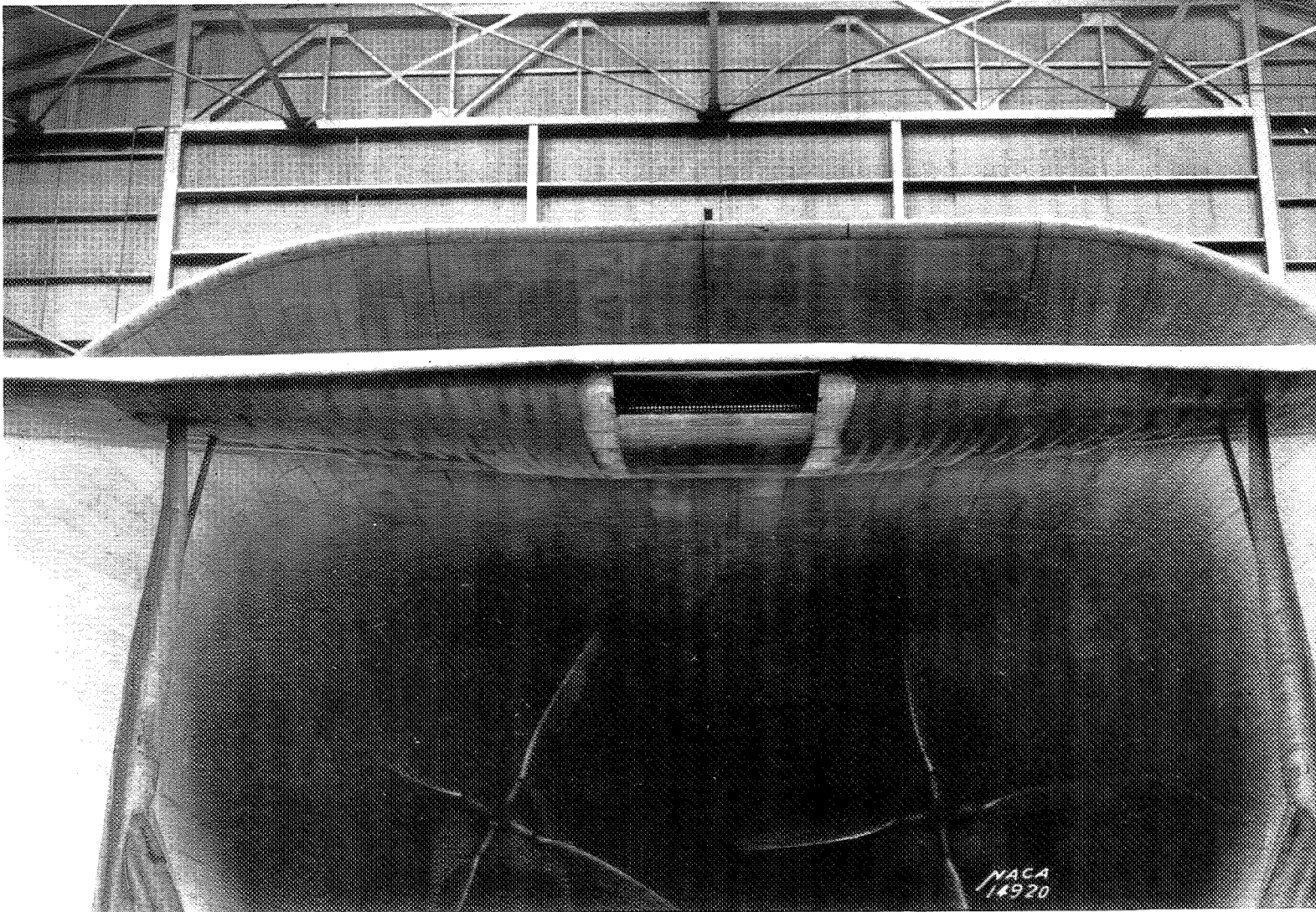
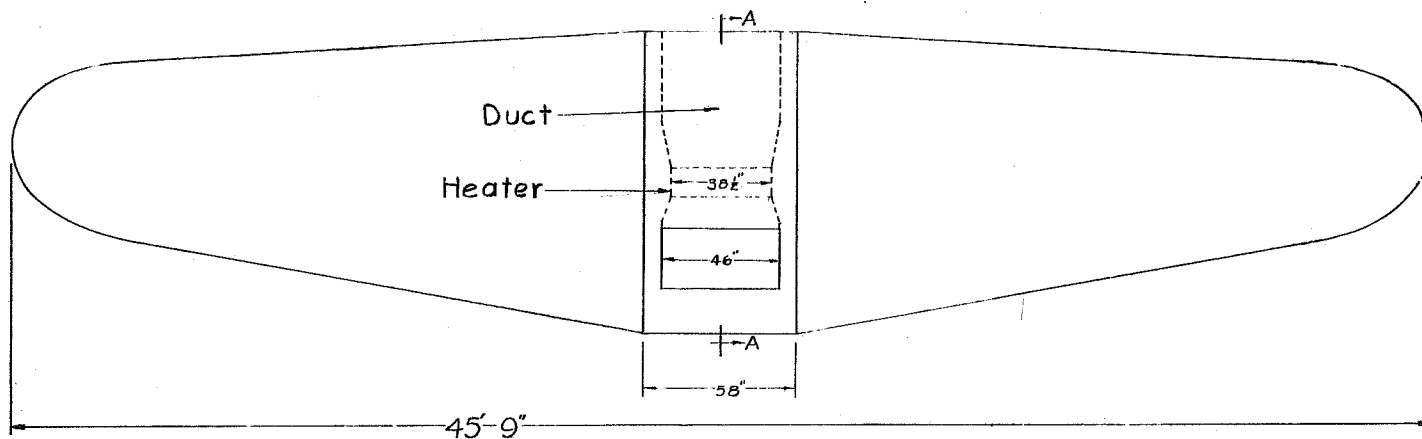
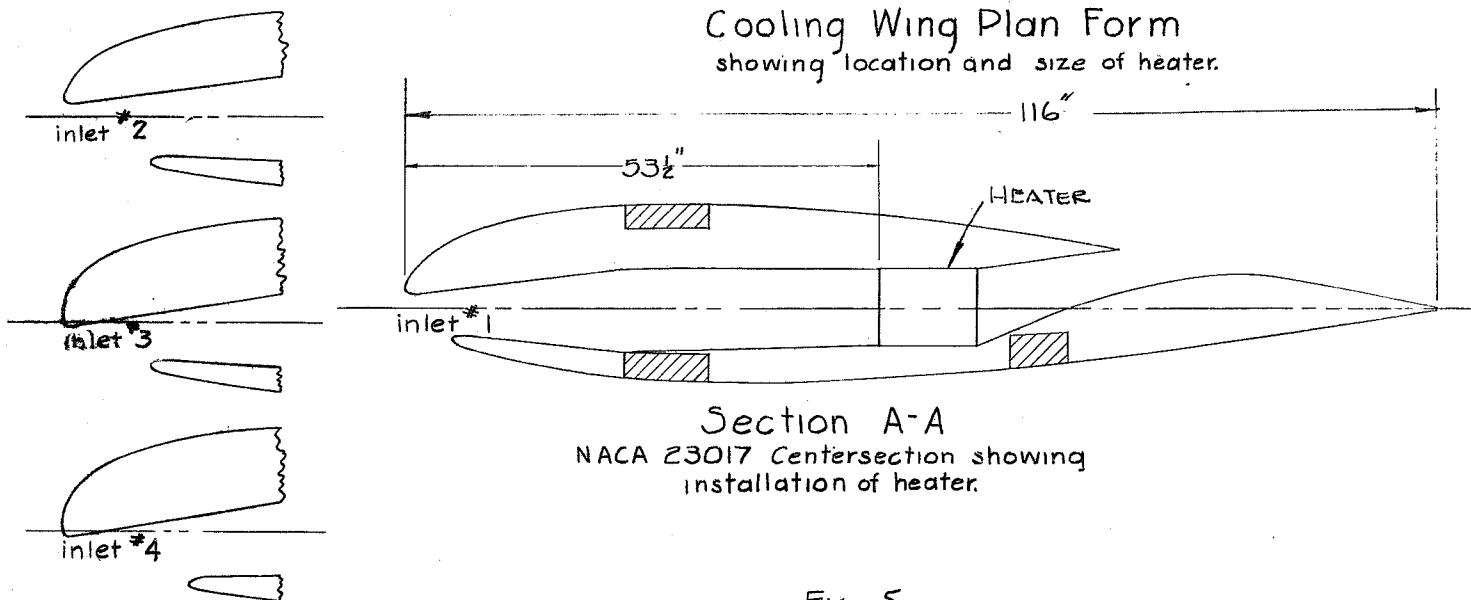


Figure 4.- Photograph of cooling wing in full-scale tunnel.

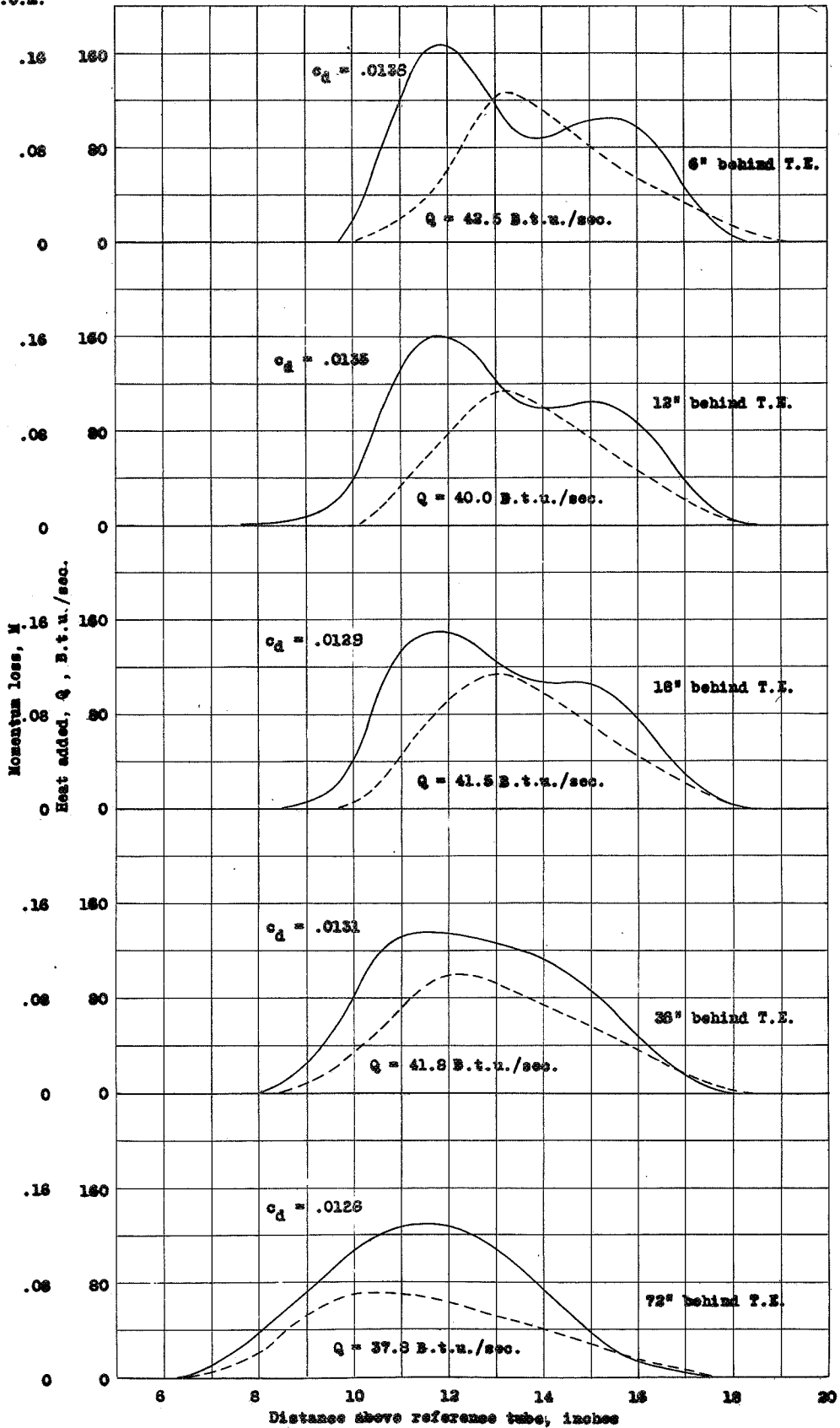


Cooling Wing Plan Form  
showing location and size of heater.



Section A-A  
NACA 23017 Centersection showing  
installation of heater.

FIG. 5



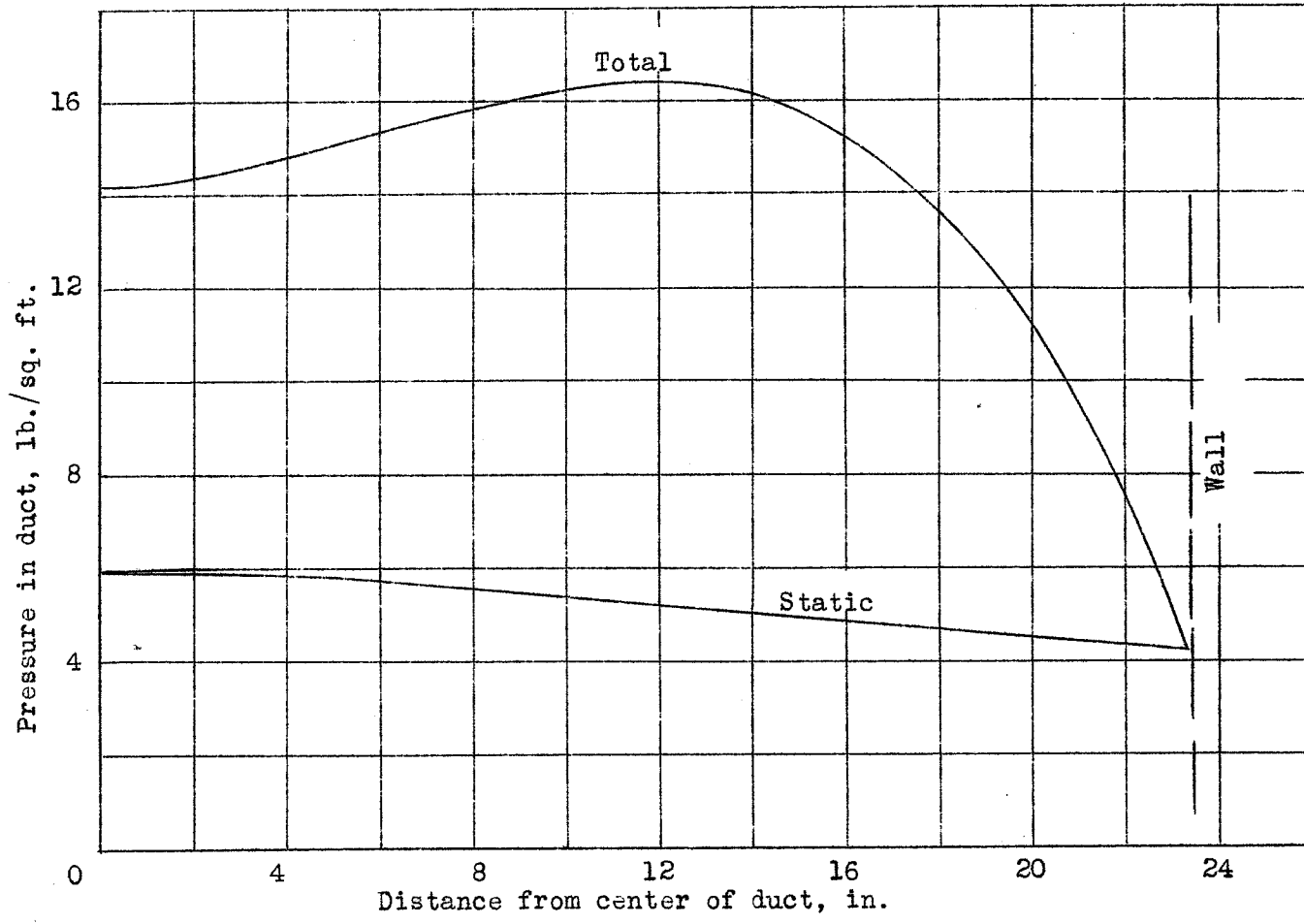


Figure 7.

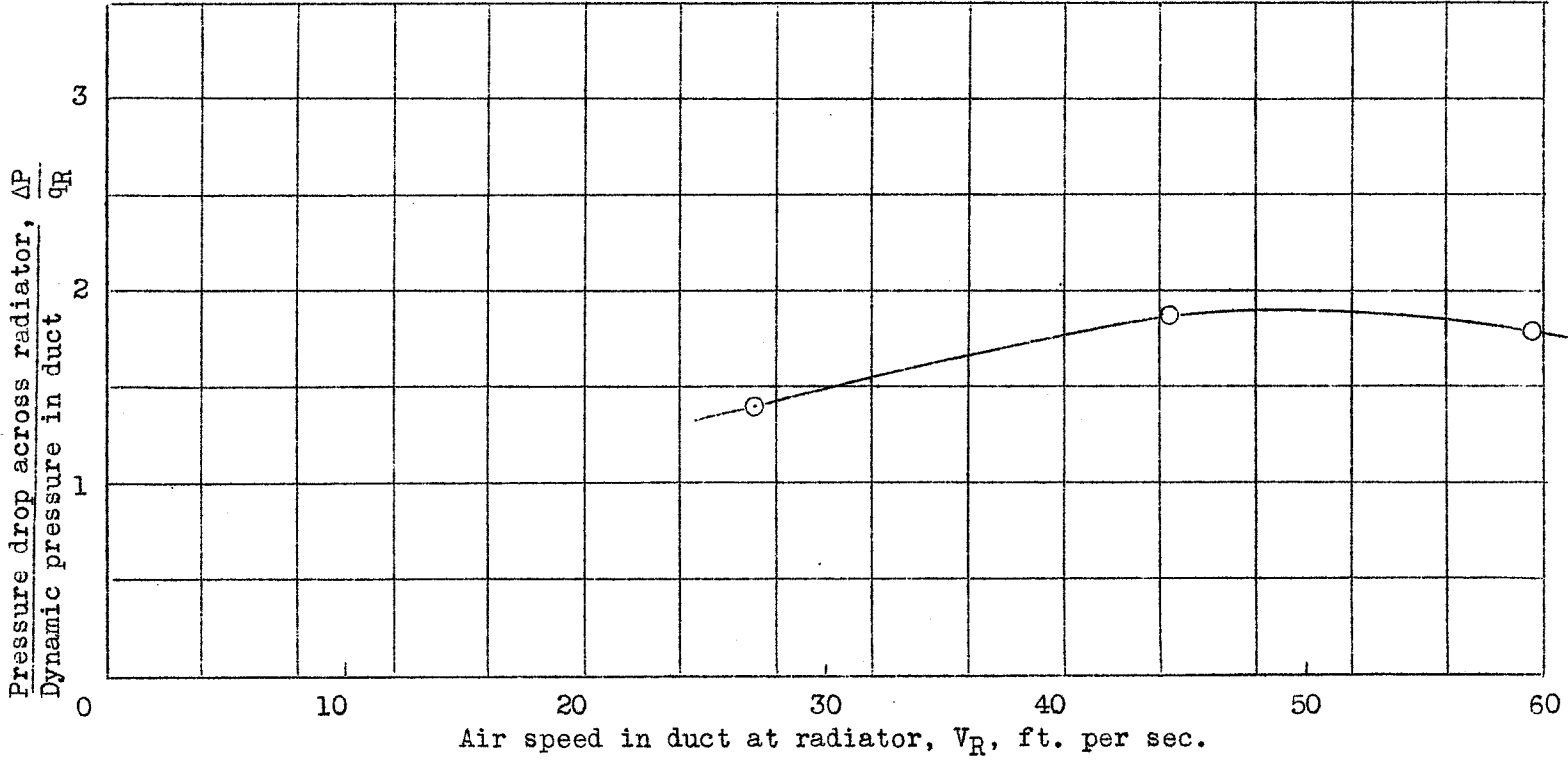


Figure 8.