

# Lorentz Invariance Violation and the Observed Spectrum of Ultrahigh Energy Cosmic Rays

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## Abstract

There has been much interest in possible violations of Lorentz invariance, particularly motivated by quantum gravity theories. It has been suggested that a small amount of Lorentz invariance violation (LIV) could turn off photomeson interactions of ultrahigh energy cosmic rays (UHECRs) with photons of the cosmic background radiation and thereby eliminate the resulting sharp steepening in the spectrum of the highest energy CRs predicted by Greisen Zatsepin and Kuzmin (GZK). Recent measurements of the UHECR spectrum reported by the HiRes and Auger collaborations, however, indicate the presence of the GZK effect. We present the results of a detailed calculation of the modification of the UHECR spectrum caused by LIV using the formalism of Coleman and Glashow. We then compare these results with the experimental UHECR data from Auger and HiRes. Based on these data, we find a best fit amount of LIV of  $4.5^{+1.5}_{-4.5} \times 10^{-23}$ , consistent with an upper limit of  $6 \times 10^{-23}$ . This possible amount of LIV can lead to a recovery of the cosmic ray spectrum at higher energies than presently observed. Such an LIV recovery effect can be tested observationally using future detectors.

*Key words:* cosmic rays; Lorentz invariance; quantum gravity

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## 1 Introduction

Because of their extreme energy and isotropic distribution, it is believed that UHECRs are extragalactic in origin. After the discovery of the cosmic back-

ground radiation (CBR), Greisen [1] and Zatsepin and Kuzmin [2] pointed out that photomeson interactions should deplete the flux of cosmic rays with energies above  $\sim 50$  EeV. One of us [3], using data on the energy dependence of the photomeson production cross section, then made a quantitative calculation of this “GZK effect” deriving the mean photomeson energy loss attenuation length for protons as a function of proton energy. These results indicated that the attenuation length of a proton with an energy greater than 100 EeV is less than 100 Mpc, which is much less than the visible radius of the universe. Thus, what is sometimes referred to as the GZK “cutoff” is not a true cutoff, but a suppression of the ultrahigh energy cosmic ray flux arising from a limitation of the proton propagation length through the cosmic background radiation owing to energy losses.

From time to time there have been reports in the literature of the detection of giant air shower events from primaries with energies above the GZK suppression energy (trans-GZK events) (*e.g.*, Refs. [4] – [6]). Such events have stimulated suggestions that a violation of Lorentz invariance or a modification of the Lorentz transformation relations at ultrahigh energies could result in a nullification of the GZK effect [7],[8]. Most significantly, the AGASA group reported 11 events above the GZK suppression energy [6], increasing the interest in the possibility of such new physics [9]. See Ref.[10] for a recent review of this topic.

However, a reanalysis of the AGASA data (unpublished) has resulted in cutting their originally reported number of trans-GZK events by half. More importantly, the HiRes [11] and Auger groups [12], with larger exposures, have very recently claimed to have found a GZK suppression effect. Motivated by these new results, we have undertaken new detailed calculations of the effect of a very small amount of Lorentz invariance violation (LIV) on the spectrum of UHECRs at Earth. We present our results here and compare them with the HiRes and Auger data separately.

## 2 Violating Lorentz Invariance

With the idea of spontaneous symmetry breaking in particle physics came the suggestion that Lorentz invariance (LI) might be weakly broken at high energies. Some of the more recent motivation for LIV has been in relating it to possible Planck scale phenomena that could lead to astrophysically observable consequences [13]. The idea that Planck scale physics may lead to a natural abrogation of the GZK effect has been of particular interest, since this would lead to a direct observational test. Significant fluxes of UHECRs at trans-GZK energies, could be the result of a very small amount of LIV [14] – [18]. Such a test would have important implications for some quantum gravity and large

extra dimension models, since those models may predict very small amount of LIV.

Although no true quantum theory of gravity exists, it is natural to tie LIV to various quantum gravity models. A few examples of such work can be found in Refs. [15] – [20]. For more references, we refer to an excellent review by Mattingly [21]. A data table of constraints on LIV and CPT violation parameters within the framework of the “Standard Model Extension” model [22] has recently been given by Kostelecky and Russell [23].

In this paper, we reinvestigate the observational implications of the possible effect of a very small amount of LIV, *viz.*, that cosmic rays could indeed reach us after originating at distances greater than 100 Mpc without undergoing large energy losses from photomeson interactions. We considered this topic before in a more simplistic manner [24] when there was a clear discrepancy between the AGASA group data [6] and the earlier HiRes data. However, as discussed above, the observational situation has changed and now requires a more detailed approach. We therefore undertook a detailed calculation of the modification of the UHECR spectrum caused by LIV using the formalism of Coleman and Glashow and the kinematical approach originally given by Alfaro and Palma [18] in the context of the Loop Quantum Gravity model [25],[26]. (See also Ref. [27].) Then, by comparing our results with the observational UHECR data we can place a quantitative limit on the amount of LIV. We also discuss how a small amount of LIV that is consistent with the observational data can still lead to a recovery of the cosmic ray flux at higher energies than presently observed.

### 3 LIV Framework

Coleman and Glashow have proposed a simple formulation for breaking LI by a small first order perturbation in the free particle Lagrangian [14]. This formalism has the advantages of (1) simplicity, (2) preserving the  $SU(3) \otimes SU(2) \otimes U(1)$  standard model of strong and electroweak interactions, (3) having the perturbative term in the Lagrangian to consist of operators of mass dimension 4 that thus preserves power counting renormalizability, and (4) being rotationally invariant in a preferred frame that can be taken to be the rest frame of the 2.7 K cosmic background radiation. This formalism has proven useful in exploring astrophysical data for testing LIV [14],[19],[28].

To accomplish this, Coleman and Glashow start with the free particle Lagrangian

$$\mathcal{L} = \partial_\mu \Psi^* \mathbf{Z} \partial^\mu \Psi - \Psi^* \mathbf{M}^2 \Psi \quad (1)$$

where  $\Psi$  is a column vector of  $n$  fields with  $U(1)$  invariance and the positive Hermitian matrices  $\mathbf{Z}$  and  $\mathbf{M}^2$  can be transformed so that  $\mathbf{Z}$  is the identity and  $\mathbf{M}^2$  is diagonalized to produce the standard theory of  $n$  decoupled free fields.

They then add a leading order perturbative, Lorentz violating term constructed from only spatial derivatives so that

$$\mathcal{L} \rightarrow \mathcal{L} + \partial_i \Psi \epsilon \partial^i \Psi, \quad (2)$$

where  $\epsilon$  is a dimensionless Hermitian matrix that commutes with  $\mathbf{M}^2$  so that the fields remain separable and the resulting single particle energy-momentum eigenstates go from eigenstates of  $\mathbf{M}^2$  at low energy to eigenstates of  $\epsilon$  at high energies.

The Lorentz violating perturbative term shifts the poles of the propagators, resulting in free particle dispersion relations of the form

$$E^2 = \vec{p}^2 + m^2 + \epsilon \vec{p}^2. \quad (3)$$

These can be put in the standard form for the dispersion relations

$$E^2 = \vec{p}^2 c_{MAV}^2 + m^2 c_{MAV}^4, \quad (4)$$

by shifting the renormalized mass by the small amount  $m \rightarrow m/(1 + \epsilon)$  and shifting the velocity from  $c$  ( $=1$ ) by the amount  $c_{MAV} = \sqrt{1 + \epsilon} \simeq 1 + \epsilon/2$ .

Since the group velocity is given by

$$\frac{\partial E}{\partial |\vec{p}|} = \frac{|\vec{p}|}{\sqrt{|\vec{p}|^2 + m^2 c_{MAV}^2}} c_{MAV} \rightarrow c_{MAV} \text{ as } |\vec{p}| \rightarrow \infty, \quad (5)$$

Coleman and Glashow thus identify  $c_{MAV}$  as the maximum attainable velocity of the free particle.

Using this formalism, it becomes apparent that, in principle, different particles can have different maximum attainable velocities (MAVs) resulting from the individually distinguishable eigenstates of the  $\epsilon$  matrix. These various MAVs can all be different from  $c$  as well as different from each other. Hereafter, we denote the MAV of a particle of type  $i$  by  $c_i$  and the difference

$$c_i - c_j = \frac{\epsilon_i - \epsilon_j}{2} \equiv \delta_{ij} \quad (6)$$

There are other popular formalisms that are inspired by quantum gravity models or by speculations on the nature of space-time at the Planck scale,  $1/M_{Pl} \simeq 1.5 \times 10^{-35}$  m, where  $M_{Pl} = 1/\sqrt{G} \simeq 1.2 \times 10^{19}$  GeV. Such formalisms, in the context of effective field theory, can be expressed by postulating Lagrangians containing operators of dimension  $\geq 5$  with suppression factors as multiples of  $M_{Pl}$  [19],[29]. This leads to dispersion relations having a series of smaller and smaller terms proportional to  $p^{n+2}/M_{Pl}^n \simeq E^{n+2}/M_{Pl}^n$ , with  $n \geq 1$ . However, in relating LIV to the observational data on UHECRs, we find it useful to use the simpler formalism of Coleman and Glashow. Given the limited energy range of the UHECR data relevant to the GZK effect, this formalism can later be related to possible Planck scale phenomena and quantum gravity models of various sorts.

We now consider the photomeson production process near threshold where single pion production dominates,

$$p + \gamma \rightarrow N + \pi. \quad (7)$$

Using the normal Lorentz invariant kinematics, the energy threshold for photomeson interactions of UHECR protons of initial laboratory energy  $E$  with low energy photons of the CBR with laboratory energy  $\omega$  is determined by the relativistic invariance of the square of the total four-momentum of the proton-photon system. This relation, together with the threshold inelasticity relation  $E_\pi = [m/(M + m)]E$  for single pion production, yields the threshold conditions for head on collisions in the laboratory frame. In terms of the pion energy for single pion production at threshold

$$4\omega E_\pi = \frac{m^2(2M + m)}{M + m}, \quad (8)$$

where  $M$  is the rest mass of the proton and  $m$  is the rest mass of the pion [3].

If LI is broken so that  $c_\pi > c_p$ , it follows from equations (3), (6) and (8) that the threshold energy for photomeson production is altered because the square of the four-momentum is shifted from its LI form so that the threshold condition becomes

$$4\omega E_\pi = \frac{m^2(2M + m)}{M + m} + 2\delta_{\pi p} E_\pi^2 \quad (9)$$

Equation (9) is a quadratic equation with real roots only under the condition

$$\delta_{\pi p} \leq \frac{2\omega^2(M + m)}{m^2(2M + m)} \simeq \omega^2/m^2. \quad (10)$$

Defining  $\omega_0 \equiv kT = 2.35 \times 10^{-4}$  eV, equation (10) can be rewritten

$$\delta_{\pi p} \leq 3.23 \times 10^{-24} (\omega/\omega_0)^2. \quad (11)$$

If LIV occurs and  $\delta_{\pi p} > 0$ , photomeson production can only take place for interactions of CBR photons with energies large enough to satisfy equation (11). Single photon photomeson production takes is dominated by the  $\Delta$  resonance and takes place close to the interaction threshold. This fact, together with equation (9) implies that under some conditions photomeson interactions leading to GZK suppression can occur for “lower energy” UHE protons interacting with relatively higher energy CBR photons on the Wien tail of the Planck spectrum, but such interactions for higher energy protons, which would normally interact with photons having smaller values of  $\omega$ , will be forbidden. Thus, the observed UHECR spectrum may exhibit the characteristics of GZK suppression near the normal GZK threshold, but the UHECR spectrum can “recover” at higher energies owing to the possibility that photomeson interactions at higher proton energies may be forbidden.

#### 4 Kinematics

We now consider a detailed quantitative treatment of this possibility, *viz.*, *GZK coexisting with LIV*. We first give the kinematical relations needed to perform our calculations in the presence of a small violation of Lorentz invariance. We will denote quantities in the proton rest frame by a prime and quantities in the cms system of the proton-photon collision by an asterisk. Quantities in the laboratory frame are left unprimed. Following equations (3) and (6), we denote

$$E^2 = p^2 + 2\delta_a p^2 + m_a^2 \quad (12)$$

where  $\delta_a$  is the difference between the MAV for the particle  $a$  and the speed of light in the low momentum limit ( $c = 1$ ).

The cms energy of particle  $a$  is then given by

$$\sqrt{s_a} = \sqrt{E^2 - p^2} = \sqrt{2\delta_a p^2 + m_a^2} \quad (13)$$

where, of course, we must have the condition  $s_a \geq 0$ . It is important to note that, owing to LIV, in the cms where  $p = 0$  the particle will not generally be

at rest because

$$v = \frac{\partial E}{\partial p} \neq \frac{p}{E}. \quad (14)$$

We follow Ref. [18] in defining the square of the *total* rest energy in the cms by

$$s \equiv E_{tot}^2 - p_{tot}^2 \quad (15)$$

Then denoting  $s_p$  to be the square of the initial proton energy in the system where the initial proton momentum is zero, it follows that

$$s = 2\sqrt{s_p}\epsilon + s_p \quad (16)$$

where we now define  $\epsilon$  as the energy of the photon in this system.

Now let us obtain the expression for the modified inelasticity,  $K$ , for the photopion producing reaction  $p + \gamma \rightarrow N + \pi$ . Since the inelasticity is defined by the fraction of the total energy carried away by the pion, we can relate the energy of the emerging proton and pion to the total energy in the laboratory system (essentially the initial energy of the proton) by

$$\begin{aligned} E_\pi &= K_\theta E_p \\ E_N &= (1 - K_\theta) E_p \end{aligned} \quad (17)$$

where  $K_\theta$  is the inelasticity for a given  $\theta$  which is the angle between the momentum vectors of the photon and the proton in the laboratory system. In order to solve for the inelasticity, we calculate the cms energy of the nucleon in two different ways. On one hand, we can use the Lorentz transformation of the laboratory nucleon energy to relate it to the cms energy:

$$\begin{aligned} E_N &= \gamma^*(E_N^* + \beta^* p_N^* \cos \theta) \\ &= \gamma^*(E_N^* + \beta^* \sqrt{E_N^{*2} - s_N(E_N)} \cos \theta), \end{aligned} \quad (18)$$

where the Lorentz factor for the cms frame is the ratio of the total laboratory energy  $E_p + \omega \approx E_p$  to the total cms energy which is given by equation (13) and where we now define  $\omega$  to be the observed energy of the CBR photon in the laboratory system..

On the other hand, we can derive the cms energy of the nucleon from the threshold conditions by replacing the masses of the particles with their rest

energies as prescribed by equation (13). This yields the relationship

$$2\sqrt{s}E_N^* = s + s_N - s_\pi \quad (19)$$

where the quantities  $s_N$  and  $s_\pi$  can be determined from equation (13) and are given by

$$\begin{aligned} s_\pi &= \delta_\pi (K_\theta E_{p_i})^2 + m_\pi^2 \\ s_N &= \delta_N [(1 - K_\theta) E_{p_i}]^2 + m_p^2. \end{aligned} \quad (20)$$

Here we have replaced  $p$  with  $E$  since we can exchange momentum for energy, given the high Lorentz factor. We can now combine equations (18) and (19) to yield a transcendental equation for  $K_\theta$ :

$$\begin{aligned} (1 - K_\theta)\sqrt{s} &= \frac{(s + s_N(K_\theta) - s_\pi(K_\theta))}{2\sqrt{s}} \\ &+ \beta \sqrt{\frac{(s + s_N(K_\theta) - s_\pi(K_\theta))^2}{4s} - s_N \cos \theta}. \end{aligned} \quad (21)$$

The total inelasticity,  $K$ , will be an average of  $K_\theta$  with respect to the angle between the proton and photon momenta,  $\theta$ :

$$K = \frac{1}{\pi} \int_0^\pi K_\theta d\theta. \quad (22)$$

The primary effect of LIV on photopion production is a reduction of phase space allowed for the interaction. This results from the limits on the allowed range of interaction angles implied by equations (21) and (22). As the pion rest energy grows, the cosine term in equation (21) becomes larger. For collisions with  $\theta < \pi/2$ , kinematically allowed solutions become severely restricted. The modified inelasticity that results is the key in determining the effects of LIV on photopion production. The inelasticity rapidly drops for higher incident proton energies.

As shown in Ref.[14], in order to modify the effect of photopion production on the UHECR spectrum above the GZK energy we must have  $\delta_\pi > \delta_p$ . It is shown in Figure 10 of Ref. [30] that for most of the allowed parameter space near threshold  $\delta_\pi$  can be as much as an order of magnitude greater than  $\delta_p$ . Therefore, in this paper we will assume that  $\delta_\pi \gg \delta_p$  at or near threshold. This assumption is also made in Ref. [18]. We will thus take  $\delta_{\pi p} \simeq \delta_\pi \equiv \delta$ . We have numerically determined that the dependence of our results on the  $\delta_{\pi p}$



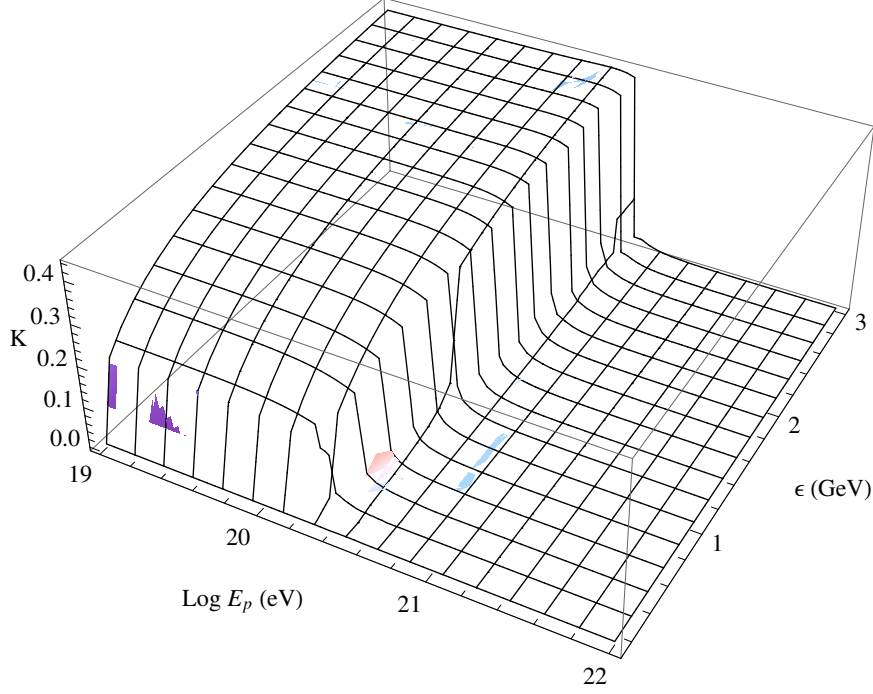


Fig. 1. The calculated proton inelasticity modified by LIV for  $\delta_{\pi p} = 3 \times 10^{-23}$  as a function of CBR photon energy and proton energy.

parameter dominates over that on the  $\delta_p$  parameter, as concluded in Ref. [14]. The effect of taking a value of  $\delta_p$  comparable to  $\delta_\pi$  on the UHECR spectrum will be presented in a future paper.

Figure 1 shows the calculated proton inelasticity modified by LIV for a value of  $\delta_{\pi p} = 3 \times 10^{-23}$  as a function of both CBR photon energy and proton energy. Other choices for  $\delta_{\pi p}$  yield similar plots. The principal result of changing the value of  $\delta_{\pi p}$  is to change the energy at which LIV effects become significant. For a choice of  $\delta_{\pi p} = 3 \times 10^{-23}$ , there is no observable effect from LIV for  $E_p$  less than  $\sim 2 \times 10^{20}$  eV. Above this energy, the inelasticity precipitously drops as the LIV term in the pion rest energy approaches  $m_\pi$ .

With this modified inelasticity, the proton energy loss rate by photomeson production is given by

$$\frac{1}{E} \frac{dE}{dt} = -\frac{\omega_0 c}{2\pi^2 \gamma^2 \hbar^3 c^3} \int_{\eta}^{\infty} d\epsilon \epsilon \sigma(\epsilon) K(\epsilon) \ln[1 - e^{-\epsilon/2\gamma\omega_0}] \quad (23)$$

where  $\eta$  is the photon threshold energy for the interaction in the cms and  $\sigma(\epsilon)$  is the total  $\gamma$ -p cross section with contributions from direct pion production, multipion production, and the  $\Delta$  resonance.

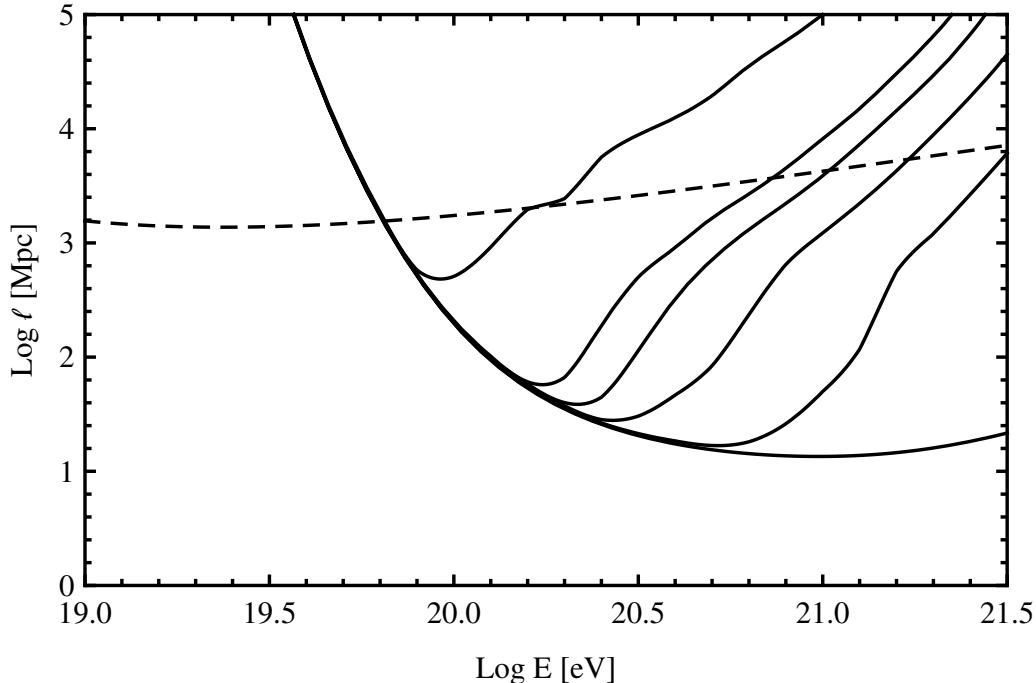


Fig. 2. The calculated proton attenuation lengths as a function proton energy modified by LIV for various values of  $\delta_{\pi p}$  (solid lines), shown with the attenuation length for pair production unmodified by LIV (dashed lines). From top to bottom, the curves are for  $\delta_{\pi p} = 1 \times 10^{-22}, 3 \times 10^{-23}, 2 \times 10^{-23}, 1 \times 10^{-23}, 3 \times 10^{-24}, 0$  (no Lorentz violation).

The corresponding proton attenuation length is given by  $cE/(dE/dt)$ . This attenuation length is plotted in Figure 2 for various values of  $\delta_{\pi p}$  along with the unmodified pair production attenuation for comparison. We do not explore the effects of modifying pair production through LIV in this paper.

## 5 UHECR Spectra with LIV and Comparison with Present Observations

We will start our calculation of LIV modified UHECR spectra by assuming power-law source spectra for the UHECRs that are chosen to fit the UHECR data below 60 EeV. We then consider the propagation of high energy protons, including energy losses resulting from cosmological redshifting, pair production and pion production through interactions with CBR photons.

We shall assume for this calculation a flat  $\Lambda$ CDM universe with a Hubble constant of  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , taking  $\Omega_\Lambda = 0.7$  and  $\Omega_m = 0.3$ . The energy loss owing to redshifting for a  $\Lambda$ CDM universe is then given by

$$-(\partial \log E / \partial t)_{redshift} = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}. \quad (24)$$

The attenuation length for protons against pair production is given by

$$-(\partial \log E / \partial t)_{\gamma p} \equiv r_{\gamma p} = r_\pi(E) + r_{e^+e^-}(E), \quad (25)$$

The attenuation lengths,  $\ell = cE/r(E)$ , for protons against energy loss by both pion production, with and without LIV, are shown together with that for pair production in Figure 2. The CBR photon number density increases as  $(1+z)^3$  and the CBR photon energies increase linearly with  $(1+z)$ . The corresponding energy loss for protons at any redshift  $z$  is thus given by

$$r_{\gamma p}(E, z) = (1+z)^3 r[(1+z)E]. \quad (26)$$

We take the photomeson loss rate,  $r_\pi(E)$ , by updating [3] using the latest cross sections listed in the Particle Data Group (<http://pdg.lbl.gov>) and in Ref. [31]. We take the pair-production loss rate,  $r_{e^+e^-}(E)$  from Ref. [32].

We calculate the initial energy,  $E_i(z)$ , at which a proton is created at a redshift  $z$  whose observed energy today is  $E$  following the methods detailed in Refs. [33] and [34]. We neglect the effect of possible small intergalactic magnetic fields on the paths of these ultrahigh energy protons and assume that they will propagate along straight lines from their source. The total flux of emitted particles from a volume element  $dV = R^3(z)r^2 dr d\Omega$  from redshift  $z$  and distance  $r$  with measured energy  $E$  is given by

$$J(E)dE = \frac{q(E_i, z)dE_i dV}{(1+z)4\pi R_0^2 r^2}. \quad (27)$$

We assume that the average UHECR volume emissivity is given by  $q(E_i, z) = K(z)E_i^{-\Gamma}$ .

We will assume a source evolution  $q(E_i, z) \propto (1+z)^\zeta$  with  $\zeta = 3.6$ , out to a maximum redshift of 2.5. This assumption corresponds to a redshift evolution that is proportional to the star formation rate. Our results on LIV are insensitive to the evolution model assumed because evolution does not affect the shape of the UHECR spectrum near the GZK cutoff energy [24,33]. At higher energies where the attenuation length may again become large owing to an LIV effect, we find the effect of evolution to be less than 10% when compared to the no-evolution case ( $\zeta = 0$ ).

Since  $R_0 = (1+z)R(z)$  and  $R(z)dr = cdt$ , by integrating equation (27), one obtains

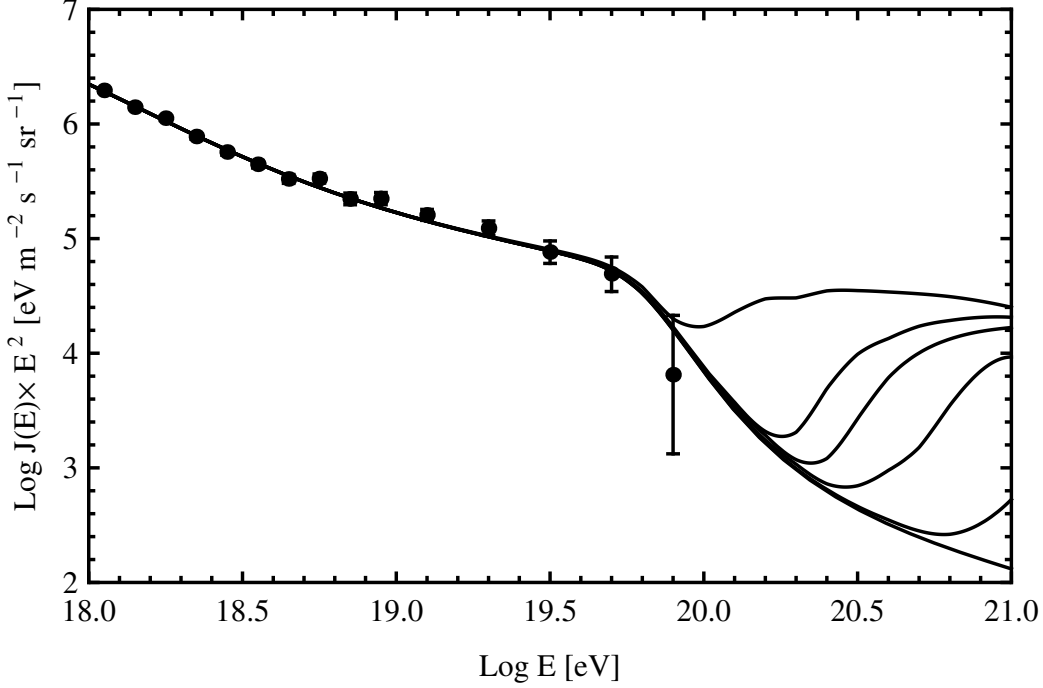


Fig. 3. Comparison of the HiRes II data with calculated spectra for various values of  $\delta_{\pi p}$ . From top to bottom, the curves give the predicted spectra for  $\delta_{\pi p} = 1 \times 10^{-22}, 3 \times 10^{-23}, 2 \times 10^{-23}, 1 \times 10^{-23}, 3 \times 10^{-24}, 0$  (no Lorentz violation).

$$J(E) = \frac{3cK(0)}{8\pi H_0} E^{-\Gamma} \int_0^{z_{max}} \frac{(1+z)^{(\zeta-1)}}{\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}} \left(\frac{E_i}{E}\right)^{-\Gamma} \frac{dE_i}{dE} dz. \quad (28)$$

In this expression,  $K(0)$  is determined by fitting our final calculated spectrum to the observational UHECR data [24] assuming  $\Gamma = 2.55$ , which is consistent with the Auger data below 100 EeV.

The results are shown in Figures 3 and 4.

## 6 Discussion of Results

It has been suggested that a small amount of Lorentz invariance violation (LIV) could turn off photomeson interactions of ultrahigh energy cosmic rays (UHECRs) with photons of the cosmic background radiation and thereby eliminate the resulting sharp steepening in the spectrum of the highest energy CRs predicted by Greisen Zatsepin and Kuzmin (GZK). Recent measurements of the UHECR spectrum reported by the HiRes [11] and Auger [12] collaborations, however, indicate the possible presence of a GZK effect.

In order to determine the implications for the search for Lorentz invariance

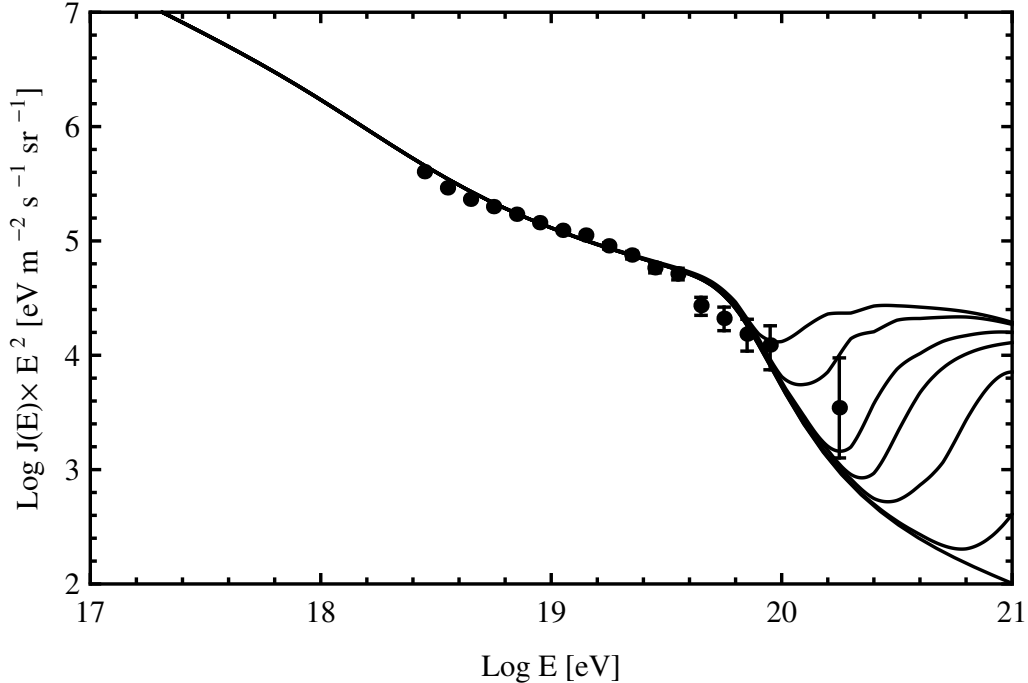


Fig. 4. Comparison of the Auger data with calculated spectra for various values of  $\delta_{\pi p}$ . From top to bottom, the curves give the predicted spectra for  $\delta_{\pi p} = 1 \times 10^{-22}, 6 \times 10^{-23}, 4.5 \times 10^{-23}, 3 \times 10^{-23}, 2 \times 10^{-23}, 1 \times 10^{-23}, 3 \times 10^{-24}, 0$  (no Lorentz violation).

violation at ultrahigh energies from the analysis of the air shower events observed by HiRes and AGASA, we undertook a detailed analysis of the spectral features produced by modifications of the kinematical relationships caused by LIV at ultrahigh energies. In our analysis, we calculate modified UHECR spectra for various values of the Coleman-Glashow parameter,  $\delta_{\pi p}$ , defined as the difference between the maximum attainable velocities of the pion and the proton produced by LIV. We then compare our results with the experimental UHECR data and thereby place limits on the amount of LIV as defined by the  $\delta_{\pi p}$  parameter.

Our results show that the amount of presently observed GZK suppression in the UHECR data is consistent with the possible existence of a small amount of LIV. In order to quantify this, we determined the value of  $\delta_{\pi p}$  that results in the smallest  $\chi^2$  for the modeled UHECR spectral fit using the observational data above the GZK energy. We find this value to be  $4.5 \times 10^{-23}$ . We then determined the range of acceptable values for  $\delta_{\pi p}$ . This was done by computing the probability of getting a  $\chi^2$  value at least as small as the  $\chi^2$  value determined from the fit. We rejected  $\delta_{\pi p}$  values outside of the confidence level associated with  $1\sigma$ . We thus obtained a best-fit range of  $\delta_{\pi p} = 4.5_{-4.5}^{+1.5} \times 10^{-23}$ , corresponding to an upper limit on  $\delta_{\pi p}$  of  $6 \times 10^{-23}$ , as shown in Figure 4.

The HiRes spectral data (see Figure 3) do not go to high enough energy to

quantitatively constrain LIV. We also note that the Auger spectrum, being consistent with no obvious pair-production feature, does not constrain LIV for the pair-production interaction.

A small LIV effect can be distinguished from a higher energy component produced by so-called top-down models because the latter predict relatively large fluxes of UHE photons and neutrinos as well as a significant diffuse GeV background flux that could be searched for by the Fermi  $\gamma$ -ray space telescope. The Pierre Auger Observatory collaboration has provided observational upper limits on the UHE photon flux that have already disfavored top-down models [35]. The upper limits from Auger indicate that UHE photons at best make up only a small percentage of the total UHE flux. This contradicts predictions of top-down models that the flux of UHE photons should be larger than that of UHE protons (See Ref. [9] for a review).

As opposed to the predictions of the top-down models, the LIV effect cuts off UHE pion production at the higher energies and consequent UHE neutrino and photon production from UHE pion decay. LIV would also not produce a GeV photon flux.

It is also possible that the apparent modified GZK suppression in the data may be related to an overdensity of nearby sources related to the local supergalactic enhancement [3].<sup>1</sup> More and better data will be required in order to resolve this question. An LIV effect can be distinguished from a local source enhancement by looking for UHECRs at energies above  $\sim 200$  EeV, as can be seen from Figures 3 and 4. This is because the small amount of LIV that fits the observational UHECR spectra can lead to a recovery of the cosmic ray flux at higher energies than presently observed. Searching for such an effect will require obtaining a data set containing a much higher number of UHECR air shower events.

In the future, such an increased number of events may be obtained. The Auger collaboration has proposed to build an “Auger North” array that would be seven times larger than the present southern hemisphere Auger array (<http://www.augernorth.org>). Further into the future, space-based telescopes designed to look downward at large areas of the Earth’s atmosphere as a sensitive detector system for giant air-showers caused by trans-GZK cosmic rays [36]. We look forward to these developments that may have important implications for fundamental high energy physics.

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<sup>1</sup> A correlation with nearby AGN has been hinted at in the Auger data [37]. However, the HiRes group has found no significant correlation [38].

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