FRACTURE MECHANICS FOR COMPOSITES
STATE OF THE ART AND CHALLENGES

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For laminated composite materials, interlaminar fracture mechanics has proven useful for characterizing the onset and growth of delaminations. To fully understand this failure mechanism, the total strain energy release rate, G_T, the mode I component due to interlaminar tension, G_I, the mode II component due to interlaminar sliding shear, G_{II}, and the mode III component, G_{III}, due to interlaminar scissoring shear, need to be calculated. In order to accurately predict delamination onset or growth for two-dimensional problems, these calculated G components are compared to interlaminar fracture toughness properties experimentally measured over a range from pure mode I loading to pure mode II loading.

It is state of the art to determine a quasi static mixed-mode fracture criterion by plotting the interlaminar fracture toughness, G_c , versus the mixed-mode ratio, G_{II}/G_T, obtained from data generated using pure Mode I Double Cantilever Beam (DCB) (G_{II}/G_T=0), pure Mode II End Notched Flexure (ENF) (G_{II}/G_T=1), and Mixed Mode Bending (MMB) tests of varying ratios. A curve fit of these data is performed to determine a mathematical relationship between G_c and G_{II}/G_T. Failure is expected when, for a given mixed mode ratio G_{II}/G_T, the calculated total energy release rate, G_T, exceeds the interlaminar fracture toughness, G_c. Although several different types of test specimens have also been suggested for the measurement of the mode III interlaminar fracture toughness property, an interaction criterion incorporating the scissoring shear, however, has not yet been established and remains a challange.

The methodology described above has been extended to predict fatigue delamination onset life but to date a standard only exists for the Mode I DCB test. Interlaminar fracture mechanics has also been used to characterize the extension or growth of delaminations when subjected to fatigue loading. In analogy with metals, delamination growth rate can therefore be expressed as a power law function. However, the exponent is typically high for composite materials compared to metals and standards for the measurement of fatigue delamination growth have not yet been established and remain a challange.

Today a variety of methods are used to compute the strain energy release rate based on results obtained from finite element analysis. The virtual crack closure technique (VCCT) is widely used for computing strain energy release rates based on results from continuum (2D) and solid (3D) finite element analyses to provide results on the mode separation required when using the mixed-mode fracture criterion. Although the original publication on VCCT dates back more than a quarter century state of the art techniques typically require geometric non-linear finite element analyses with additional post processing routines that are currently not an integral part in most commercial codes. ABAQUS recently announced the release of a new add-on for ABAQUS 6.5 called VCCT for ABAQUS which is a first step in making computational fracture mechanics for composite available and attractive to a larger user community. Other large commercial finite element codes such as MSC NASTRAN or ANSYS, which are the frequently used in industry, however, do not offer any choice for calculating mixed mode energy release rates today. The implementation of methods to compute mixed mode energy release rates into these codes remains a challenge.

To date interlaminar fracture mechanics has proven useful for characterizing the onset of delaminations in composites and has been used with limited success primarily to investigate onset in fracture toughness specimens and laboratory size coupon type specimens. Future acceptance of the methodology by industry and certification authorities however, requires the validation and verification of the methodology and successful demonstration on structural level. The objective of this presentation is to demonstrate the state-of-the-art in the areas of delamination characterization, interlaminar fracture mechanics analysis tools and demonstrate the application on structural level for which a panel was selected which is reinforced with stringers. Full implementation of Interlaminar Fracture Mechanics (ILFM) in design however remains a challenge and requires a continuing development effort of codes to calculate energy release rates and advancements in delamination onset and growth criteria under mixed mode conditions.

COMPUTATIONAL FRACTURE MECHANICS FOR COMPOSITES
STATE OF THE ART AND CHALLENGES[1]

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ABSTRACT

Interlaminar fracture mechanics has proven useful for characterizing the onset of delaminations in composites and has been used with limited success primarily to investigate onset in fracture toughness specimens and laboratory size coupon type specimens. Future acceptance of the methodology by industry and certification authorities however, requires the successful demonstration of the methodology on the structural level. In this paper, the state-of-the-art in fracture toughness characterization, and interlaminar fracture mechanics analysis tools are described. To demonstrate the application on the structural level, a panel was selected which is reinforced with stringers. Full implementation of interlaminar fracture mechanics in design however remains a challenge and requires a continuing development effort of codes to calculate energy release rates and advancements in delamination onset and growth criteria under mixed mode conditions.

1. BACKGROUND

Many composite components in aerospace structures are made of flat or curved panels with co-cured or adhesively bonded frames and stiffeners. Over the last decade a consistent step-wise approach has been developed which uses experiments to detect the failure mechanism, computational stress analysis to determine the location of first matrix cracking and computational fracture mechanics to investigate the potential for delamination growth. Testing of thin skin stiffened panels designed for aircraft fuselage applications has shown that bond failure at the tip of the frame flange is an important and very likely failure mode. Debonding also occurs when a thin-gage composite fuselage panel is allowed to buckle in service. A methodology based on fracture mechanics [1] has proven useful for characterizing the onset and growth of delaminations in composites and has been used with limited success to investigate delamination onset and debonding in simple laboratory coupon type specimens [2, 3]. Future acceptance of a fracture mechanics methodology by industry and certification authorities however, requires the successful demonstration of the methodology on structural level.

The objective of this paper is to demonstrate the state-of-the-art in the areas of delamination characterization, interlaminar fracture mechanics analysis tools and demonstrate the application on the structural level for which a panel was selected which is reinforced with stringers. The advances required in all three areas in order to reach the level of maturity desired for implementation of this methodology for design and certification of composite components are highlighted.

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2. METHODOLOGY

2.1. Interlaminar Fracture Mechanics

Interlaminar fracture mechanics has proven useful for characterizing the onset and growth of delaminations [1, 4-6]. When using fracture mechanics, the total strain energy release rate, G_T, the mode I component due to interlaminar tension, G_I, the mode II component due to interlaminar sliding shear, G_{II}, and the mode III component, G_{III}, due to interlaminar scissoring shear, as shown in Figure 1, are calculated along the delamination. The calculated G_I, G_{II}, and G_{III} components are then compared to interlaminar fracture toughness values in order to predict delamination onset or growth. Today, the interlaminar fracture toughness values are determined experimentally over a range of mode mixities from pure mode I loading to pure mode II loading [7-10].

A quasi static mixed-mode fracture criterion is determined by plotting the interlaminar fracture toughness, G_c, versus the mixed-mode ratio, G_{II}/G_T. The fracture toughness data is generated experimentally using pure Mode I (G_{II}/G_T=0) Double Cantilever Beam (DCB), pure Mode II (G_{II}/G_T=1) four point End Notched Flexure (4ENF), and Mixed Mode Bending (MMB) tests of varying ratios as shown in Figure 2 for a carbon/epoxy material. A failure criterion – as shown in Figure 2 - was suggested by Benzeggah and Kenane [11] using a simple mathematical relationship between G_c and G_{II}/G_T

$$G_c = G_{Ic} + \left(G_{IIc} - G_{Ic}\right) \cdot \left(\frac{G_{II}}{G_T}\right)^{\eta} . \qquad (1)$$

In this expression, G_{Ic} and G_{IIc} are the experimentally-determined fracture toughness data for mode I and II as shown in Figure 2. The factor η was determined by a curve fit using the Levenberg-Marquardt algorithm in KaleidaGraph™. Failure is expected when, for a given mixed mode ratio G_{II}/G_T, the calculated total energy release rate, G_T, exceeds the interlaminar fracture toughness, G_c.

In order to predict delamination onset or growth for three-dimensional problems, however, the entire failure surface G_c=G_c (G_I, G_{II}, G_{III}) as shown in Figure 3 is required. Although several specimens have been suggested for the measurement of the mode III interlaminar fracture toughness property [12, 13], an interaction criterion incorporating the scissoring shear has not yet been established and remains a challenge. Currently, the edge-cracked torsion test (ECT) is being considered for standardization [14-16].

The methodology has been extended to predict fatigue delamination onset life [17-19]. To date, a standard only exists for the Mode I DCB test [20] although mixed-mode onset data have been generated [21]. Interlaminar fracture mechanics has also been used to characterize the extension or growth of delaminations when subjected to fatigue loading [22]. In analogy with metals, delamination growth rate can therefore be expressed as a power law function. However, the exponent is typically high for composite materials compared to metals [23, 24]. To date, standards for the measurement of fatigue delamination growth have not yet been established but

development is currently being discussed in standard developing organizations[3] and few results have been published [25]. The discussion of load history effects and spectrum loading on delamination growth are in its infancy [26]. In view of the uncertainties related to the high exponents of the delamination grow rate, it has been suggested to design to levels below a threshold stain energy release rate to ensure no delamination growth.

2.2. Analysis Tools

Several methods have emerged in the past for computing strain energy-release-rates for delamination growth in a wide variety of composite structures. The methods are primarily based on analytical, closed form solutions or finite element analysis.

One method – described in detail in reference [27] - is based on a sublaminate analysis which treats portions of a laminate as higher-order plates. The plates may be stacked such that the displacements and tractions are identical at the shared interfaces. By assuming a constant cross-section in one dimension, the resulting systems of governing differential equations can be solved in closed form. The plates may be coupled end-to-end, allowing complex structures to be modeled in a manner similar to finite element analysis. The method has been implemented into a commercial software (SUBLAM) that allows the computation of mixed-mode strain energy release rates. Through superposition, general loading conditions can be applied, and coupling with general purpose finite element codes can be accomplished.

In the past decade, Davidson et. al. developed two-dimensional and three-dimensional crack tip elements [28-30]. The crack tip elements provide analytical solutions for strain energy release rates and mode mix using plate theory-based near-tip forces and moments. These latter quantities may often be obtained from global, undamaged finite element models of the structure of interest. In comparison to finite element modeling, the crack tip element analyses provide a significant computational advantage for predicting energy release rates.

Several methods are documented in the literature to compute the strain energy release rate based on results obtained from finite element analysis. The *finite crack extension method* [31, 32] requires two complete analyses. In the model, the crack gets extended for a finite length prior to the second analysis. The method provides one global total energy release rate as global forces on a structural level are multiplied with global deformations to calculate the energy available to advance the crack. The *virtual crack extension method* [33-42] requires only one complete analysis of the structure to obtain the deformations. The total energy release rate or *J-integral* is computed locally at the crack front, and the calculation only involves an additional computation of the stiffness matrix of the elements affected by the virtual crack extension. The method yields the total energy release rate as a function of the direction in which the crack was extended virtually, yielding information on the most likely growth direction. Modifications of the method have been suggested in the literature to allow the mode separation for two-dimensional

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[3] The Composite Materials Handbook MIL-17; http://www.mil17.org/
ASTM International, Committee D30 on Composite Materials; http://www.astm.org/
European Structural Integrity Society (ESIS), TC4: Polymers and Polymer Composites; http://www.esisweb.org/

analysis [43, 44]. An *equivalent domain integral method* which can be applied to both linear and nonlinear problems and additionally allows for mode separation was proposed in [45, 46]. A comprehensive overview of different methods used to compute energy release rates is given in [47]. New methods to compute the strain energy release rate based on results obtained from finite element analysis have also been published recently [48-50].

2.2.1 Virtual Crack Closure Technique

For delaminations in laminated composite materials where the failure criterion is highly dependent on the mixed-mode ratio (as shown in Figure 2), the virtual crack closure technique (VCCT) has been widely used for computing energy release rates [51, 52]. Results based on continuum (2-D) and solid (3-D) finite element analyses provide the mode separation required when using the mixed-mode fracture criterion.

The mode I, and mode II components of the strain energy release rate, G_I, G_{II} are computed as shown in Figure 4 for a 2-D four-node element as an example of VCCT. The terms X'_i , Z'_i are the forces at the crack tip at nodal point i and u'_ℓ, w'_ℓ and u'_{ℓ^*}, w'_{ℓ^*} are the displacements at the corresponding nodal points ℓ and ℓ^* behind the crack tip. For geometric nonlinear analysis where large deformations may occur, both forces and displacements obtained in the global coordinate system need to be transformed into a local coordinate system (x', z') which originates at the crack tip as shown in Figure 4. The local crack tip system defines the tangential (x', or mode II) and normal (z', or mode I) coordinate directions at the crack tip in the deformed configuration. The extension to 3-D is straightforward and the total energy release rate G_T is calculated from the individual mode components as $G_T = G_I + G_{II} + G_{III}$, where $G_{III} = 0$ for the two-dimensional case shown in Figure 4.

Although the original publication on VCCT dates back a quarter century [51], the virtual crack closure technique has been used mainly by scientists in universities, research institutions and government laboratories and is usually implemented in their own specialized codes or used in post-processing routines in conjunction with general purpose finite element codes. Until recently, FRANC2D, developed and owned by the Cornell Fracture Group (CFG) at Cornell University, has been the only publicly available, highly specialized finite element code that uses the virtual crack closure technique [53, 54]. To date, the virtual crack closure technique has not been implemented into the other large commercial general-purpose finite element codes such as the MSC line of products, ANSYS® or PERMAS. Only ABAQUS® started to offer VCCT for ABAQUS® in 2005 as a separate addition to their ABAQUS® Standard software [55]. The implementation allows for the calculation of the mixed-mode energy release rate at the crack tip in 2D models and along the delamination front in shell and 3D solid models. Crack and delamination propagation analysis are possible based on different failure criteria including the one shown in equation (1).

Lately, the HyperSizer® bonded joint analysis capability, which is based on analytical (non-FEA) formulations has been extended to include the virtual crack closure technique for predicting crack growth [56]. New methods to compute mixed-mode energy release rates suitable for the application with the p-version of the finite element method have also recently been developed [57]. Some modified and newly developed formulations also allow applications

independent of the finite element analysis and are suitable for boundary element analysis [54, 58].

2.2.2 A Global/Local Shell 3D Modeling Technique

Built-up structures are traditionally modeled and analyzed using plate or shell finite elements to keep the modeling and computational effort affordable. Computed mixed mode strain energy release rate components, however, depend on many variables such as element order and shear deformation assumptions, kinematic constraints in the neighborhood of the delamination front, and continuity of material properties and section stiffness in the vicinity of the debond when delaminations or debonds are modeled with plate or shell finite elements [59-61]. These problems may be avoided by using three-dimensional models. Since many layers of brick elements through the thickness are often necessary to model the individual plies, however, the size of finite element models required for accurate analyses may become prohibitively large.

For detailed modeling and analysis of the delaminations, the shell/3D modeling technique will reduce the modeling time since existing plate or shell models may be modified to shell/3D models. This is a considerable advantage compared to the creation of an entirely new three-dimensional finite element model. The technique will also reduce computational time because only a relatively small section of interest needs to modeled with solid elements keeping the number of unknowns small. The technique combines the accuracy of the full three-dimensional solution with the computational efficiency of a plate or shell finite element model and has been demonstrated for various applications such as fracture toughness characterization specimens [62] and on the coupon level for the skin/stringer separation specimen [63, 52]

3. STRINGER STIFFENED PANEL

For the demonstration of a fracture mechanics methodology on the structural level, a stringer stiffened panel as shown in Figure 5, was selected and analyzed. The square panel is made of carbon/epoxy tape and reinforced with three stringers made of carbon/epoxy plain weave fabric. The stiffened panel is bolted to a steel picture frame and subjected to pure in-plane shear loading. During manufacturing, an artificial defect had been placed at the termination of the center stiffener. Sufficient shear loading causes the panel to buckle as shown in Figure 5. The resulting out-of-plane deformation causes skin/stringer separation at the location of the initial defect as shown in the detail of the deformed mesh in Figure 5. A total of eight delamination lengths between 81.9 and 355.6 mm were modeled. The initial length corresponds to the length of the insert used to create an initial defect at the termination of the center stringer. Additional lengths were chosen to study the change in energy release rate distribution across the width of the stringer with increasing delamination length. The mixed-mode strain energy release rates were calculated using the virtual crack closure technique across the width of the stringer foot. A failure index was calculated by correlating the results with the mixed-mode failure criterion of the graphite/epoxy material.

3.1 Finite Element Model

The global model representing the steel loadframe, the graphite tape/epoxy panel and the graphite fabric/epoxy stringers was created using standard S4 shell elements available in the finite element software ABAQUS®. To accurately model earlier tests, which were performed under constant displacement control, uniform displacements *u,v* were applied at one corner node to introduce shear as shown in Figure 5. The inplane displacements *u,v* were suppressed at the diagonally opposite corner, and the out of plane displacements *w* were suppressed along all four edges across the entire width of the steel load frame.

At the stringer termination, shell elements representing the foot of the stringer and the panel were removed from the original shell model. The shell elements used to model the flange web and hat were not removed. A local 3D FE-model containing a straight delamination front was generated using C3D8I solid brick elements and inserted into the global shell model as shown in the detail of Figure 5. The local 3D model consists of an intact section and a delaminated section with a fine mesh around the delamination front. The plane of delamination is located at the bondline between stringer foot and the panel. The delamination was modeled as a discrete discontinuity using two unconnected nodes with identical coordinates, one on each side of the delamination. A refined mesh as shown in Figure 5 was used along the stringer boundary in order to capture edge effects. Using the finite sliding option available in ABAQUS®, contact was modeled between the delaminated surfaces to avoid interpenetration during analysis. The local 3D model was connected with the shell model using the shell to solid coupling option in ABAQUS® which allows the connection between non-conforming shell and solid models. For the entire investigation of the panel buckling, the ABAQUS® geometric nonlinear analysis procedure was used. Additional modeling details are described in reference [64].

3.2 Calculation of Mixed-Mode Strain Energy Release Rates and Failure Indices

The virtual crack closure technique (VCCT) – discussed earlier - was used to calculate the mode contributions G_I, G_{II} and G_{III}, the total energy release rate $G_T=G_I+G_{II}+G_{III}$, as well as the mixed mode ratios G_S/G_T along the delamination front across the width of the stringer for all delamination lengths modeled. Here, G_S denotes the sum of the in-plane shearing components $G_{II}+G_{III}$. For two-dimensional analyses, where $G_{III}=0$, this definition is equal to the commonly used definition of the mixed mode ratio, G_{II}/G_T. For three-dimensional analysis, which also yields results for the scissoring mode G_{III}, the modified definition of G_S is introduced since a mixed-mode failure criterion, which accounts for all three modes is currently not available.

For each nodal point along the delamination front, the critical energy release rate G_c was calculated from the mixed mode failure criterion for graphite/epoxy (Figure 2)

$$G_c = 207.7 + 1126.8 \cdot \left(\frac{G_S}{G_T} \right)^{4.46} . \qquad (2)$$

for the computed mixed-mode ratio G_S/G_T at each point. Subsequently, the failure index G_T/G_c was determined with the assumption that delamination propagation occurs for

$$\frac{G_T}{G_c} \geq 1. \tag{3}$$

For all delamination lengths modeled, the computed failure indices were calculated for every fifth load increment plus the final increment and plotted versus the dimensionless coordinate s across the width of the stringer b (see detail in Figure 5)

$$s(y) = \frac{y - y_0}{b}; \quad 0.0 \leq s \leq 1.0. \tag{4}$$

At the left edge of the stringer, the nodal point coordinates are equal to $y=y_0$ which yields s=0.0, and the right edge nodal point coordinates are equal to $y=y_b$ which results in s=1.0 as depicted in Figure 5.

The calculated failure index for delamination length a=81.9 mm is shown in Figure 6 for selected analysis increments only. The failure index peaked at the edges (s=0.0 and s=1.0) with an additional peak around the center (s~0.5) underneath the stringer. Early in the analysis (small increment numbers), which corresponds to small applied displacements (u=v), the failure index G_T/G_c is well below unity across the entire width. This result indicates that the delamination is not going to grow. With increasing load, the failure index approaches unity first near one edge where failure is expected to initiate. Generally, for the next load increment, the index is well above unity across the entire width. For the longer delaminations (a=101.6 mm and a=355.6 mm) which are associated with a different global buckling pattern, the distribution across the width changes, the failure index peaks in the center underneath the stringer web and is reduced toward the edges as discussed in detail in reference [64].

A different way to visualize the results is to plot the critical displacement, i.e. the applied displacement (u or v) for which G_T/G_c=1.0, at the center of the specimen (s=0.5) versus the delamination length as shown in Figure 7. The critical displacements for delamination lengths of a=81.9 mm, 88.9 mm, and 94.9 mm, were almost identical. For the longer delaminations modeled (a=101.6 mm up to 355.6 mm), the critical displacements are significantly lower, which suggests that rapid delamination progress is to be expected once the delamination starts to propagate for a critical applied displacement. Analysis and result details are described in reference [64].

CONCLUDING REMARKS

For laminated composite materials, fracture mechanics has proven useful for characterizing the onset of delaminations in composites. To fully understand this failure mechanism, the mixed-mode strain energy release rates need to be calculated and compared to interlaminar fracture toughness properties experimentally measured over a range from pure mode I loading to pure mode II loading.

It is state of the art to determine mode I fracture toughness using Double Cantilever Beam (DCB) and mode II fracture toughness using End Notched Flexure (ENF) tests. The

Mixed Mode Bending (MMB) tests is used to determine the fracture toughness of varying mixed-mode ratios. Although several different types of test specimens have also been suggested for the measurement of the mode III interlaminar fracture toughness property, an interaction criterion incorporating the scissoring shear, has not yet been established and remains a challange.

The methodology described above has been extended to predict fatigue delamination onset life, but to date a standard only exists for the Mode I DCB test. Interlaminar fracture mechanics has also been used to characterize the extension or growth of delaminations when subjected to fatigue loading. Standards for the measurement of fatigue delamination growth have not yet been established and remain a challenge.

Today, a variety of methods are used to compute the strain energy release rate based on results obtained from finite element analysis. The virtual crack closure technique (VCCT) is widely used for computing mixed-mode strain energy release rates based on results from finite element analyses. Currently, VCCT has only been implemented into the commercial finite element software ABAQUS® whereas other large commercial finite element codes do not offer any choice for calculating mixed mode energy release rates today. The implementation of methods to compute mixed mode energy release rates into these codes remains a challenge.

To date, interlaminar fracture mechanics has been used with limited success primarily to investigate onset in fracture toughness specimens and laboratory size coupon type specimens. Future acceptance of the methodology by industry and certification authorities requires the validation and verification of the methodology and successful demonstration on the structural level. The skin/stringer separation of a graphite/epoxy composite panel reinforced with three stringers and subjected to pure shear loading was analyzed to demonstrate the state-of-the-art application on the structural level. Full implementation of Interlaminar Fracture Mechanics in design, however remains a challenge and requires advancements in delamination onset and growth criteria under mixed mode conditions and continuing development effort of codes to calculate energy release rates.

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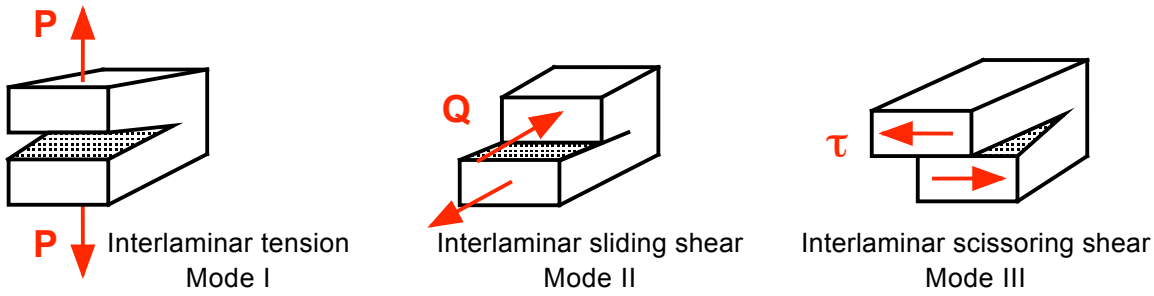

Figure 1: *Fracture Modes*.

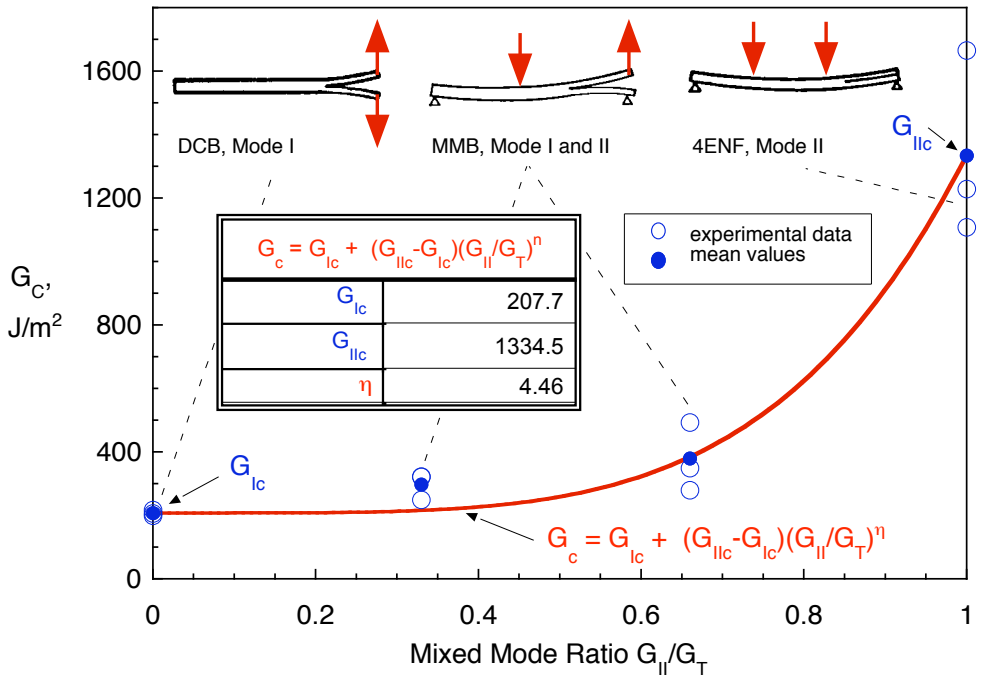

Figure 2: *Mixed-mode fracture criterion for a toughened carbon/epoxy.*

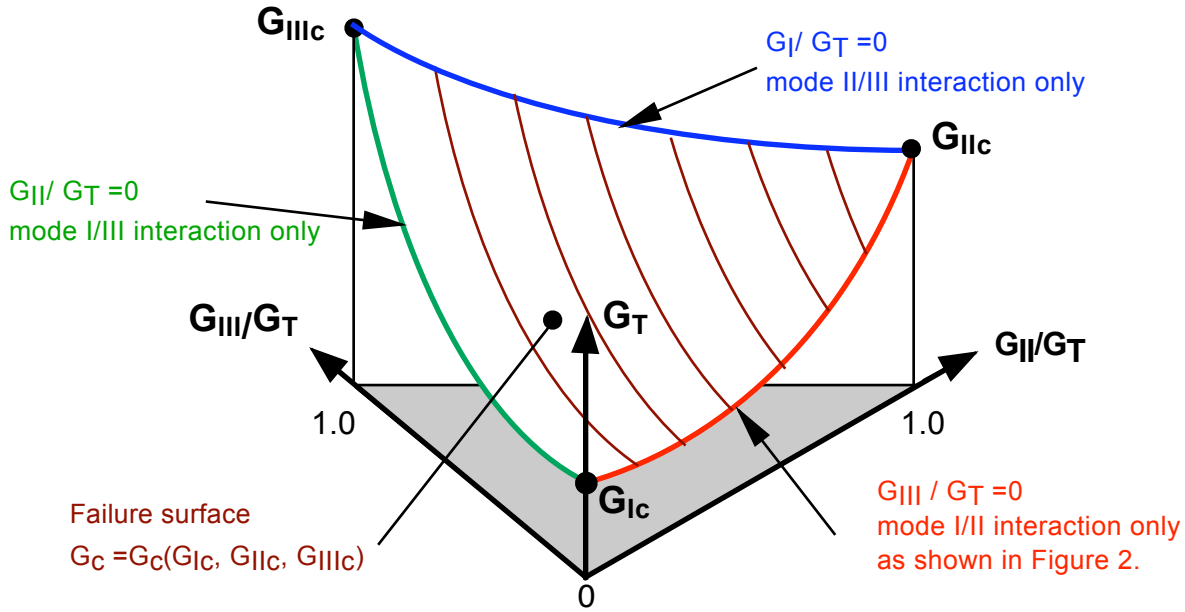

Figure 3: *Mixed mode failure criterion for modes I, II and III.*

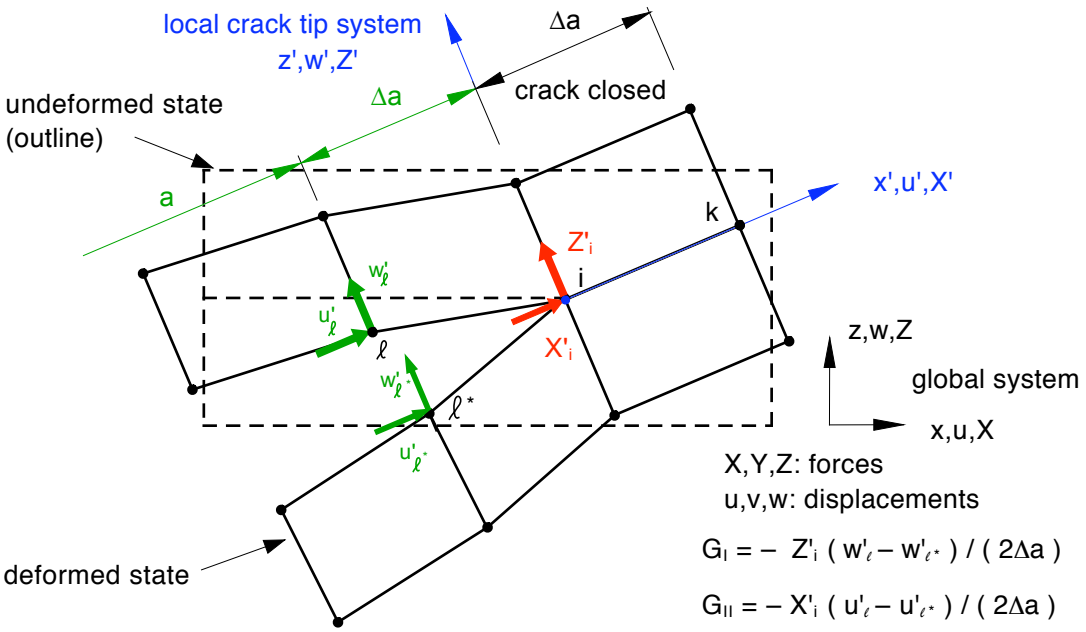

Figure 4: *Virtual Crack Closure Technique (VCCT).*

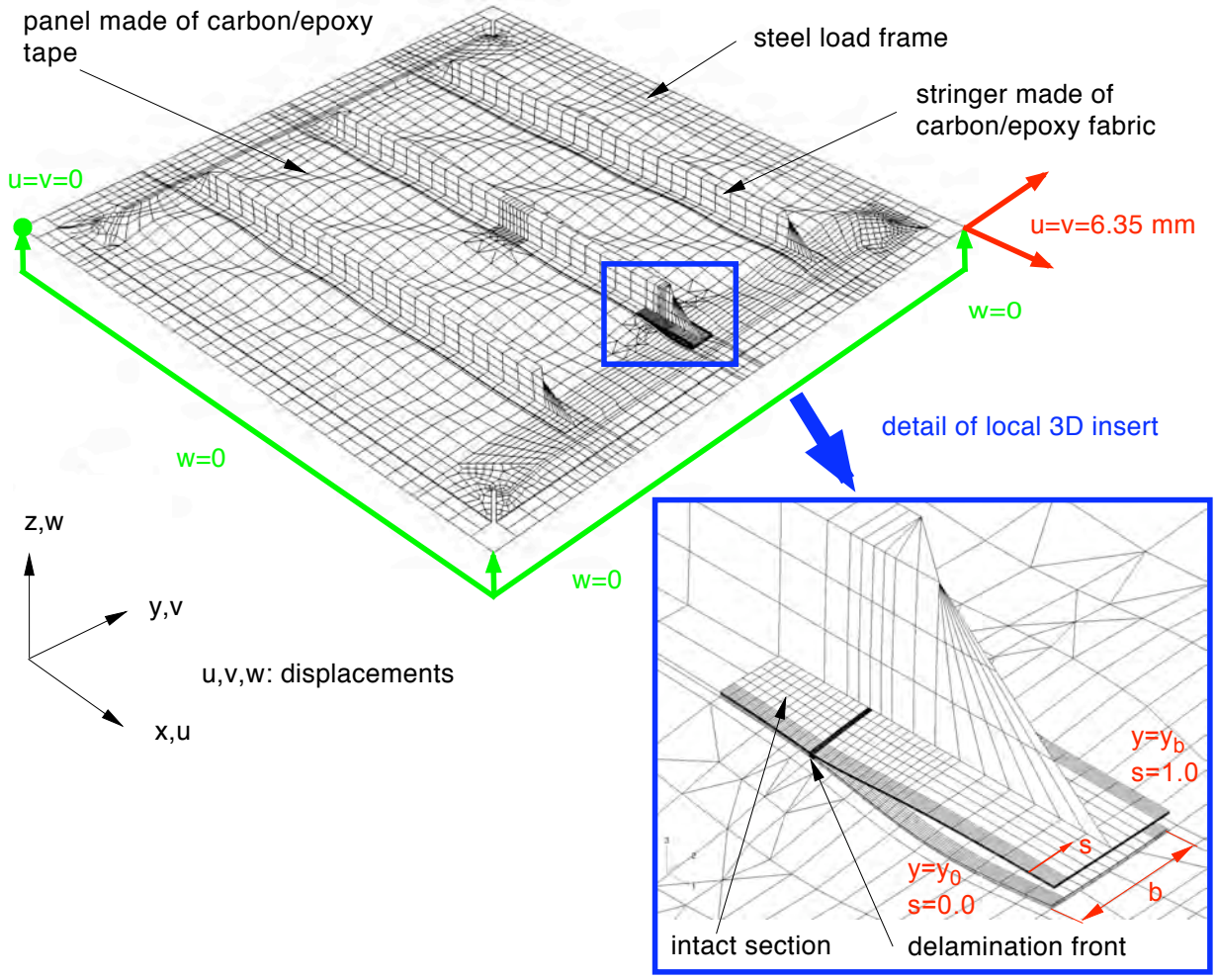

Figure 5: *Finite Element model of stiffened panel (1016 mm x 1016 mm) and load frame.*

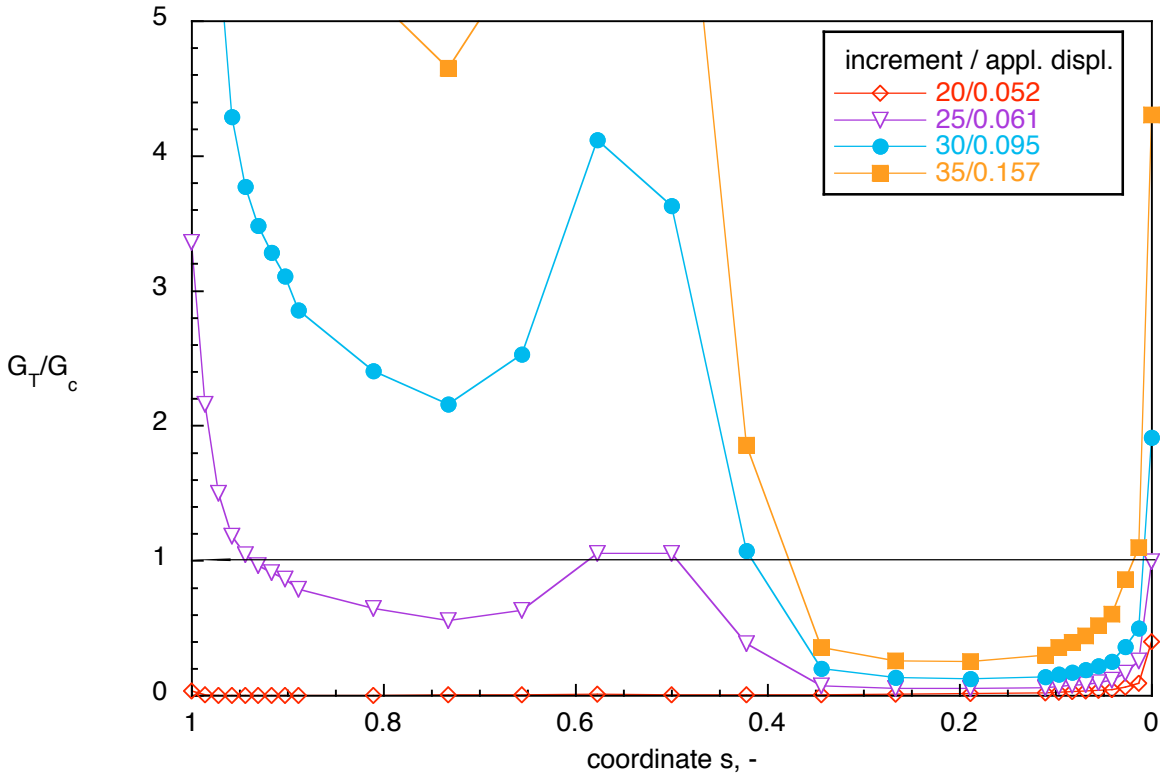

Figure 6. *Computed failure index across the width of the stringer for selected increments for delamination length a=81.9 mm.*

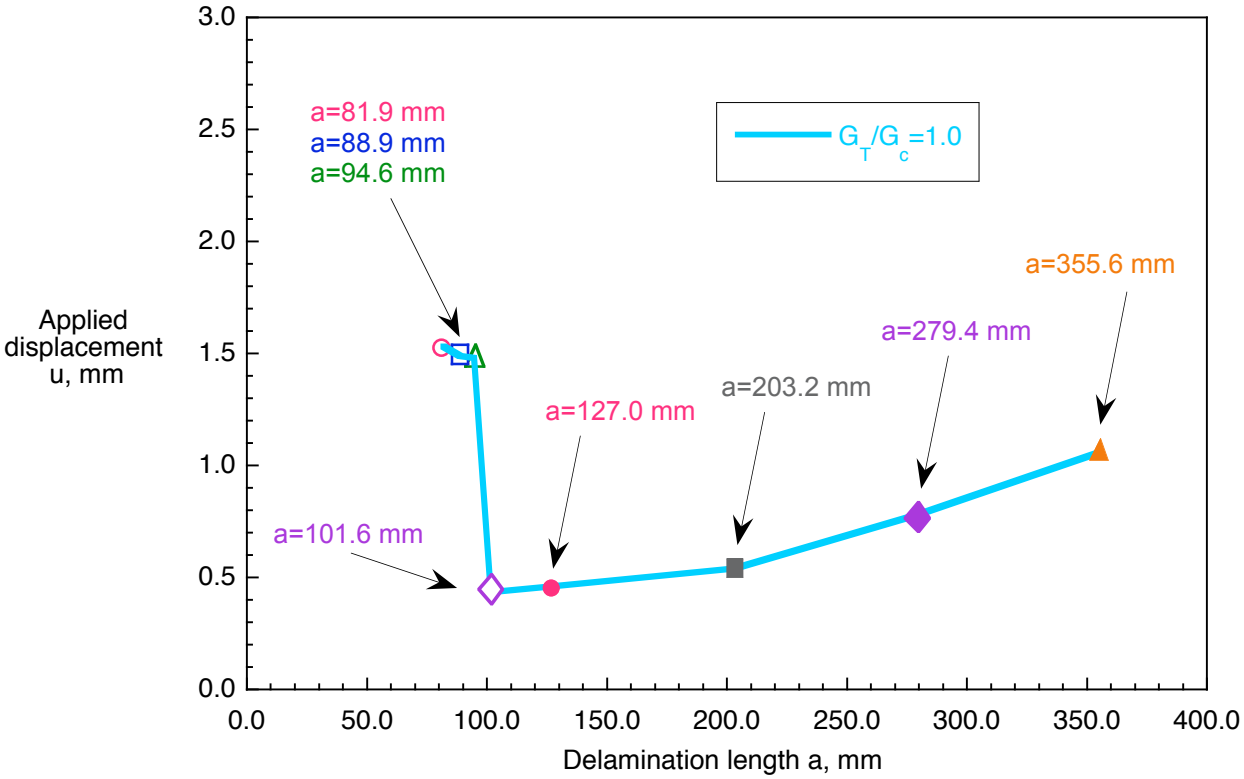

Figure 7. *Applied external displacement at delamination onset for different delamination lengths modeled.*