Structure Computation of Quiet Spike™ Flight-Test Data During Envelope Expansion

Sunil L. Kukreja*

NASA Dryden Flight Research Center, Edwards Air Force Base, California 93523-0273

System identification or mathematical modeling is used in the aerospace community for development of simulation models for robust control law design. These models are often described as linear time-invariant processes. Nevertheless, it is well known that the underlying process is often nonlinear. The reason for using a linear approach has been due to the lack of a proper set of tools for the identification of nonlinear systems. Over the past several decades, the controls and biomedical communities have made great advances in developing tools for the identification of nonlinear systems. These approaches are robust and readily applicable to aerospace systems. In this paper, we show the application of one such nonlinear system identification technique, structure detection, for the analysis of F-15B Quiet Spike™ aeroservoelastic flight-test data. Structure detection is concerned with the selection of a subset of candidate terms that best describe the observed output. This is a necessary procedure to compute an efficient system description that may afford greater insight into the functionality of the system or a simpler controller design. Structure computation as a tool for black-box modeling may be of critical importance for the development of robust parsimonious models for the flight-test community. Moreover, this approach may lead to efficient strategies for rapid envelope expansion, which may save significant development time and costs. The objectives of this study are to demonstrate via analysis of F-15B Quiet Spike aeroservoelastic flight-test data for several flight conditions that 1) linear models are inefficient for modeling aeroservoelastic data, 2) nonlinear identification provides a parsimonious model description while providing a high percent fit for cross-validated data, and 3) the model structure and parameters vary as the flight condition is altered.

I.   Introduction

SYSTEM identification, or black-box modeling, is a critical step in aircraft development, analysis, and validation for flight worthiness. The development and testing of aircraft typically takes many years and requires considerable expenditure of limited resources. One reason for lengthy development time/costs is assuming that the underlying system is linear and invariant throughout the flight envelope. This assumption is related to having an inadequate knowledge of an appropriate model type or structure to use for parameter estimation. Selection of an insufficient model structure may lead to difficulties in parameter estimation, giving estimates with significant biases and/or large variances [1]. This often complicates control synthesis or renders it infeasible. The power of using structure-detection techniques as a tool for model development (i.e., black-box modeling) is that it can provide a parsimonious system description that can describe complex aeroelastic behavior over a large operating range. Consequently, this provides models that can be more robust and therefore reduce development time.

Moreover, when studying aeroelastic systems, it may not be practical to assume that the exact model structure is well known a priori. In aerospace systems analysis, one of the main objectives is not only to estimate system parameters but to gain insight into the structure of the underlying system. Therefore, structure computation is of significant relevance and importance to modeling and design of aircraft and aerospace vehicles. Structure computation may indicate deficiencies in an analytical model and could lead to improved modeling strategies and also provide a parsimonious black-box system description for control synthesis [1–3].

For linear systems modeling, a commonly used approach for determining model structure is the minimum description length (MDL) proposed by Rissanen [4]. MDL was specifically developed to overcome some of the inconsistencies of Akaike's information criterion [5]; that is, its variance does not tend to zero for larger sample sizes N.

Recently, the bootstrap method has been shown to be a useful tool for structure detection of nonlinear models [6–8]. The bootstrap is a numerical method for estimating parameter statistics that requires few assumptions [9]. The conditions needed to apply bootstrap to least-squares estimation are quite mild: namely, that the errors be independent, identically distributed, and have zero mean.

In this paper, we investigate whether 1) a linear or nonlinear model best represents the observed data and 2) the system is invariant during envelope expansion (varying Mach number). The data analyzed in this study are from the F-15B Quiet Spike™ flight-test program, which was a collaborative effort between Gulfstream Aerospace Corporation (Savannah, Georgia) and NASA Dryden Flight Research Center [10–12]. The results show that

1) Linear models are inefficient for modeling aeroservoelastic data.

2) Nonlinear identification provides a parsimonious model description while providing a high percent fit for cross-validated data.

3) The model structure and parameters vary as the flight condition is altered.

II.   NARMAX Model Form

The dynamic behavior of many nonlinear systems can be represented as a discrete polynomial that expands the present output value in terms of present and past values of the input signal and past values of the output signal [13–15]. A system modeled in this form is popularly known as a NARMAX (nonlinear autoregressive, moving-average exogenous) model and is linear in the parameters.

Recently, Kukreja and Brenner [16] showed that NARMAX identification is well suited to describing aeroelastic phenomena. The NARMAX structure is a general parametric form for modeling nonlinear systems [14]. This structure describes both the stochastic

and deterministic components of nonlinear systems. Many nonlinear systems are a special case of the general NARMAX structure [17]. In this paper, we focus on a special class of NARMAX models: nonlinear polynomial models. The polynomial NARMAX structure models the input–output relationship as a nonlinear difference equation of the form

$$z(n) = f^l[z(n-1), \ldots, z(n-n_z), u(n), \ldots, u(n-n_u), e(n-1), \ldots, e(n-n_e)] + e(n) \quad (1)$$

where f denotes a nonlinear mapping, l is the order of the nonlinearity, u is the controlled or exogenous input, z is the measured output, and e is the uncontrolled input or innovation. This nonlinear mapping may include a variety of nonlinear terms, such as terms raised to an integer power, products of present and past inputs, past outputs, or cross terms. In general, the nonlinear mapping f can be described by a wide variety of nonlinear functions such as sigmoids or splines [17,18]. This system description encompasses many forms of linear and nonlinear difference equations that are linear in the parameters.

Identifying a NARMAX model requires two things: 1) structure detection and 2) parameter estimation. Structure detection can be divided into model-order selection and selecting which parameters to include in the model. We consider model-order selection as part of structure detection because, theoretically, there are an infinite number of candidate terms that could be considered initially. Establishing the model order, then, limits the choice of terms to be considered. For the NARMAX model, the system order is defined to be an ordered tuple as

$$O \doteq [n_u \quad n_z \quad n_e \quad l] \quad (2)$$

where n_u is the maximum lag on the input, n_z the maximum lag on the output, n_e the maximum lag on the error, and l is the maximum nonlinearity order. Note that for nonpolynomial NARMAX models, l may be simply replaced by a nonlinear mapping of some specified class. In this paper, we assume that the system order is known.

III. Structure Detection

The structure-detection problem is that of selecting the subset of candidate terms that best describe the output. Therefore, the parametrization of a system is still further reduced by determining which of the components are required. A binomial coefficient is defined as

$$\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{k!} = \frac{n!}{k!(n-k)!}$$

where $k!$ is the factorial of k, $k! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (k-1) \cdot k$. The maximum number of terms in a NARMAX model with n_z, n_u, and n_e dynamic terms and lth-order nonlinearity is

$$p = \binom{n_u + n_z + n_e + 1 + l}{l} \quad (3)$$

As a result, the number of candidate terms becomes very large for even moderately complex models, making structure detection difficult. We define the maximum number of terms p as the number of candidate terms to be initially considered for identification. Because of the excessive parameterization (the curse of dimensionality), the structure-detection problem often leads to computationally intractable combinatorial optimization problems.

IV. Time Series

The data considered in this paper are time series because the input signal $u(n)$ is assumed to be zero or constant. Time-series analysis is used when inputs are not available to the experimenter or when it is unclear which signals are inputs and which are outputs [19]. Models arising from times-series data can have several unique forms

[1,20,21]. In our treatment of the data, the ARMA and NARMA model classes are of practical significance.

This special case of the general NARMAX model [Eq. (1)] can be written as

$$z(n) = f^l[z(n-1), \ldots, z(n-n_z), e(n-1), \ldots, e(n-n_e)] + e(n) \quad (4)$$

where we redefine the model order for this model set as

$$O = [n_z \quad n_e \quad l] \quad (5)$$

The maximum number of candidate terms in a model [Eq. (4)] with n_z and n_e dynamic terms and lth-order nonlinearity is

$$p = \binom{n_z + n_e + l}{l} \quad (6)$$

Note that ARMA models can be estimated using the Yule–Walker equations or the instrumental variable estimator to avoid estimating the MA part [1,20,21]. This is the approach taken in this paper. For a NARMA model, the NMA part must be modeled. For nonlinear systems, output additive noise can produce multiplicative terms between input, output, and itself. To compute unbiased parameter estimates, a noise model (i.e., NMA) needs to be estimated [22].

V. Structure-Detection Methods

With the model types defined for the flight-test data available for analysis, we describe two approaches applicable to these model classes. The first is appropriate for autoregressive (AR) models and the second is appropriate for NARMA models.

A. Minimum Description Length

A commonly used technique in linear system identification to determine model structure is MDL [4]. MDL was specifically developed to overcome some of the inconsistencies of Akaike's information criterion [5]; that is, its variance does not tend to zero for larger sample sizes N.

The number of parameters necessary to reproduce an observed sequence $\{z_1, \ldots, z_N\}$ of a time series depends on the model and parameters assumed to have generated the data [4]. The MDL technique finds the model that minimizes the description length and thereby computes an estimate of model order [4].

Binary prefix codes are used to encode data strings. These data strings can be made up of symbols, parameters, numbers, etc. It is known that the average length of a code word is bounded by Shannon's theorem [4]. Therefore, it is possible to write [4]

$$\sum_x p(x)L(x) \geq -\sum_x p(x) \log p(x) \quad (7)$$

where $L(x)$ is the length of the code word (i.e., length of parameter vector θ), and $p(x)$ is the probability of x. It is also possible to write

$$L(z|x, \theta) = -\log p(z|x, \theta) \quad (8)$$

where $L(z|x, \theta)$ is known as the log-likelihood function (to be maximized). Let $\hat{\theta}$ denote the value of the parameter that maximizes the likelihood and thus minimizes the parameter vector length (i.e., code-word length) $L(z|x, \theta)$. Because $\hat{\theta}$ can only be encoded up to a certain precision, the code-word length $L(z|x, \theta)$ becomes longer than the desired minimum $L(z|x, \hat{\theta})$, given noise considerations. Let the precision be $\delta = 2^{-q}$, where q is the number of bits used for encoding the parameter. It is possible to save on the code-word length if q is small. However, the result is a loss in precision. The optimal precision depends on the size of the observed data via $-\log \delta = 0.5 \log N$ and hence the total code-word length for k parameters is given by the MDL:

$$\text{MDL}(k) = -\log[\text{maximized likelihood}] + \tfrac{1}{2}k \log N \quad (9)$$

which, for an AR(n_z) model, gives

$$\text{MDL}\,(n_z) = \log[\text{maximized likelihood}] + \frac{n_z}{N}\log N \qquad (10)$$

B. Bootstrap

Recently, the bootstrap has been shown to be a useful tool for structure detection of nonlinear models [8]. The bootstrap is a numerical method for estimating parameter statistics that requires few assumptions [9]. The conditions needed to apply bootstrap to least-squares estimation are quite mild: namely, that the errors be independent, identically distributed, and have zero mean.

The bootstrap is a technique to randomly reassign observations that enables reestimates of parameters to be computed. This randomization and computation of parameters is done numerous times and treated as repeated experiments. In essence, the bootstrap simulates a Monte Carlo analysis. For structure computation, the bootstrap method is used to detect spurious parameters: those parameters for which the estimated values cannot be distinguished from zero.

Application of an appropriate ℓ_2 estimator to measured data gives the model response $\hat{\mathbf{Z}}$, residuals $\hat{\boldsymbol{\varepsilon}}$, and parameter estimate $\hat{\boldsymbol{\theta}}$. The bootstrap assumes that these residuals $\hat{\boldsymbol{\varepsilon}} = [\hat{\varepsilon}_1, \hat{\varepsilon}_2, \ldots, \hat{\varepsilon}_N]$ arise from an unknown distribution $\mathcal{D}$. By randomly resampling these residuals, with replacement, it is possible to generate a resampled version of the prediction errors $\hat{\boldsymbol{\varepsilon}}^* = [\hat{\varepsilon}_1^*, \hat{\varepsilon}_2^*, \ldots, \hat{\varepsilon}_N^*]$, for which the distribution estimates $\mathcal{D}$. The resampling procedure for each $\hat{\varepsilon}_i^*$ involves randomly selecting from $\hat{\boldsymbol{\varepsilon}}$ with an equal probability associated with each of the N elements. For example, a possible resampled version of the errors for $N = 5$ is $\hat{\boldsymbol{\varepsilon}}^* = [\hat{\varepsilon}_4, \hat{\varepsilon}_1, \hat{\varepsilon}_4, \hat{\varepsilon}_2, \hat{\varepsilon}_3]$. The star notation indicates that $\hat{\boldsymbol{\varepsilon}}^*$ is not the calculated error $\hat{\boldsymbol{\varepsilon}}$, but rather a resampled version of it. These resampled errors are added to the model response to generate a bootstrap replication of the original data:

$$\mathbf{Z}^* = \boldsymbol{\Psi}_{ze}\hat{\boldsymbol{\theta}} + \hat{\boldsymbol{\varepsilon}}^* \qquad (11)$$

A new bootstrapped parameter estimate $\hat{\boldsymbol{\theta}}^*$ is obtained from this bootstrapped data $\mathbf{Z}^*$. This procedure is repeated B times to provide a set of parameter estimates from the B bootstrap replications:

$$\hat{\boldsymbol{\Theta}}^* = \left[\hat{\boldsymbol{\theta}}_1^*, \ldots, \hat{\boldsymbol{\theta}}_B^*\right] \qquad (12)$$

Parameter statistics can then be easily computed from $\hat{\boldsymbol{\Theta}}^*$ by forming percentile intervals at a chosen level of significance α.

Two methods related to the bootstrap are genetic algorithms and particle filters. They are both resampling procedures for which the goal is to estimate model parameters and distribution, mainly for the nonlinear and non-Gaussian case.

Genetic algorithms are related to subset selection that may be loosely considered a type of Monte Carlo approach. However, as with subset selection, genetic algorithms fail to produce an optimal model set. Genetic algorithms are known to be computationally demanding, can lead to premature convergence on poor solutions, and have a tendency to converge toward local optima (and in some cases to arbitrary points) [23–26].

Particle filters implement a resampling method that tries to achieve the same goal as the genetic algorithm of estimating a sequence of hidden parameters based only on observed data. Particle filtering has been useful in a variety of areas but is difficult to implement for problems such as regressor selection, due to computational complexity leading to intractable problems for moderately complex full-model forms [27–30].

Structure detection can provide useful process insights that can be used in subsequent development or refinement of physical models. Therefore, in the sequel, we investigate the applicability of MDL and the bootstrap to experimental aircraft data. Specifically, MDL and bootstrap methods are used as structure-detection tools to assess whether the 1) underlying data are best described by a linear time-invariant or nonlinear model and 2) model structure is invariant during envelope expansion.

VI. Experimental F-15B Quiet Spike Data

The MDL and bootstrap technique were applied to experimental flight-test data from the F-15B Quiet Spike project by Gulfstream Aerospace Corporation (Savannah, Georgia) and NASA Dryden Flight Research Center [10–12]. The data analyzed for this study used the structural accelerometer response output of the Quiet Spike boom tip when fully extended.

In this study, we only analyzed data from cases in which the Mach number was varied to be able to quantify whether model structure and parameters vary as the flight condition parameter is altered. If data were analyzed from cases in which more than one flight condition parameter was altered, one could not assess whether the dynamics and nonlinear effects vary due to Mach number or a combination of conditions.

A. Procedures

Flight data were gathered during subsonic, transonic, and supersonic flutter clearance of the F-15B Quiet Spike (see Fig. 1). This paper considers accelerometer data measured during pilot-induced pitch raps at Mach 0.85, 0.95, and 1.40 at 12,192 m (40,000 ft). The output was taken as the response of an accelerometer mounted near the Quiet Spike boom tip (see Fig. 2). Data were sampled at 400 Hz. For analysis, the recorded flight-test data were decimated by a factor of 2, resulting in a final sampling rate of 200 Hz.

For identification of a linear model, an arbitrarily large AR model of fiftieth order ($n_z = 50$) was posed for estimation and the MDL technique was used to determine the optimal linear model. For identification of a nonlinear model, a model order with fourth-order output and error dynamics and second-order nonlinearity, $O = [4, 4, 2]$, was used. A model with fourth-order dynamics was selected because it has been observed that aeroservoelastic structures are well defined by a fourth-order linear time-invariant system [31–33]. The nonlinearity order was chosen as second-order because empirical results showed that models of higher nonlinear order were not efficient to describe the data sets available for analysis. This gave a full-model description with 45 candidate terms. The nonlinear model was identified by applying the bootstrap approach. For the bootstrap method, $B = 100$ bootstrap replications were generated to assess the distribution of each parameter. For all three techniques, each parameter was tested for significance at the 95% confidence level.

For both linear and nonlinear identification, Table 1 shows the number of data points available for each flight condition. The estimation data sets were from accelerometer response measurements of the primary sensor on the boom tip and the cross-validation data sets were of the backup sensor at the same location.

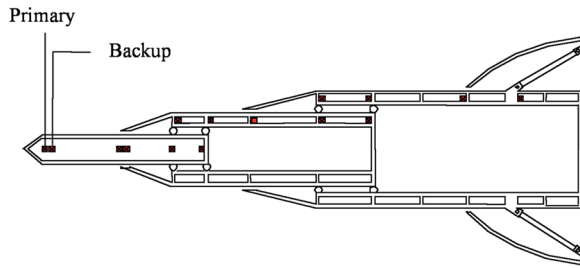**Fig. 1   Flight-test article in extended configuration.**

Fig. 2 Quiet Spike sensor locations.

B. Results

The results of identifying the F-15B Quiet Spike data are presented. Figure 3 shows the output data sets used for this analysis. The data represent structural accelerometer response (primary sensor boom tip) used to compute the system structure.

Equations (13–15) depict the model structure computed by the bootstrap method:

Mach 0.85, altitude 40,000 ft (12,192 m),

$$z(n) = \hat{\theta}_1 z(n-1) + \hat{\theta}_2 z(n-2) + \hat{\theta}_3 z(n-3) + \hat{\theta}_4 z^2(n-4)$$
$$+ \hat{\theta}_5 \varepsilon(n-1) + \hat{\theta}_6 \varepsilon(n-2) + \hat{\theta}_7 \varepsilon(n-3)$$
$$+ \hat{\theta}_8 z(n-4)\varepsilon(n-4) + \hat{\theta}_9 \varepsilon^2(n-4) + \varepsilon(n) \qquad (13)$$

Mach 0.95, altitude 40,000 ft (12,192 m),

$$z(n) = \hat{\vartheta}_1 z(n-1) + \hat{\vartheta}_2 z(n-2) + \hat{\vartheta}_3 z(n-3)$$
$$+ \hat{\vartheta}_4 z(n-3)z(n-4) + \hat{\vartheta}_5 z^2(n-4) + \hat{\vartheta}_6 \varepsilon(n-1)$$
$$+ \hat{\vartheta}_7 \varepsilon(n-2) + \hat{\vartheta}_8 \varepsilon(n-3) + \hat{\vartheta}_9 z(n-3)\varepsilon(n-4)$$
$$+ \hat{\vartheta}_{10} z(n-4)\varepsilon(n-3) + \hat{\vartheta}_{11} \varepsilon(n-3)\varepsilon(n-4)$$
$$+ \hat{\vartheta}_{12} z(n-4)\varepsilon(n-4) + \hat{\vartheta}_{13} \varepsilon^2(n-4) + \varepsilon(n) \qquad (14)$$

Mach 1.40, altitude 40,000 ft (12,192 m),

$$z(n) = \hat{\beta}_1 z(n-1) + \hat{\beta}_2 z(n-2) + \hat{\beta}_3 z(n-3)$$
$$+ \hat{\beta}_4 z(n-1)z(n-4) + \hat{\beta}_5 \varepsilon(n-1) + \hat{\beta}_6 \varepsilon(n-2)$$
$$+ \hat{\beta}_7 \varepsilon(n-3) + \hat{\beta}_8 z(n-1)\varepsilon(n-4) + \hat{\beta}_9 z(n-4)\varepsilon(n-1)$$
$$+ \hat{\beta}_{10} \varepsilon(n-1)\varepsilon(n-4) + \varepsilon(n) \qquad (15)$$

Equations (13–15) represent the computed model structure for flight conditions Mach 0.85, 0.95, and 1.40, respectively. The computed model structures are represented by a combination of linear and nonlinear lagged-output terms and contain 9, 13, and 10 terms for Mach 0.85, 0.95, and 1.40, respectively. Hence, the bootstrap technique successfully produced a parsimonious model description from the full set of 45 candidate terms.

For AR (linear) model identification using MDL to compute structure, the estimated models were of order $n_z = 42, 44$, and 46 for Mach 0.85, 0.95, and 1.40, respectively. These models are not shown because they are simply a dynamic expansion of the output up to the order stated. However, for cross-validation data, we show the model

Table 1 Data points available at each flight condition

Altitude 12,192 m (40,000 ft)			
Mach number	0.85	0.95	1.40
Estimation data	572	400	504
Validation data	572	400	504

fit of these linear models compared with the cross-validation fit obtained with the NARMA models (see Fig. 4).

Figure 4 shows the predicted output for the cross-validation data sets for the identified structures; Fig. 4a displays Eq. (13) and $AR(n_z = 42)$, Fig. 4b displays Eq. (14) and $AR(n_z = 44)$, and Fig. 4c displays Eq. (15) and $AR(n_z = 46)$). Each panel displays the full time history of the predicted output of the linear and nonlinear models superimposed on top of the measured output. For Mach 0.85
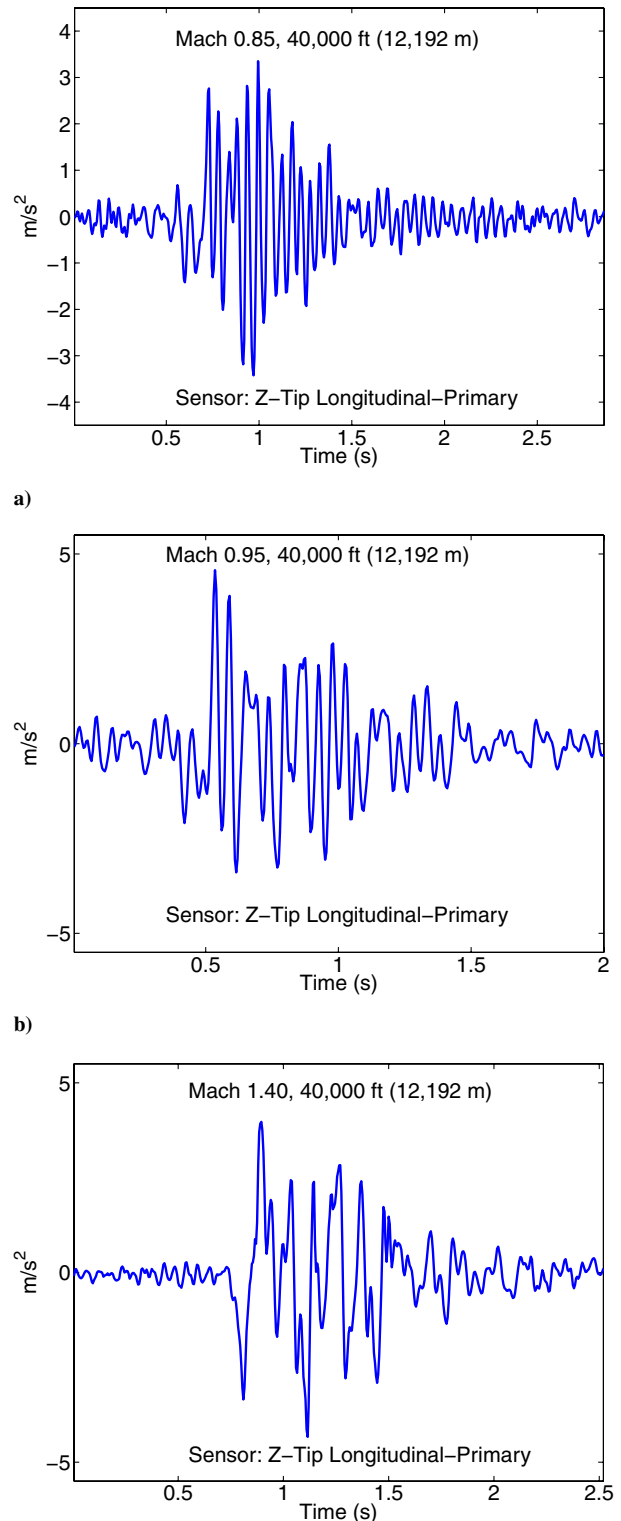

a)

b)

c)

Fig. 3 Estimation data: recorded structural accelerometer response to stick raps.

(Fig. 4a), the predicted outputs of the linear and nonlinear models account for over 91 and 95% of the measured output variance, respectively. For Mach 0.95 (Fig. 4b), the predicted output of the linear and nonlinear models account for over 92 and 97% of the measured output variance, respectively. For Mach 1.40 (Fig. 4c), the predicted output of the linear and nonlinear models account for over 91 and 96% of the measured output variance, respectively.
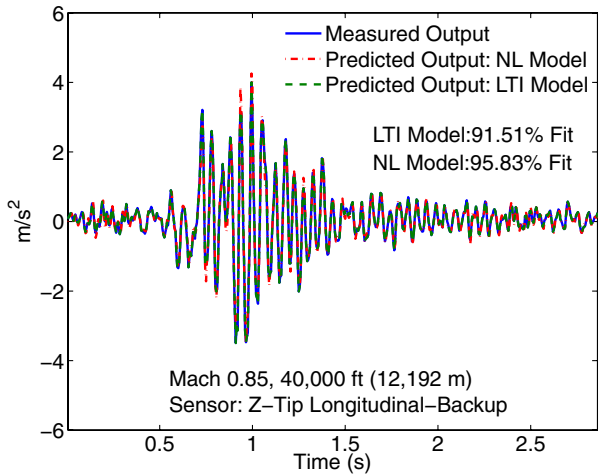
a)
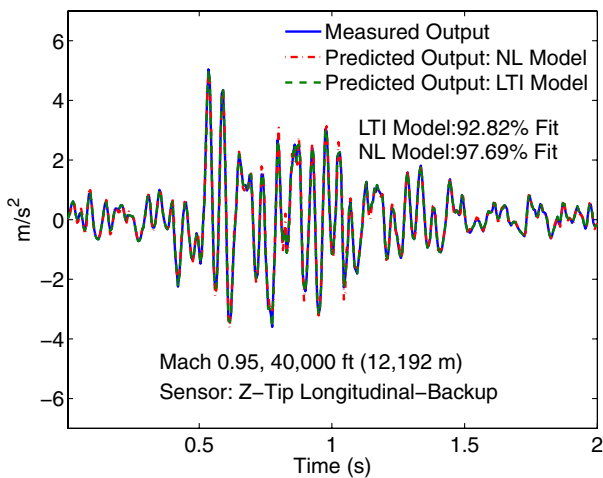
b)
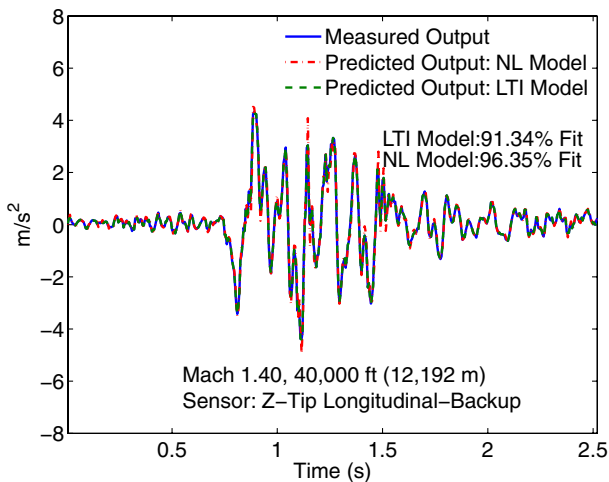
c)

Fig. 4 Cross-validation data: predicted linear and nonlinear model accelerometer response of z-tip longitudinal sensor superimposed on top of measured accelerometer output.

The results demonstrate that although the AR models contain many more terms to explain the underlying process, they still offer a lower percent fit compared with the nonlinear model at the cost of model complexity (higher order), which often leads to more complex control synthesis. The nonlinear models contain only a few terms and were capable of explaining a larger percent of the output variance. For these data sets, the results show that linear models are inefficient for accurate modeling of aeroservoelastic data. These results show that a nonlinear identification approach offers a parsimonious system description while providing a high percent fit for cross-validated data. Moreover, the results illustrate the need to vary model structure for different flight conditions.

VII. Discussion

Experimental results demonstrate that structure computation as a tool for black-box modeling may be useful for the analysis of dynamic aircraft data. The bootstrap successfully reduced the number of regressors posed to aircraft aeroelastic data, yielding a parsimonious model structure for each data set. Additionally, these parsimonious structures were capable of predicting a large portion of the cross-validation data collected on a backup sensor at a similar location. However, for linear analysis, the MDL approach was not able to reduce the model order (structure) as well and yielded a more complex system description. Although these linear models have higher complexity (degrees of freedom), they provided a model-predictive capability that explained a smaller percent of the observed output variance. This find indicates that a linear model may not be appropriate to describe aeroservoelastic data. A higher percent fit offered by the parsimonious nonlinear models suggests that the identified structures and parameters explain the data well. Using percent fit alone as an indicator of model goodness could lead to incorrect interpretations of model validity. Nevertheless, in many cases, for nonlinear models, this may be the only indicator that is readily available.

In this work, the results show that although the linear dynamics remained invariant for all flight conditions available for analysis, the nonlinear dynamics changed as the Mach number increased. For Mach 0.85, the model [Eq. (13)] displayed a mildly nonlinear process, which physically makes sense because the airflow is mainly subsonic. When the Mach number was increased to 0.95, the model [Eq. (14)] demonstrated a richer nonlinear dynamic description, which is likely due to embedded shock formations in the transonic regime. For Mach 1.40, the model [Eq. (15)] displayed a mildly nonlinear process again, which physically makes sense, because in this regime, the shocks become fixed. It is difficult to make definitive comments on the underlying physics responsible for this behavior without extensive analysis of different flight conditions. The important points to note are that this study suggests that

1) Nonlinear models are appropriate to describe the dynamics behavior of advanced aircraft.

2) Models describing aircraft dynamics vary with flight condition.

This suggests that nonlinear modeling may afford a robust and parsimonious system description compared with a larger operating regime and that models used for prediction (e.g., control) should not be invariant for all flight conditions.

For this study, only a polynomial mapping with fourth-order output and error lag was used as a basis function to explain the nonlinear behavior of the F-15B Quiet Spike data. Clearly, different basis functions and a higher dynamic order (lag order) should be investigated to determine if another basis can produce accurate model predictions with reduced complexity. Moreover, further studies are necessary to evaluate whether the model structure is invariant under different operating conditions, such as altitude, and model parameterizations.

This study illustrates the usefulness of structure detection as an approach to compute a parsimonious model of a highly complex nonlinear process, as demonstrated with experimental data of aircraft aeroelastic dynamics. Moreover, analysis of flight-test data can provide useful process insights that can be used in subsequent development or refinement of physical models. In particular,

morphological models are based on assumptions (e.g., these effects are important and those are negligible), which may be incorrect [34,35]. A structure computation approach to model identification may help uncover such surprises.

VIII.   Conclusions

Results show that linear models are inefficient for modeling aeroservoelastic data and that nonlinear identification provides a parsimonious model description while providing a high percent fit for cross-validated data. Moreover, the results demonstrate that model structure and parameters vary as the flight condition varies. These results may have practical significance in the analysis of aircraft dynamics during envelope expansion and could lead to more efficient control strategies. In addition, this technique could allow greater insight into the functionality of various systems dynamics by providing a quantitative model that is easily interpretable.

References

[1]   Ljung, L., *System Identification: Theory for the User*, 2nd ed, Prentice–Hall, Upper Saddle River, NJ, 1999.

[2]   Harris, C. J., and Billings, S. A., *Self Tuning and Adaptive Control: Theory and Applications*, 2nd ed., Peter Peregrinus, London, 1985.

[3]   Zhou, K., Doyle, J. C., and Glover, K., *Robust and Optimal Control*, 1st ed., Prentice–Hall, Englewood Cliffs, NJ, 1999.

[4]   Rissanen, J., "Modelling by Shortest Data Description," *Automatica*, Vol. 14, No. 5, 1978, pp. 465–471.
      doi:10.1016/0005-1098(78)90005-5

[5]   Akaike, H., "A New Look at the Statistical Model Identification," *IEEE Transactions on Automatic Control*, Vol. 19, No. 6, 1974 pp. 716–723.

[6]   Kukreja, S. L., "A Suboptimal Bootstrap Method for Structure Detection of Nonlinear Output-Error Models," Proceedings of the 13th IFAC Symposium on System Identification, Vol. 13, Elsevier, Amsterdam, Aug. 2003, pp. 1566–1571.

[7]   Kukreja, S. L., and Brenner, M. J., "Nonlinear Black-Box Modelling of Aeroelastic Systems Using Structure Detection: Application to F/A-18 Data," *Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 2, 2007, pp. 557–564.
      doi:10.2514/1.20835

[8]   Kukreja, S. L., Galiana, H. L., and Kearney, R. E., "A Bootstrap Method for Structure Detection of NARMAX Models," *International Journal of Control*, Vol. 77, No. 2, 2004, pp. 132–143.
      doi:10.1080/00207170310001646264

[9]   Efron, B., "Computer and the Theory of Statistics: Thinking the Unthinkable," *SIAM Review*, Vol. 21, No. 4, 1979, pp. 460–480.
      doi:10.1137/1021092

[10] Howe, D., "Improved Sonic Boom Minimization with Extendable Nose Spike," 43rd AIAA Aerospace Sciences Meeting and Exhibit, Reno, NV, AIAA Paper 2005–1014, Jan. 2005.

[11] Simmons, F., and Freund, D., "Morphing Concept for Quiet Supersonic Jet Boom Mitigation," 43rd AIAA Aerospace Sciences Meeting and Exhibit, Reno, MV, AIAA Paper 2005–1015, Jan. 2005.

[12] Simmons, F., and Freund, D., "Wing Morphing for Quiet Supersonic Jet Performance-Variable Geometry Design Challenges for Business Jet Utilization," 43rd AIAA Aerospace Sciences Meeting and Exhibit, Reno, NV, AIAA, Paper 2005–1017, Jan. 2005.

[13] Kortmann, M., and Unbehauen, H., "Structure Detection in the Identification of Nonlinear Systems," *Traitement du Signal*, Vol. 22, No. 1, 1988, pp. 5–25.

[14] Leontaritis, I. J., and Billings, S. A., "Input-Output Parametric Models for Nonlinear Systems Part 1: Deterministic Nonlinear Systems," *International Journal of Control*, Vol. 41, No. 2, 1985, pp. 303–328.
      doi:10.1080/0020718508961129

[15] Leontaritis, I. J., and Billings, S. A., "Input-Output Parametric Models for Nonlinear Systems Part 1: Stochastic Nonlinear Systems,"
      International Journal of Control, Vol. 41, No. 2, 1985, pp. 329–344.
      doi:10.1080/0020718508961130

[16] Kukreja, S. L., and Brenner, M. J., "Nonlinear Aeroelastic System Identification with Application to Experimental Data," *Journal of Guidance, Control, and Dynamics*, Vol. 29, No. 2, 2006, pp. 374–381.
      doi:10.2514/1.15178

[17] Chen, S., and Billings, S. A., "Representations of Nonlinear Systems: The NARMAX Model," *International Journal of Control*, Vol. 49, No. 3, 1989, pp. 1013–1032.

[18] Billings, S. A., and Chen, S., "Extended Model Set, Global Data and Threshold Model Identification of Severely Nonlinear Systems," *International Journal of Control*, Vol. 50, No. 5, 1989, pp. 1897–1923.
      doi:10.1080/00207178908953473

[19] Verhaegen, M., and Dewilde, P., "Subspace Model Identification Part 1: The Output-Error Statespace Model Identification Class of Algorithms," *International Journal of Control*, Vol. 56, No. 5, 1992, pp. 1187–1210.
      doi:10.1080/00207179208934363

[20] Brillinger, D. R., *Time Series: Data Analysis and Theory*, 2nd ed., Classics in Applied Mathematics, Vol. 36, Society for Industrial and Applied Mathematics, Philadelphia, 2001.

[21] Hamilton, J. D., *Time Series Analysis*, 1st ed., Princeton Univ. Press, Princeton, NJ, 1994.

[22] Kukreja, S. L., "A Suboptimal Bootstrap Method for Structure Detection of Nonlinear Output-Error Models with Application to Human Ankle Dynamics," *International Journal of Control*, Vol. 78, No. 12, 2005, pp. 937–948.
      doi:10.1080/00207170500199627

[23] Barricelli, N. A., "Symbiogenetic Evolution Processes Realized by Artificial Methods," *Methods*, Vol. 1, 1957, pp. 143–182.

[24] Fraser, A., *Computer Models in Genetics*, 1st ed., McGraw–Hill, New York, 1970.

[25] Kjellström, G., "Optimization of Electrical Networks with Respect to Tolerance Costs," *Ericsson Technics*, Vol. 3, No. 3, 1970, pp. 157–175.

[26] Whitley, D., "A Genetic Algorithm Tutorial," *Statistics and Computing*, Vol. 4, 1970, pp. 65–85.

[27] Arulampalam, M. S., Maskell, S., Gordon, N., and Clapp, T., "A Tutorial on Particle Filters for Online Nonlinear/Non-Gaussian Bayesian Tracking," *IEEE Transactions on Signal Processing*, Vol. 50, No. 2, 2002, pp. 174–188.
      doi:10.1109/78.978374

[28] Douc, R., Guillin, A., and Najim, J., "Moderate Deviations for Particle Filtering," *Annals of Applied Probability*, Vol. 15, No. 1B, 2005, pp. 587–614.
      doi:10.1214/105051604000000657

[29] Gordon, N. J., Salmond, D. J., and Smith, A. F. M., "Novel Approach to Nonlinear/Non-Gaussian Bayesian State Estimation," *IEE Proceedings F, Radar and Signal Processing*, Vol. 140, No. 2, 1993, 107–113.

[30] Higuchi, T., "Monte Carlo Filter Using the Genetic Algorithm Operators," *Journal of Statistical Computation and Simulation*, Vol. 59, No. 1, 1997, pp. 1–23.

[31] Baldelli, D. H., Chen, P. C., Liu, D. D., Lind, R., and Brenner, M., "Nonlinear Aeroelastic Modeling by Block-Oriented Identification," 45th AIAA/ASME/ASCE/AHS/ Structures, Structural Dynamics and Materials Conference, Palm Springs, CA, AIAA Paper 2004–1938, Apr. 2004.

[32] Gawronski, W., *Balanced Control of Flexible Structures*, 1st ed., Springer–Verlag, London, 1996.

[33] Smith, R. S., "Model Validation for Robust Control: An Experimental Process Control Application," *Automatica*, Vol. 31, No. 11, 1995, pp. 1637–1647.
      doi:10.1016/0005-1098(95)00093-C

[34] Pearson, R. K., "Nonlinear Input/Output Modelling," *Journal of Process Control*, Vol. 5, No. 4, 1995, pp. 197–211.
      doi:10.1016/0959-1524(95)00014-H

[35] Pottmann, M., and Pearson, R. K., "Block-Oriented NARMAX Models with Output Multiplicities," *AIChE Journal*, Vol. 44, No. 1, 1998, pp. 131–140.
      doi:10.1002/aic.690440114