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AIAA 90-2801

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AIAA Atmospheric Flight Mechanics Conference
August 20 - 22, 1990 / Portland, OR

OPTIMAL INPUT DESIGN FOR AIRCRAFT PARAMETER ESTIMATION USING DYNAMIC PROGRAMMING PRINCIPLES

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Abstract

A new technique was developed for designing optimal flight test inputs for aircraft parameter estimation experiments. The principles of dynamic programming were used for the design in the time domain. This approach made it possible to include realistic practical constraints on the input and output variables. A description of the new approach is presented, followed by an example for a multiple input linear model describing the lateral dynamics of a fighter aircraft. The optimal input designs produced by the new technique demonstrated improved quality and expanded capability relative to the conventional multiple input design method.

Symbols

$\mathbf{D}$	dispersion matrix
$E\{\cdot\}$	expectation operator
$\mathbf{F}$	system dynamics matrix
g	acceleration due to gravity, m/sec^2
$\mathbf{G}$	control matrix
$\mathbf{H}$	observation matrix
J	cost function
$\mathbf{M}$	information matrix
N	total number of sample times
p	roll rate, rps
r	yaw rate, rps
$\mathbf{R}$	measurement noise covariance matrix
$\mathbf{S_i}$	i^{th} discrete sensitivity matrix
$\mathbf{u}(t)$	m-dimensional control vector at time t
U	airspeed, m/sec
$\mathbf{x}(t)$	n-dimensional state vector at time t
$\mathbf{y}(i)$	q-dimensional output vector
$\mathbf{y_m}(i)$	measured output vector
β	sideslip angle, rad
δ_a	aileron deflection, rad
δ_{ij}	Kronecker delta
δ_r	rudder deflection, rad
Δt	sampling interval
η_k	k^{th} output amplitude constraint
μ_j	j^{th} input amplitude constraint
$\boldsymbol{\theta}$	p-dimensional parameter vector
ϕ	roll angle, rad
σ_k	Cramer-Rao bound for parameter k
$\boldsymbol{\upsilon}(i)$	gaussian white noise random vector
ζ_k	Cramer-Rao bound goal for parameter k
$\mathbf{0}$	zero vector

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Introduction

Aircraft flight tests designed specifically for the purpose of parameter estimation are generally motivated by one or more of the following objectives:

1. The desire to correlate aircraft model parameter estimates from wind tunnel experiments with estimates obtained from flight test data.

2. Refinement of the parameter estimates for the aircraft model for puposes of control system analysis and design.

3. Accurate prediction of the response of the aircraft using the mathematical model, including flight simulation.

4. Aircraft acceptance testing.

The achievement of any of the above objectives involves many factors, including the selection of instrumentation and signal conditioning, flight test operational procedure, input design, aircraft model determination, and the parameter estimation algorithm.

In this work, a new computational procedure was developed for the design of optimal flight test input signals for parameter estimation experiments. For the most part, the other considerations in the flight test design manifest themselves in the detail of the input design problem formulation. The fundamental principles and procedures regarding the input design remain unaltered.

Input designs for aircraft parameter estimation experiments are evaluated by examining the Cramer-Rao lower bounds on the parameter standard errors, which are a function of the information content in the aircraft response to a given input[1]. These Cramer-Rao bounds are the theoretical lower limits for parameter standard errors using an asymptotically unbiased and efficient estimator, such as maximum likelihood. Comparisons using the Cramer-Rao bounds isolate the merits of the input design from the merits of the parameter estimation algorithm used to extract the aircraft model parameter estimates from the flight data.

Past studies of optimal input design in the time domain usually formulated the problem as a fixed time variational calculus problem, using some norm of the information matrix or its inverse as the cost function, and imposing an energy constraint on the input to indirectly implement practical input and output amplitude constraints.[1,2] An approach by Mehra[2] analyzed the problem in the frequency domain, but there is some difficulty in properly translating results in the frequency domain into a realistic input design in the time domain. Chen[3] developed the first time domain design method which discarded the fixed time assumption, producing a suboptimal iterative technique using Walsh functions.

The new approach to optimal input design for parameter estimation flight tests described here departs from previous approaches by embodying the following capabilities and characteristics as part of the problem formulation:

1. Multiple input design capability.

2. Practical input constraints, including maximum input amplitudes, control system dynamics, input spectrum high frequency limitations, and, in cases where a human pilot must realize the designed input, pilot implementation and coordination constraints.

3. Output amplitude constraints, which can be specified a priori as a function of considerations for validity of the aircraft model whose parameters are to be estimated from the flight data, and the safety of the test aircraft and pilot during flight test operations.

4. Global minimization of the required flight test time, subject to the conditions of the problem formulation, so that results from expensive and limited flight test resources can be maximized.

5. Single pass solution.

The next section describes the problem formulation. Following this is a description of the solution method which uses the principles of dynamic programming. Several example input designs using the new technique for the lateral dynamics of a fighter aircraft are then given. Finally, some concluding remarks are included.

Problem Statement

For aircraft parameter estimation experiments, typically a linear perturbation model structure is assumed. The flight test inputs are perturbations about trim to ensure that the system response can be adequately modelled by such a structure. The assumed model is given by

$$\dot{\mathbf{x}}(t) = \mathbf{F}(\boldsymbol{\theta})\,\mathbf{x}(t) + \mathbf{G}(\boldsymbol{\theta})\,\mathbf{u}(t)\,, \qquad \mathbf{x}(0) = \mathbf{0} \tag{1}$$

$$\mathbf{y}(t) = \mathbf{H}(\boldsymbol{\theta})\,\mathbf{x}(t) \tag{2}$$

$$\mathbf{y_m}(i) = \mathbf{y}(i) + \boldsymbol{\upsilon}(i) \quad i = 1, 2, \ldots, N \tag{3}$$

where the measurement noise $\boldsymbol{\upsilon}(i)$ is assumed gaussian with

$$E\{\boldsymbol{\upsilon}(i)\} = 0 \qquad \text{and} \qquad E\left\{\boldsymbol{\upsilon}(i)\,\boldsymbol{\upsilon}^T(j)\right\} = \mathbf{R}\,\delta_{ij} \tag{4}$$

Constraints were imposed on all input amplitudes and selected output amplitudes. These constraints arise from the practical considerations that control input amplitudes are limited by mechanical stops, flight control software limiters, or linear control effectiveness. Selected output variable amplitudes must be limited to ensure validity of the assumed linear model form and also to ensure safety during the flight test. The constraints are given by

$$\left| u_j(t) \right| \le \mu_j \quad \forall\, t, \; j = 1, 2, \ldots, m \tag{5}$$

$$|y_k(t)| \le \eta_k \quad \forall t,\ k \in (1,2,\ldots,q) \tag{6}$$

where μ_j and η_k are positive constants. The minimum achievable values for the parameter standard errors using an asymptotically unbiased and efficient estimator are given by the square root of the diagonal elements of the so-called dispersion matrix, **D**. The dispersion matrix is defined as the inverse of the information matrix **M**, the latter being a measure of the information content in the data from an experiment. The expressions for these matrices are

$$\mathbf{M} = \left[\sum_{i=1}^{N} \left(\frac{\partial y(i)}{\partial \theta} \right)^T \mathbf{R}^{-1} \left(\frac{\partial y(i)}{\partial \theta} \right) \right] \tag{7}$$

$$\mathbf{D} = \mathbf{M}^{-1} \tag{8}$$

where the sensitivities are computed from

$$\frac{d}{dt}\left[\frac{\partial x}{\partial \theta_j}\right] = \frac{\partial \mathbf{F}}{\partial \theta_j} x + \mathbf{F}\frac{\partial x}{\partial \theta_j} + \frac{\partial \mathbf{G}}{\partial \theta_j} u \quad j=1,2,\ldots,p \tag{9}$$

$$\frac{\partial y}{\partial \theta_j} = \frac{\partial \mathbf{H}}{\partial \theta_j} x + \mathbf{H}\frac{\partial x}{\partial \theta_j} \tag{10}$$

The above equations follow from equations (1) - (2) and the assumed analyticity of $x(t)$.

Generally, required accuracies for the parameters can be specified a priori by the end users of the parameter estimates. These values represent goals for the Cramer-Rao bounds of each model parameter. The optimization problem is then : *choose the input which minimizes the time to achieve the a priori desired accuracies on the parameters.* This approach obviates the need for parameter weighting required by fixed test time approaches, and also maximizes the effectiveness of limited and expensive flight test time.

As posed, this optimization problem is difficult to solve in general. At this point, considerations particular to optimal input design for aircraft parameter estimation problems were invoked in order to limit the allowable control set to square wave inputs only. Among these considerations were analytic work for similar problems which indicated that the optimal input should be of the "bang-bang" type[3], the input capabilities of human pilots, and previous flight test evaluations which demonstrated that square wave type inputs were superior to sinusoidal type inputs for parameter estimation experiments, largely due to their wider frequency spectrum[1].

For the above reasons, and to make the optimization problem tractable, input forms were limited to square waves only; i.e., only full positive, full negative, or zero amplitude were allowed for any control at any time. With this restriction, the problem becomes a high order combinatorial problem involving output amplitude constraints, which is well-suited to solution by the method of dynamic programming, as described next.

Solution Methodology

For purposes of illustration, assume that two output amplitudes are constrained. The allowable output space at any given time then can be represented by a plane region whose borders correspond to the output amplitude constraints, as shown in figure 1. The plane region is divided into discrete output space boxes. Time is divided into discrete steps called stages. The constrained outputs of the system are examined at every discrete time, which are separated by one stage time. Feasible outputs at any time must be contained in one of the discrete output space boxes. Starting at the initial condition box in output space, all possible controls (full positive, full negative, or zero amplitude) are applied over one stage time and the consequences of each control possibility are computed. These consequences include the system outputs from integrating system dynamic equations of motion (equations (1) and (2)) and a cost associated with each particular control possibility. In general, this results in several reachable boxes in feasible output space at the next time stage. Any control which takes the output outside feasible output space is dropped from consideration for inclusion as part of the optimal control sequence. This implements the output amplitude constraints. For each time stage, the reachable output space is computed as the result of all possible control inputs starting at the reachable output space boxes found for the preceding time stage.

Thus, one can picture a complicated network of connections between output space boxes separated by one time stage. After only a few stages, several boxes in output space at a particular time can be reached by more than one input sequence. The preferred sequence is that which results in the lowest cost. This input sequence is saved and associated with that particular output space box at that time, while the other (inferior) control sequences which reach the same box in output space at that time are discarded.

The cost function was chosen as the square of the Euclidean distance between the point in parameter hyperspace corresponding to the current Cramer-Rao bounds computed from the information matrix to the rectangular parallelepiped in parameter space which represented the goal values of the parameters. The cost function can be expressed as

$$J = \sum_{k=1}^{p} (\sigma_k - \zeta_k)^2 \quad \forall k \text{ with } \sigma_k > \zeta_k \tag{11}$$

An illustration of the cost value for two parameters in three example cases A,B, and C, is shown in figure 2. The boundaries of the parallelepiped in parameter space are the goal values for the Cramer-Rao bounds. The location of points A,B, and C would be determined by the computed Cramer-Rao bounds associated with, say, three different inputs, or perhaps the same input sequence at three different times. At any time, the Cramer-Rao bounds were obtained from a sequential computation of the additional information (and thus the additional available parameter accuracy) resulting from the application of a particular control possibility over one stage time.

In order to compute the cost for any candidate control at any time, it is necessary to sequentially compute the dispersion matrix, from which the Cramer-Rao bounds for each parameter may be computed as the square root of the corresponding diagonal elements. First define the discrete sensitivity matrix

$$S_i = \frac{\partial y(i)}{\partial \theta} \tag{12}$$

Then the sequential calculation of the dispersion matrix, due to Chen[3], is given by the relation

$$D_{i+1} = D_i - D_i S^T_{i+1}\left[S_{i+1} D_i S^T_{i+1} + R\right]^{-1} S_{i+1} D_i \tag{13}$$

Only those parameters whose Cramer-Rao bound goal is not yet achieved contribute to the cost. Bellman's principle of optimality[4] was used to choose the optimal control sequence to any box in reachable output space at any time stage, based on the cost function given above. The first time stage where the cost became zero for some box in output space (all Cramer-Rao bound goals attained) was designated the minimum time solution, and the input sequence to reach that output space box at that stage was designated the optimal input sequence.

Several modifications which take advantage of the structure of the optimal input problem for aircraft parameter estimation were made to reduce memory requirements. In addition, sophistications were added that adjusted the input possibilities at certain time stages in order to account for control system dynamics, limitations on high frequencies in the input, and practical implementation and pilot coordination constraints, the latter being especially important for multiple input designs. Details of these features and other aspects of the solution method may be found in reference 5.

Example

The lateral dynamics of an advanced fighter aircraft in level flight at 10,000 m altitude and an airspeed of 179.72 m/sec. may be represented by the dynamic system and measurement model specified by equations (1) - (3) where

$$\mathbf{x} = [\beta, p, r, \phi]^T \quad , \quad \mathbf{u} = [\delta_a, \delta_r]^T \tag{14}$$

$$\mathbf{F} = \begin{bmatrix} Y_\beta & 0 & -1 & -g/U \\ L_\beta & L_p & L_r & 0 \\ N_\beta & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{15}$$

$$\mathbf{G} = \begin{bmatrix} 0 & Y_{\delta_r} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \\ 0 & 0 \end{bmatrix} \tag{16}$$

$$\mathbf{R} = \begin{bmatrix} 0.000361 & 0 & 0 & 0 \\ 0 & 0.04 & 0 & 0 \\ 0 & 0 & 0.0064 & 0 \\ 0 & 0 & 0 & 0.0059 \end{bmatrix} \tag{17}$$

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

subject to the following amplitude constraints on input and output variables

$$|\delta_a| \leq 0.07 \text{ radians} \quad \forall t \tag{19}$$

$$|\delta_r| \leq 0.07 \text{ radians} \quad \forall t \tag{20}$$

$$|\beta| \leq 0.15 \text{ radians} \quad \forall t \tag{21}$$

$$|\phi| \leq 1.0 \text{ radians} \quad \forall t \tag{22}$$

The a priori values of the model parameters are given in Table 1. The above model does not correspond to any existing fighter aircraft or instrumentation system. The nominal values used are for demonstration purposes only, although the model structure is quite generally applicable.

A common practical input design procedure employs doublets in the rudder and aileron with frequencies close to the damped natural frequency of the aircraft Dutch roll mode. This type of design is shown in figure 3. A first order lag with time constant 0.1 second was implemented to model pilot and control system actuation delay. Input amplitude was adjusted to 0.07 rad so that output amplitude constraints were satisfied. Cramer-Rao bounds associated with this 10 second input design appear in column 3 of Table 1.

The optimal input design technique was used with the same problem formulation, a priori parameter values, input and output constraints, measurement noise covariance, and control system dynamics. In order to facilitate comparison of the results, the input design technique was used to produce a fixed time design by setting the goals for all Cramer-Rao bounds to zero, and limiting the run of the input design program to a maximum of 10 seconds. The resulting input design and constrained output responses appear in figure 4. The Cramer-Rao bounds were lowered for all parameters (compare columns 3 and 4 of Table 1), with all output amplitude constraints satisfied.

In figure 5, the optimal input design was performed for the same problem using a larger (0.1 rad) input amplitude constraint. In this case, a minimum time solution was computed. The goals for the Cramer-Rao bounds were the accuracies associated with the doublet inputs taken from column 3 of Table 1. The same accuracy or better for each parameter relative to the doublet inputs case was obtained in a shorter total test time of 8.7 seconds, as shown in column 5 of Table 1.

For all input designs presented thus far, the control inputs were sequenced; that is, only one control was moved at a time. This feature is helpful when human pilots must implement the input design. For cases when a computerized system can realize the inputs, the requirement for sequenced control inputs can be relaxed. This case is shown in figure 6, using the same input amplitude as in figure 5. Again, a fixed time solution was computed by setting all goals for the Cramer-Rao bounds to zero with a 10 second maximum test time. Columns 5 and 6 of Table 1 show that a gain of at least 25% in accuracy for all parameters except those associated with the Y force was achieved for a 10 second total test time by relaxing the control sequencing requirement. All optimal input designs excited the aircraft response as much as possible to obtain small Cramer-Rao bounds, but still kept the output response within the imposed amplitude constraints.

Concluding Remarks

Optimal input design for aircraft parameter estimation experiments was done in the time domain using the principles of dynamic programming to achieve specific goals for Cramer-Rao bounds in a time optimal fashion. Optimization in the time domain allowed various practical aspects of the input design to be incorporated in a straightforward manner. Bellman's principle of optimality was enforced so that the designed input was globally time optimal, subject to the imposed constraints on the input form, the output amplitude constraints, the dynamic and measurement model, and the discretization of both time and the constrained output variable amplitudes in the dynamic programming solution.

The quality and expanded capability of the optimal input design technique was exhibited by application to a fourth order multiple input system with restrictive output amplitude constraints.

The optimal input design described here has potential for producing practical, optimal solutions for input design problems of current interest in aircraft parameter estimation flight experiments. Some of these problems are:

1. Multiple input designs.
2. Pilot input coordination/implementation.
3. Restrictive output amplitude constraints.
4. Input designs for control augmented aircraft.

The approach outlined in this report is capable of handling any or all of the above problems, rendering a globally time optimal input design solution in a single pass calculation scheme.

The algorithm is well suited to trade-off studies for flight test input design. For example, the effect of changes in measurement noise characteristics, Cramer-Rao bound goals, input amplitude constraints, output amplitude constraints, or control system dynamics may be evaluated in terms of total test time required, or achievable parameter accuracies.

Acknowledgments

This research was conducted at the NASA Langley Research Center under the support of NASA Cooperative Agreement NCC1-29.

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Table 1 - Input Design Results

Parameter	Parameter Value	Cramer-Rao Bounds Figure 3	Figure 4	Figure 5	Figure 6
Y_β	–0.1095	0.0493	0.0447	0.0422	0.0383
$Y_{\delta r}$	0.0219	0.0231	0.0201	0.0216	0.0187
L_β	–14.424	0.4954	0.3220	0.4202	0.2761
L_p	–1.2039	0.0903	0.0626	0.0702	0.0490
L_r	0.9029	0.3990	0.2249	0.3000	0.1442
$L_{\delta a}$	–16.828	0.7133	0.4606	0.6317	0.4029
$L_{\delta r}$	2.404	0.2883	0.2358	0.2299	0.1726
N_β	2.864	0.1031	0.0491	0.0617	0.0284
N_p	–0.009	0.0171	0.0107	0.0124	0.0059
N_r	–0.2241	0.0786	0.0493	0.0543	0.0363
$N_{\delta a}$	–0.358	0.1320	0.0826	0.0946	0.0539
$N_{\delta r}$	–1.790	0.0516	0.0298	0.0378	0.0271
Max. Amp. (rad)		0.07	0.07	0.10	0.10
Total Time (sec)		10.0	10.0	8.7	10.0

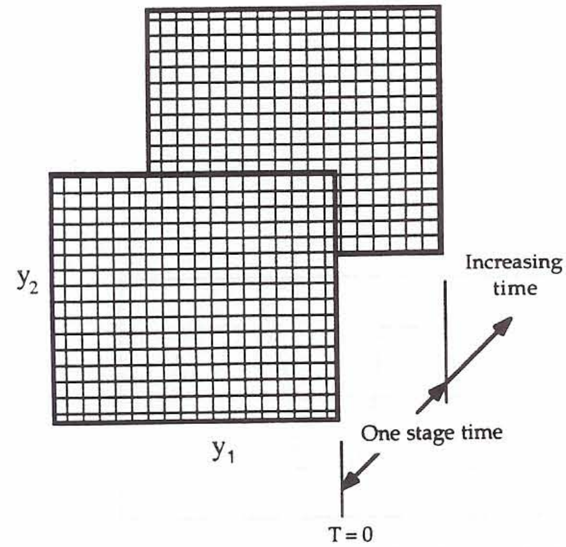

Figure 1 - Output space for discrete stage times

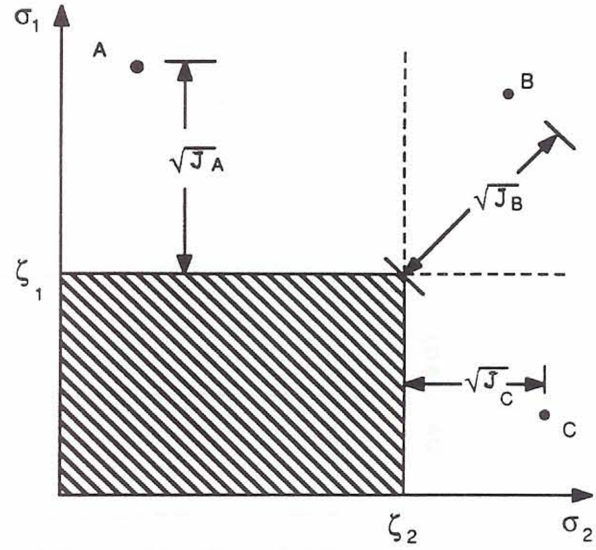

Figure 2 - Cost function for two parameters

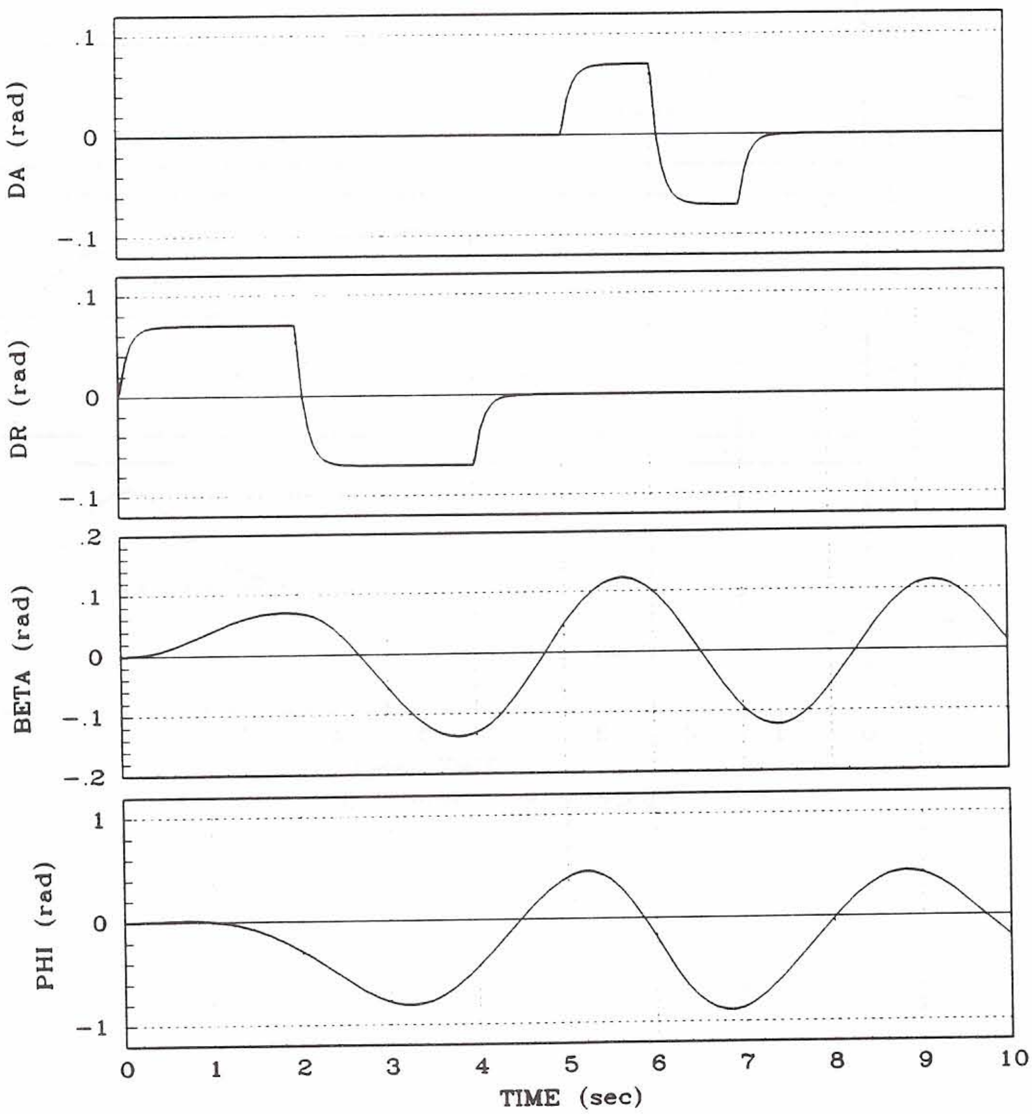

Figure 3 – Doublet Inputs

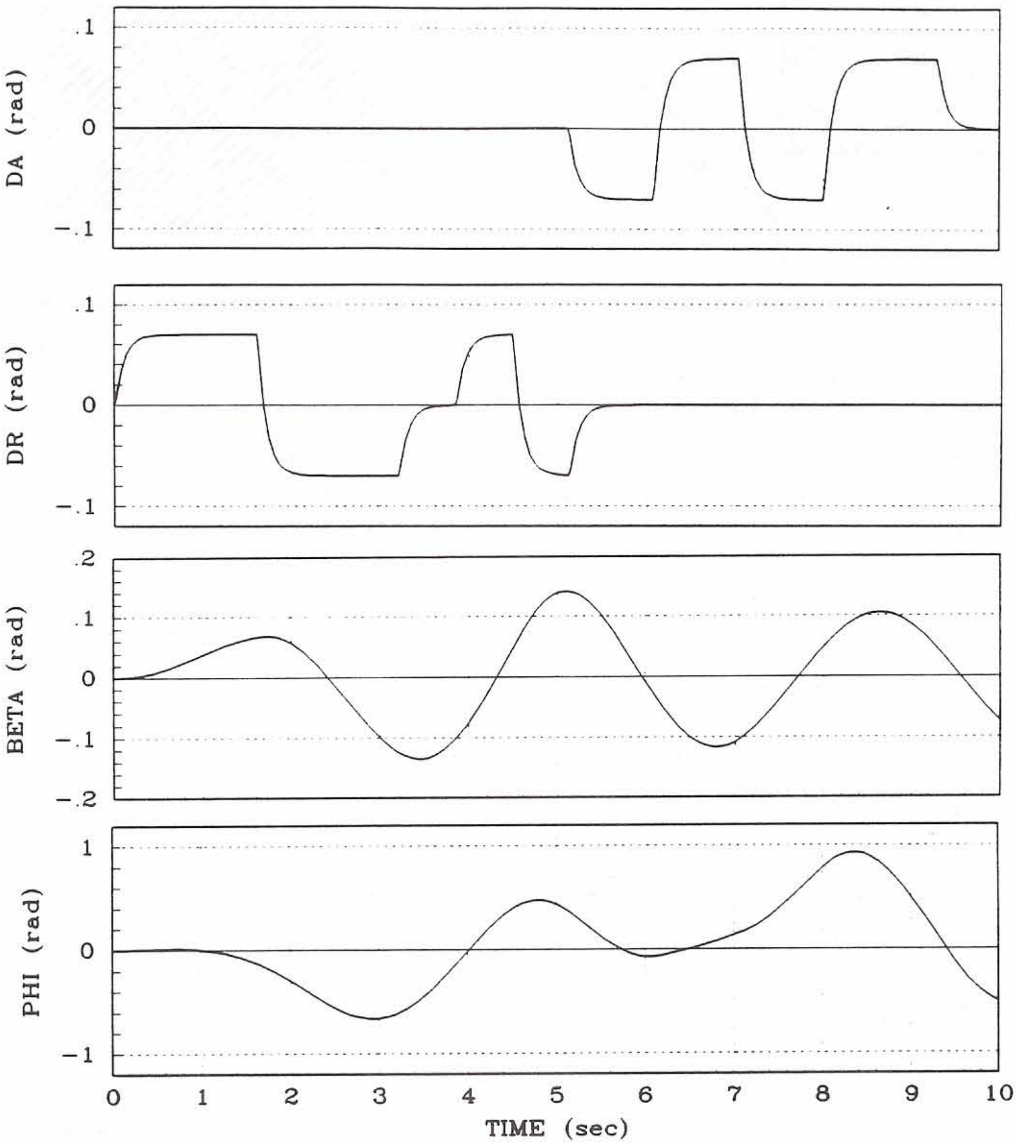

Figure 4 – Optimal Inputs

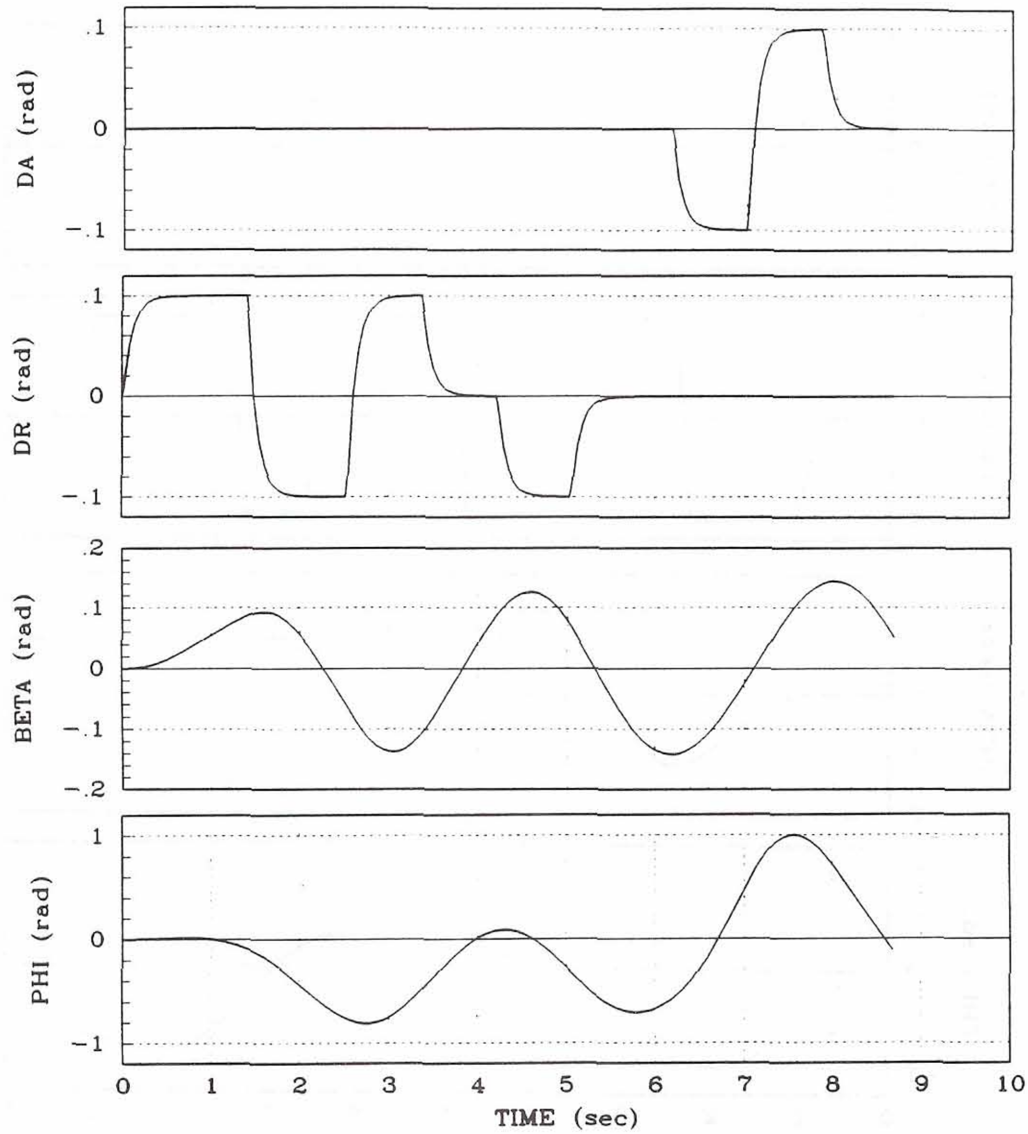

Figure 5 – Optimal Inputs, Increased Amplitude

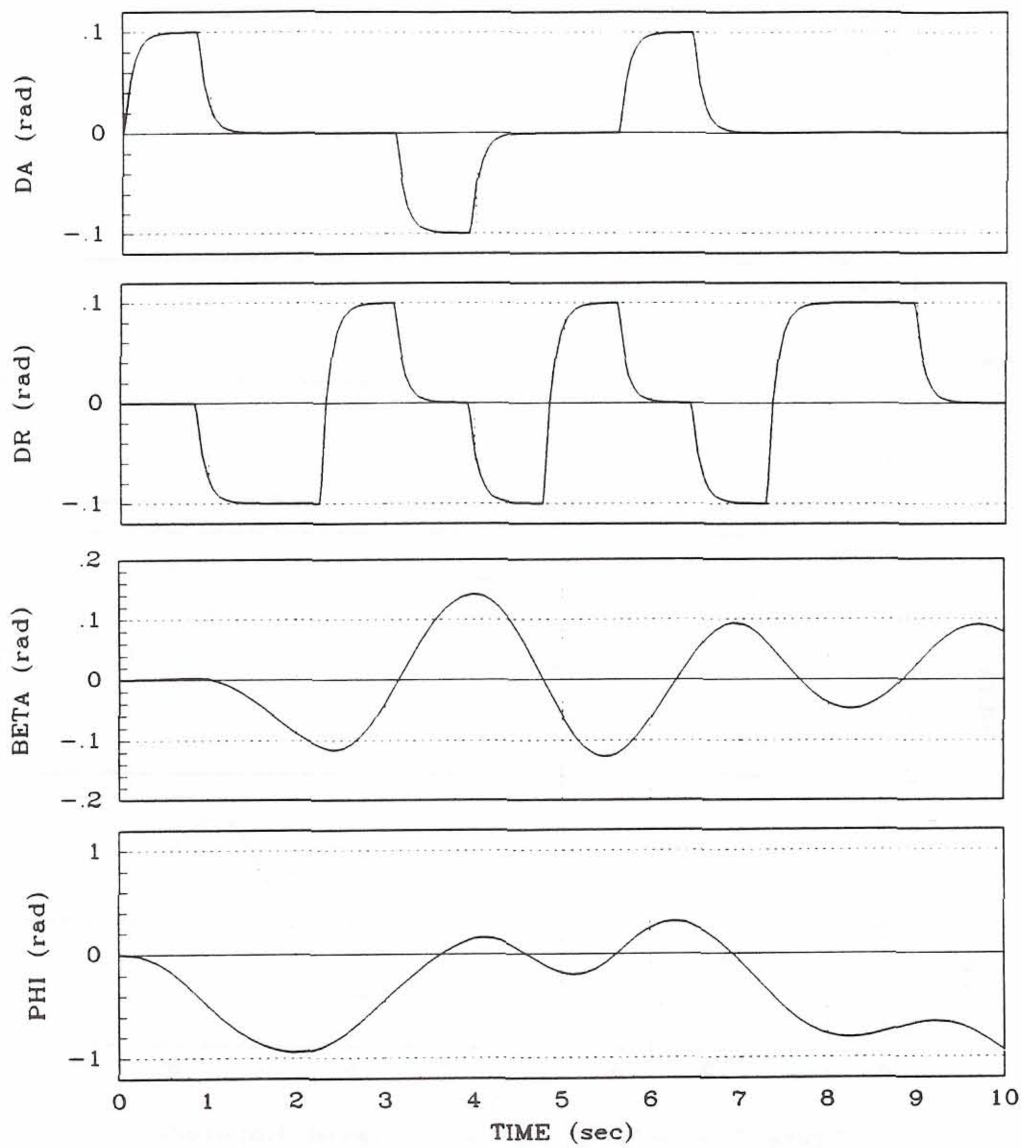

Figure 6 – Optimal Inputs, Increased Amp., No Control Sequencing