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Comparison of Nonlinear Filtering Techniques for Lunar Surface Roving Navigation

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Abstract

Leading up to the Apollo missions the Extended Kalman Filter, a modified version of the Kalman Filter, was developed to estimate the state of a nonlinear system. Throughout the Apollo missions, Potter's Square Root Filter was used for lunar navigation. Now that NASA is returning to the Moon, the filters used during the Apollo missions must be compared to the filters that have been developed since that time, the Bierman-Thornton Filter (UD) and the Unscented Kalman Filter (UKF). The UD Filter involves factoring the covariance matrix into UDU^T and has similar accuracy to the Square Root Filter; however it requires less computation time. Conversely, the UKF, which uses sigma points, is much more computationally intensive than any of the filters; however it produces the most accurate results. The Extended Kalman Filter, Potter's Square Root Filter, the Bierman-Thornton UD Filter, and the Unscented Kalman Filter each prove to be the most accurate filter depending on the specific conditions of the navigation system.

Introduction

The Kalman Filter, which was originally developed on the eve of the Apollo Space program, has been proven to be a very powerful aerospace navigation tool. The navigation system used throughout the Apollo missions consisted of an inertial frame referenced optical sensor and a modification to the Kalman Filter, which was developed by James H. Potter while working with Dr. Richard Battin at the MIT Instrumentation Laboratory (ref. 5). Since the Kalman Filter was first used in space applications, there have been several modifications and attempts to make the filter more accurate and more efficient. Now that NASA is preparing to return to the Moon each of these filters must be compared in order to determine which will be most effective and practical.

The Kalman Filter, which originally appeared in a 1960 publication by Dr. Rudolph Kalman, "A New Approach to Linear Filtering and Prediction Problems," was studied and further developed by Dr. Stanley Schmidt and the Dynamics Analysis Branch at the NASA Ames Research Center (ARC) (ref. 3). The goal of the Dynamics Analysis Branch was to develop a navigation system for the Apollo missions (ref. 4). The Kalman Filter was originally designed for linear systems with continuous measurements; however for lunar navigation, a nonlinear filter with discrete time measurements was needed. As a result, the Extended Kalman Filter (EKF) was developed at ARC. The EKF makes the system linear around the current estimated state rather than the reference state. Once the EKF was tested it became evident that it would be very useful, as it was comparably accurate to previously used, nonlinear filters but greatly reduced the need for computer memory and computation time (ref. 4).
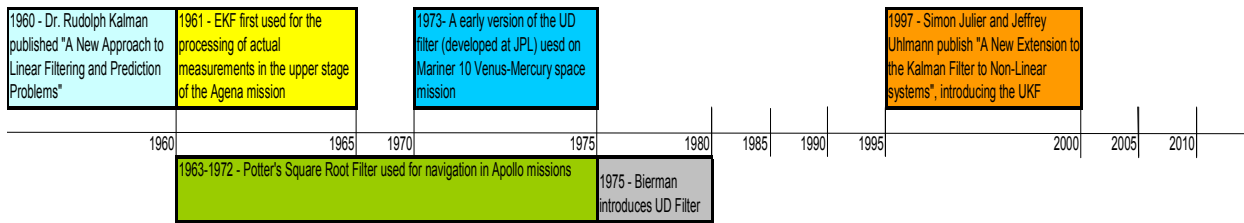
Figure 1.—Timeline of Kalman Filter History.

Several modifications to the EKF have been developed, attempting to increase the filter's stability while decreasing the computer time, shown in figure 1. Two of these modifications are Potter's Square Root Filter, used throughout the Apollo program and the Bierman-Thornton UD Filter, used on the Mariner 10 Venus-Mercury mission in 1973 (ref. 4). More recently, in 1997, a paper titled, "A New Extension of the Kalman Filter to Nonlinear Systems" was published by Simon J. Julier and Jeffrey K. Uhlmann (ref. 2). The UKF has smaller linearization errors than the Kalman Filter, when applied to a nonlinear system and does not require the calculation of partial derivative matrices, reducing computation time.

The Kalman Filter, along with its modifications, has proven to be very effective in space navigation applications since the 1960s. When NASA returns to the Moon, expanding our horizons farther than ever before, one of the modifications of the Kalman Filter will once again be instrumental to the lunar navigation system.

Problem Definition

The purpose of the study presented here was to determine which of the EKF, Potter's Square Root Filter, the UD Filter, or the UKF is the most effective filter for the lunar roving navigation system. The major aspects that determine the effectiveness of a filter include the reduction of error, consistent results, and the decrease of computation time. The lunar navigation roving profile examined in the study is a 10 km northbound track with a 5 km/hr constant velocity, starting at –89 °N, 0 °E.

Each of these filters was compared in four different navigation systems, each with 20 different noise runs, which each last for a period of 7200 sec, with measurements taken on a 1 sec period. The specific time period over which these filters were compared is illustrated in figures 2(a) and (b). Figure 2(a) represents the starting orientation of the two satellites, while figure 2(b) shows the ending orientation of the two satellites, both figures shown in Moon-centered Moon-fixed axes.

The first navigation system consists of a single Lunar Relay Satellite (LRS), shown in figure 3(a). In this system the rover receives two-way radiometric measurements in parallel with one-way measurements. This configuration eliminates clock and frequency bias. The second navigation system has all the components of the first navigation system, and additionally receives one-way measurements from a second LRS when it is in view, shown in figure 3(b). The third navigation system, shown in figure 3(c), also has all the components of the first navigation system, but also receives one-way measurements from both of the Lunar Communication Terminals (LCT). The fourth and final navigation system has all the components of the second navigation system, and in addition receives one-way measurements from both LCTs, shown in figure 3(d) (ref. 7).
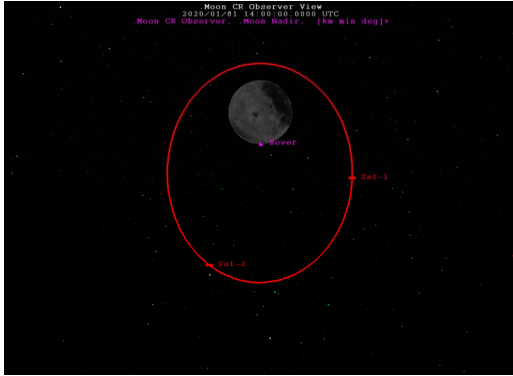
Figure 2(a).—Position of Rover, Sat 1, and Sat 2 at the start of Time Period Two.

Figure 2(b).—Position of Rover, Sat 1, and Sat 2 at the end of Time Period Two.
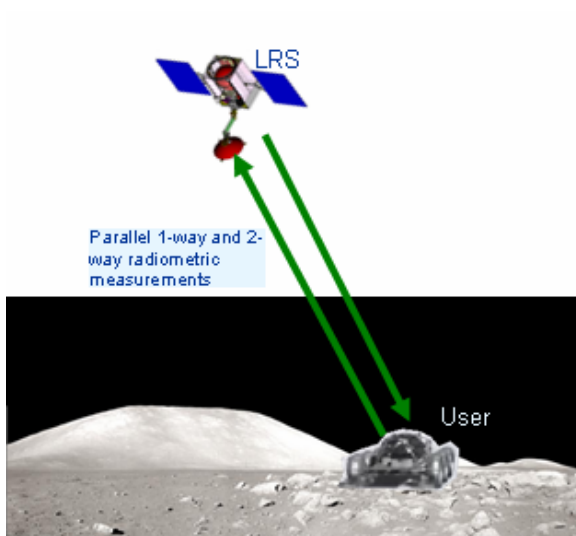

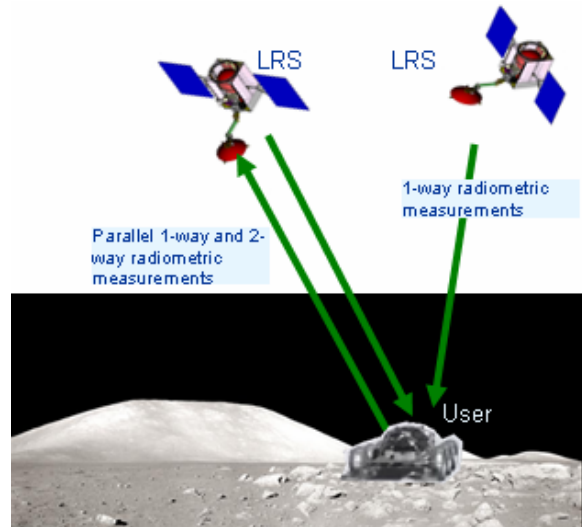Figure 3(a).—Navigation System One.
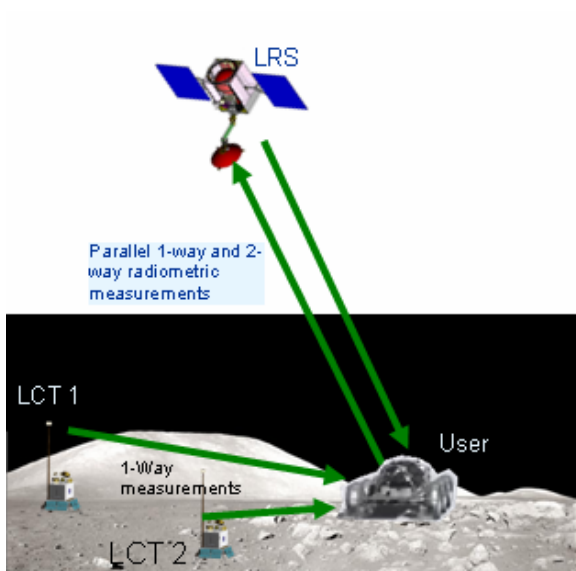

Figure 3(b).—Navigation System Two.

Figure 3(c).—Navigation System Three.

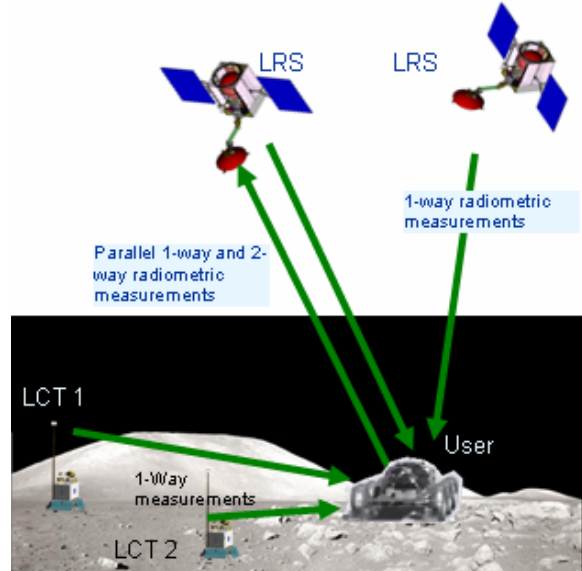
Figure 3(d).—Navigation System Four.

Approach/Method

Kalman Filter

The Kalman Filter consists of a time update and a measurement update step, which are propagated to estimate the mean and covariance of a linear system (ref. 6). The filter can begin with $\hat{x}_k^-$, if all the measurements before, but not including the measurements at time k, are available; or $\hat{x}_k^+$ if all the measurements including the measurement at time k, are available. Equations (1) and (2) correspond to the time update portion of the filter. Equation (3) corresponds to the Kalman Gain. Equations (4) and (5) correspond to the measurement update portion of the filter.

$$\hat{x}_k^- = F_{k-1}\hat{x}_{k-1}^+ + G_{k-1}u_{k-1} \tag{1}$$

$$P_k^- = F_{k-1}P_{k-1}^+ F_{k-1}^T + Q_{k-1} \tag{2}$$

$$K_k = P_k^- H_k (H_k P_k^- H_k^T + M_k R_k M_k)^{-1} \tag{3}$$

$$P_k^+ = (I - K_k H_k)P_k^- \tag{4}$$

$$\hat{x}_k^+ = \hat{x}_{k-1}^- + K_k(y_k - H_k(\hat{x}_{k-1}^-)) \tag{5}$$

Extended Kalman Filter

The EKF, a modified form of the Kalman Filter, was developed for nonlinear applications such as navigation. This filter varies from the Kalman Filter because it makes the nonlinear system linear around the current estimate of the Kalman Filter (ref. 1). Equations (6) through (10) correspond in the same manner as equations (1) through (5) do for the Kalman Filter.

$$\hat{x}_k^- = f_{k-1}(\hat{x}_{k-1}^+, u_{k-1}, 0) \tag{6}$$

$$P_k^- = F_{k-1}P_{k-1}^+ F_{k-1}^T + Q_{k-1} \tag{7}$$

$$K_k = P_k^- H_k^T (H_K P_k^- H_k^T + M_k R_k M_k)^{-1} \tag{8}$$

$$P_k^+ = (I - K_k H_k)P_k^- (I - K_k H_k)^T \tag{9}$$

$$\hat{x}_k^+ = \hat{x}_{k-1}^- + K_k(y_k - h_k(\hat{x}_{k-1}^-, 0)) \tag{10}$$

Potter's Square Root Filter, the UD Filter and the UKF, are each modifications to the EKF routine, developed in an attempt to become more accurate and consistent, and to reduce computer round off errors.

Potter's Square Root Filter

Potter's Square Root Filter uses the Cholesky factor of the covariance matrix (S) in place of the covariance matrix (P). The Cholesky factor is taken from a positive definite, symmetric matrix such that $P = SS^T$. In order to avoid computer round off error that may occur in the filter, the Cholesky of P is taken at the beginning of each run and then propagated though each time step using the Householder transformation. In contrast to the Kalman Filter and the EKF, the Square Root Filter is calculated serially, meaning that each measurement, for $i = 1,...,n$, enters the filter serially, instead of one batch of measurements, for each time step. The updated measurement portion of the filter is shown in equations (11) through (21). Equations (11) and (12) represent the pre-measurement initialization. Equations (13) through (18) represent the serial processing of measurements. Equations (19) and (20) represent the post measurement output.

$$\hat{x}_{0k}^+ = \hat{x}_k^- \tag{11}$$

$$S_{0k}^+ = chol(P_k^+) \tag{12}$$

$$\Phi_i = S_{i-1,k}^{+T} H_{ik}^T \tag{13}$$

$$a_i = 1/(\Phi_i^T \Phi_i + R_{ik}) \tag{14}$$

$$\gamma_i = 1/(1 \pm (a_i R_{ik})^{1/2}) \tag{15}$$

$$S_{ik}^+ = S_{i-1,k}^+ (I - a_i \gamma_i \Phi_i \Phi_i^T) \tag{16}$$

$$K_{ik} = \Phi_i^T S_{ik}^+ \Phi_i \tag{17}$$

$$\hat{x}_{ik}^+ = \hat{x}_{ik-1}^- + K_{ik}(y_{ik} - h_{ik}(\hat{x}_{ik-1}^-,0)) \tag{18}$$

$$S_k^+ = S_{nk}^+ \tag{19}$$

$$\hat{x}_k^+ = \hat{x}_{nk}^+ \tag{20}$$

Bierman-Thornton UD Filter

The UD Filter has similar accuracy to Potter's Square Root Filter, but requires less computation time (ref. 6). Like the Square Root Filter, measurements are processed serially. To initialize this filter the covariance matrix P^+ is decomposed into U^+, D^+, and U^{+T} factors, where U is a unit triangular matrix and D is a diagonal matrix with positive diagonal elements. Equations (21) through (25) correspond to the time update portion of the filter. Equations (26) through (29) correspond to the serial processing of the measurements to update the covariance terms. Equations (30) and (31) represent the post measurement output.

$$\hat{x}_k^- = f_{k-1}(\hat{x}_{k-1}^+, u_{k-1}, 0) \tag{21}$$

$$W = [F * U^+, I] \tag{22}$$

$$W^T = [U^{+T} * F^T; I] \tag{23}$$

$$D = [D^+, 0; 0, Q] \tag{24}$$

$$P^- = WDW^T \tag{25}$$

$$\alpha_i = H_i P_{i-1} H_i^T + R_i \tag{26}$$

$$\overline{UDU}^T = \{D_{i-1} - (1/\alpha_i)[(D_{i-1}U_{i-1}^T H_i^T)(D_{i-1}U_{i-1}^T H_i^T)^T]\} \tag{27}$$

$$U^+ = U^- \overline{U} \tag{28}$$

$$D^+ = \overline{D} \tag{29}$$

$$P^+ = U_n D_n U_m^T \tag{30}$$

$$\hat{x}_k^+ = \hat{x}_{k-1}^- + K_k(y_k - h_k(\hat{x}_{k-1}^-, 0)) \tag{31}$$

Unscented Kalman Filter

The UKF is different than the other filters because is much more computationally intensive, in an attempt to be more accurate (ref. 2). The UKF is more accurate because it does not attempt to make a nonlinear system linear. The unscented transform was developed from the ideas that it is easier to perform a nonlinear transformation on a single point than a probability function and that it is possible to find a group of single points in state space whose sample probability density function approximates the true probability density function of the state vector (ref. 6).

During the time update portion of the filter, twice as many sigma points as the length of the measurement vector are chosen so that all together they have the same mean and covariance of $\hat{x}$ (ref. 2). The known nonlinear function is applied to each sigma point. The resulting transformed vectors are use to get a good estimate of the true mean and covariance. Equations (32) to (36) calculate the sigma points and the time updated state vector and covariance matrix, based off of the sigma points. Equations (37) to (40) calculate the estimated measurements from the sigma points, the measurement covariance matrix, and the measurement-state cross-covariance matrix. Equations (41) to (43) process the Kalman gain matrix, and the post measurement outputs.

$$\hat{x}_{k-1}^i = \hat{x}_{k-1}^+ + \overline{x}^{(i)} \qquad i = 1,..,2n \tag{32}$$

$$\overline{x}^{(i)} = \sqrt{(nP_{k-1}^+)_i^T} \tag{33}$$

$$\overline{x}^{(n+1)} = (\sqrt{nP_{k-1}^+})_i^T \tag{34}$$

$$\hat{x}_k^- = (1/2n)\sum_{i=1}^{2n} \hat{x}_k^i \tag{35}$$

$$P_k^- = (1/2n)\sum_{i=1}^{2n} (\hat{x}_k^{(i)} - \hat{x}_k^-)(\hat{x}_k^{(i)} - \hat{x}_k^-)^T + Q_{k-1} \tag{36}$$

$$\hat{y}_k^{(i)} = h(\hat{x}_k^{(i)}, t_k) \tag{37}$$

$$\hat{y}_k = (1/2n)\sum_{i=1}^{2n} \hat{y}_k^{(i)} \tag{38}$$

$$P_y = (1/2n)\sum_{i=1}^{2n} (\hat{y}_k^{(i)} - \hat{y}_k)(\hat{y}_k^{(i)} - \hat{y}_k)^T + R_k \tag{39}$$

$$P_{xy} = (1/2n)\sum_{i=1}^{2n} (\hat{x}_k^i - \hat{x}_k^-)(\hat{y}_k^{(i)} - \hat{y}_k)^T \tag{40}$$

$$K_k = P_{xy} P_y^{-1} \tag{41}$$

$$\hat{x}_k^+ = \hat{x}_k^- + K_k(y_k - \hat{y}_k) \tag{42}$$

$$P_k^+ = P_k^- - K_k P_y K_k^T \tag{43}$$

Results

Results are reported out into four sections comparing each of the four filters described above. Results are plotted into four subplots, corresponding to each of the four navigation system options explored. The x axis corresponds to the time axis, in seconds, for the 2 hr roving profile. The y axis corresponds to the error, in meters, between the filter output and the real trajectory, plotted in log scale. Each of the 20 noise profile runs are plotted within each subplot.

Extended Kalman Filter

The estimated position error calculated by the EKF for each of the four systems ranged from 1 m down to 1 cm, and was most commonly on the order of one decimeter, shown in figure 4. System One produced the most consistent results, while they other systems each had a greater range of error. The error in both System Three and System Four decreased with time throughout the 7200 time steps.

Potter's Square Root Filter

The Square Root Filter produced a similar range of error to that of the EKF, from 1 m to 1 cm, illustrated in figure 5. However, the Square Root Filter's results were more concentrated towards the 1 m end of that range and had inconsistent results of less than 1 cm of error. Unlike the EKF, none of the systems displayed a decline in error with time. System One was the most consistent system and its steadiness improved through each run, but all the systems were less consistent than in the EKF.
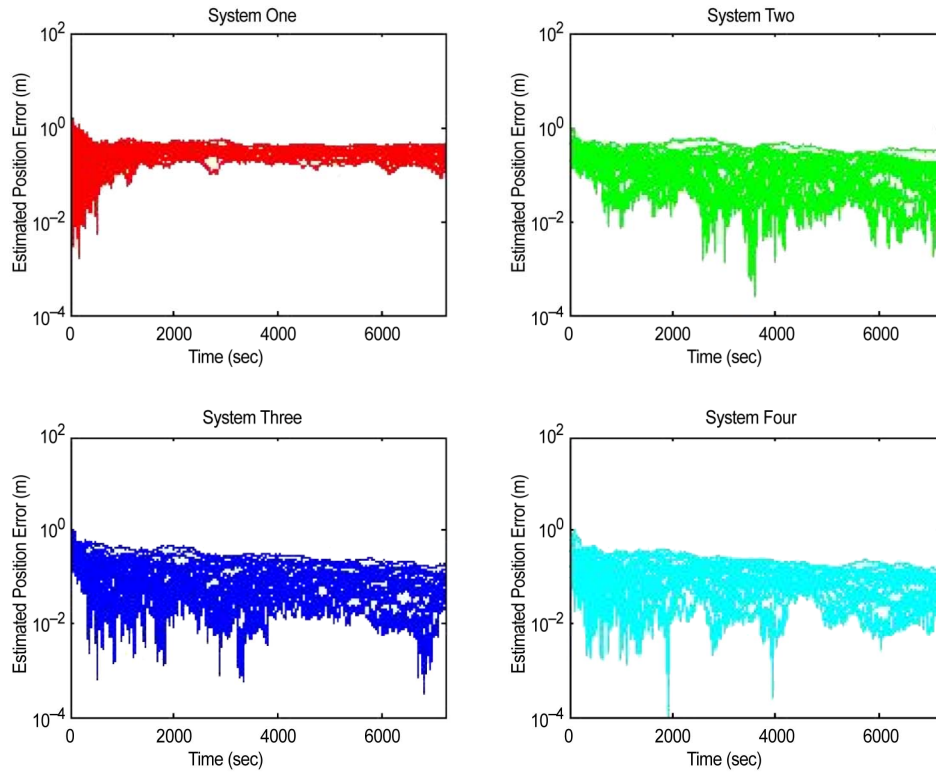
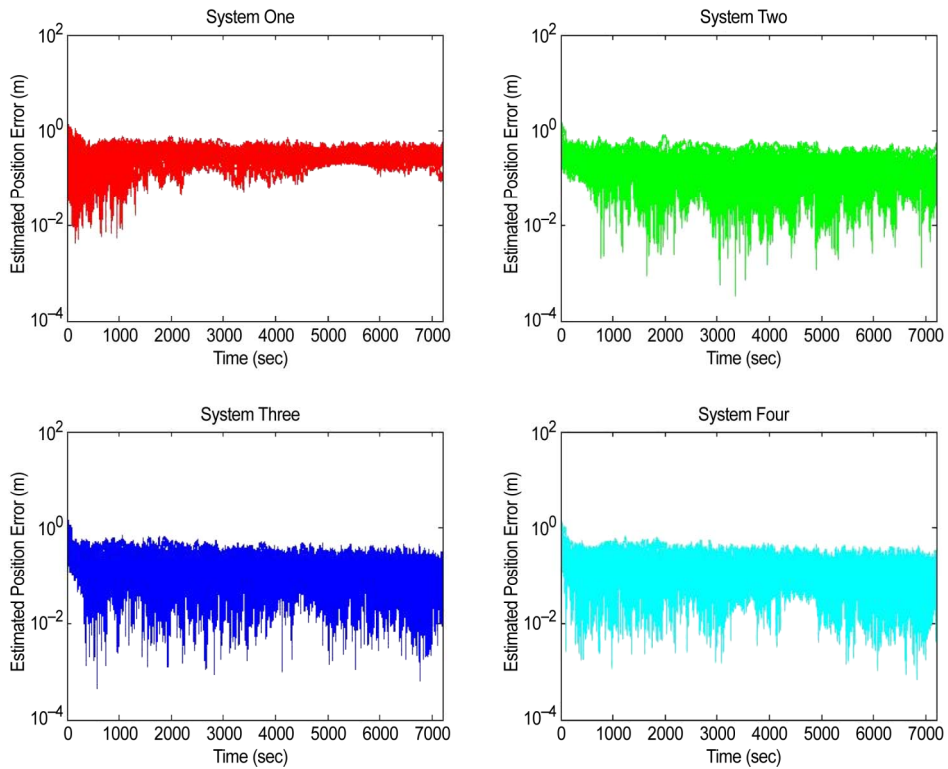Figure 4.—Estimated Position Error versus Time for the EKF.

Figure 5.—Estimated Position Error versus Time for the Square Root Filter.

Bierman-Thornton UD Filter

All four of the systems in the UD Filter displayed a decline in error with time as seen in figure 6. The range of the results of the UD Filter was slightly larger than that of the previous two filters extending just above 1 m of error and with occasional results well below 1 cm of error. However the majority of the runs seem to calculate less error than that of the Square Root Filter. System One was initially quite consistent but quickly decomposed around time step 2000. Systems Three and Four produced the best results from this filter because they were most consistent and declined with the steepest slope.

Unscented Kalman Filter

The UKF estimated error in a similar range of the UD Filter, shown in figure 7. Although the range was larger than that of the EKF, the results were concentrated around a smaller error than the EKF. All of the systems demonstrated a reduction in error throughout each of the runs, with Systems Three and Four declining at the sharpest angle. System One was the least consistent system peaking just after 2000 sec and then declining from there.
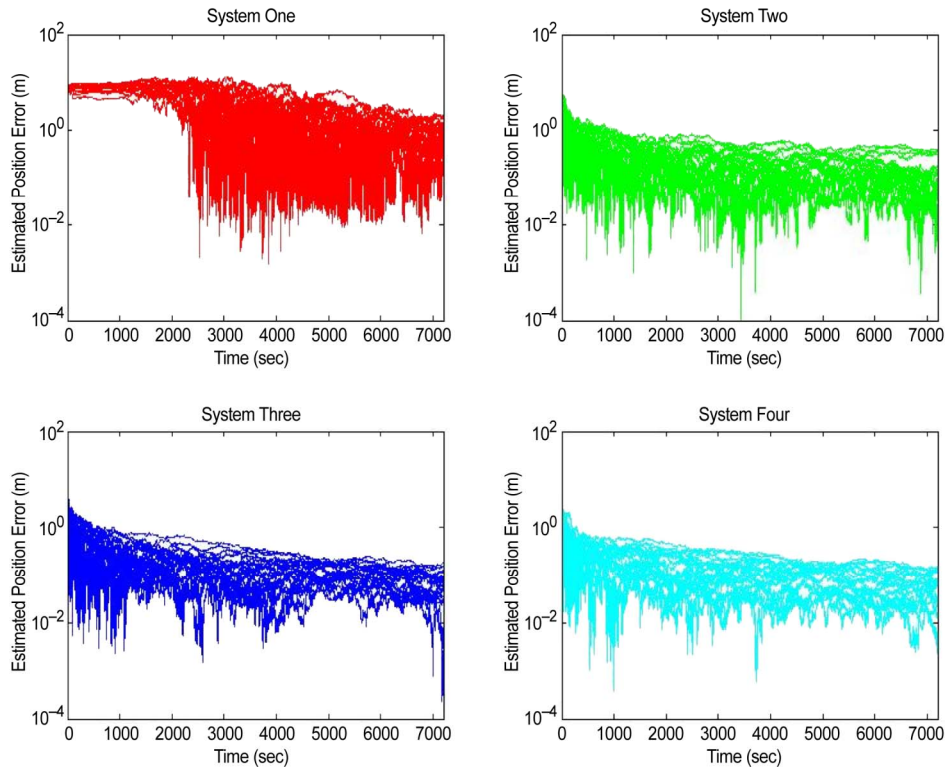

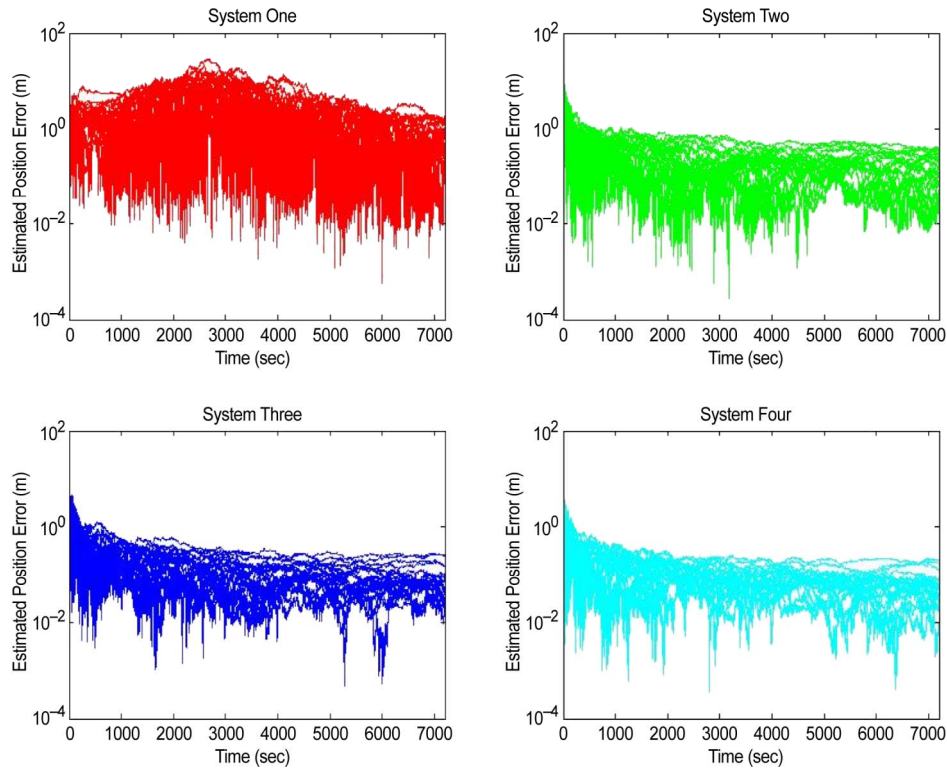Figure 6.—Estimated Position Error versus Time for the UD Filter.

Figure 7.—Estimated Position Error versus Time for the UKF.

In figures 8 to 11, the root mean square of the estimation error over twenty runs is plotted against time, comparing the four filter variations for each navigation system. System One proved to be the anomaly as the UD Filter and the UKF performed very poorly. The EKF and Square Root Filter both produced a fairly constant error between a decimeter and a meter. The UD and UKF both attained errors as high as 10 m and then declined to less than 1 m with time. The other three systems had more in common. The EKF performed surprisingly well compared to the other filters. The EKF always produced an error similar to the best of the other filters and was sometimes the most accurate of all the filters. The Square Root Filter produced a fairly constant error of just over a decimeter in each of the systems. The UD Filter began with slightly greater error than the Square Root Filter and decreased in error so that by the end the error was equal to (in System Two) and less than (Systems Three and Four) that of the Square Root Filter. The UKF behaved in a similar fashion to that of the UD Filter beginning with a greater error and then decreasing to a smaller error. In System Four the UKF was the most accurate filter, under a decimeter of error, for nearly the entire run, with the only exceptions being at the very beginning and the very end of the run.

Computation Time

The bar graph shown in figure 12 compares the computation time of the four filtering routines. The EKF was computationally the fastest running filter, taking about 744 sec to complete all twenty runs. The next fastest filter was the UD Filter in 960 sec, just slightly faster than the Square Root Filter in 965 sec. The UKF involved the greatest number of computations and therefore took about 4190 sec to run. Since each run consisted of 7200 time steps, each equivalent to one second, which works out to 144 000 sec per time period, each of the filters was working much faster than real time. All of these times were measured on Q34 XPPro MultiPlatform Load Intel Core2 CPU 6400 at 2.13GHz, 1 GB of RAM computer.
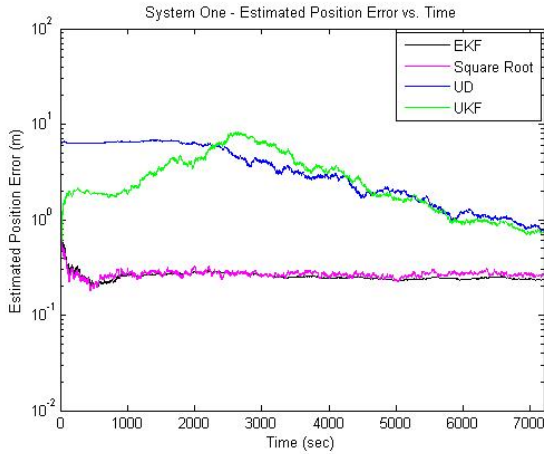
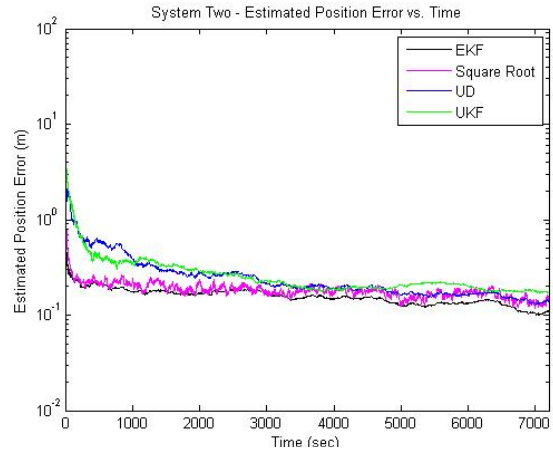Figure 8.—Estimated Position Error versus Time for System One.

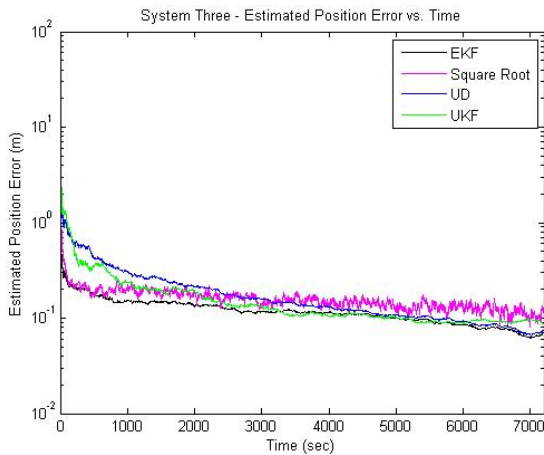Figure 9.—Estimated Position Error versus Time for System Two.

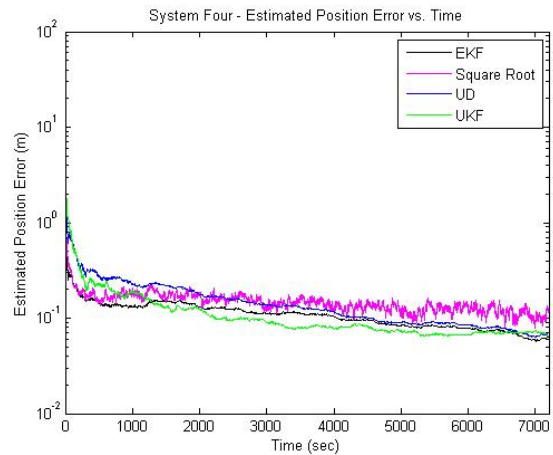Figure 10.—Estimated Position Error versus Time for System Three.

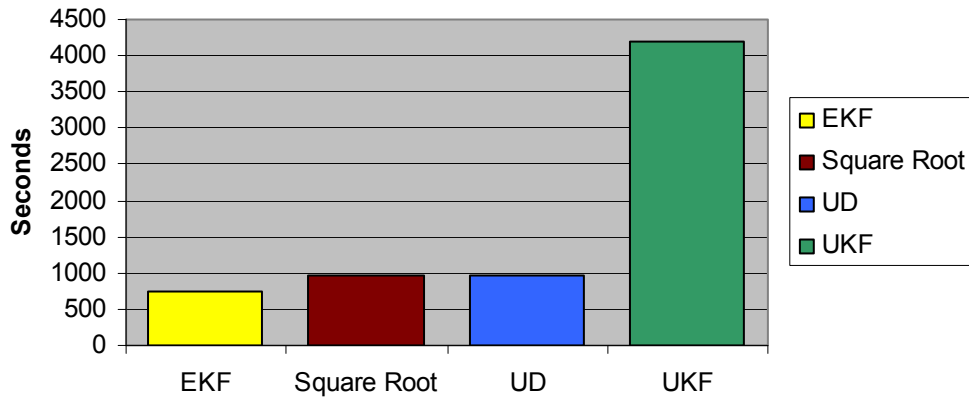Figure 11.—Estimated Position Error versus Time for System Four.

Figure 12.—Computation Time of the EKF, Square Root Filter, UD Filter, and UKF

Conclusion

Each of the Square Root Filter, UD Filter, and the UKF produce more accurate results than the EKF at certain times. The first navigation system, which only received radiometric measurements from one LRS, produced large errors with the UD Filter and the UKF. These errors can be attributed to the fact that there are simply not enough measurements in this system to produce an accurate estimate. Theoretically, the Square Root Filter and the UD Filter should produce results on the same order of magnitude of accuracy. The results show that the UD Filter started with similar accuracy of the Square Root Filter but became increasingly more accurate throughout each run. In addition, the UD Filter required less computation time than the Square Root Filter. Therefore, the UD Filter is more efficient than the Square Root Filter. In System Four the UKF was the most accurate filter for almost the entire run. The error calculated by the UKF continued to decreases in a similar manner to that of the UD Filter. The UKF did not require the calculation of the partial derivatives of the H matrix; however it was still much more computationally intensive than all of the other filters because it performed calculations for each of the sigma points (there are twice as many sigma points as the length of the measurement vector) (ref. 2). As a result the UKF was by far the slowest running filter. Even though it was slowest, it still completed the calculations for the time period in about 4190 sec, much faster than the real time of 144,000 sec. Therefore, with further investigation, the UKF may prove to be the most efficient filter for lunar navigation.

There is a great deal more that must be considered before a conclusive decision can be made with regards to which filter will be the most efficient for lunar navigation. Additional techniques that are currently being considered involve the components and design of the navigation system, the use of inertial measurements in the filter, as well as other filter types, including the Square Root UKF. The Kalman Filter, along with the filters that have been developed from Kalman Filter theory, have each proven that they are and will continue to be useful tools in navigation.

References

1. Grewal, M.S., Weill, L.R., and Andrews, A.P. (2001). *Global Positioning Systems, Inertial Navigation and Integration*. New York: John Riley and Sons.
2. Julier, S.J. and Uhlmann, J.K. (1997). "A New Extension of the Kalman Filter to Nonlinear Systems." Retrieved June 8, 2007 at http://www.cs.unc.edu/~welch/kalman/media/pdf/Julier1997_SPIE_KF.pdf
3. Kalman, R.E. (1960). "A new approach to linear filtering and prediction problems." Retrieved August 1, 2007 from the ASME Journal of basic Engineering: http://www.cs.unc.edu/~welch/kalman/media/pdf/Kalman1960.pdf
4. McGee, L.A. Schmidt, S.F. National Aeronautics and Space Administration (1985*) Discovery of the Kalman Filter as a Practical Tool for Aerospace and Industry*. Retrieved June 5, 2007, from NASA technical reports server: http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19860003843_1986003843.pdf
5. National Aeronautics and Space Administration. (1965, September 1). *Apollo Navigation Ground and Onboard Capabilities.* Retrieved June 11, 2007, from NASA technical reports server: http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19690020854_1969020854.pdf
6. Simon, D. (2006). *Optimal State Estimation: Kalman, H Infinity, and Nonlinear Approaches*. New York: John Riley and Sons.
7. Space Communication Architecture Working Group—NASA. (2006, May 15). *NASA Space Communication and Navigation Architecture Recommendations for 2005-2030.* Retrieved August 1, 2007 from: https://www.spacecomm.nasa.gov/spacecomm/Modules/document_open_direct.cfm?docpath=%400W0JR%5D%5DKTYBS%3F%25%5D%3C%3ACU5%29AG%23%29%21%5B9X%2C%3E%2A%5FIQ8WX%3FP%2D%228%20%0A

8. Welch, B.W., Connolly, J.W., and Sands, O.S. National Aeronautics and Space Administration (2007, February). *Dilution-of-Precision-Based Lunar Surface Navigation System Analysis Using Earth-Based Assets.* Retrieved June 5, 2007 from NASA technical reports server: http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/20070018936_2007018375.pdf

9. Van der Merwe, R. and Wan, E.A. (2001). "The Square-Root Unscented Kalman Filter for State and Parameter-Estimation" [Electronic version]. *IEEE,* 3461-3464.

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13. SUPPLEMENTARY NOTES

14. ABSTRACT
Leading up to the Apollo missions the Extended Kalman Filter, a modified version of the Kalman Filter, was developed to estimate the state of a nonlinear system. Throughout the Apollo missions, Potter's Square Root Filter was used for lunar navigation. Now that NASA is returning to the Moon, the filters used during the Apollo missions must be compared to the filters that have been developed since that time, the Bierman-Thornton Filter (UD) and the Unscented Kalman Filter (UKF). The UD Filter involves factoring the covariance matrix into UDU^T and has similar accuracy to the Square Root Filter; however it requires less computation time. Conversely, the UKF, which uses sigma points, is much more computationally intensive than any of the filters; however it produces the most accurate results. The Extended Kalman Filter, Potter's Square Root Filter, the Bierman-Thornton UD Filter, and the Unscented Kalman Filter each prove to be the most accurate filter depending on the specific conditions of the navigation system.

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Moon; Navigation; Surface navigation; Doppler navigation; Radio navigation; Satellite navigation systems; Positioning; State estimation; Kalman filters

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