On Flow Stagnation in a Tube Radiator

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An analysis of the physical process for occurrence of flow stagnation in a space tube-radiator is performed and the mechanism and mathematic description for the flow stagnation are presented. Two causes for pressure drop unbalance between tubes of the radiator are identified: non-uniform cooling environment and different local flow resistances between the tubes. This analysis provides a theoretical basis for experimental simulations of the flow stagnation in a ground-based lab as well as two suggested methods to experimentally simulate flow stagnation. Criteria for the flow stagnation, depending on the viscosity data regressive polynomial, are derived from the extreme condition of the pressure drop in colder tubes. A preliminary numerical calculation is conducted for a space tube-radiator model which confirms the physical and mathematical analyses. The prediction by the criteria for flow stagnation in the tube-radiator model coincides with the numerical calculation result.

Nomenclature

B	=	radiator panel width
b	=	radiator panel width spanning a tube
c_p	=	specific heat
D	=	inner diameter of radiator tube
h	=	heat transfer coefficient
k	=	thermal conductivity
L	=	length of radiator panel
$\dot{M}$	=	total mass flow rate in radiator
$\dot{m}$	=	mass flow rate in tube(s)
n	=	number of tubes
Δp	=	pressure drop
q	=	heat flux
T	=	temperature
x	=	coordinate along tube
$\alpha, \beta, \chi, \delta$	=	coefficient in viscosity data regressive polynomial
ε	=	emission coefficient of radiator surface
μ	=	dynamic viscosity
σ	=	Stefan-Boltzmann constant
ξ	=	emission efficiency
η, ζ, Φ	=	variables in Equation 16

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Subscripts

1	=	section 1
2	=	section 2
b	=	bulk
c	=	critical state
e	=	end state
f	=	friction
i	=	indicium of tubes
in	=	inlet
m	=	minimum
out	=	outlet
u	=	unstable state
v	=	valve
w	=	wall
∞	=	environment

I. Introduction

FLUID tube radiators have been widely applied in space missions and must operate over a wide range of thermal conditions. For crewed missions, operating fluids are selected based on safety considerations as well as low freezing points. Often the fluid selected has a strong temperature-viscosity dependence. Non-uniform heat transfer by the radiator with these types of fluids may result in flow stagnation, which can significantly reduce the heat rejection from the radiator to environment. The decrease in radiator performance because of the flow stagnation may or may not be desirable. In some cases, such as the Apollo spacecraft during low power phases of the mission and the long periods of docked operations at Skylab, the flow stagnation of radiators was used to reduce heat loss. However, flow stagnation has not proven to be an issue for other space applications[1]. Obviously, it is very important to have a good understanding of the physics of flow stagnation and develop pertinent mathematical models to predict when flow stagnation occurs.

Hahn and Reavis[1] have suggested a mathematical model to analyze the flow stagnation in fluid tube radiators. They obtained a result for the header outlet temperature versus the initial outlet temperature having a similar curve pattern to the result obtained using Thermal Desktop®. Their analysis did not give the mechanism of the stagnation.

In this paper, based on an analysis of the fluid flow and heat transfer processes in a tube radiator, the occurrence condition of flow stagnation is proposed. The mechanism and mathematical description for the flow stagnation are presented. The analysis in the present paper provides a theoretical basis for experimental simulation of flow stagnation in a parallel tube-radiator. Criteria for the flow stagnation, depending on the viscosity data regressive polynomial, are derived from the extreme condition of the pressure drop in colder tubes. A preliminary numerical calculation is performed to prove the prediction by the criteria.

II. Basic Equations Governing the Flow and Thermal Process

A balance between the thermal radiation from the radiator to environment (e.g. deep space $T_\infty \sim 3$ K); the heat provided by the fluid flow in the tubes of the radiator; and the convective heat transfer in the tubes from the bulk flow to the wall of the tubes determines the relationship between the temperature distributions of the working fluid and the mass flow rate. The thermal process of the flow is governed by the following equations:

$$b_i \sigma \xi \int_0^L \varepsilon (T_{w,i}^4 - T_\infty^4) dx = \dot{m}_i c_p (T_{in} - T_{out,i}) \tag{1}$$

$$\dot{m}_i c_p (T_{in} - T_{out}) = \pi D_i \int_0^L h_i (T_{b,i} - T_{w,i}) dx \tag{2}$$

$$\dot{M} = \sum_1^n \dot{m}_i \tag{3}$$

where subscript i indicates the parameters referring to an individual tube; b is the radiator panel width spanning a tube; σ is Stefan-Boltzmann constant; ξ is the emission efficiency; ε is the emission coefficient of the radiator surface; T_w is the local wall temperature of the tube; T_∞ is the environment temperature; x is the coordinate along the tube; $\dot{m}$ is the flow rate in a tube; c_p is the specific heat of the working fluid in the tube; T_{in} and T_{out} are the inlet and outlet temperatures of the fluid; D is the tube inner diameter; h is the heat transfer coefficient from the tube wall to the bulk flow in the tube; L is the tube length; $\dot{M}$ is the total mass flow rate in the radiator; n is the number of tubes of the radiator in total; and T_b is the fluid local bulk temperature in a tube. T_b can be estimated by

$$T_{b,i}(x) = T_{in} - \frac{b_i \sigma \xi \int_0^x \varepsilon (T_{w,i}^4 - T_\infty^4) dx}{\dot{m}_i c_p} \tag{4}$$

Since the tube diameter used in the radiators is very small and the flow in the tubes is laminar, the pressure drop in each tube can be calculated by

$$\Delta p_i = \frac{128 \dot{m}_i \int_0^L \mu_i dx}{\rho \pi D_i^4} + \Delta p_{v,i} \tag{5}$$

where Δp_v is the local pressure drop created by local flow resistance elements other than the tube, such as valve or elbow.

For a radiator consisting of a set of parallel tubes fed by a common manifold on the inlet and collected by a similar manifold at the outlet, the pressure is balanced through self-adjustment at the end manifolds. Neglecting pressure drop within the manifolds, the pressure drop in each of the tubes should be equal to the pressure drop between the two end manifolds, expressed by

$$\Delta p = \Delta p_i \tag{6}$$

III. Mechanism of Flow Stagnation: Pressure Drop Unbalance in Radiator

Under the ideal conditions of uniform cooling and identical flow resistances through all tubes (i.e., no manufacturing variability between tubes); the viscosity increases uniformly as the environmental temperature drops and theoretically no stagnation should occur. The reason is that the pressure balance between the tubes can be reached in the end manifolds through self-adjustment until a very high temperature differential between inlet and outlet is reached. However, if the radiator is placed in a non-uniform thermal environment or the radiator has non-uniform heat transfer performance and/or different local flow resistances existing in different tubes, flow stagnation would occur in some tubes at low mass flow rates. In this case, the flow temperature may be much higher than the freezing point of the working fluid.

In fact, it is Eq. 6 that should be taken as the basic criterion for flow stagnation. When and only when the flows in all tubes meet the condition expressed by Eq. 6, the radiator does not stall. Once the pressure drop in one or several tubes increases and cannot balance with the pressure drop in other tubes through the self-adjustment, the flow stagnation will occur in the tube(s).

It can be seen from Eq. 5 that only two factors affect on the pressure drop balance: the flow friction resistance and the local flow resistance, described by the first and the second term in the right side of Eq. 5, respectively. It implies that only change of either the flow friction resistance or the local flow resistance is able to cause a pressure instability such that Eq. 6 no longer holds.

A. Flow stagnation Induced by Non-uniform Cooling Environment

The strong temperature-viscosity dependence results in a non-monotonic curve of pressure-drop versus mass flow rate, which has a pressure-drop reversion. As the tube gets colder, the minimum obtainable pressure drop occurs at a higher mass flow rate. For a given geometry of the radiator, the pressure drop formed by friction resistance in a tube is a function of mass flow rate and viscosity of the working fluid, which is expressed as $\Delta p = f(\dot{m}, \mu)$. On the other hand, the viscosity is a function of bulk temperature distribution of the working fluid in the tube which can be determined by solving Eqs. 1 - 6. For different temperature distributions in a tube, the relationship between pressure drop and mass flow rate can be qualitatively shown in Fig. 1, where

$$\int_0^L \mu_1(T_1)dx < \int_0^L \mu_2(T_2)dx < \int_0^L \mu_3(T_3)dx < ......$$

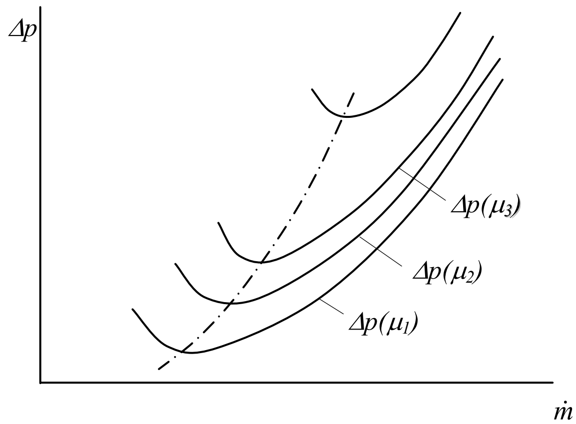

Figure 1. Schematic of pressure drop at different cooling conditions (not scaled).

Assuming that no local resistance exists in the tubes and one section of the radiator experiences a different thermal environment than remaining area. For example, imagine a radiator panel along a curved spacecraft where half of the radiator faces deep space (T_∞ = 3 K) while the other half faces earth (T_∞ = 243 K). Both sections must meet the pressure drop balance expressed by

$$\frac{\dot{m}_1\int_0^L \mu_1 dx}{n_1\rho\pi D^4} = \frac{\dot{m}_2\int_0^L \mu_2 dx}{n_2\rho\pi D^4} \quad (7)$$

where the subscript 1 and 2 refer to the section 1 and section 2, respectively, n is the number of tubes in each relevant section. The mass conservation Eq. 3 reduces to

$$\dot{M} = \dot{m}_1 + \dot{m}_2 \quad (8)$$

As the total mass flow rate decreases, the mass flow rate in the two sections decreases as well. The pressure drop produced by the flow friction resistance in section 1, Δp_1, decreases to the minimum and then increases, while the decrease of the mass flow rate in section 2 would further reduce the pressure drop in section 2. It means that the pressure at the outlet of section 2 would be larger than the pressure at the outlet of section 1, which forces the fluid at the outlet manifold from section 2 towards section 1 to reduce the flow rate in section 1 and increase in section 2, as shown in Fig. 2. Consequently, a new balance of pressure drop between the two sections is reached, at which the flow rate in section 1 decreases, but flow rate in section 2 finally increases rather than decreases, as shown in Fig. 3. This new balance is unstable because it is not possible to find another balance state if the total flow rate decreases further. The new unstable balance has to satisfy simultaneously the pressure drop balance and mass conservation conditions:

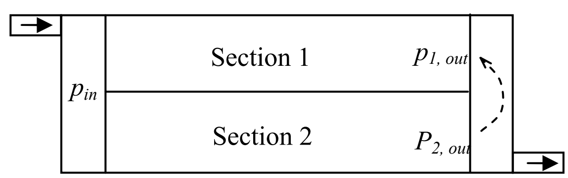

Figure 2. Unbalance pressure at outlet manifold

$$\Delta p_{1,u} = \Delta p_{2,u} \quad (9)$$

$$\dot{m}_u - \dot{m}_m = (\dot{m}_{2,u} - \dot{m}_{2,m}) + (\dot{m}_{1,u} - \dot{m}_{1,m}) \quad (10)$$

At the unstable balance state any small decrease of the total flow rate will lead to a spiral:

$$\dot{m}_1\Downarrow \Rightarrow \Delta p_1 \Uparrow \Rightarrow \dot{m}_2 \Uparrow \text{ and } \Delta p_2 < \Delta p_1$$

As a result, section 1 of the radiator stalls in which the flow rate $\dot{m}_1$ essentially goes to zero, while the flow rate in section 2, $\dot{m}_2$, jumps to $\dot{m}_{2,e}$, which is determined by

$$\dot{m}_{2,e} = \dot{m}_{1,u} + \dot{m}_{2,u} \quad (11)$$

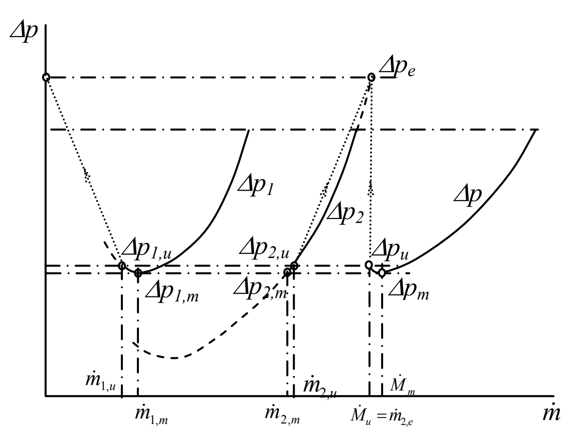

Figure 3. Pressure-drop in different sections at different cooling environments.

A preliminary calculation for the realistic working fluid mixture of 50% propylene-glycol and 50% water demonstrates this pattern of pressure-drop changes. It is found that the difference between $\Delta p_{1,m}$ and $\Delta p_{1,u}$; between $\Delta p_{2,m}$ and $\Delta p_{2,u}$ and consequently between Δp_m and Δp_u is very small and thus can be ignored. More importantly, after the flow rate in section 1 reaches the minimum value, $\dot{m}_{1,m}$, the radiator barely attains a new balance state satisfying Eqs. 9 and 10 as the total flow rate decreases further, i.e., the unstable balance state may not exist at all. Therefore, taking $\dot{m}_{1,m}$ as the critical mass flow rate to judge the occurrence of flow stagnation is reasonable.

B. Flow stagnation Induced by Additional Local Flow Resistance

As mentioned above, if all sections of the radiator face exactly the same environment, no flow stagnation will occur. However, if a slightly higher local flow resistance is added to a section of the radiator, a flow stagnation would occur in this section. The mechanism of the stagnation is similar as in the case of non-uniform cooling environment. The pressure drop pattern is shown in Fig. 4.

Because of the uniform cooling environment the pressure drop formed by flow friction resistance in all tubes of the radiator is the same and depicted by the red line in Fig. 4. Once a realistic local flow resistance, such as a valve, a fitting, or even manufacturing variability between tubes is added, the mass flow rate is redistributed. The mass flow rate in section 1 is reduced and so the pressure drop formed by flow friction resistance decreases, depicted by the dash line coinciding with the red line. In the meantime, mass flow rate and the pressure drop in section 2 increases, depicted by the line partially coinciding with the red line. The total pressure drop in section 1 is the sum of the flow friction resistance and the valve resistance depicted by the shadow region. The pressure drop pattern in the radiator in this case becomes similar to the pattern in the case of the radiator in a non-uniform cooling environment. The flow stagnation will occur in section 1 when the total flow rate decreases to a certain value. The same analysis can be made as in the last section. The critical mass flow rate in section 1 $\dot{m}_{1,c}$ occurs at the minimum point of the Δp_1-$\dot{m}$ curve, as shown in Fig. 4.

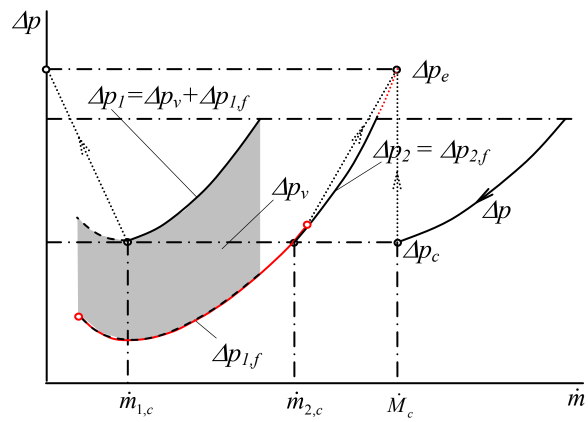

Figure 4. Pressure drop in different sections at uniform cooling environment with an additional local flow resistance in section 1.

It is of interest to note that the flow stagnation only depends on the pressure drop balance, no matter what cooling mode is used. Therefore, the initial lab experiments to simulate the flow stagnation can be conducted using convection cooling instead of radiation.

IV. Criterion of Flow Stagnation in Radiator

The pattern of pressure-drops shown in Figs. 3 and 4 indicate that $\frac{\partial \Delta p_1}{\partial \dot{m}_1} = 0$ can be used to predict flow stagnation. The following derivation gives a simple criterion:

$$\frac{\partial \Delta p_1}{\partial \dot{m}_1} = \frac{\partial}{\partial \dot{m}_1}\left[\frac{128\dot{m}_1}{\rho n_1 \pi D^4}\int_0^L \mu_1 dx\right] = \frac{128}{\rho n_1 \pi D^4}\left[\int_0^L \mu_1 dx + \dot{m}_1 \frac{\partial}{\partial \dot{m}_1}\int_0^L \mu_1 dx\right] = 0 \tag{12}$$

As a first approach, $\mu_1 = \alpha T_1^2 + \beta T_1 + \chi$ is assumed. For a 50/50 propylene-glycol and water mixture it is found that the regressive polynomial has coefficients of $\alpha = 1.145540793\text{x}10^{-5}$, $\beta = -0.00711425$, and $\chi = 1.10752655$ for temperatures between 268 and 318K. When compared to experimental data, the maximum error of the polynomial is about 6% for temperatures between 268 and 298 K and 23.3% for temperatures between 298 and 318 K.

A thermal energy balance gives $T_1 = T_{in} - n_1\pi Dq_1 x/(\dot m_1 c_p)$, where $T_{\rm in}$ is the entrance temperature of the working fluid of the radiator, q_1 is the heat flux transferred by the radiator in section 1 that is assumed as uniform along the tubes. Therefore, from Eq. 12 we have

$$\int_0^L\left[\alpha\left(T_{in}-\frac{n_1\pi Dq_1x}{\dot m_1c_p}\right)^2+\beta\left(T_{in}-\frac{n_1\pi Dq_1x}{\dot m_1c_p}\right)+\chi\right]dx+\dot m_1\int_0^L\left[2\alpha\left(T_{in}-\frac{n_1\pi Dq_1x}{\dot m_1c_p}\right)\left(\frac{n_1\pi Dq_1x}{\dot m_1^2c_p}\right)+\beta\left(\frac{n_1\pi Dq_1x}{\dot m_1^2c_p}\right)\right]dx=0 \qquad (13)$$

Rearranging the terms, Eq. 13 gives

$$\int_0^L \mu_{in}dx=\int\alpha\left(\frac{n_1\pi Dq_1x}{\dot m_1c_p}\right)^2dx \qquad (14)$$

where $\mu_{\rm in}$ is the viscosity of the working fluid at the entrance temperature. Equation 14 directly gives the critical mass flow rate in section 1:

$$\dot m_{1,C}=\frac{n_1\pi Dq_1L}{c_p}\sqrt{\frac{\alpha}{3\mu_{in}}} \qquad (15)$$

which can be taken as the criterion to predict when flow stagnation will occur.

A more accurate prediction may be given by cubic correlation of viscosity as follows:
Let $\mu_1 = \alpha T_1^3+\beta T_1^2+\chi T+\delta$, the similar derivation gives

$$\left.\begin{aligned}
\dot m_{1,C}&=\frac{n_1\pi Dq_1}{c_p\Phi}\\
\Phi&=\left(\sqrt{\frac{\varsigma\eta^3}{27}+\frac{\varsigma^2}{4}}-\frac{\eta^3}{27}-\frac{\varsigma}{2}\right)^{1/3}-\left(-\sqrt{\frac{\varsigma\eta^3}{27}+\frac{\varsigma^2}{4}}-\frac{\eta^3}{27}-\frac{\varsigma}{2}\right)^{1/3}-\frac{\eta}{3}\\
\eta&=-\frac{6\alpha T_{in}+2\beta}{3\alpha L}\\
\varsigma&=\frac{2\mu_{in}}{\alpha L^3}
\end{aligned}\right\} \qquad (16)$$

The cubic regressive polynomial of viscosity data, $\mu_1 = \alpha T_1^3+\beta T_1^2+\chi T_1+\delta$, reduces the error somewhat. For 50/50 propylene-glycol and water mixture this cubic polynomial gives $\alpha = -264.60606049\times10^{-9}$, $\beta =$ 244.17114604x10^{-6}, $\chi = -0.07521945$, $\delta = 7.73975331$ with a maximum error of 2% between 268 and 303 K and 5.7% between 303 and 318 K, respectively.

It is interesting to note that both equations Eq.15 and 16 only contain the nonlinear terms of the corresponding viscosity regressive polynomial.

V. Preliminary Numerical Calculation Results

A preliminary numerical calculation for a typical space radiator model used by Hahn and Reavis[1] has been performed with relevant dimensions of the model shown in Fig. 5. The model uses the following: D=1.7526x10^{-3} m; B = 1.778 m; L = 2.667 m; n = 20; k_f = 0.35 W/m-K; ρ = 1062 kg/m^3; c_p = 3436 J/kg-K; σ = 5.670x10^{-8} W/m^2-K^4; ε = 0.92; $\mu = 1.448\mu_f$ in which μ_f is calculated by the viscosity-temperature correlation given by Hahn and Reavis in Equation (A2)[1]. The radiator is assumed to face two different radiation environments. Half of the panel of the radiator (section 1, which spans 10 tubes, faces an environment corresponding to deep space (3 K) while the other half panel (section 2) faces to an environment of 243 K.

The numerical calculation based on Eqs. 1 - 6 shows that for the given inlet temperature of 314 K, with the total mass flow rate reduced to 0.016 kg/s, flow stagnation occurs in section 1, as shown in Fig. 6, where Δp, Δp_1, and Δp_2 are the pressure drop between the inlet and outlet of the radiator, section 1 and 2 respectively. $\dot M$, $\dot m_1$, and $\dot m_2$ are the mass flow rates in the radiator and each section. It can be seen that Fig 6 is quite similar to Fig.3, confirming the analysis in Section III.

For this case, the flow stagnation occurs in section 1 at $\dot m_1 = 0.0053$Kg/s, but the minimum flow rate in section 1 should be between 0.0053 and 0.0063 Kg/s, which usually is taken as the critical mass flow rate,

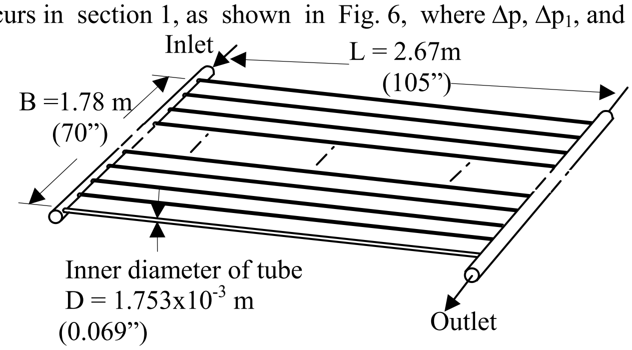

Figure 5. Geometry of a space radiator model

say $\dot{m}_{1,c} = 0.0058$Kg/s. Actually, this is a peculiar case, in which after the pressure drop in section 1 reverses, the new balance state satisfying Eqs. 9 and 10 exists, as mentioned above.

The preliminary calculation results confirm the analysis in Section III for the mechanism of flow stagnation. On the other hand, the criterions suggested in Section IV are also coincident with the numerical calculation. The prediction by Eq. 15 gives $\dot{m}_{1,c,pre} = 0.0038$ Kg/s which is 34.5% less than the calculation result while Eq. 16 predicts $\dot{m}_{1,c,pre} = 0.0066$ Kg/s, higher 13.8% than the calculation result.

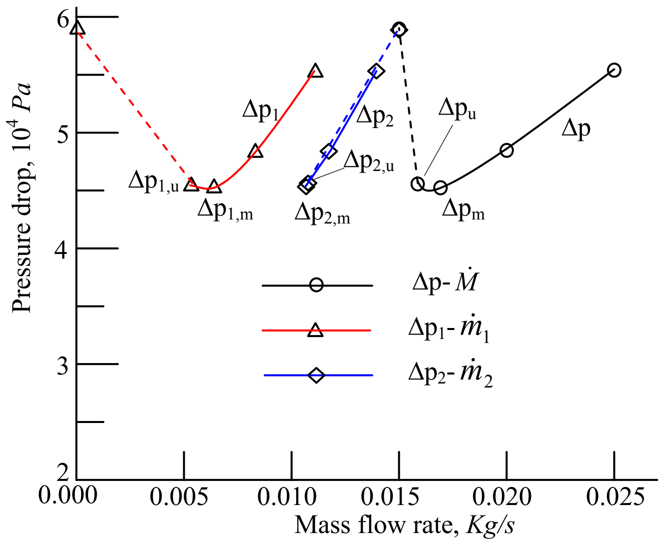

Figure 6. Pressure drop versus mass flow rate

VI. Discussion

The above analysis shows that flow stagnation can occur either when the radiator is exposed to a non-uniform cooling environment or additional local flow resistance in a section of the radiator. This analysis provides a theoretical basis for experimental simulation of a space radiator.

The analysis also reveals two practical methods to experimentally simulate the flow stagnation:

1. Create different cooling conditions for different sections of the radiator through adjusting the coolant temperature and/or flow rate.
2. Create different local flow resistances in different sections of the radiator through adjusting control valves in the fluid tubes.

The prediction for the flow stagnation by Eq. 15 or 16 gives a direction to the experiment arrangement and operations.

VII. Conclusion

Based on the analysis and the preliminary numerical calculation results we can confidentially conclude that

1. In order for stagnation to occur, either the radiator must experience a non-uniform cooling environment or different local flow resistances between tubes.
2. Experiments to simulate the flow stagnation in lab on earth can be performed using convection cooling mode instead of radiation cooling mode.
3. Flow stagnation occurs in the section experiencing the colder environment or having higher local flow resistance when the total mass flow rate reduces to a certain value.
4. The practical criterion for flow stagnation can be derived from the extreme condition of pressure drop in colder section of the radiator, which is described by $\frac{\partial \Delta p_1}{\partial \dot{m}_1} = 0$.
5. Predictions for the flow stagnation by the criteria expressed by Eq. 15 or 16 with the corresponding regressive polynomial of viscosity data are in agreement with the numerical calculation results.

References

[1]Hahn, N. and Reavis, G., "Investigation into the Understanding of Flow Stagnation in Fluid Tube Radiators," *AIAA-2007-0529, 45th AIAA Aerospace Sciences Meeting and Exhibit, January 8-11, 2007, Reno, Nevada.*