

# Lightning Charge Retrievals: Dimensional Reduction, LDAR Constraints, and a First Comparison w/LIS Satellite Data.

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# SDMetrics Entry - Presentations

**Title:** Lightning Charge Retrievals: Dimensional Reduction, LDAR Constraints, and a First Comparison w/LIS Satellite Data

**Presenters:** W. J. Koshak, E. P. Krider, N. Murray, D. J. Boccippio

**Conference Name:** Course Presentation to UAB Students at Laboratory for Global Health Observation

**Location:** UAB, Birmingham, AL

**Conference Start Date:** 11/27/2007

**Conference End Date:** 11/27/2007

**Date Presented:** 11/27/2007

**Conference Proceedings to Follow:** No

**Organization:** VP61

# What is an Inverse Problem?

Unknown

$\vec{f}_t$

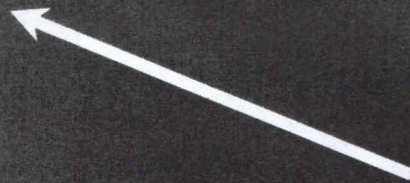


Measurements

$\vec{g}$



Law





# First Inversion Scientist?



Hunger ...

Tracks ...

Invert ...

Decide to Follow ...

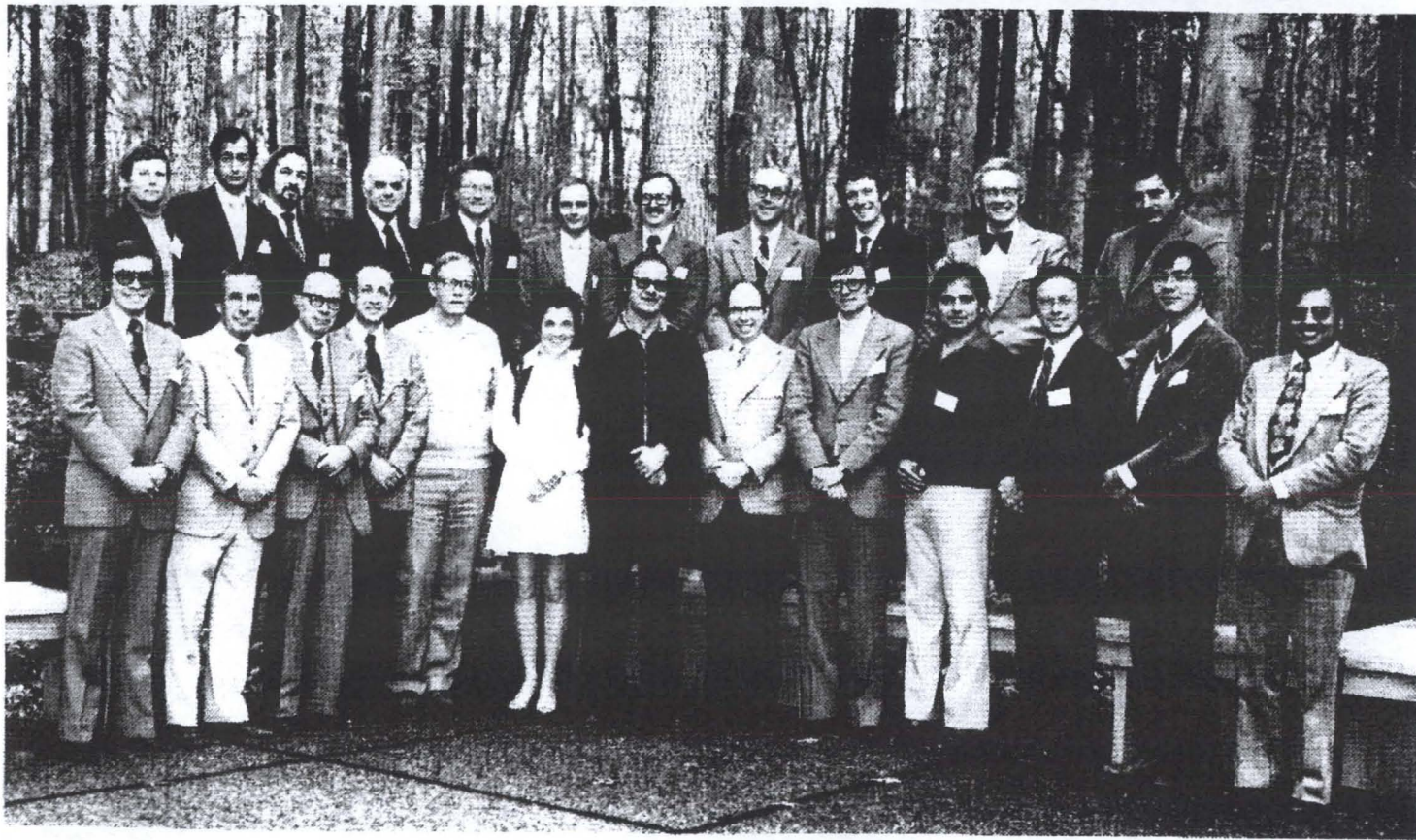
Find ...

Kill ...

Eat !



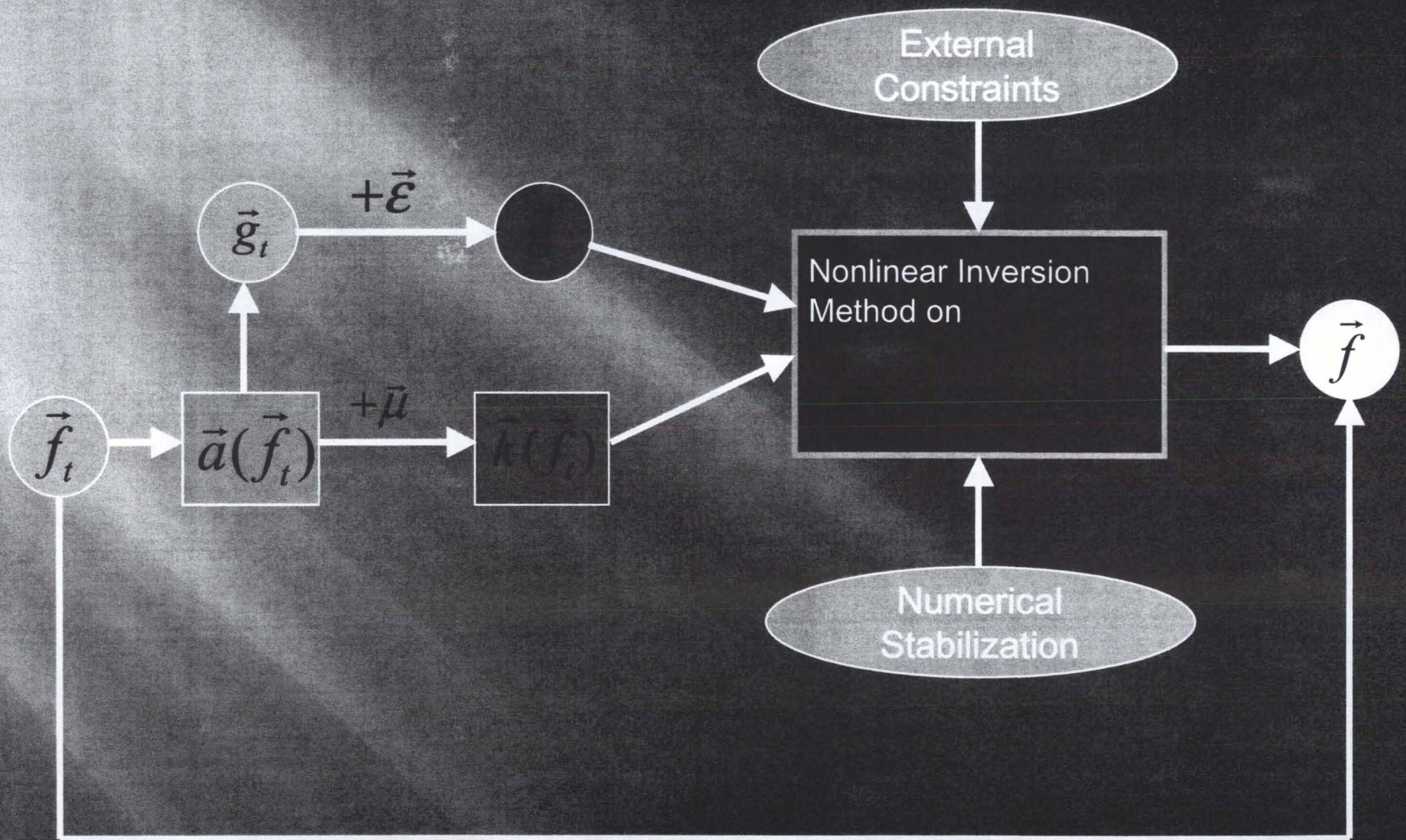
# 1<sup>st</sup> International Interactive Workshop on Inverse Methods 1976



Workshop Speakers and Chairmen: (Left to right) Front row: M. P. McCormick (Associate Chairman), Langley Research Center; S. Twomey, U. Arizona; L. Kaplan, U. Chicago; M. Chahine, JPL/Cal. Tech; H. van de Hulst, U. Leiden, Netherlands; C. Whitney, C. S. Draper Lab; E. Westwater, NOAA/WPL; D. Staelin, MIT; B. Conrath, Goddard SFC; J. Kuriyan, UCLA; J. Gille, NCAR; W. Chu, Old Dominion U.; A. Deepak (Chairman), Old Dominion U. Second row: J. Lenoble, U. de Lille, France; B. Herman, U. Arizona; A. Fymat, JPL/Cal Tech; J. King, AFGI; A. Green, U. Florida; H. Malchow, C. S. Draper Lab; W. Irvine, U. Massachusetts; H. Fleming, NOAA/NESS; C. Rodgers, U. Oxford, UK; C. Mateer, Atmos. Environ. Serv., Canada; T. Pepin, U. Wyoming.

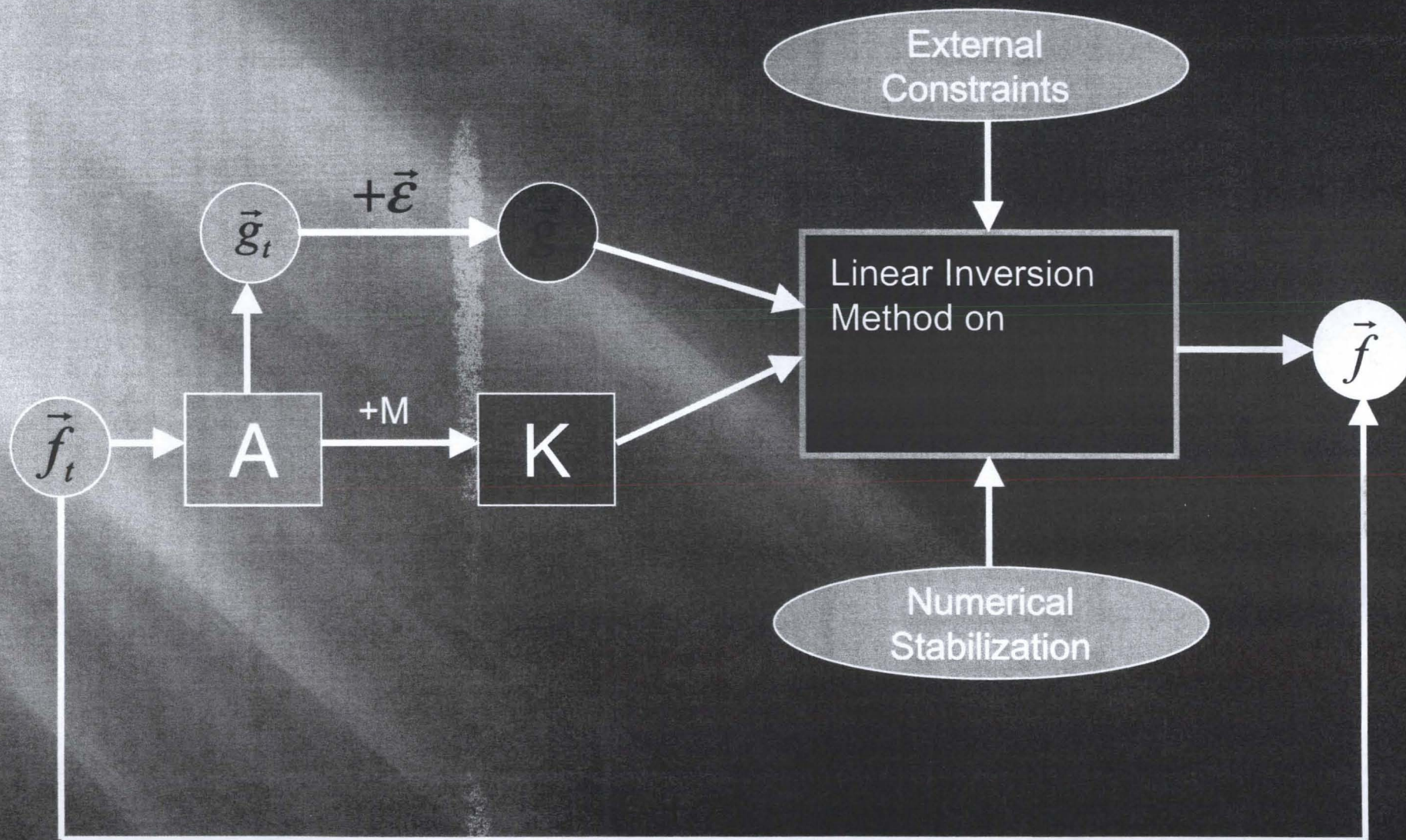


# General Inversion:



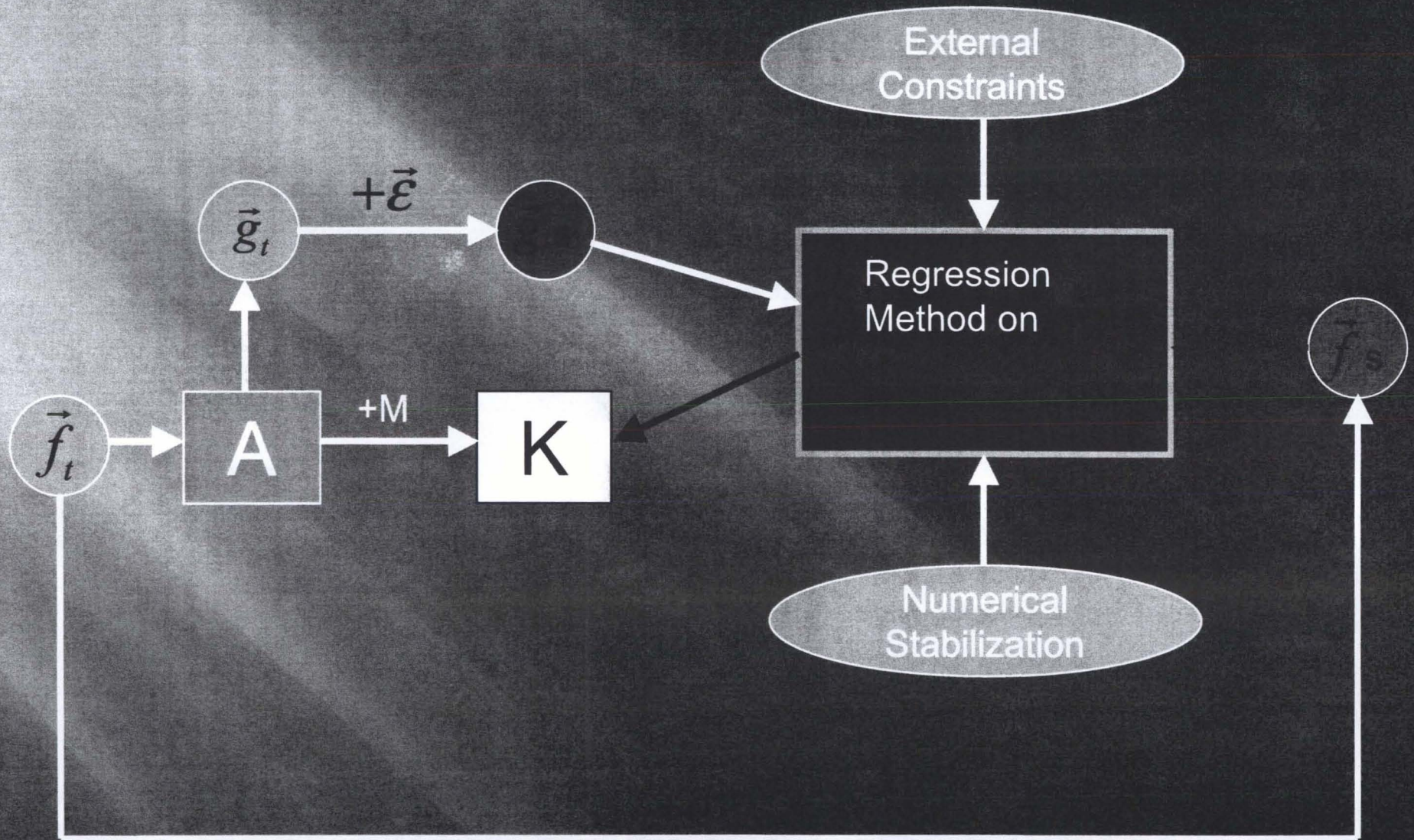


# Linear Inversion:



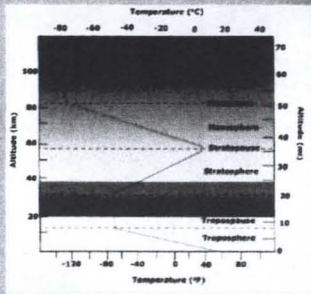


# Regression:



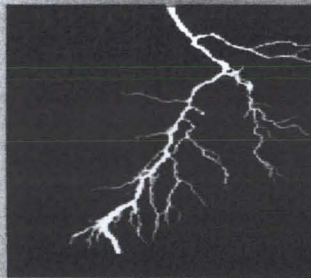


# Examples

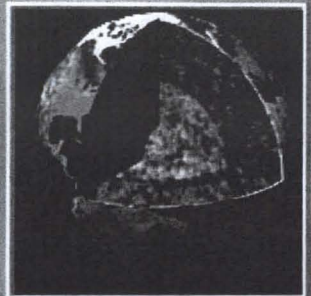


Temperature Sounding  
(IR Spectrometer)

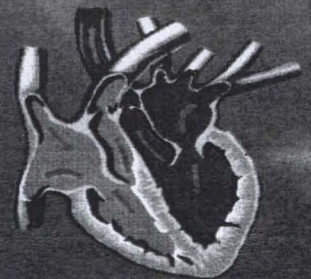
... Many More!



Lightning Charge  
(Electric Field Change)



Earth Interior Density  
(Sound Wave Propagation)



Cardiac Parameters  
(Surface Electric Potential)

I.P

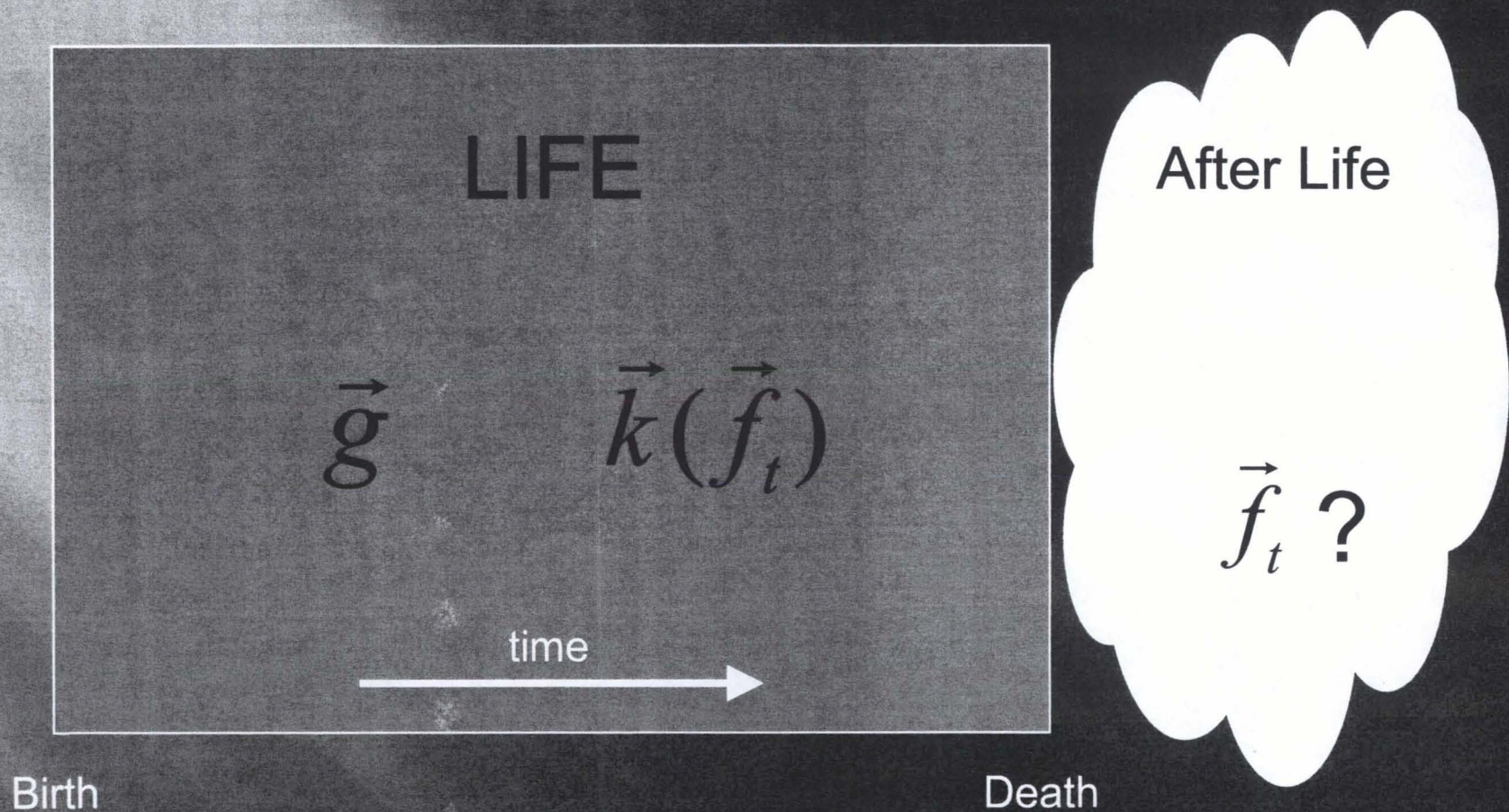
Inverse  
Problems

At the intersection of Mathematics and Physics, inverse problems are concerned with the determination of the cause from the effect.

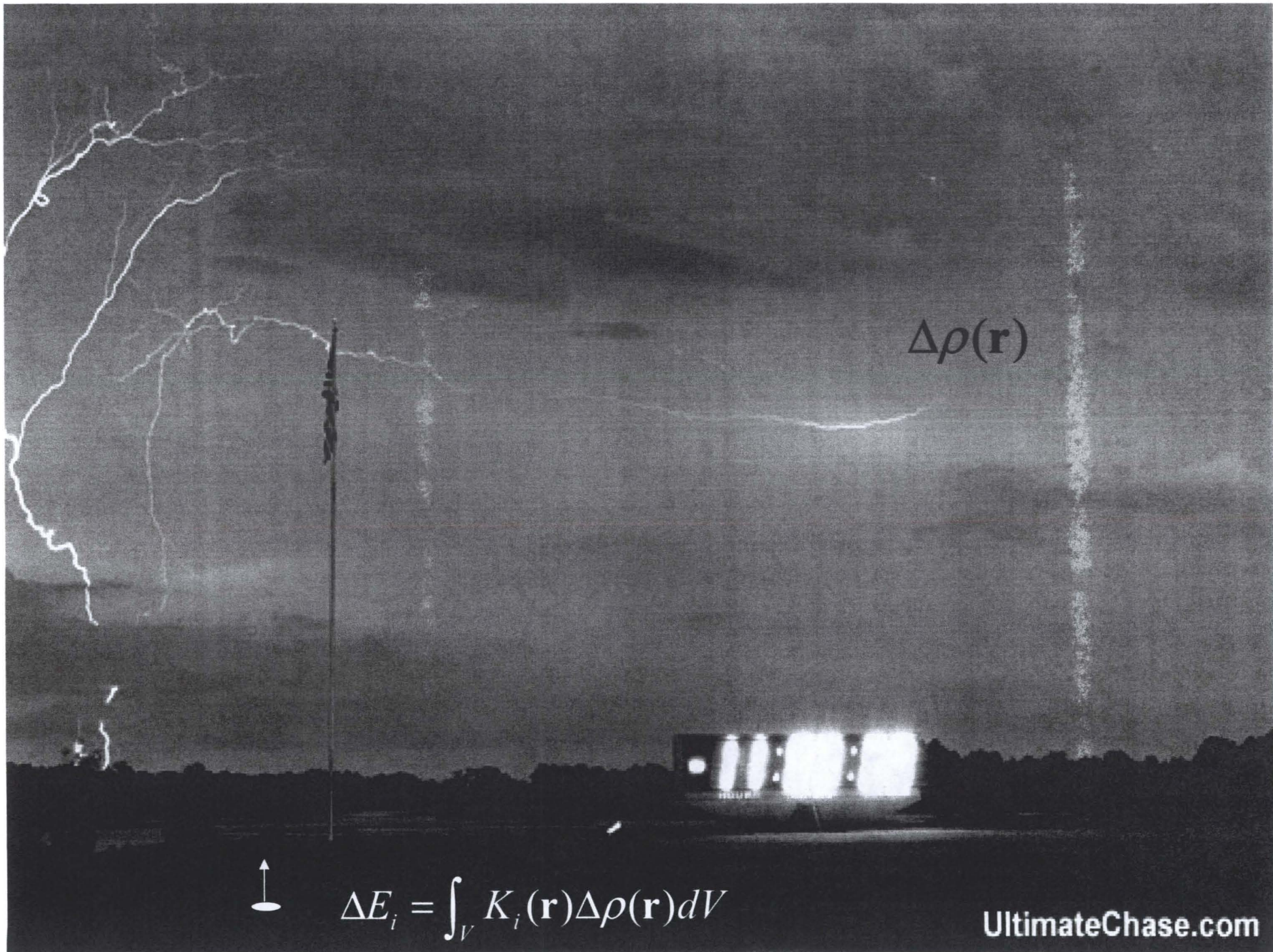
Department of Physics



# We are ALL Inverters







$\Delta\rho(\mathbf{r})$

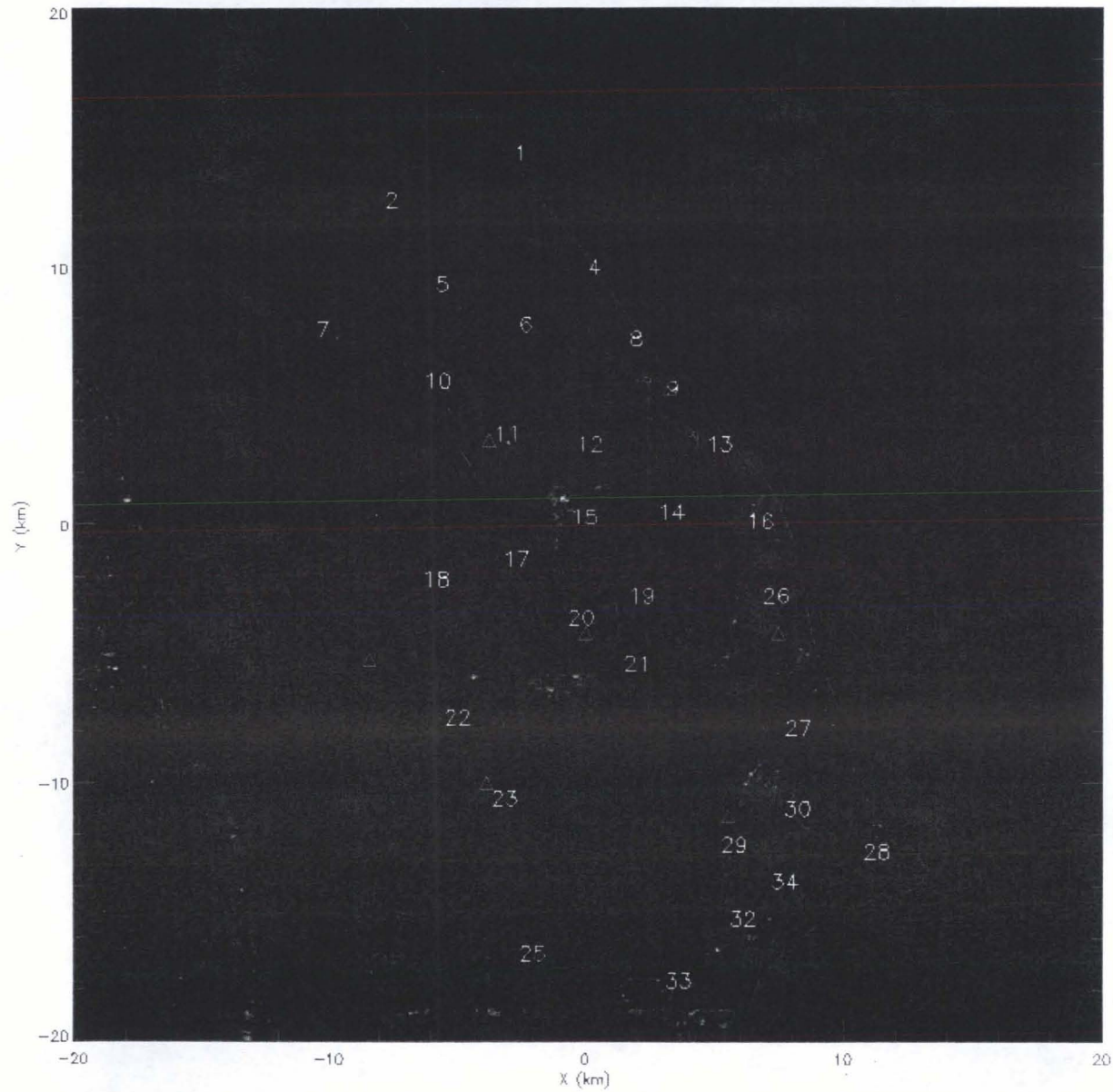


$$\Delta E_i = \int_V K_i(\mathbf{r}) \Delta\rho(\mathbf{r}) dV$$

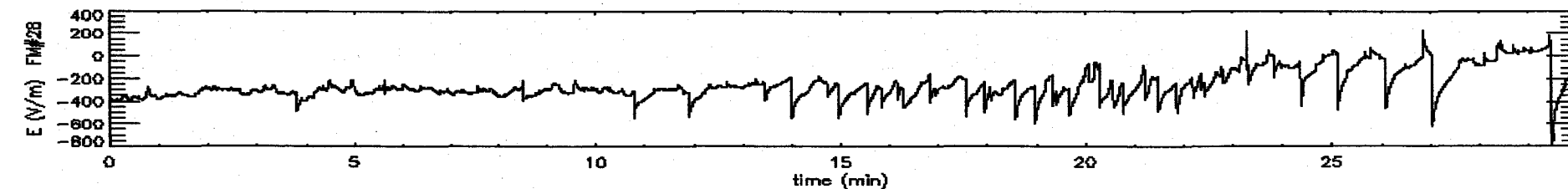
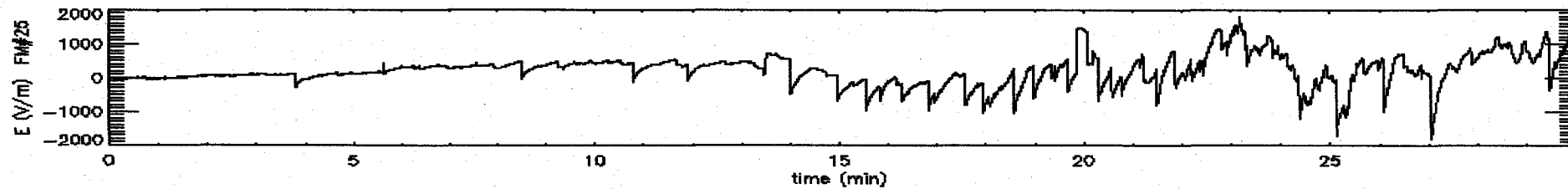
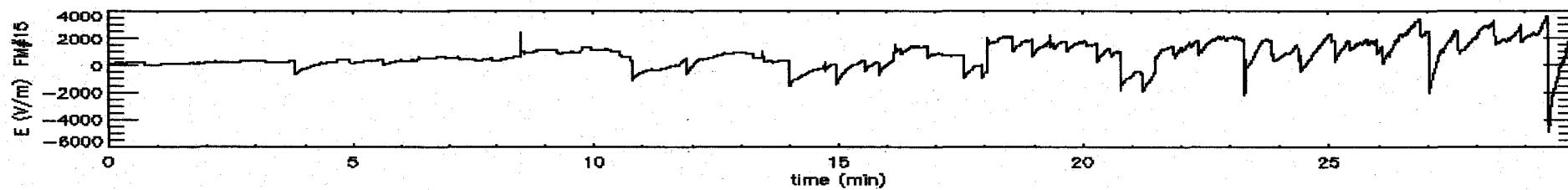
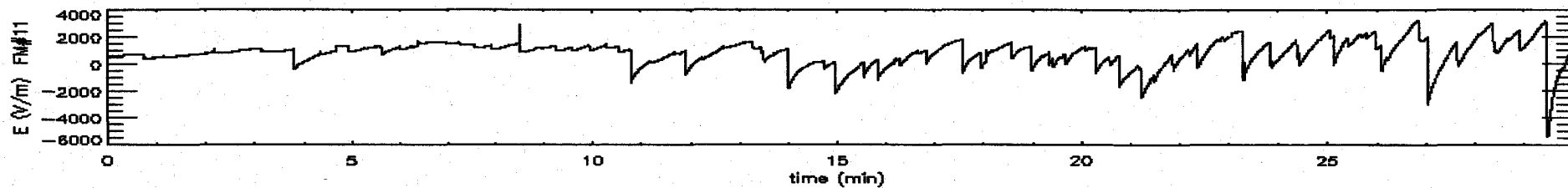
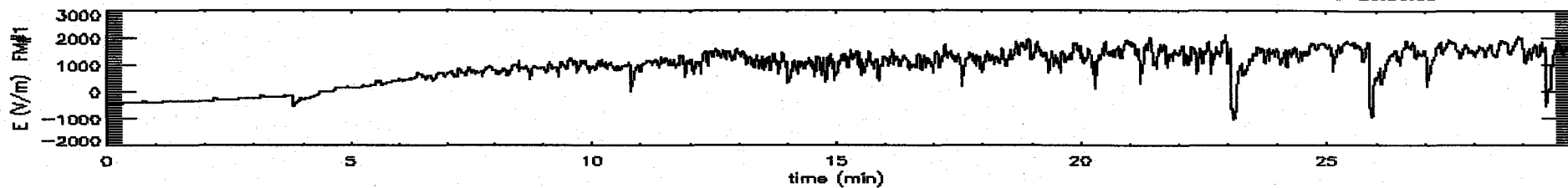
UltimateChase.com



# KSC Field Mill Network









# Chi-Squared Goodness-of-Fit

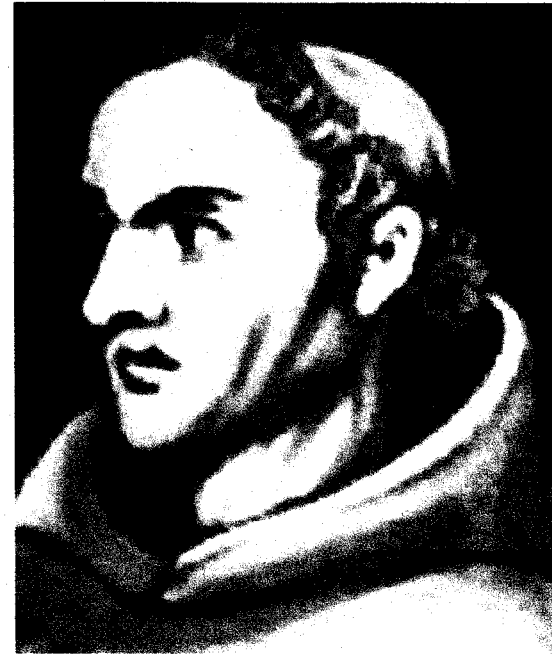
$$\chi^2(\mathbf{a}) = \frac{1}{m-n} \sum_{i=1}^m \frac{(M_i(\mathbf{a}) - \Delta E_i)^2}{\sigma_i^2}$$

$$\mathbf{a} = (a_1, \dots, a_n)$$



# Ockham's Razor

"Pluralitas non est ponenda  
sine neccesitate"



Circa 1288-1348



## **NEEDS** (necessitate)

- Air
- Water
- Food
- Minimal Shelter/Clothing
- Medical Care
- Love

## **WANTS** (Pluralitas)

- A/C
- SUV
- Jet Ski
- Designer Clothing
- Boat/Yacht
- Video Camera
- Gourmet Foods
- Swimming Pools
- Sporting Goods
- Stereos
- Plastic Surgery
- Bungee Jumping
- Vacations
- Opulent Home(s)
- 
- etc.
-



# Jacobson and Krider 1976

## Electrostatic Field Changes Produced by Florida Lightning

ELIZABETH A. JACOBSON AND E. PHILIP KRIDER

*Institute of Atmospheric Physics, The University of Arizona, Tucson 85721*

(Manuscript received 10 June 1975, in revised form 13 September 1975)

### ABSTRACT

The electrical behavior of thunderstorms triggered by local heating and sea-breeze convergence, a low pressure disturbance, and a weak frontal passage has been studied at the NASA Kennedy Space Center, Florida. A nonlinear least-squares minimization procedure has been developed to describe changes in the total electrostatic field produced by lightning in terms of point charge models for the cloud charge distributions. The results of this analysis indicate that discharges to ground usually neutralize cloud charges in the range from  $-10$  to  $-40$  C. The computed charge altitudes for Florida are somewhat higher than for other geographical locations, 6 to 9.5 km, but the corresponding ambient air temperatures,  $-10$  to  $-34^{\circ}\text{C}$ , are similar. A large fraction of the discharges to ground show total field changes which are small or even reversed in polarity within 3 km of the discharges. An analysis of these cases suggests that ground discharges often neutralize a small positive charge, 0.5 to  $\pm$  C at altitudes of 1 to 3 km, in addition to the larger negative charge higher in the cloud.

### 1. Introduction

The NASA Kennedy Space Center (KSC) has constructed and is currently operating a large network of ground-based electric field mills, in order to identify clouds which might be an electrical hazard to space vehicles prior to and during launch. These instruments have been installed to minimize the chance of lightning disturbances such as those which occurred during the launch of Apollo 12 (Krider *et al.*, 1974). The KSC network represents a unique facility for the study of lightning and thunderstorms not only because of its size and the quality of its data acquisition system, but also because it is located in Florida, a region of high thunderstorm frequency which has received little previous study. In this paper, we present and analyze typical electric fields produced by thunderstorms and lightning at KSC. Data were obtained during two storms in 1973 using a single field mill and for a number of storms in 1974 using 21 instruments.

### 2. Instrumentation

A map showing the locations of the field mill sites at KSC is given in Fig. 1. Each field mill contains eight vertically oriented stator sectors which are alternately covered and uncovered by a grounded rotor turning at 1800 rpm. The differential voltage between the covered and uncovered stators is filtered to remove high-frequency components, amplified, and rectified using a synchronous sample and hold circuit. The resulting signal is passed through a low-pass filter with a 0.1 s time constant to remove ac components and

provides both the amplitude and polarity of the external electric field. The 0.1 s time constant is too slow to time-resolve individual return strokes in discharges to ground, but is more than adequate to resolve an entire flash. The analog output of each field mill is transmitted to a central receiving station, where it is

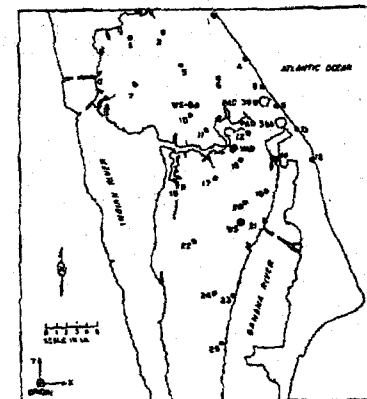
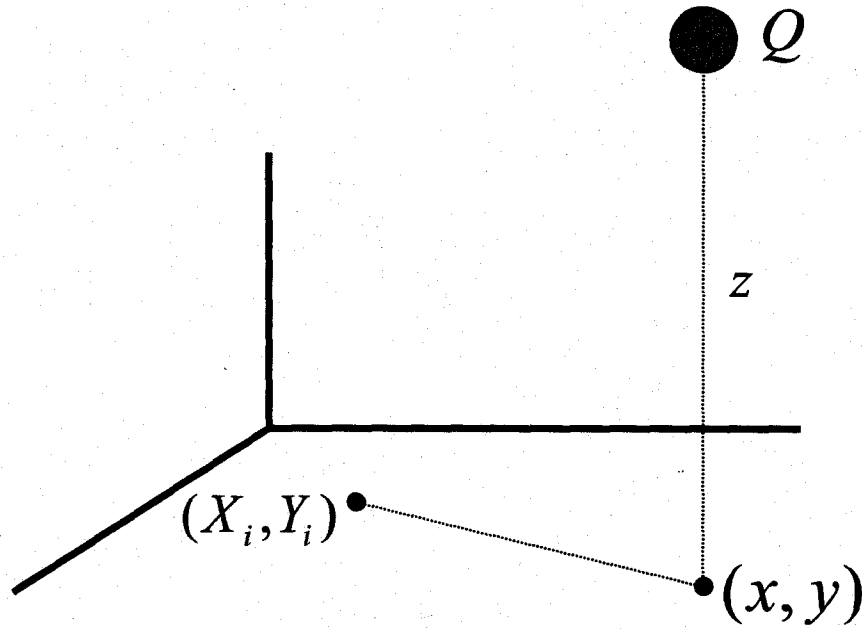


FIG. 1. The locations of the electric field measuring sites at the NASA Kennedy Space Center, Florida. WS is the site of the KSC weather office, and WS-B the location of the X-band radar.



# 1-Charge Model (4 parameters)



$$M_{1i}(\mathbf{a}_1) = \frac{-Qz}{2\pi\epsilon_o[(X_i - x)^2 + (Y_i - y)^2 + z^2]^{3/2}},$$

$$\mathbf{a}_1 = (x, y, z, Q)$$

# An Analysis of the Charge Structure of Lightning Discharges to Ground

PAUL R. KREHBIEL, MARK BROOK, AND ROY A. MCCRORY<sup>1</sup>

New Mexico Institute of Mining and Technology, Socorro, New Mexico 87801

Sources of charge for the individual strokes of four multiple-stroke flashes to ground have been determined, using measurements of the electrostatic field change obtained at eight locations on the ground beneath the storm. The resulting charge locations have been compared to 3-cm radar measurements of precipitation structure in the storm. The field changes of individual strokes were found to be reasonably consistent with the lowering to ground of a localized or spherically symmetric charge in the cloud. The centers of charge for successive strokes of each flash developed over large horizontal distances within the cloud, up to 8 km, at more or less constant elevation between the  $-9^{\circ}$  and  $-17^{\circ}\text{C}$  environmental (clear air) temperature levels. Comparison with the radar measurements has shown that the discharges developed through the full horizontal extent of the precipitating region of the storm and appeared to be bounded within this extent. In one instance where cellular structure of the storm was apparent, the strokes selectively discharged regions where the precipitation echo was the strongest. Vertical extent of the stroke charge locations was small in comparison with the vertical extent of the storm. The field changes in the intervals between strokes have been found to exhibit many of the features which Malan and Schoonland used to infer that ground flashes discharge a nearly vertical column of charge in the cloud. This and other evidence is used to show that their observations, which were made at a single station, could instead have been of horizontally developing discharges. The interstroke field changes have been analyzed using a point dipole model and found to correspond to predominantly horizontal charge motion that was closely associated with the ground stroke sources for the flashes. The interstroke activity served effectively to transport negative charge in the direction of earlier stroke volumes and often persisted in the vicinity of an earlier stroke volume, while subsequent strokes discharged more distant regions of the cloud. Long-duration field changes that sometimes preceded the first stroke of a flash have been analyzed and found to correspond to a series of vertical and horizontal breakdown events within the cloud, prior to development of a leader to ground. These events were associated in part with the negative charge region that became the source of the first stroke and effectively transported negative charge away from the first stroke charge volume and from the charge volumes of subsequent strokes. Several continuing current discharges were found also to progress horizontally within the cloud and sustained currents in the range of 580 A to less than 50 A. The continuing current field changes were consistently better fitted by the monopole charge model than the field changes of discrete strokes within the same flash.

## Krehbiel, Brook, and McCrory 1979

### 1. INTRODUCTION

Wilson [1916] first suggested that simultaneous measurements 'at a sufficiently large number of points' of the electrostatic field change caused by lightning could be used to obtain a measure of the discharge, and he proceeded to specify the equations relating the field change at the ground to the height and distance of simple one- and two-point charge models. Although it was not feasible for him to make such measurements, he approximated them in this and a later study [Wilson, 1920] by measuring the electric field change as a function of distance from various lightning discharges. From this he was able to obtain estimates of the average electric moment destroyed by a lightning flash and of the height and magnitude of the charge(s) involved.

The first multiple-station lightning study was conducted in New Mexico by Workman et al. [1942], who made time-resolved measurements of the electric field change caused by lightning at eight stations over an area 6 km in diameter. The data were used to compute center of charge locations for the individual strokes of discharges to ground and for complete intracloud discharges. Both the negative and positive charges were found to lie within a limited range of ambient cloud temperature, between  $-5^{\circ}$  and  $-25^{\circ}\text{C}$ , and were distributed horizontally within the storm. Evidence of cellular electric structure was identified which followed the motion of the storm and was associated with rain sheets below cloud base.

Correspondence with simple point charge models was obtained for about half of the discharges studied. Later Reynolds and Neill [1955] repeated the study of Workman et al. with improved instrumentation, using 11 field measuring stations over an area 10 km in diameter. Unfortunately, rain effects and saturation of the instruments limited their ability to analyze data recorded from storms over the stations. Discharges which they were able to study gave results similar to those found by Workman et al., except that intracloud discharges were found to be oriented more vertically than those in the earlier study, and evidence of a possible lower positive charge was inferred.

Other multistation lightning studies have utilized time-resolved measurements of the electric field change at less than four locations to investigate the relative locations of charges involved in successive strokes of discharges to ground [Hacking, 1954; Takeuti, 1966; Ogawa and Brook, 1969] or have utilized total field change measurements to determine charge locations for flashes as a whole, primarily of ground discharges [Workman and Halzer, 1942; Barnard, 1951; Tamura, 1958; Hasekeyama, 1958; Jacobson and Krider, 1976]. The latter studies, made at widely different geographical locations, found the lightning charges to be located in a temperature regime similar to that found in New Mexico.

In the present paper we report results of lightning electric field measurements made at eight locations on the ground beneath a thunderstorm. The measurements were of sufficient accuracy and time resolution to allow the charge sources of individual strokes of ground discharges to be located and hence the charge 'structure' of the flash to be determined. At the same time, high-resolution radar measurements of the pre-

<sup>1</sup> Now at U.S. Environmental Protection Agency, Environmental Monitoring Systems Laboratory, Las Vegas, Nevada 89114.

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0148-0227/79/006C-1335\$01.00

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## 4-Station Solution (Krehbiel et al., 1979)

**START:** 
$$|\Delta E_i| = \frac{|Q|_z}{2\pi\epsilon_o [(X_i - x)^2 + (Y_i - y)^2 + z^2]^{3/2}} .$$

**Algebra & Differencing Stations GIVES:**

$$\begin{bmatrix} 2(X_1 - X_2) & 2(Y_1 - Y_2) & U_{12} \\ 2(X_1 - X_3) & 2(Y_1 - Y_3) & U_{13} \\ 2(X_1 - X_4) & 2(Y_1 - Y_4) & U_{14} \end{bmatrix} \begin{bmatrix} x \\ y \\ \eta \end{bmatrix} = \begin{bmatrix} X_1^2 + Y_1^2 - X_2^2 - Y_2^2 \\ X_1^2 + Y_1^2 - X_3^2 - Y_3^2 \\ X_1^2 + Y_1^2 - X_4^2 - Y_4^2 \end{bmatrix} ,$$

$U_{ij}$  's depend on the  $\Delta E$ s

$$|Q|_z = \frac{1}{2} \eta^{3/2}$$

# Overdetermined Fixed Matrix (OFM) Approach

**START:** 
$$|\Delta E_i| = \frac{|Q|z}{2\pi\epsilon_o [(X_i - x)^2 + (Y_i - y)^2 + z^2]^{3/2}}$$

**Raise to -2/3 power, Algebra & NO Differencing of Stations GIVES:**

$$\begin{bmatrix} d_1 \\ \cdot \\ \cdot \\ d_m \end{bmatrix} = \begin{bmatrix} (X_1^2 + Y_1^2) & -2X_1 & -2Y_1 & 1 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ (X_m^2 + Y_m^2) & -2X_m & -2Y_m & 1 \end{bmatrix} \begin{bmatrix} w \\ wx \\ wy \\ wr^2 \end{bmatrix} \equiv \mathbf{T}\mathbf{s},$$

$$d_i \equiv |\Delta E_i|^{-2/3}, \quad w \equiv \left( \frac{2\pi\epsilon_o}{|Q|z} \right)^{2/3}, \quad r^2 \equiv x^2 + y^2 + z^2.$$

**Extraction:**

$$x = s_2/s_1$$

$$y = s_3/s_1$$

$$z = (s_1s_4 - s_2^2 - s_3^2)^{1/2}/s_1$$

$$Q = (2\pi\epsilon_o\Delta E_1) / \left[ (s_1^2s_4 - s_1s_2^2 - s_1s_3^2)^{1/2} |\Delta E_1| \right], \quad \text{where } \mathbf{s} = (\mathbf{T}^T\mathbf{T})^{-1}\mathbf{T}^T\mathbf{d}$$



# 4-Station vs. OFM

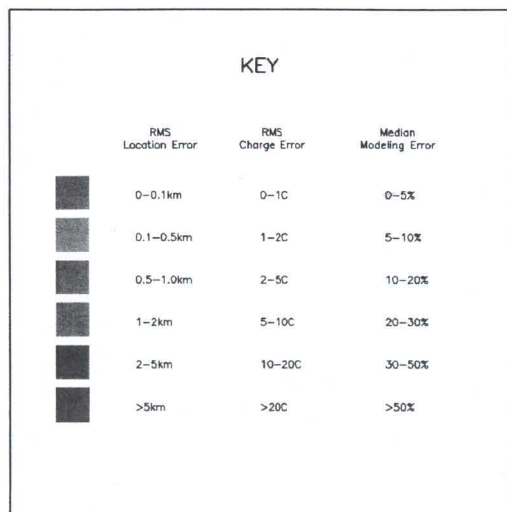
Topic	4-Station	OFM
# stations	4	$m$
Withstand errors?	No	Yes (least squares)
Fixed Matrix?	No	Yes
Understand eigenvalues?	No	Yes
Computationally efficient?	No	Yes
Bipolar Retrievals?	No	Yes
Replaces Chi-Square Analyses?	No	Yes

360,000  
monopoles  
retrieved

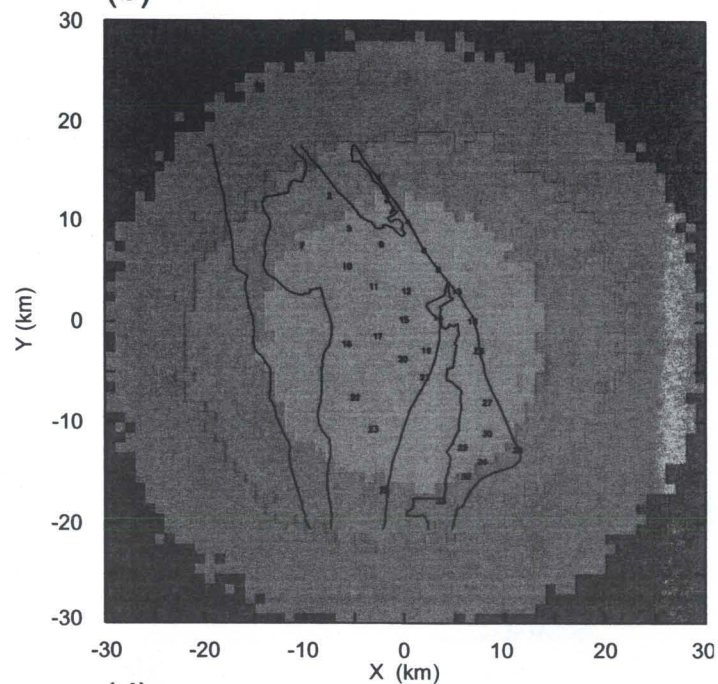
96,179  
ruptured

456,179  
analyzed

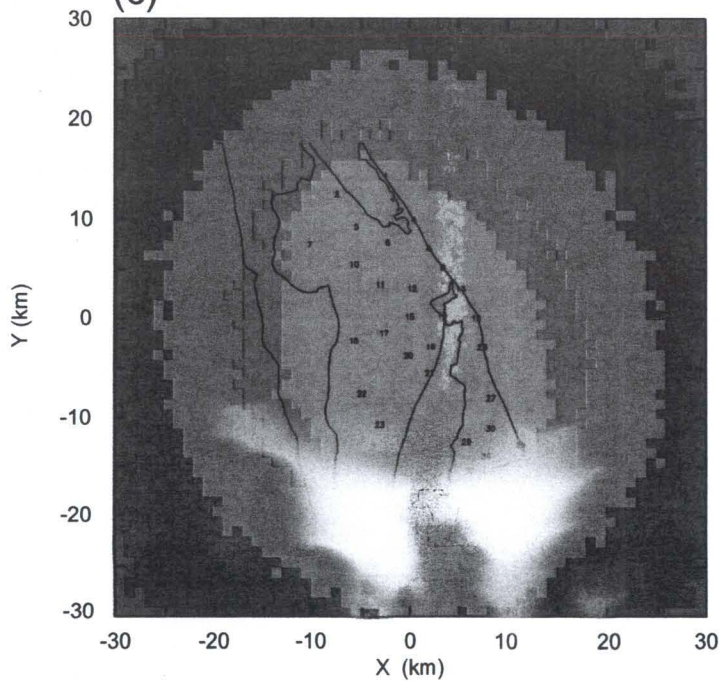
(a)



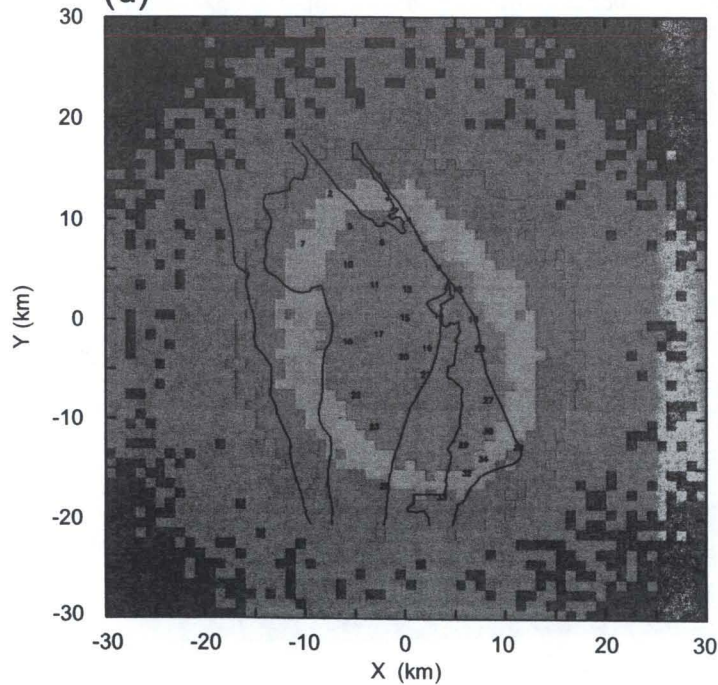
(b)



(c)



(d)



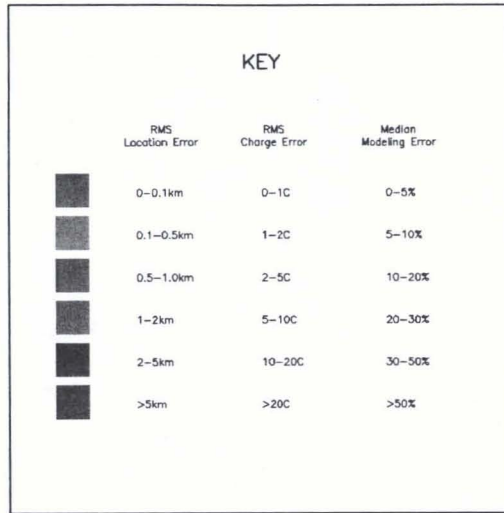


360,000  
monopoles  
retrieved

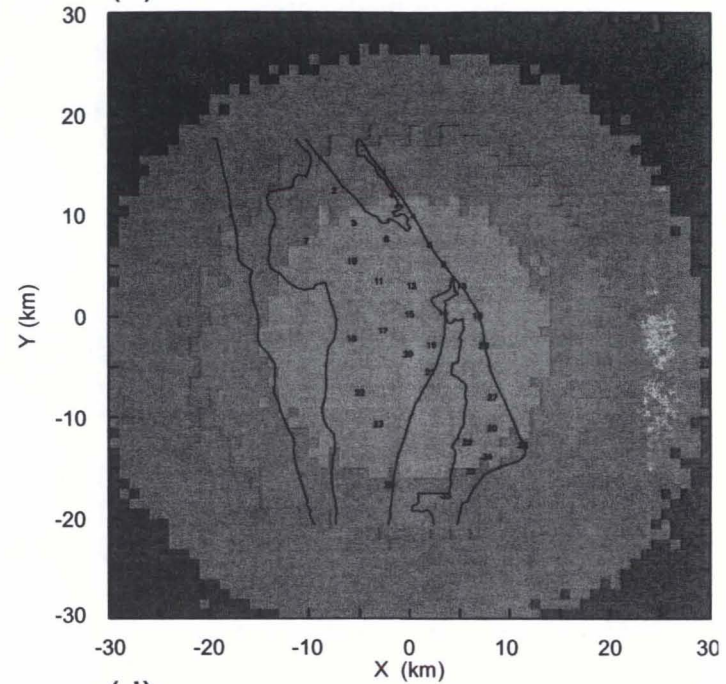
198  
ruptured

360,198  
analyzed

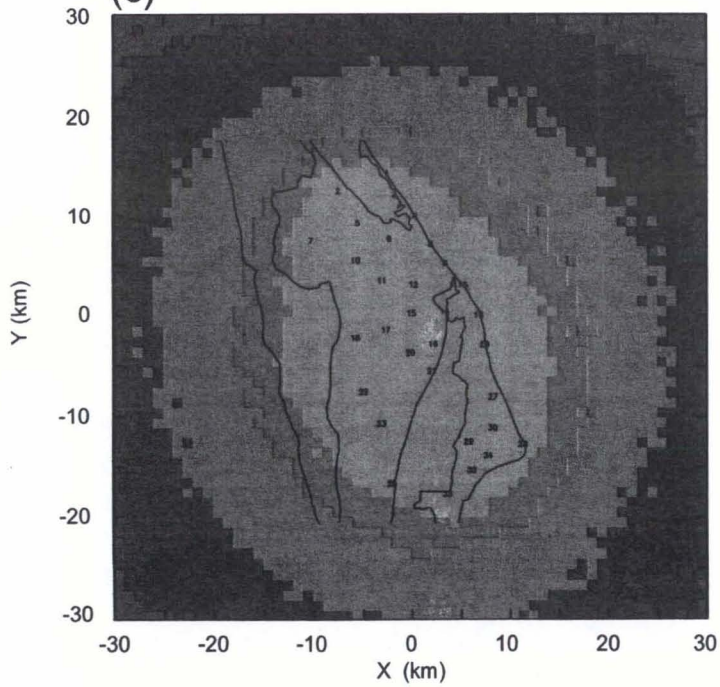
(a)



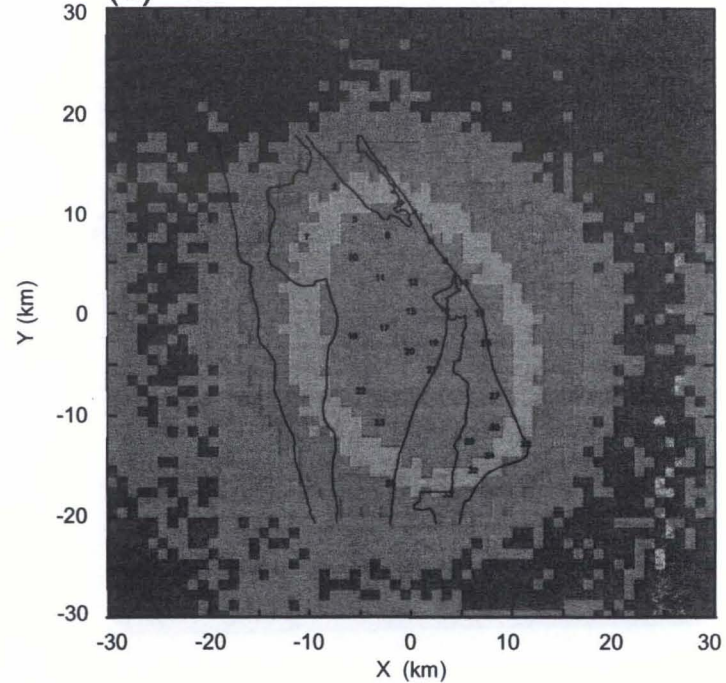
(b)



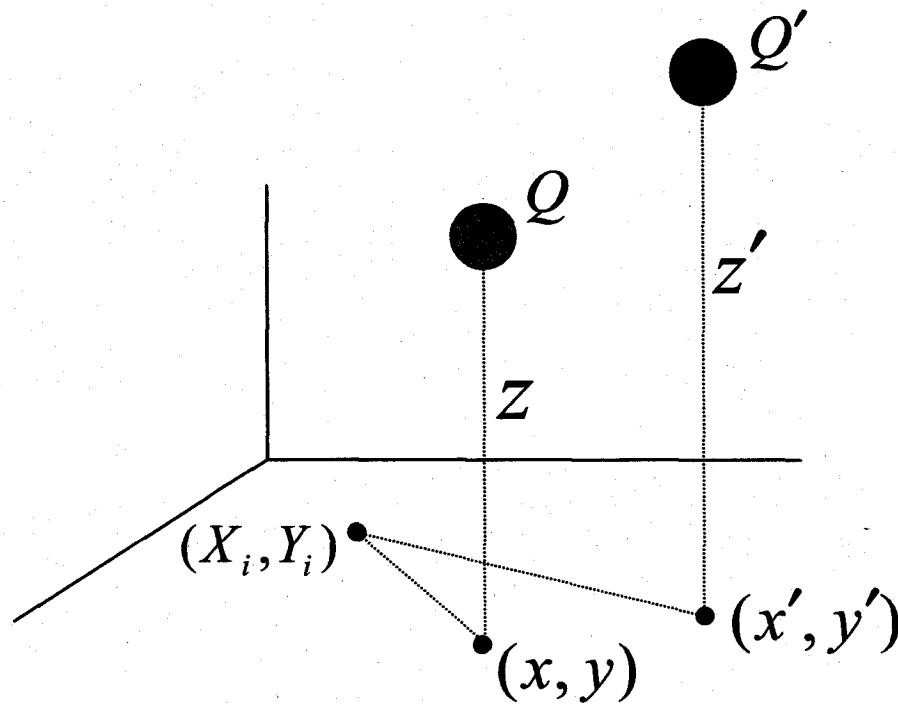
(c)



(d)



## 2-Charge Model (8 Parameters)

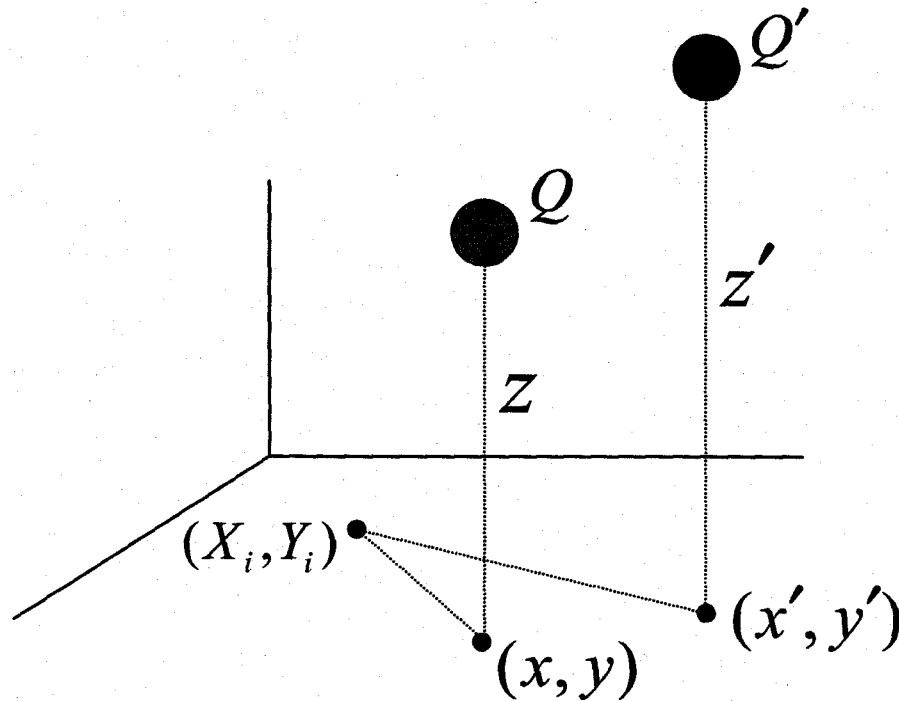


$$M_{2i}(\mathbf{a}_2) = \frac{-1}{2\pi\epsilon_0} \left[ \frac{Qz}{[(X_i - x)^2 + (Y_i - y)^2 + z^2]^{3/2}} + \frac{Q'z'}{[(X_i - x')^2 + (Y_i - y')^2 + z'^2]^{3/2}} \right],$$

$$\mathbf{a}_2 = (x, y, z, Q, x', y', z', Q')$$



# DR Model (2 Charges but 4 Parameters!)



$$D_i(\mathbf{a}_1) = M_{1i}(\mathbf{a}_1) + \Upsilon_i(\mathbf{a}_1) ,$$

$$\Upsilon_i(\mathbf{a}_1) \equiv M_{1i}(\mathbf{a}'_1(\mathbf{a}_1)) = \frac{-Q'(\mathbf{a}_1)z'(\mathbf{a}_1)}{2\pi\epsilon_0 \left[ (X_i - x'(\mathbf{a}_1))^2 + (Y_i - y'(\mathbf{a}_1))^2 + (z'(\mathbf{a}_1))^2 \right]^{3/2}} ,$$

$$\mathbf{a}_1 = (x, y, z, Q)$$

# DR Chi-Squared

$$\chi^2(\mathbf{a}_1) = \frac{1}{m-4} \sum_{i=1}^m \frac{(D_i(\mathbf{a}_1) - \Delta E_i)^2}{\sigma_i^2}$$

$$\mathbf{a}_1 = (x, y, z, Q)$$



Scan Slab that Intersects  
Negative Charge Region

$a'_1 = (x', y', z', Q')$

$a_1 = (x, y, z, Q)$

Thundercloud

8 km

6 km

Lower C

OFM

Field Sensor

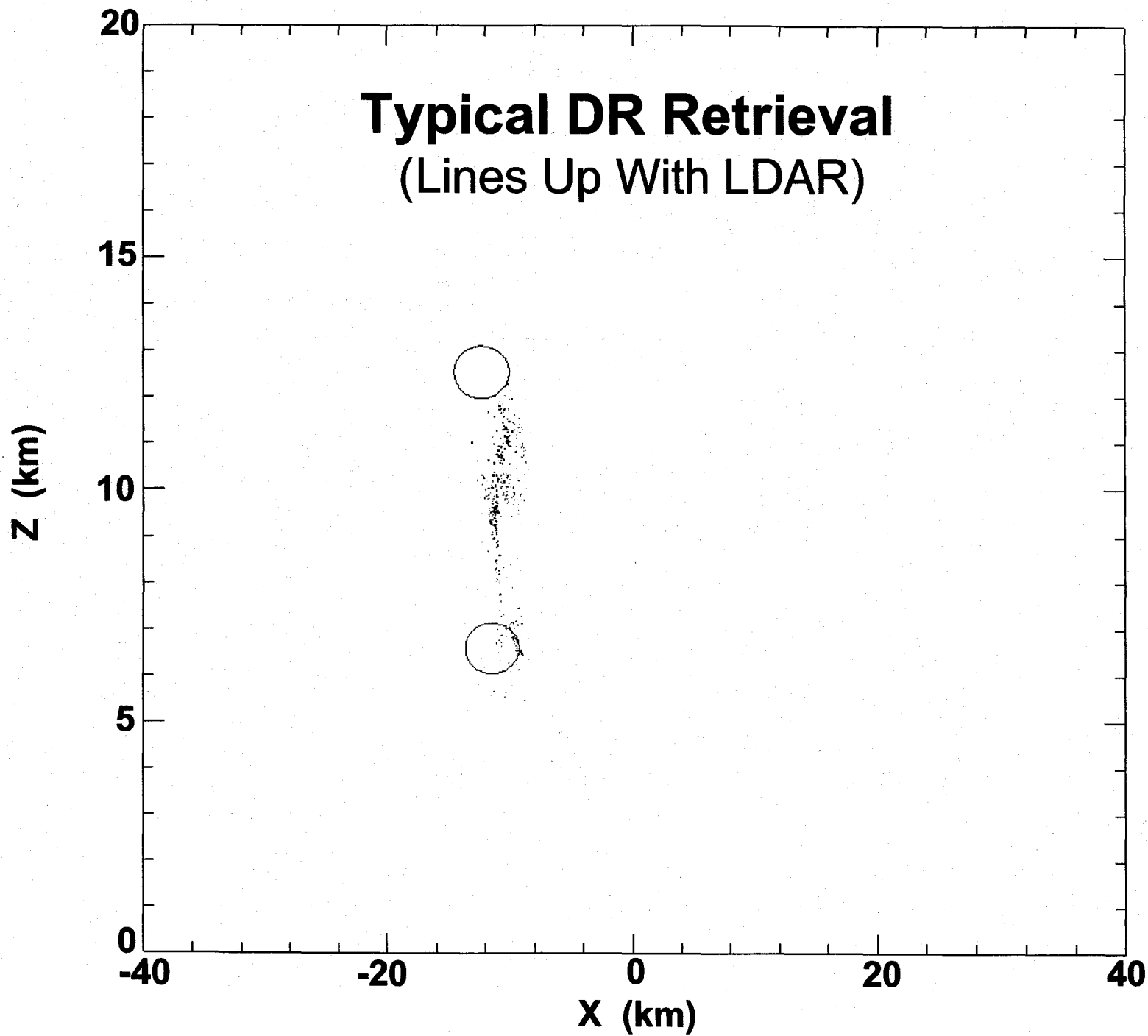
$\Delta E_i - M_{1i}(a_1) = \text{Residual Field Change}$

$x$

$y$

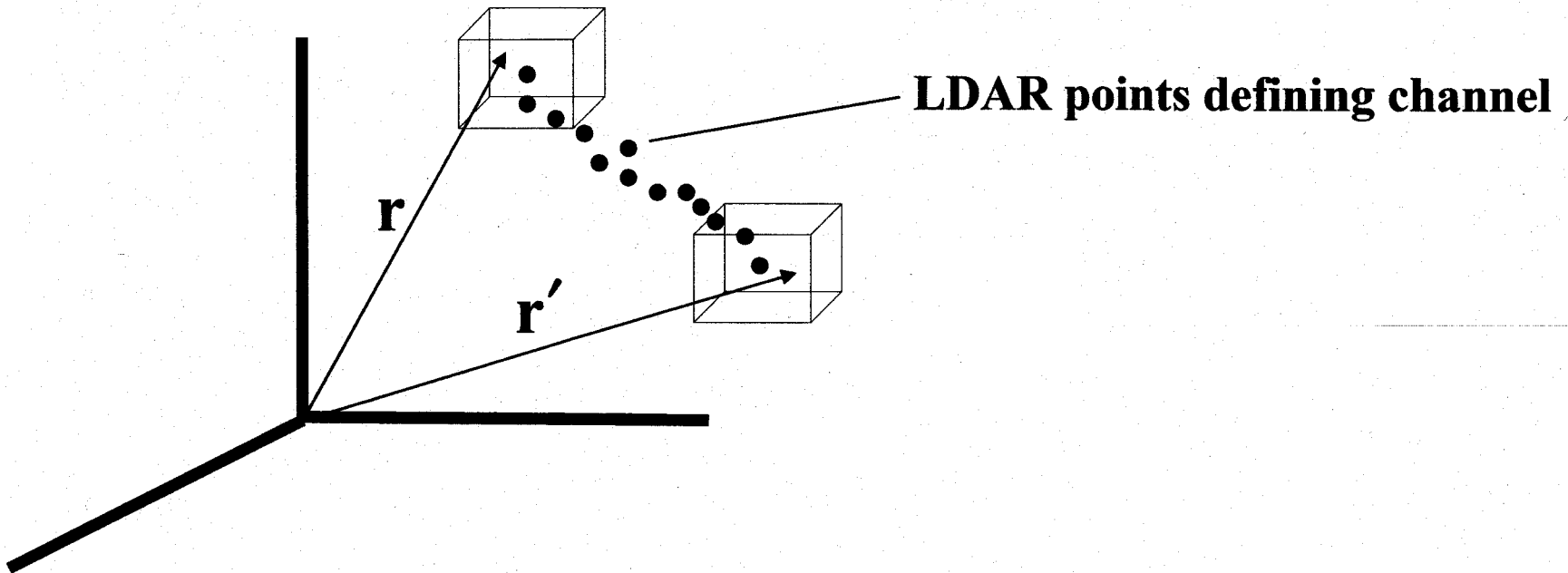
$z$

**(a)**





# LDAR-Constrained Charge Solutions



$$\begin{bmatrix} Q \\ Q' \end{bmatrix} = (\mathbf{K}^T \mathbf{K})^{-1} \mathbf{K}^T \mathbf{g}.$$

Notes:

Q-step (1,10)

deltaE/Charge Data

LDAR Data

open files

Eb = 200 kV/m

Eb = 300 kV/m

Eb = 400 kV/m

Eb = 450 kV/m

Eb = 100000 kV/m

period

filter

ranges

ticks

VIEW: 4 panel

VIEW: North

VIEW: Time

VIEW: Down

VIEW: East

VIEW: West

VIEW: Values

VIEW: Criteria

Charge Density

Energy Type

Histogram (Daily)

Histogram (All)

make png

quit

Q-next

Q-prev

Q-stop

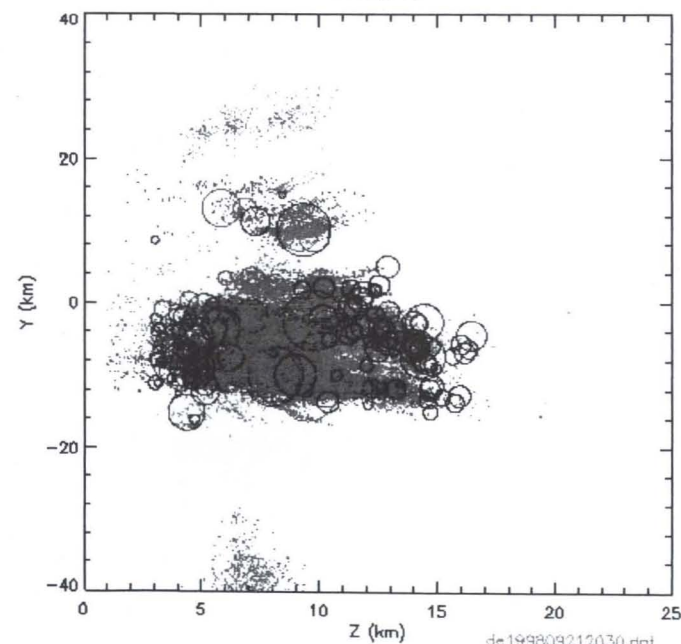
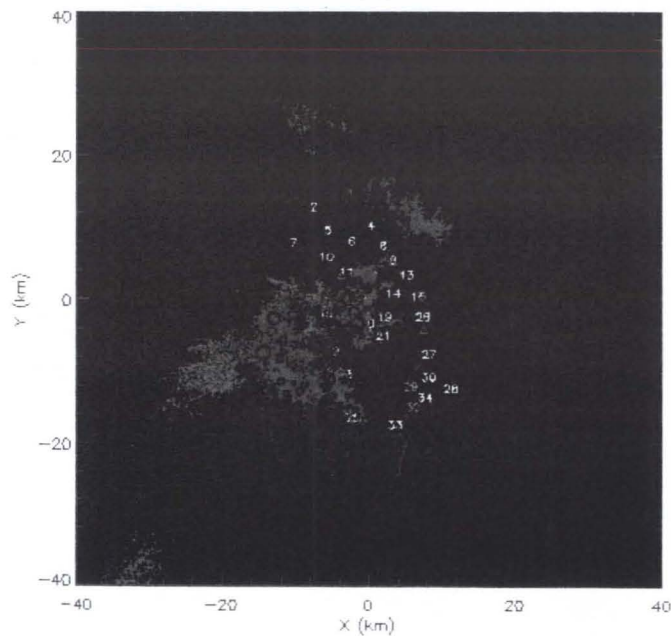
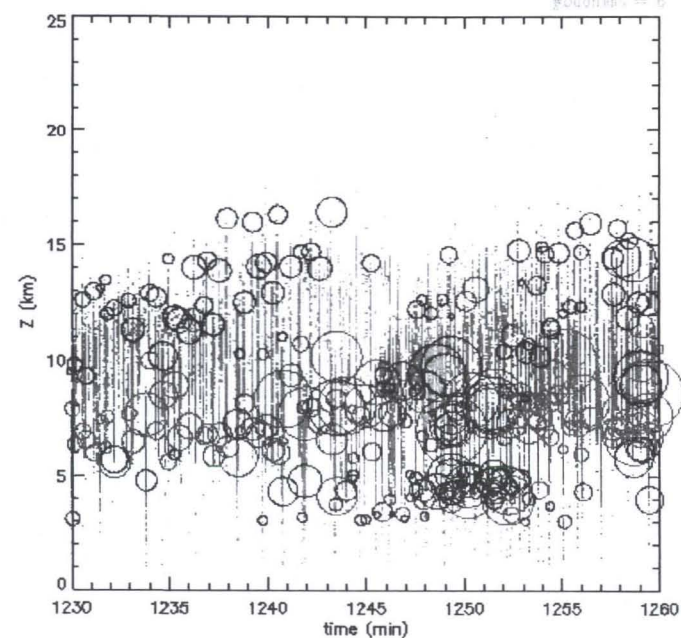
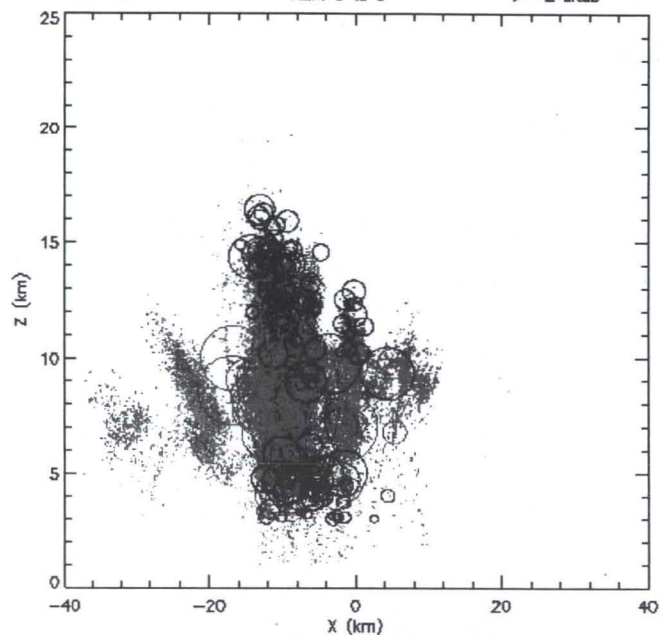
XY-zoom in

XY-zoom out

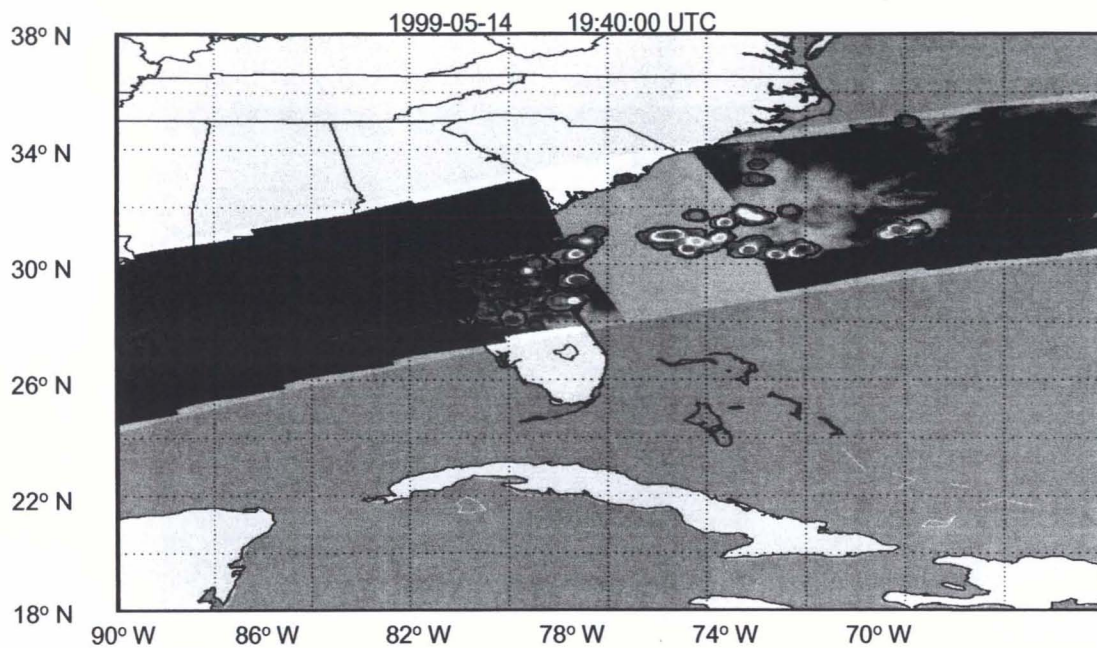
next -->

NP = 117 FILTER: X: -40 to 40 Eb = 400 kV/m  
 ntf = 0 Y: -40 to 40 trigger = 3  
 nvf = 0 Z: 1 to 25 damin = 100  
 naf = 37 X2R: 0 to 5 >= 2 sites

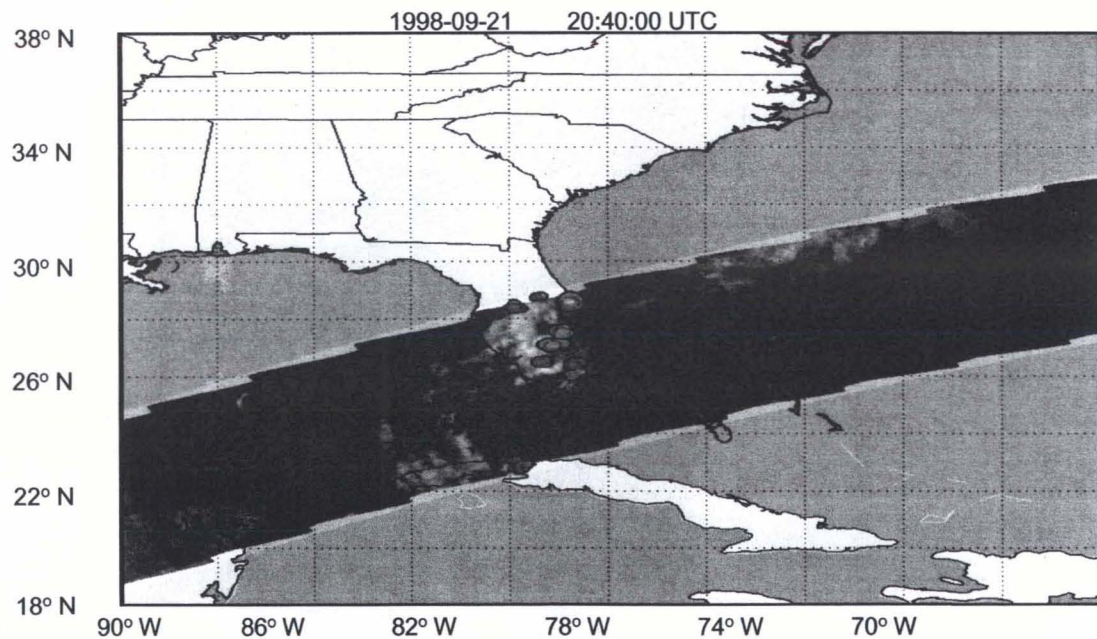
nLDAR=58917 9-21-98/264  
 period: 20:30:00.000 - 21:00:00.000



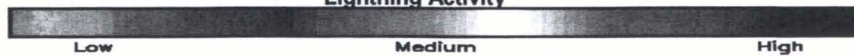
(a)



(b)



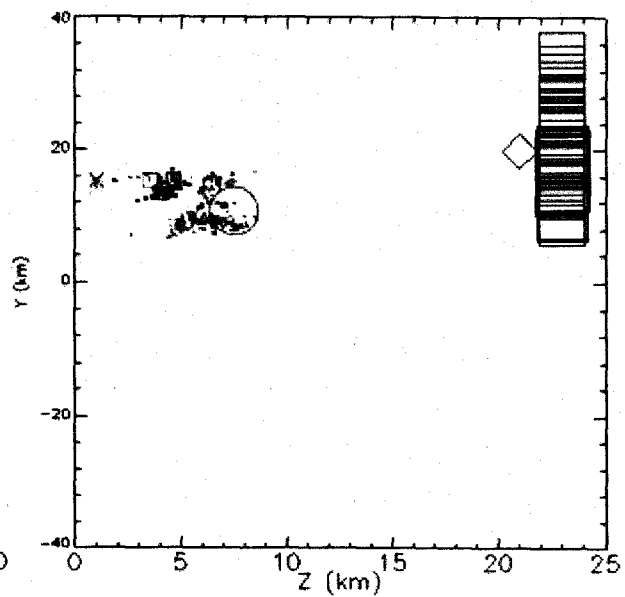
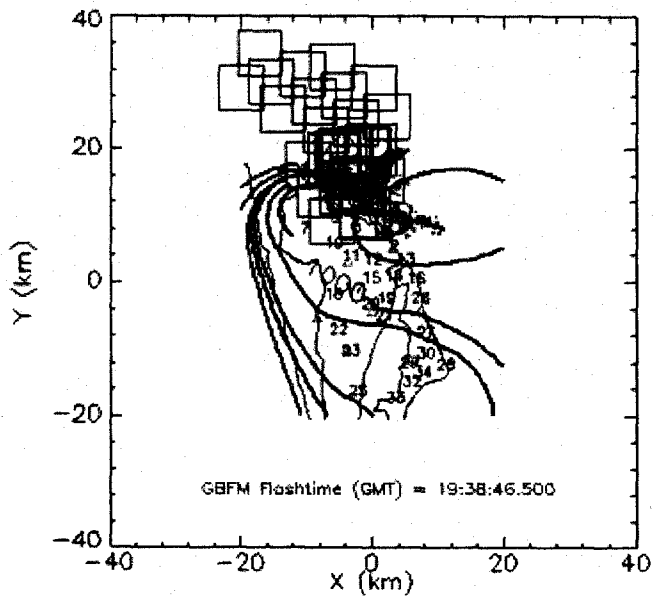
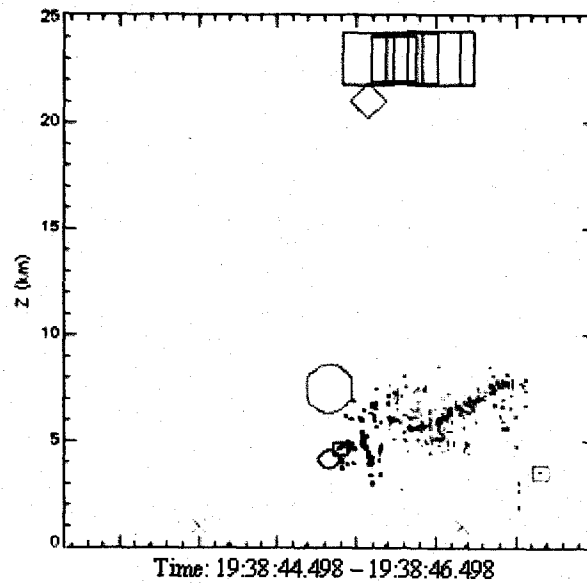
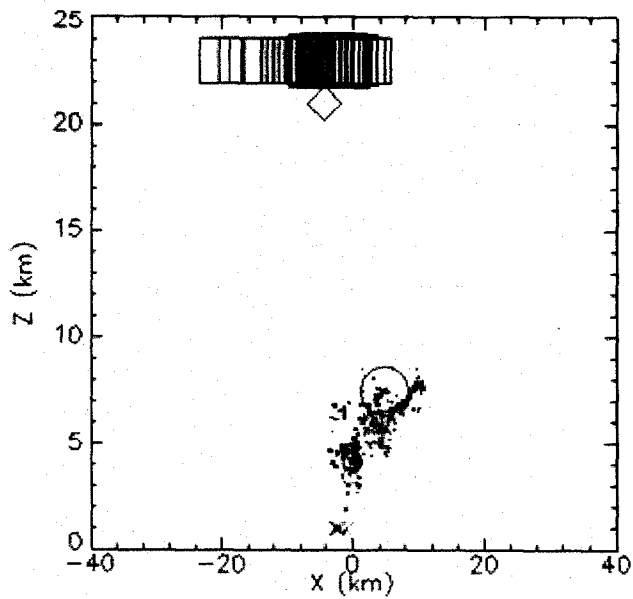
Lightning Activity





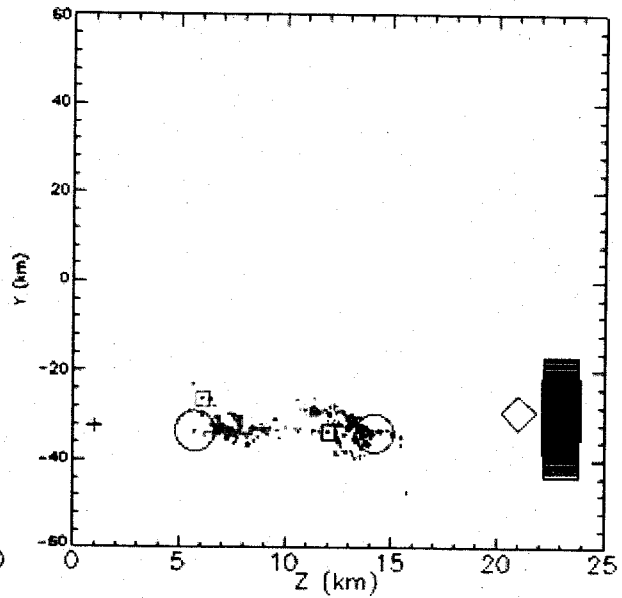
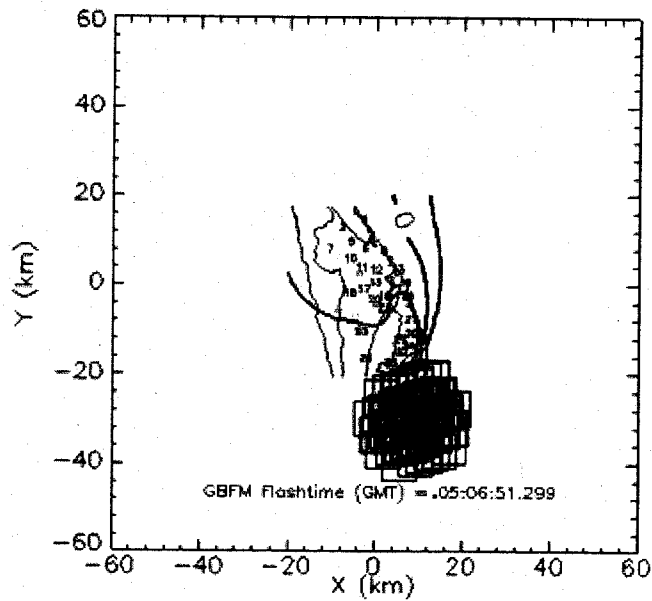
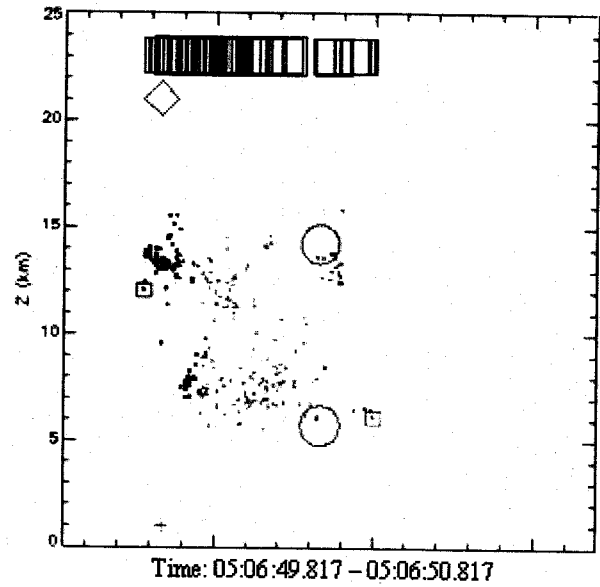
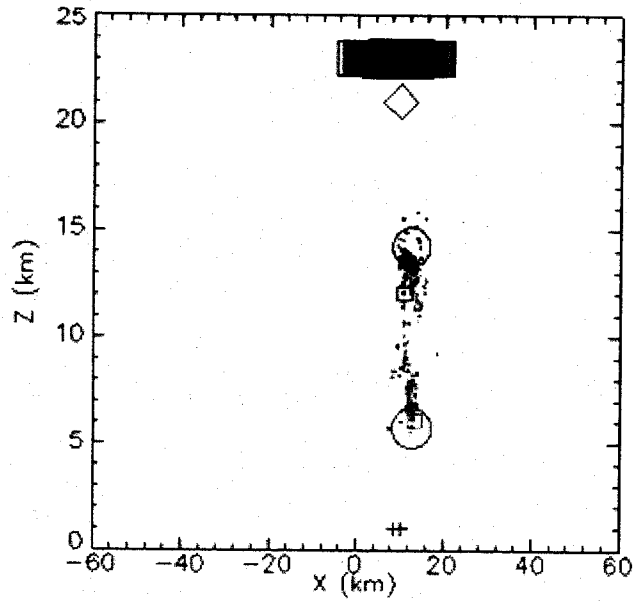
May 14, 1999 (Day 134)

$Q=56.168, Q'=-8.209$



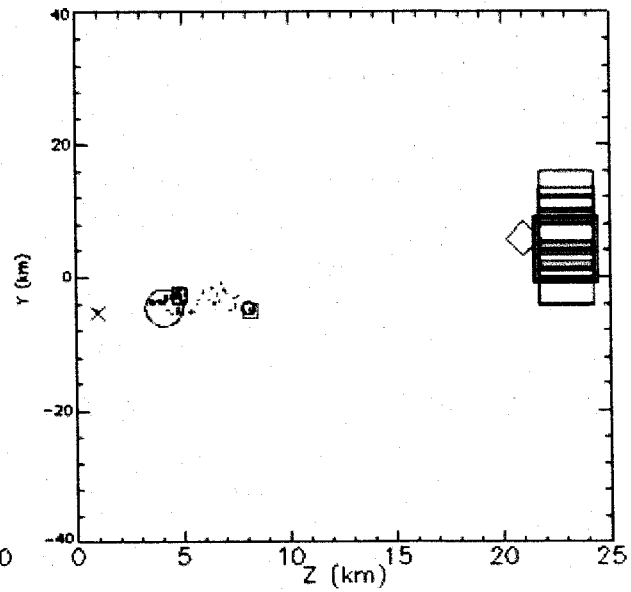
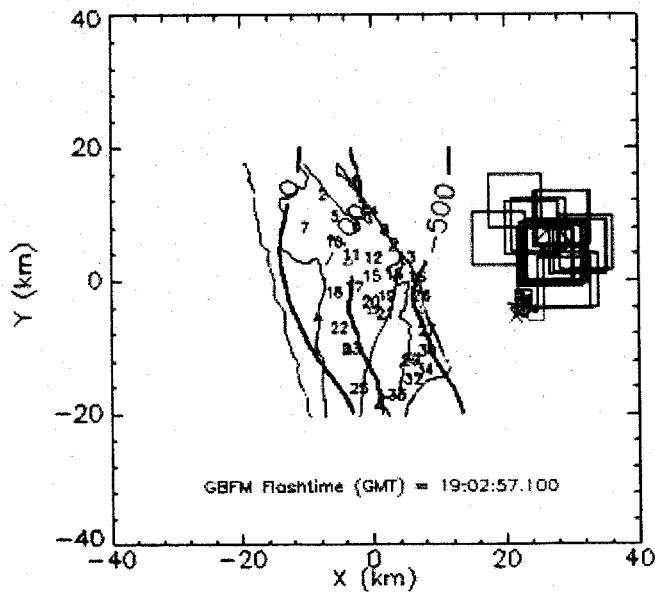
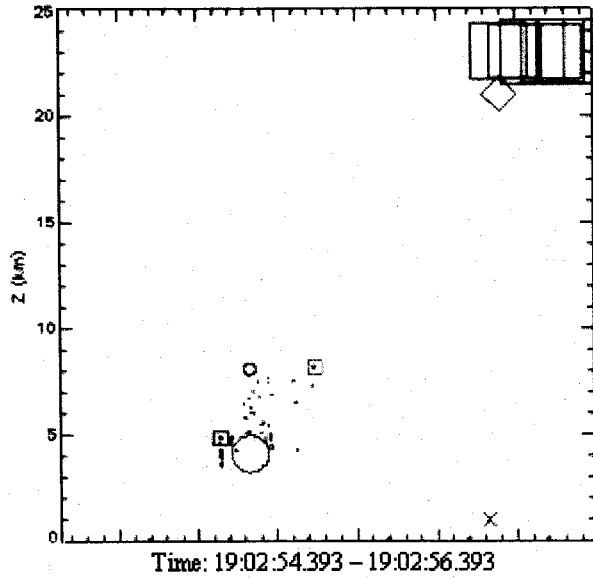
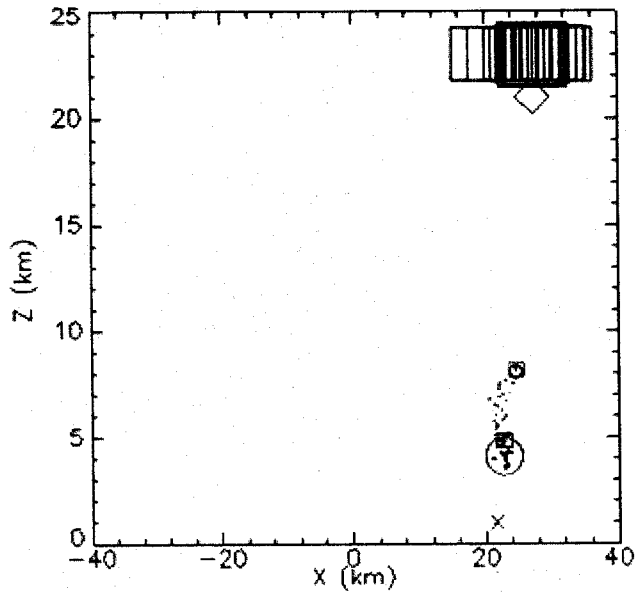
June 11, 1999 (Day 162)

Q=40.072, Q'=-36.123



June 29, 1999 (Day 180)

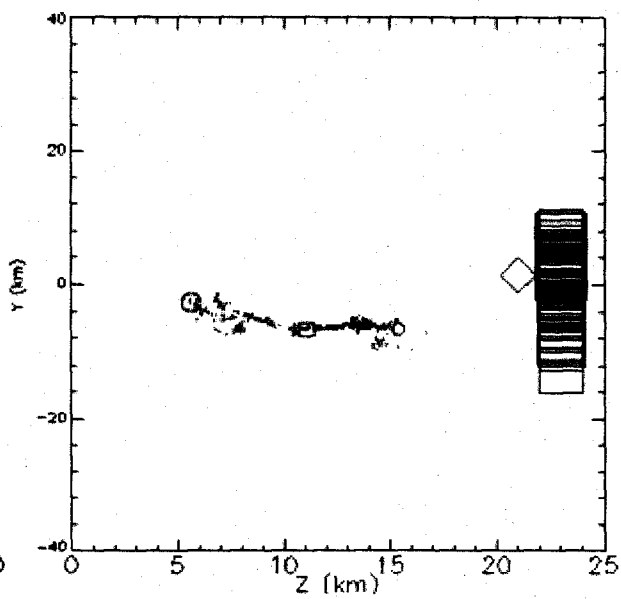
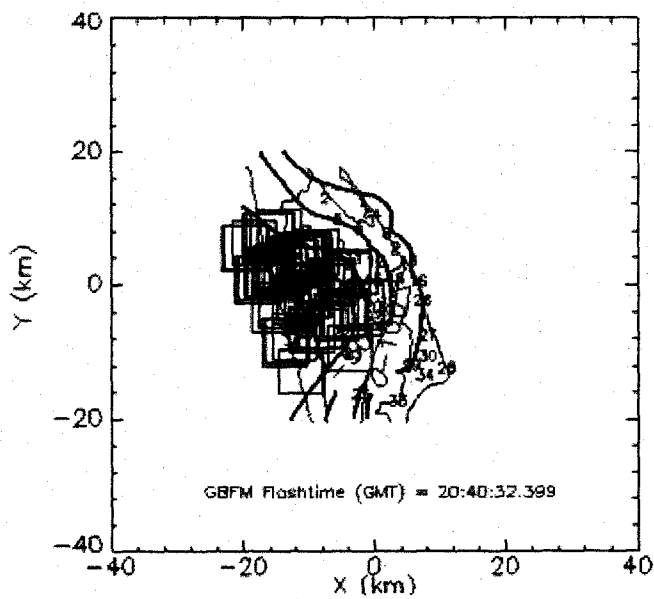
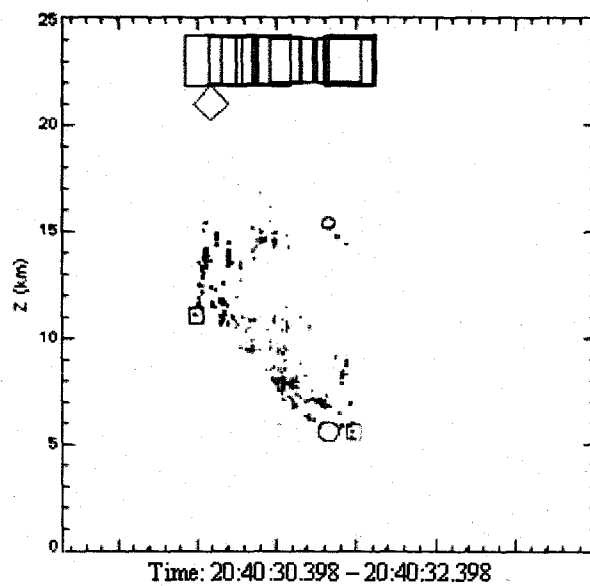
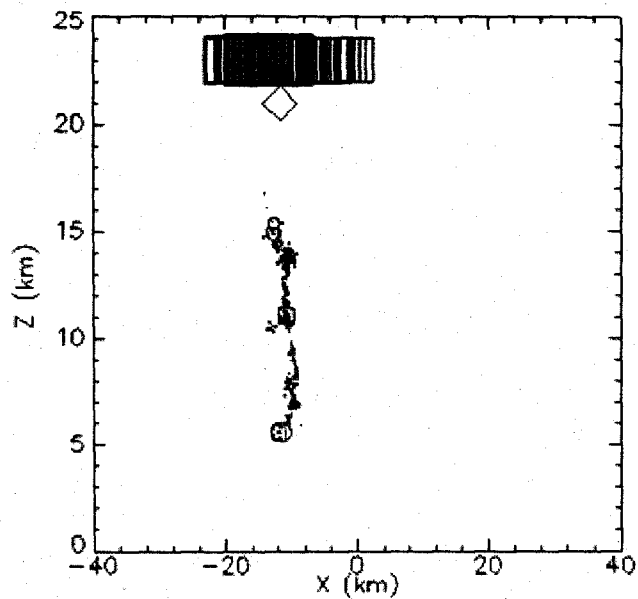
$Q=33.633$ ,  $Q'=-3.760$





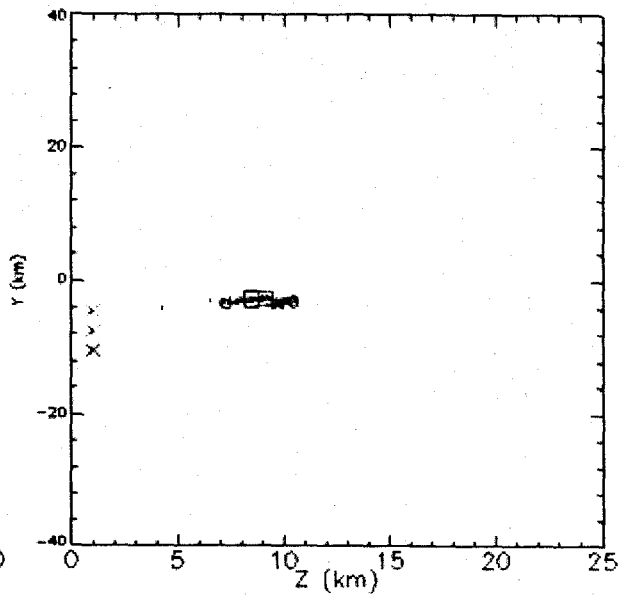
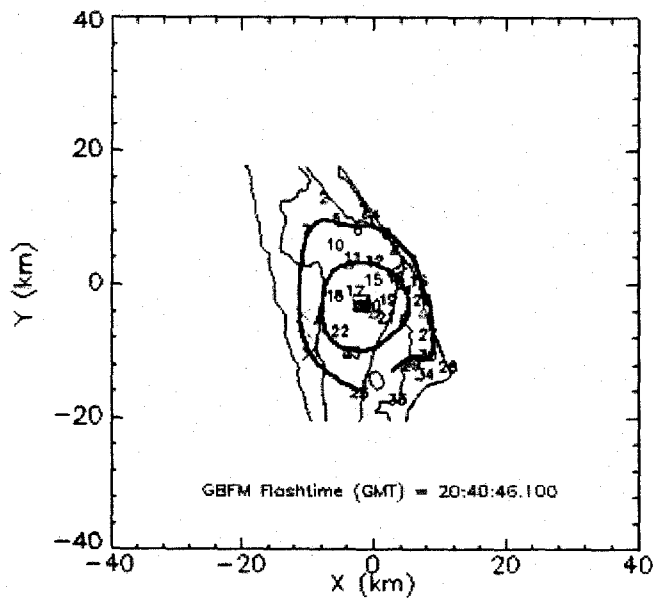
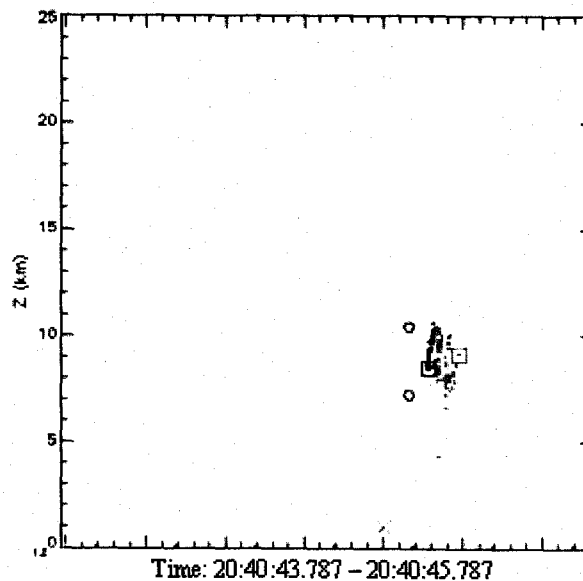
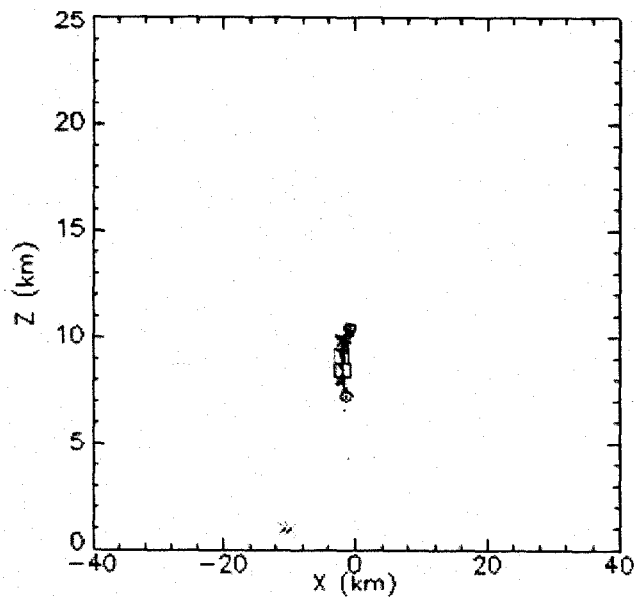
September 21, 1998 (Day 264)

$Q=9.317$ ,  $Q'=-3.337$



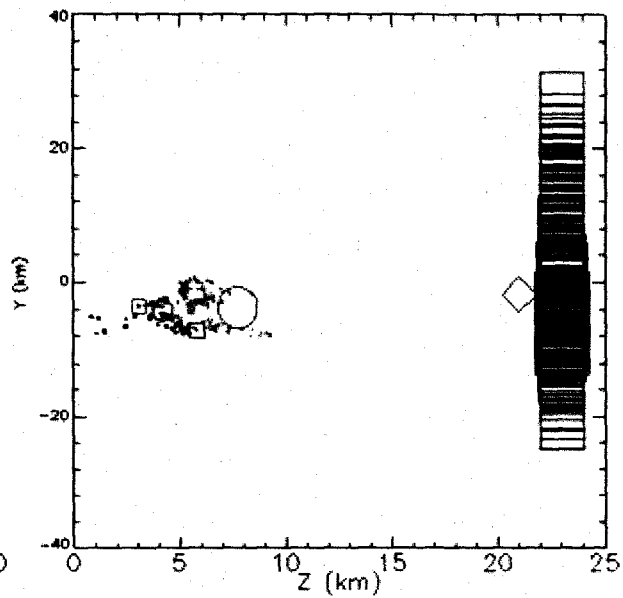
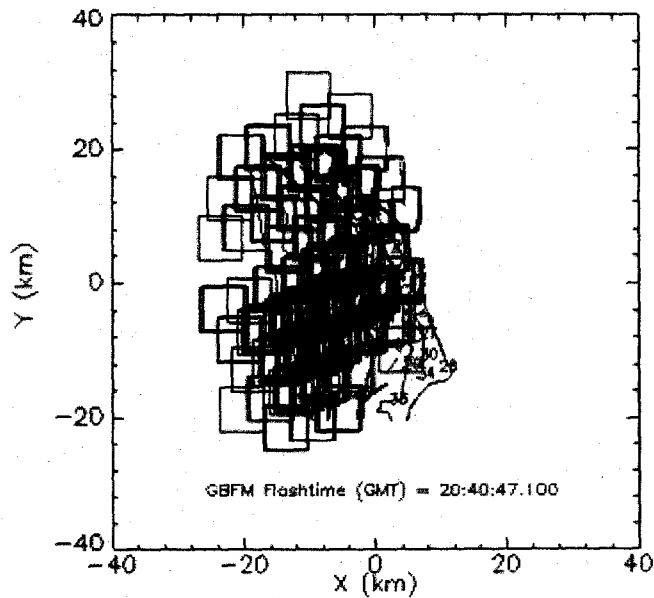
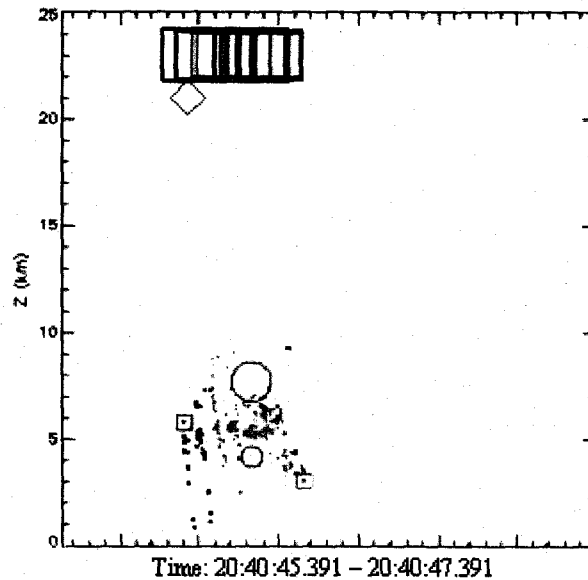
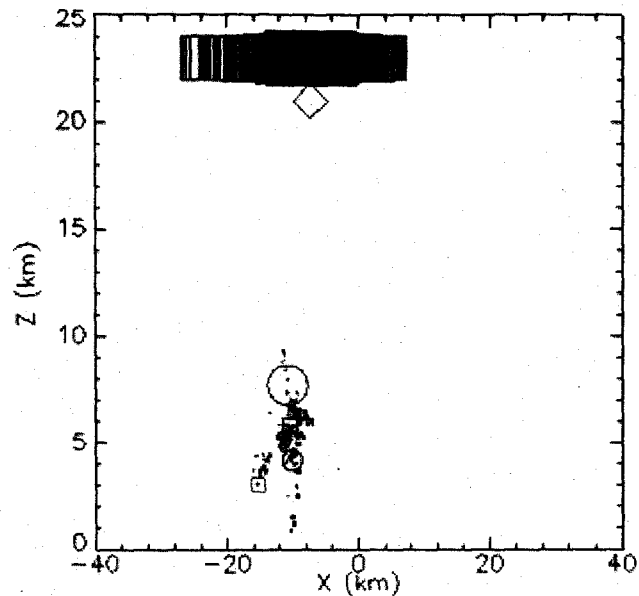
September 21, 1998 (Day 264)

Q=2.462, Q'=-2.412



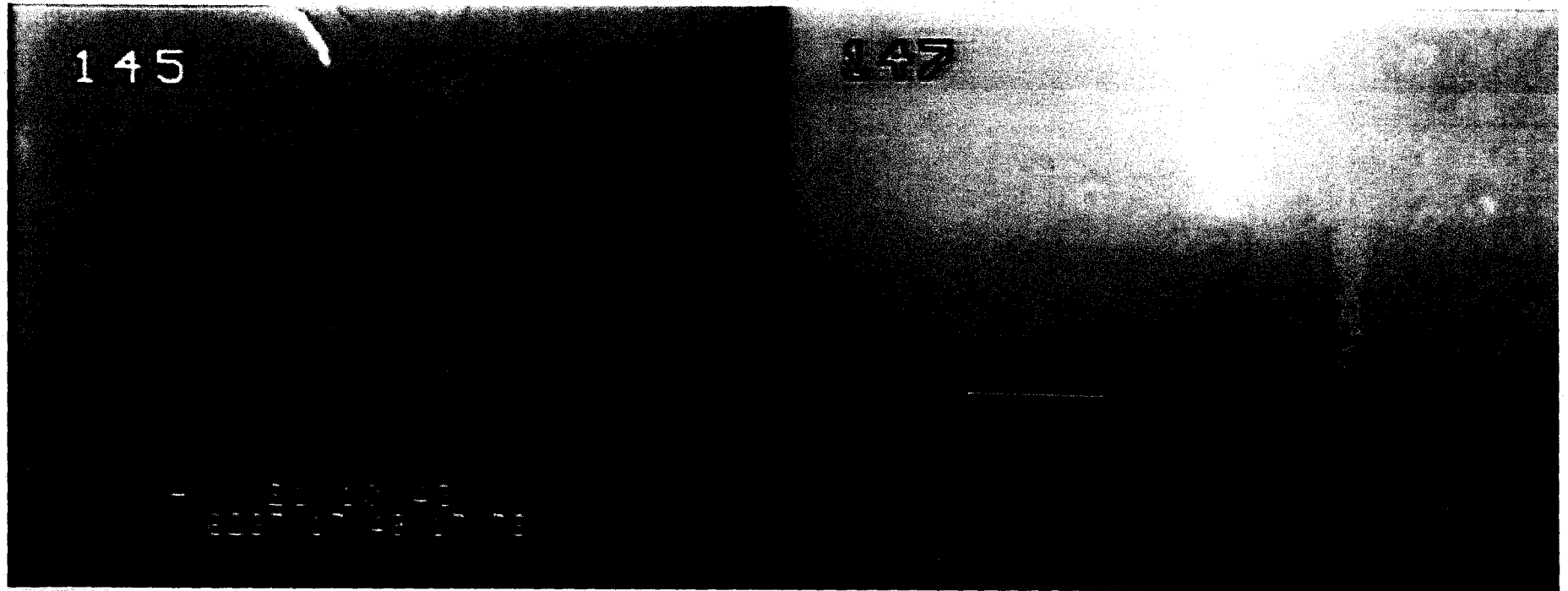
September 21, 1998 (Day 264)

Q=39.719, Q'=-10.397





# Strike to Launch Pad 39B (STS-115)

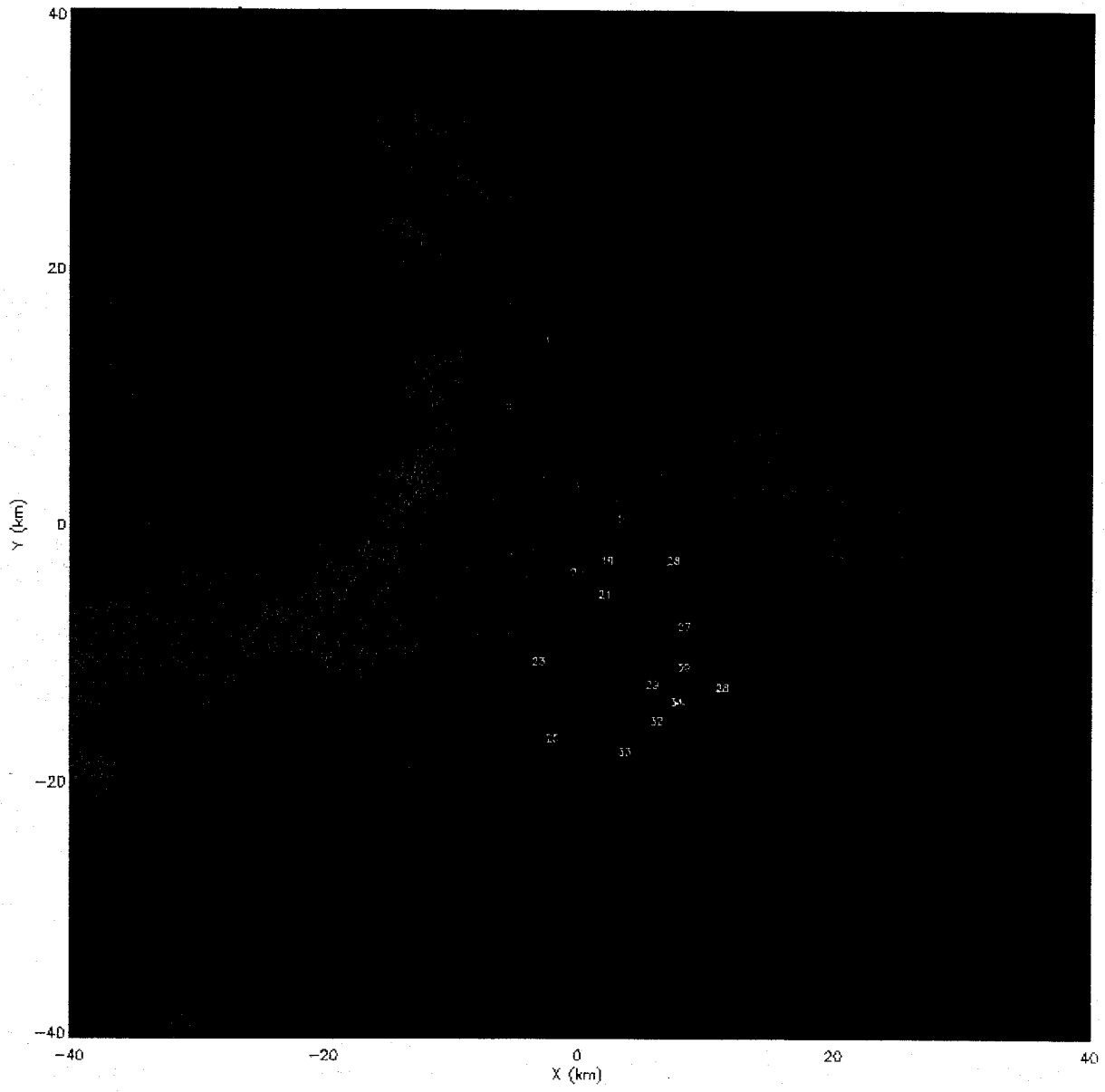


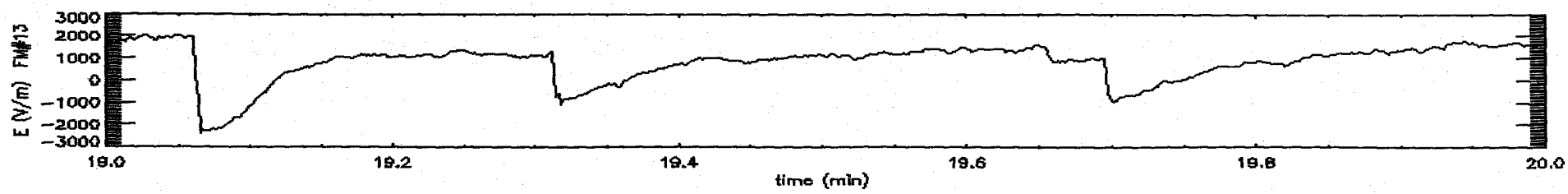
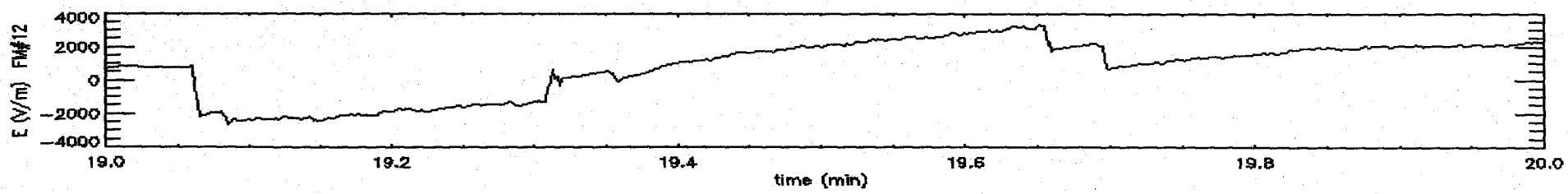
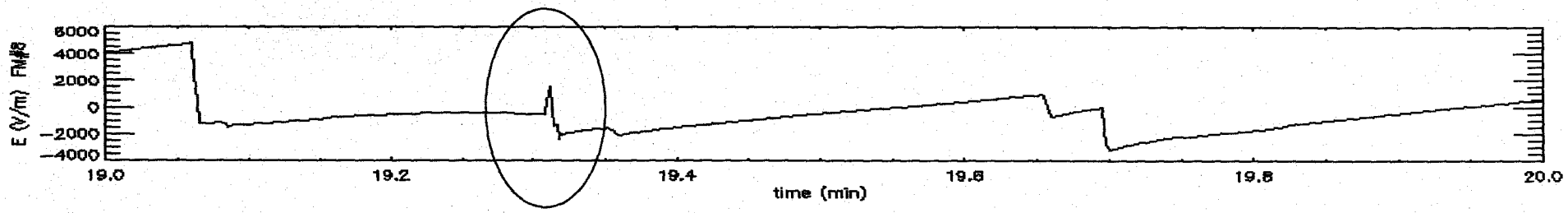
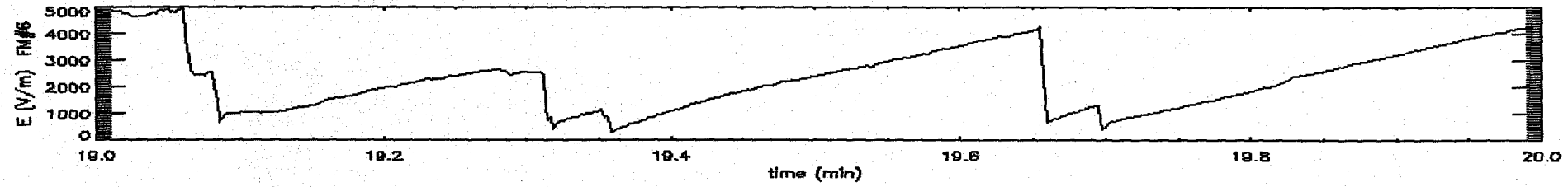
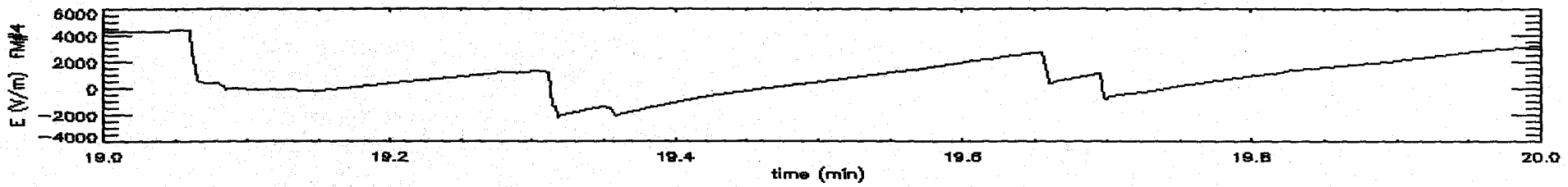
(August 25, 2006 @ 17:49:17.78 GMT)

FILTER: X: -40 to 40  
Y: -40 to 40  
Z: 1 to 25

nLDAR=471387

period: 17:11:00.000 - 18:11:00.000



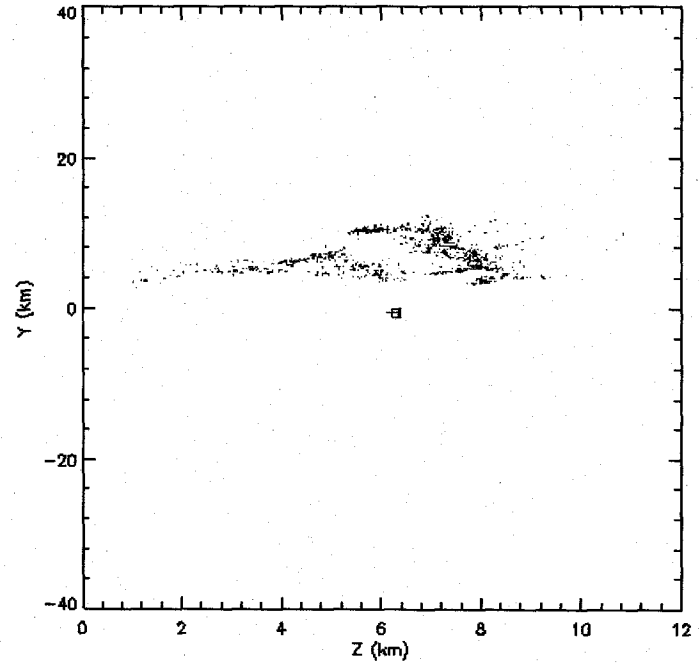
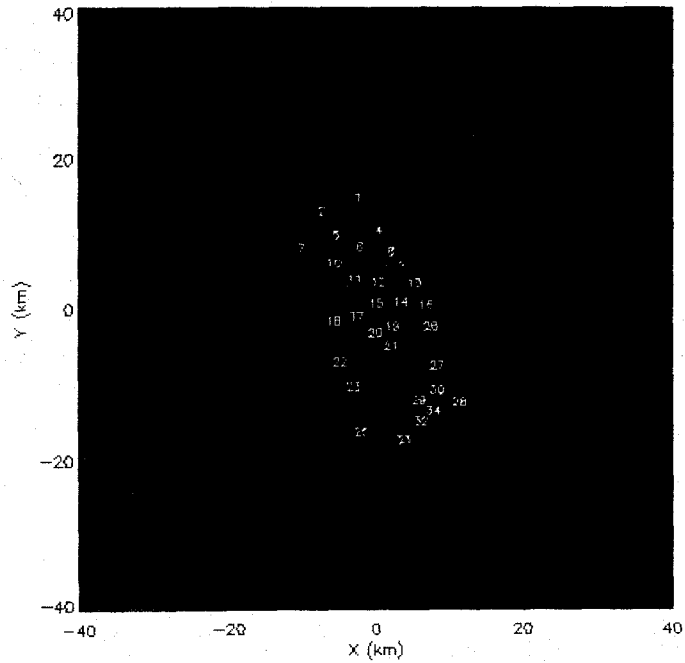
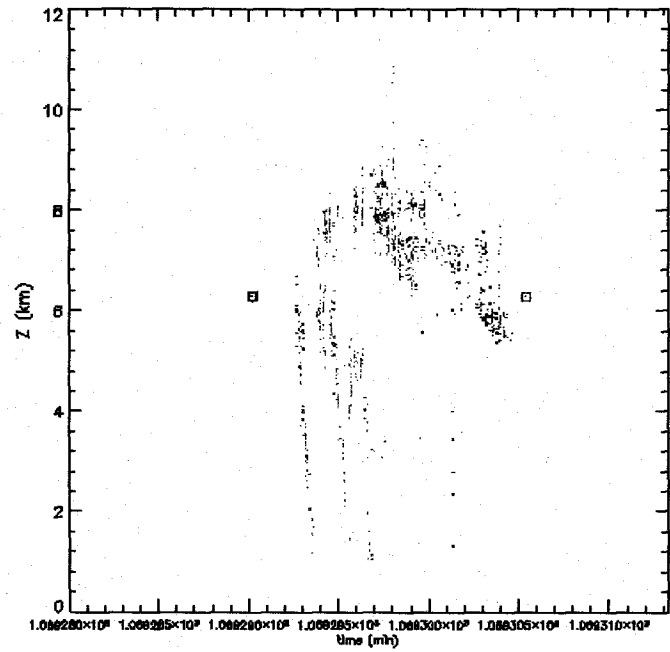
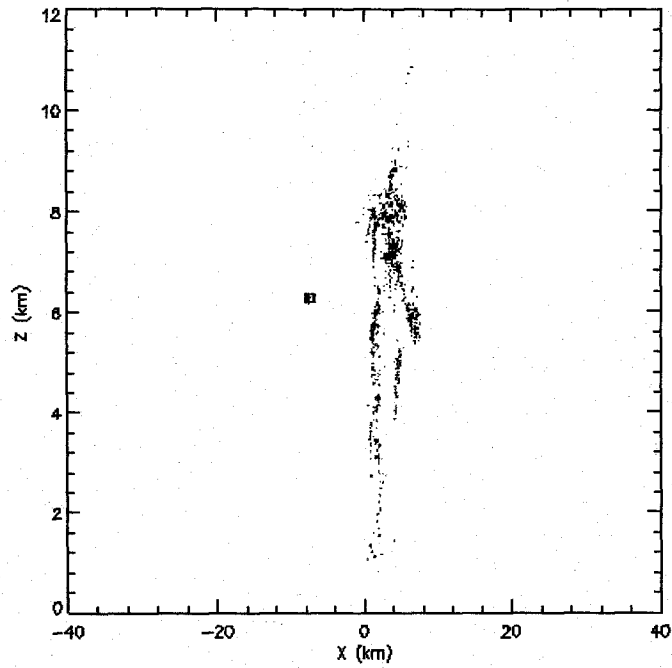


FILTER: X: -40 to 40  
Y: -40 to 40  
Z: 1 to 25

nLDAR=1084

137

period: 17:48:16.797 - 17:49:18.797





*Thank  
You !*