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# Lunar Ascent and Rendezvous Trajectory Design 

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# LUNAR ASCENT AND RENDEZVOUS TRAJECTORY DESIGN 

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#### Abstract

The Lunar Lander Ascent Module (LLAM) will leave the lunar surface and actively rendezvous in lunar orbit with the Crew Exploration Vehicle (CEV). For initial LLAM vehicle sizing efforts, a nominal trajectory, along with required delta-V and a few key sensitivities, is very useful. A nominal lunar ascent and rendezvous trajectory is shown, along with rationale and discussion of the trajectory shaping. Also included are ascent delta- V sensitivities to changes in target orbit and design thrust-to-weight of the vehicle. A sample launch window for a particular launch site has been completed and is included. The launch window shows that budgeting enough delta-V for two missed launch opportunities may be reasonable. A comparison between yaw steering and on-orbit plane change maneuvers is included. The comparison shows that for large plane changes, which are potentially necessary for an anytime return from mid-latitude locations, an onorbit maneuver is much more efficient than ascent yaw steering. For a planned return, small amounts of yaw steering may be necessary during ascent and must be accounted for in the ascent delta- V budget. The delta- V cost of ascent yaw steering is shown, along with sensitivity to launch site latitude. Some discussion of off-nominal scenarios is also included. In particular, in the case of a failed Powered Descent Initiation burn, the requirements for subsequent rendezvous with the Orion vehicle are outlined.


## INTRODUCTION

As outlined in the Vision For Space Exploration, the Constellation program has been charged with developing the vehicles necessary to expand the envelope of the human domain outside of low Earth orbit. The fleet of vehicles that fall under the Constellation umbrella includes a lander that descends to the lunar surface and later returns the crew to lunar orbit at the conclusion of surface operations. The Lunar Lander (previously known as LSAM) is envisioned as a two stage vehicle, composed of a Descent Module (DM) and an Ascent Module (AM). The DM provides the propulsive capability needed to deliver the combined AM/DM stack to the lunar surface. Following lunar surface operations, the AM lifts off (leaving the DM on the surface) and performs an active rendezvous to deliver the crew to the Orion vehicle for the return journey.

## FLIGHT PROFILE

## Ascent

The lunar ascent profile, shown in Figure 1, begins with a 100-meter vertical rise phase ${ }^{1}$, which lasts approximately 10 seconds. The vertical rise is followed by a Single Axis Rotation (SAR) maneuver. The SAR logic calculates a single-axis time optimal rotation from the initial attitude to the final attitude, given a final attitude command (yaw, pitch, and roll) and limits on angular velocity and angular acceleration ( $5.0 \mathrm{deg} / \mathrm{sec}$ and $15.0 \mathrm{deg} / \mathrm{sec}^{2}$ were used, respectively). The exact length of this maneuver varies from case to case, but, given these limits, it is generally on the order of 10 seconds. The SAR is followed by a Powered Explicit Guidance (PEG) ${ }^{234}$ phase that delivers the vehicle to the desired orbit. For a given constant throttle, PEG calculates the steering command to achieve the provided radius magnitude, velocity magnitude, and flight path angle targets. The solution calculated by PEG is the optimal solution, given the targets that are provided to the algorithm. An external optimizer called Nonlinear Programming Solver from Stanford's Systems Optimization Laboratory (NPSOL) was used to optimize the selection of the PEG targets, while meeting a case-specific set of constraints, with the objective of maximum mass to orbit. The constraint set consists of a minimum altitude constraint, a perilune target, an apolune target, and a yaw target for the end of the SAR maneuver.

[^0]

Figure 1: Ascent trajectory profile

## Ascent Assumptions

The following assumptions were made for the ascent analysis:

1. Launch Site: Aristarchus plateau (26 degrees $N$ latitude, 311 degrees $E$ longitude)
2. CEV inclination: 145.765 degrees (deg)
3. Apolune target $=75$ kilometers $(\mathrm{km})$
4. Perilune target $=15.24 \mathrm{~km}$
5. Only central body forces are modeled. No higher order terms are included.
6. The lunar rotation rate is modeled.
7. The environment constants are from the JPL Lunar Constants and Models document. ${ }^{5}$
8. The minimum instantaneous altitude is zero kilometers.
9. The Longitude of the Ascending Node (LAN) target is optimized for minimum ascent deltavelocity magnitude (delta-V).
10. The LL Thrust to Weight (T/W) is approximately equal to 0.35 (Earth T/W).
11. The LL will perform planar (yaw) steering during ascent in order to be in-plane with the CEV rendezvous phantom plane at main engine cutoff.

## Rendezvous

Following main engine cutoff, the AM coasts for a period of time before the rendezvous sequence begins. The AM will then perform a sequence of burns leading to rendezvous with the CEV. This paper is concerned with the rendezvous only up to and including Terminal Phase Initiation (TPI). In other words, Proximity Operations are not considered. It is assumed that docking occurs within 1 revolution (rev) following TPI. Two types of nominal rendezvous profiles are considered here: a short 1 rev solution, and a co-elliptic 2-rev solution.

Currently, the AM has a requirement that the docking must be completed within 12 hours, though a shorter time is preferred (and planned for nominal operations). The strong desire to minimize the operational lifetime of the AM is due to the fact that the consumables budget is particularly sensitive to time (and the overall mission architecture is particularly sensitive to AM mass). For this reason, more time-consuming rendezvous solutions are not of great interest for initial vehicle sizing and requirements development.

## Rendezvous Assumptions

The following assumptions were made for the rendezvous analysis:

1. Nominally, the LL performs an on-time liftoff, which requires no additional yaw steering during the ascent burn.
2. The minimum time between the coelliptic maneuver and the Terminal Phase Initiation (TPI) maneuver is 30 minutes.
3. The desired delta altitude between the LL and the CEV is 25 km at the time of TPI. ${ }^{6}$
4. As part of the TPI targeting constraints, the LL orbital transfer angle from TPI to Terminal Phase Finalization (TPF) is 130 degrees.
5. Docking occurs within 1 rev after TPI.
6. The rendezvous plan does not reflect the following operational constraints:
a. Navigation Dispersions
b. Translational Burn Errors
c. Proximity Operations
d. Lighting
e. Tracking
f. Orbital Maintenance
g. Vehicle Attitude Maneuvers
h. Crew Timeline

## NOMINAL ASCENT \& RENDEZVOUS

## Simulation and Modeling

The ascent and rendezvous work is modeled using two different simulations. The simulation used for ascent is called Simulation and Optimization of Rocket Trajectories (SORT). SORT is a 3-DOF simulation originally developed for Space Shuttle ascent. It has been adapted to model a number of other flight phases, including Earth entry, Mars entry and ascent, and Lunar descent and ascent.

The rendezvous work was performed using the Flight Analysis System (FAS). FAS is used for the initial stages of flight planning for on-orbit trajectories. It consists of different modules that are linked using data base arrays in order to generate on-orbit functions such as rendezvous, lighting, and ground tracks. FAS has been used for Space Shuttle, Lunar, and Mars flight planning. It requires a vehicle state vector to initialize the simulation. The following nominal ascent state vector at insertion (shown in Table 1) was used to initialize the simulation.

Table 1: Nominal Insertion Vector

| Description | Value | Unit |
| :--- | :---: | :---: |
| Radius magnitude | 5750195.5 | ft |
| Inertial velocity magnitude | 5533.0615 | $\mathrm{ft} / \mathrm{s}$ |
| Inertial flight path angle | 0.034803938 | deg |
| Time of insertion (from liftoff) | 420.42283 | sec |
| Geodetic latitude of launch site | 26.0 | deg |
| Geographic longitude of launch site | 311.0 | deg |
| Geodetic latitude of insertion | 29.641459 | deg |
| Geographic longitude of insertion | 300.604763 | deg |

## Nominal Ascent Trajectory

The nominal ascent delta-V (to insertion) is $1833 \mathrm{~m} / \mathrm{s}$.

## 1-Revolution Rendezvous Maneuver Description

A 3-burn rendezvous maneuver sequence is defined, where the terminal phase of the rendezvous occurs on the first rev after launch from the lunar surface.

After insertion into lunar orbit, the LL coasts for ten minutes, at which time the LL performs a corrective combination maneuver (NCC). NCC is a Lambert targeted maneuver which is designed to create a desired differential altitude and a desired phase angle between the LL and the CEV at the first LL apsis. In addition, it will correct for any small out-of-plane errors that may have occurred during ascent.

At the first LL apolune, the TPI burn occurs. TPI is a Lambert targeted maneuver, designed to place the LL in the vicinity of the CEV, after a designated orbital transfer angle.

If necessary, the LL will perform mid-course correction burns during the transfer time. Near the end of the transfer time, the LL will perform small braking maneuvers to slow its approach rate to the CEV, finally arriving at a stable orbit condition with the CEV.

The mission sequence is presented in Table 2. Note that the total $\Delta \mathrm{V}$ for the rendezvous sequence is approximately $30 \mathrm{~m} / \mathrm{s}$. The relative motion between the LL and the CEV, starting with insertion, is shown in Figure 2.

Table 2: 1-Rev Rendezvous Sequence

| Trajectory Event | Phase ElapsedTime | Delta Velocity Magnitude of Translational Maneuver |  | Resultant Apoapsis / Periapsis |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{m} / \mathrm{s}$ | $\mathrm{ft} / \mathrm{s}$ | km | nmi |
| Liftoff Insertion <br> Corrective Combination Terminal Phase Initiation Terminal Phase Finalization Docking | 00:00:00:00 00:00:07:01 00:00:17:01 00:01:03:01 00:01:45:34 00:03:03:00 | $\begin{gathered} 1 \\ 21 \\ 8 \end{gathered}$ | $\begin{gathered} 3 \\ 69 \\ 26 \end{gathered}$ | $\begin{gathered} 74 / 15 \\ 75 / 15 \\ 101 / 74 \\ 100 / 100 \\ 100 / 100 \end{gathered}$ | $\begin{gathered} 40 / 8 \\ 41 / 8 \\ 55 / 40 \\ 54 / 54 \\ 54 / 54 \end{gathered}$ |

Total Rendezvous Delta $V=30 \mathrm{~m} / \mathrm{s}(98 \mathrm{ft} / \mathrm{s})$
Note: At the time of rendezvous initiation,
the CEV is in a $100 \times 100 \mathrm{~km}$ orbit.


Figure 2: Relative Motion between the Lunar Lander and the CEV for a 1-rev Rendezvous Sequence

## 2-Revolution Rendezvous Maneuver Description

A 4-burn rendezvous maneuver sequence is defined, where the terminal phase of the rendezvous occurs on the second rev after launch from the lunar surface.

After insertion into lunar orbit, the LL coasts to the first apolune, at which time the LL performs a corrective combination maneuver. NCC is a Lambert targeted maneuver which is designed to create a desired differential altitude and a desired phase angle between the LL and the CEV at a designated delta time or travel angle from the NCC burn (near the next LL apsis). In addition, NCC will correct for any small out-of-plane errors that may have occurred during ascent.

At the time designated for the next burn, the coelliptic maneuver places the LL into an orbit that is coelliptic with the CEV at the desired differential altitude and phase angle.

Following a coast of at least 30 minutes after the coelliptic maneuver, the TPI burn occurs. TPI is a Lambert targeted maneuver, designed to place the LL in the vicinity of the CEV, after a designated orbital transfer angle.

If necessary, the LL will perform mid-course correction burns during the transfer time. Near the end of the transfer time, the LL will perform small braking maneuvers to slow its approach rate to the CEV, finally arriving at a stable orbit condition with the CEV.

The mission sequence is presented in Table 3. Note that the total delta-V for the rendezvous sequence is approximately $31 \mathrm{~m} / \mathrm{s}$. This is consistent with the delta-V required in the $1-\mathrm{rev}$ rendezvous profile. The relative motion between the LL and the CEV, starting with insertion, is shown in Figure 3.

Table 3: 2-Rev Rendezvous Sequence

| Trajectory Event | Phase Elapsed <br> Time | Delta Velocity Magnitude of <br> Translational Maneuver |  | Resultant Apoapsis / <br> Periapsis |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | dd:hh:mm:ss | $\mathrm{m} / \mathrm{s}$ | $\mathrm{ft} / \mathrm{s}$ | km | nmi |
|  |  |  |  |  |  |
| Liftoff | $00: 00: 00: 00$ |  |  |  |  |
| Insertion | $00: 00: 07: 01$ |  |  | $74 / 15$ | $40 / 8$ |
| Corrective Combination | $00: 01: 03: 01$ | 14 | 46 | $75 / 74$ | $41 / 40$ |
| Coelliptic Maneuver | $00: 02: 00: 01$ | 1 | 3 | $75 / 75$ | $41 / 41$ |
| Terminal Phase Initiation | $00: 02: 40: 01$ | 8 | 26 | $101 / 74$ | $55 / 40$ |
| Terminal Phase Finalization | $00: 03: 22: 34$ | 8 | 26 | $101 / 100$ | $55 / 54$ |
| Docking | $00: 04: 40: 00$ |  |  | $101 / 100$ | $55 / 54$ |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

Note: At the time of rendezvous initiation,

$$
\text { the CEV is in a } 101 \times 100 \mathrm{~km} \text { orbit. }
$$



Figure 3: Relative Motion between the Lunar Lander and the CEV for a 2-rev Rendezvous Sequence

## LAUNCH WINDOW

The Lunar Ascent phase window (Figure 4) shows that LL launch opportunities occur approximately every 2 hours, due primarily to the 100 by 100 km parking orbit of the CEV. However, the rotation of the planet and perturbations to the orbit of the CEV cause the ascending node of the orbiting CEV to move over time. The Lunar Ascent plane window shows the required costs for the LL to achieve the correct node during powered ascent. Combining the two defines the launch window, which is shown for a 26 deg latitude site. The launch window information can be used to size a delta-V allocation, based on the number of desired opportunities. For example, if a 12 -hour launch window with six launch
opportunities was desired, the delta-V cost would be approximately $30 \mathrm{~m} / \mathrm{s}$. The variation of the plane window with launch site location is shown in Figure 5, with the most expensive launch window costs occurring at the mid-latitude ( $40-50$ degree) regions.

The LL launch window is determined by the overlay of the ascent plane window and the rendezvous phase window. ${ }^{7}$ For a given set of operational launch constraints, the plane window is defined to be the range of times, relative to the in-plane time, over which an ascent vehicle may launch in order to place the chaser vehicle into the required plane relative to the target vehicle. For a given set of operational rendezvous constraints, the phase window is defined to be the range of phase angles, defined at the ascent vehicle engine cutoff time, over which an ascent vehicle may launch in order to facilitate rendezvous.

Since neither the 1-rev rendezvous nor the 2-rev rendezvous has a phasing burn in its maneuver sequence, there is no rendezvous phase window associated with these rendezvous profiles. Consequently, the LL must launch on-time. However, note that, at the cost of additional rendezvous delta-V, a small phase window may be obtained. For a phase window of approximately 30 to 40 seconds, the cost will be on the order of 10 to $20 \mathrm{~m} / \mathrm{s}$. Alternatively, the appropriate phase angle for an optimum total delta-V sequence will occur approximately every two hours, which is the orbital period of the CEV.

The LL plane window is depicted in Figure 4. Since the lunar rotation rate is relatively small, the corresponding ascent performance curve is relatively shallow. For a 6 -hour launch window (plus or minus 3 hours around the in-plane point), the maximum amount of additional delta- V needed to steer into the CEV plane is about $10 \mathrm{~m} / \mathrm{s}(33 \mathrm{ft} / \mathrm{s})$. During that 6-hour period, since the orbital period of the CEV is almost 2 hours, 4 LL launch opportunities occur. Essentially, since a three- or four-impulse rendezvous maneuver sequence provides, at most, a few seconds of phase window, each launch opportunity requires an on-time liftoff. There is no launch window associated with each launch opportunity.

Note that, in general, no launch opportunity will occur at the in-plane time unless the CEV adds two or more phasing burns to its translational maneuver schedule prior to LL liftoff. The combination phasing and circularization burns would allow the CEV the opportunity to be at the appropriate phase angle at the in-plane time.

Since sites at any lunar latitude are under consideration, the variation of the plane window with launch site latitude is of interest. Figure 5 shows the plane windows for launch site latitudes from $10-90$ degrees. Since this plane window was simulated using only a central body gravity model with a 6 -minute powered flight (during which the higher order gravity terms would have a relatively small effect), these results are applicable to the negative latitudes, as well.

The steepest, and thus most expensive, curves occur at the mid-latitudes ( $40-50$ degrees latitude). Not shown here is a 45-degree latitude case in which the results were nearly identical to the 40degree case (which is the worst case shown).

As compared to the 26-degree latitude case shown in Figure 4, the 40-degree case allows less time before and after the in-plane time for a given amount of delta-V. Depending on how the phase window falls within the plane window, this may or may not mean fewer launch opportunities. However, the difference is significant enough that in most cases it will result in fewer launch opportunities.

LSAM Lunar Launch Window


Figure 4: LL launch window for launch site latitude $=26$ deg north


Figure 5: Lunar Plane Window

## ASCENT PERFORMANCE TRADES

A set of trades was run to show the sensitivity of the ascent performance to a few different design parameters. These trades include vehicle thrust-to-weight, and orbit perilune, apolune, inclination, and longitude of the ascending node. The T/W variation is particularly useful for vehicle design, since the delta- V is sensitive to $\mathrm{T} / \mathrm{W}$. The other variations are used for flight mechanics and trajectory design.

Figure 6 shows the ascent performance variation with the perilune of the orbit at insertion. The cost is approximately $2.4 \mathrm{~m} / \mathrm{s}$ of delta-V per km change in perilune. A perilune of $15.24 \mathrm{~km}(50,000 \mathrm{ft})$ was chosen for the nominal trajectory, since it is considered the lowest safe altitude. ${ }^{8}$

Figure 7 shows the ascent performance variation with the apolune of the orbit at insertion. The cost variation is $0.22 \mathrm{~m} / \mathrm{s}$ per km . The choice of apolune has very little effect on ascent performance. It does, however, affect the rendezvous trajectory design. For rendezvous trajectory design preferences, a nominal of 75 km was selected.

Figure 8 shows the ascent performance variation with initial thrust to weight ratio, in Earth G. The minimum delta-V point occurs around a $T / W=0.6$. Increases in $T / W$ cause a slight increase in delta-V. Decreases in T/W can become quite costly, particularly below $\mathrm{T} / \mathrm{W}=0.4$. The optimum point to minimize overall vehicle mass tends to be less than the minimum delta-V point, due to propulsion and structural considerations.

Figure 9 shows the variation of ascent performance with orbit inclination, from a due east launch (least expensive) to a due west launch (most expensive). The total variation is about $9 \mathrm{~m} / \mathrm{s}$, much less than what it would be at Earth (due primarily to the much slower rotation rate of the Moon). Other studies have shown that this variation is largest at equatorial launch sites (about $10 \mathrm{~m} / \mathrm{s}$ total from due east to due west). This variation is negligible at polar launch sites. The trade was run with only two different pad orientations-one ideal for a due east launch, the other ideal for a due west launch. No roll maneuver during the vertical phase was simulated, but if sufficient roll control is available, it may be used to reduce the delta- V for launches when the vehicle does not have the ideal roll on the pad to launch into the desired orbital plane.

Figure 10 shows the ascent yaw steering performance. For a ascent node target that is not the inplane node, there is an associated delta-V penalty. The variation is higher order than linear, and is not symmetric about the nominal ascent plane. The different curves show the launch site latitude variation. This information can be used to develop an ascent yaw steering budget. For example, we may want to budget $6 \mathrm{~m} / \mathrm{s}$ of delta- V to provide up to 1 degree of yaw steering globally. This amount is very close to the Apollo yaw steering budget of $20 \mathrm{ft} / \mathrm{s} .{ }^{9}$


Figure 6: Ascent Performance Variation With Perilune at Insertion


Figure 7: Ascent Performance Variation With Apolune at Insertion


Figure 8: Ascent Performance Variation With Thrust-to-Weight

## Effect of Varying Inclination on Ascent Performance (Aristarchus L.S.)



Figure 9: Ascent Performance Variation With Orbit Inclination
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Figure 10: Ascent Yaw Steering Performance

## ANYTIME RETURN: YAW STEERING VS. PLANE CHANGE

It may be necessary for the LL to launch from the Moon and rendezvous with the CEV at any time during a sortie mission with a seven-day nominal surface stay. It is assumed that the landing/launch site may be anywhere on the lunar surface. In general, for the length of the stay, the CEV will not pass directly over the launch site. One method for inserting the LL into the proper orbit for subsequent rendezvous with the CEV is to steer during ascent to achieve both the inclination and the ascending node of the CEV orbit. A second method is to perform an ascent to a combination of inclination and node chosen to minimize
plane change delta- V magnitude, followed by an on-orbit plane change maneuver. This study shows that, for the most of the length of the seven-day stay, it is significantly cheaper to utilize the second method. More importantly for mission design purposes, this study shows that the plane change method (method 2) is preferred and can be used to size the maximum delta- V needed over the seven-day stay.

## Method 1 - Ascent Yaw Steering

This method assumes that the LL will make up the difference in the ascending node between the in-plane ascent node and the current CEV longitude of ascending node, which moves to the west during the seven-day stay due to lunar rotation. The LL targets the CEV node and inclination during ascent.

This method is modeled by assuming that the CEV inclination remains constant throughout the seven-day surface stay. The target node is varied from case to case. The ascent profile is simulated and the resultant performance cost is noted. A typical way to display the performance cost curve is the delta node between the current node and the in-plane node (independent variable) versus the delta performance between the current performance cost and the in-plane performance cost (dependent variable). Another way is to have the delta time relative to the in-plane time as the independent variable. In this case, the delta node is converted to delta time by assuming that the node rate of change is equal to the rotation rate of the Moon plus the nodal precession rate of the orbit. The in-plane time (time zero) is used as the reference time.

Because the CEV inclination is assumed to be constant over the seven days, this method does not take into account the perturbative effect of the lunar gravity field on the orbital elements of the CEV. In addition, constant inclination implies that no CEV plane change occurred during the seven days.

## Method 2 - Plane Change

This method targets an inclination and ascending node combination during the ascent phase that is designed to minimize the on-orbit plane change. Following the ascent, an on-orbit plane change maneuver is performed by the LL to match the CEV orbital plane. The inclination and node combination ascent target is chosen such that the wedge angle necessary for the plane change maneuver is minimized.

This method does take into account orbit perturbations on the CEV due to the gravity field. As in the first method, it is assumed that the CEV does not perform a plane change during the seven days. Some details of how this should be modeled are still being refined, such as the fidelity of the gravity model necessary to accurately capture perturbation effects over the seven days. Performance results and recommendations will be documented in a follow-on note. However, the results contained herein are correct to the first order and adequate to support the conclusions of this study.

## Results and Discussion

Figure 11 shows the delta- V penalty cost comparison between ascent yaw steering and the plane change methods for the Aristarchus site at 26 -degrees north latitude and 311 degrees east longitude. The two plane change curves represent either a northerly or southerly heading from the launch site. They intersect at a target inclination equal to the supplement of the launch site latitude.

At the reference time equal to zero, or LL Landing, the CEV is still in-plane, so the delta- V penalty is zero. As the landing site rotates away from the CEV plane, this delta-V penalty increases until about the 72 -hour mark, then drops off to another in-plane point in the 125 - 130 -hour range. This in-plane time is dependent on the inclination of the CEV.

LSAM Ascent Steering vs Plane Change Cost Comparison


Figure 11: Ascent steering/plane change cost comparison for launch site latitude $=26 \mathrm{deg}$.
Note that for the ascent yaw steering case (red squares), the CEV maintained a constant inclination of 149.3 degrees throughout the seven days. On the other hand, orbital perturbations to the CEV during the seven days were included in the plane change curves. Thus, the difference in the in-plane time between the methods is due to the differences in the modeling.

For the three cases shown here, the inclination varies about 1 degree maximum during the stay. Previous work ${ }^{10}$ has shown that the delta-V needed for ascent varies very little with inclination - only about 10 meters per second for inclinations from 0 to 180 degrees. Thus, for only about 1 degree of inclination change, the delta performance cost for ascent is negligible.

Notice that the plane change method is significantly cheaper throughout most of the seven-day stay. Primarily, this is because, at the Moon, targeting different inclinations during ascent is much cheaper than targeting different ascending nodes. This trend holds for the other non-polar, non-equatorial cases shown in Figure 12 and Figure 13. For the 45 -degrees latitude case shown in Figure 12, the cost difference between the methods is even more significant.

Nominally, the CEV would perform a plane change maneuver such that the launch site would be in-plane at the 7 -day mark. Please note, however, that these results assume that the CEV has not yet performed the maneuver. If the CEV did perform the maneuver, the delta-V penalty at the 168 -hour mark would be zero. Similar yaw steering and plane change curves could be developed around that in-plane point, with a penalty for liftoff times either prior to or later than the in-plane time.

The data clearly show that the on-orbit plane change method is significantly cheaper than the ascent steering method. The plane change method is considered to be the preferred technique to accomplish LL/CEV rendezvous. This plane change can be performed by either vehicle for essentially the same cost. However, in case of an off-nominal scenario in which one vehicle is unable to complete the plane change maneuver, it would be advantageous for both vehicles to have the necessary plane change capability.


Figure 12: Ascent steering/plane change cost comparison for launch site latitude $=45 \mathrm{deg}$.

LSAM Ascent Steering vs Plane Change Cost Comparison


Figure 13: Ascent steering/plane change cost comparison for launch site latitude $=70 \mathrm{deg}$.

## RENDEZVOUS FOLLOWING PDI ABORT <br> \section*{Introduction}

The objective for this task is to assess the capability of the LLAM to perform a rendezvous following a Powered Descent Initiation (PDI) abort. The focus of the assessment is on the orbital mechanics involved in order for the LLAM to recover from a PDI abort. Thus, vehicle performance, in the form of the time required to rendezvous and the delta-V, is examined, rather than hardware, software, or other operational constraints. In order to adequately meet this objective, the following analysis was deemed necessary:

- Define the PDI burn trajectory for each of the assumed abort times.
- Define the preferred rendezvous maneuver sequence for each of the assumed abort times.
- Investigate the use of the height of apolune $\left(h_{a}\right)$ as an independent variable in the PDI targeting algorithm.
A total of four cases were chosen. These cases reasonably capture the spectrum of aborts, starting with a failed PDI and continuing to a case with an abort at the end of the braking burn. Each case was examined for the difficulty in achieving a lunar insertion orbit following the abort, and for the time and delta-V required to rendezvous with the CEV subsequent to insertion.
- Case 1: Abort at the time of PDI.
- Case 2: Abort at the time of PDI plus 120 seconds.
- Case 3a: Abort at the time of PDI plus 258 seconds, where the targeted apolune is equal to 100 kilometers.
- Case 3b: Abort at the time of PDI plus 258 seconds, where the targeted apolune is equal to 120 kilometers.


## Assumptions

The following assumptions were made for the rendezvous analysis:

1. Rendezvous procedures are made as independent of ground support as possible.
2. In general, from a navigation standpoint, the CEV and the LLAM know where each is, relative to the other, during the powered descent and rendezvous phases.
3. At the time of the PDI abort and throughout the rendezvous, the LLAM is fully functional.
4. The height of apolune $\left(h_{a}\right)$ is a target parameter that is used in the guidance software during the LLAM insertion burn. The value of ha is dependent on the phase angle between the LLAM and the CEV at the time of PDI abort.
5. The height of perilune $\left(h_{p}\right)$ is a target parameter that is used in the guidance software during the LLAM insertion burn. The value of $h_{p}$ is set equal to 15.24 kilometers.
6. A representative single coelliptic rendezvous maneuver sequence is defined.
7. Height Maneuver at first LLAM apsis after insertion
8. Phasing Maneuver one-half rev after the height maneuver
9. Coelliptic Maneuver 1 rev after the phasing maneuver
10. The TPI burn, which is a Lambert maneuver, approximately 1 hour after the Coelliptic Maneuver
11. The TPF burn, intersection occurring after a 150 degree transfer angle from TPI
12. The delta altitude $(\Delta \mathrm{h})$ at the coelliptic maneuver is: $\Delta \mathrm{h}=15 \mathrm{~km}$.
13. Docking will occur 1 rev after the TPI burn.
14. CEV rescue of the LLAM following a PDI abort is covered elsewhere ${ }^{11}$, and is not addressed in this analysis.
15. The rendezvous plan does not reflect the following operational constraints:
a. Navigation Dispersions
b. Translational Burn Errors
c. Proximity Operations
d. Lighting
e. Tracking
f. Orbital Maintenance
g. Vehicle Attitude Maneuvers
h. Crew Timeline

## Case 1: Abort at the time of Powered Descent Initiation

In this case, it was assumed that an abort was called before the beginning of PDI. Thus, following the abort call, the LLAM separates from the Lunar Lander Descent Module (LLDM) and remains in the post-Descent Orbit Insertion (DOI) orbit. While the LLAM is in coasting flight, the crew will prepare for a 5-burn rendezvous maneuver sequence, where the first rendezvous burn will occur at the first apolune following the abort call.

At the first apolune, the LLAM performs a height maneuver, which will serve two purposes. The height maneuver is targeted to create a desired differential altitude between the LLAM and CEV at the next perilune. Additionally, this will raise the post-DOI perilune altitude of approximately 15 km to a higher and, presumably, safer altitude during the rendezvous.

One-half rev later, at perilune, the LLAM will perform the phasing burn. In this rendezvous scheme, the phasing burn is designed to create a specified phase angle at the time of the coelliptic maneuver.

The LLAM will remain in the phasing orbit for 1 rev . At the next perilune, approximately two hours later, the coelliptic maneuver will occur. This maneuver is designed to place the LLAM in an orbit that is coelliptic to the CEV orbit. After the burn, the LLAM will have an apolune and a perilune that have a constant $\Delta \mathrm{h}$ relative to the CEV apolune and perilune, and the LLAM line of apsides will be collinear with the CEV line of apsides.

At a specified time later, the TPI burn occurs. TPI is designed to place the LLAM in the vicinity of the CEV, after a designated orbital transfer angle.

If necessary, the LLAM will perform mid-course correction burns during the transfer time. Near the end of the transfer time, the LLAM will perform small braking maneuvers to slow its approach rate to the CEV, finally arriving at a stable orbit condition with CEV.

The Case 1 mission sequence is presented in Table 4. Note that the total delta-V for the rendezvous sequence is approximately $86 \mathrm{~m} / \mathrm{s}$. The relative motion between the LLAM and the CEV, starting with insertion, is shown in Figure 14.

Table 4: Case 1 - Abort at PDI

| Trajectory Event | Phase Elapsed <br> Time | $\Delta V$ of <br> Bum | Phase <br> Angle | Delta <br> Altitude | Active <br> Vehicle | Resultant <br> ha/ $\mathbf{h p}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | dd:hh:mm:ss | $\mathrm{m} / \mathrm{s}$ | deg | km |  | km |
|  |  |  |  |  |  |  |
| PDI | $00: 00: 00: 00$ |  | -6.6 | 89 | LLDM | $101 / 15$ |
| PDI Abort | $00: 00: 00: 00$ |  | -6.6 | 89 | LLAM | $101 / 15$ |
| Insertion | $00: 00: 00: 00$ | 0 | -6.6 | 89 | LLAM | $101 / 15$ |
| Height Maneuver | $00: 00: 59: 08$ | 17 | -13.7 | -2 | LLAM | $101 / 90$ |
| Phasing Maneuver | $00: 01: 59: 29$ | 29 | -14.6 | 15 | LLAM | $235 / 90$ |
| Coelliptic Maneuver | $00: 04: 03: 07$ | 32 | 3.3 | 15 | LLAM | $90 / 84$ |
| Teminal Phase Initiation | $00: 05: 05: 47$ | 4 | 1.0 | 15 | LLAM | $105 / 83$ |
| Teminal Phase Finalization | $00: 05: 54: 57$ | 4 | 0 | 0 | LLAM | $106 / 98$ |
| Docking | $00: 07: 00: 00$ |  |  |  |  |  |
|  |  | 86 |  |  |  | $106 / 98$ |
|  |  |  |  |  |  |  |

Note: At the time of rendezvous initiation,
the CEV is in a $106 \times 98 \mathrm{~km}$ orbit.


Figure 14: Relative Motion between the LLAM and the CEV for the No PDI Abort Case

## Case 2: Abort at the time of PDI plus $\mathbf{1 2 0}$ seconds

In this case, it is assumed that an abort is called 120 seconds after PDI burn initiation. Following the abort call, the LLAM separates from the LLDM and then performs an insertion burn of approximately 340 seconds, after which the LLAM has returned to the post-DOI orbit (approximately 15 by 100 km ).

At the first apolune following insertion, the LLAM performs a height maneuver. The height maneuver is targeted to create a desired differential altitude between the LLAM and the CEV at the next perilune.

One-half rev later, at perilune, the LLAM will perform the phasing burn. In this rendezvous scheme, the phasing burn is designed to create a specified phase angle at the time of the coelliptic maneuver.

The LLAM will remain in the phasing orbit for 1 rev . Thus, at the next perilune, approximately two hours later, the coelliptic maneuver will occur. This maneuver is designed to place the LLAM in an orbit that is coelliptic to the CEV orbit. After the burn, the LLAM will have an apolune and a perilune that have a constant $\Delta \mathrm{h}$ relative to the CEV apolune and perilune, and the LLAM line of apsides will be collinear with the CEV line of apsides.

At a specified time later, the TPI burn occurs. TPI is designed to place the LLAM in the vicinity of the CEV, after a designated orbital transfer angle.

If necessary, the LLAM will perform mid-course correction burns during the transfer time. Near the end of the transfer time, the LLAM will perform small braking maneuvers to slow its approach rate to Orion, finally arriving at a stable orbit condition with the CEV.

The Case 2 mission sequence is presented in Table 5. Note that the total $\Delta \mathrm{V}$ for the rendezvous sequence is approximately $889 \mathrm{~m} / \mathrm{s}$. The relative motion between the LLAM and the CEV, starting with insertion, is shown in Figure 15.

Table 5: Case 2 - Abort at PDI + 120 sec

| Trajectory Event | Phase Elapsed <br> Time | $\Delta \mathbf{V}$ of <br> Burn | Phase <br> Angle | Delta <br> Altitude | Active <br> Vehicle | Resultant <br> ha/ $\mathbf{h p}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | dd:hh:mm:ss | $\mathrm{m} / \mathrm{s}$ | deg | km |  | km |
|  |  |  |  |  |  |  |
| PDI | $00: 00: 00: 00$ |  | -6.6 | 89 | LLDM | $101 / 15$ |
| PDI Abort | $00: 00: 02: 00$ |  | -5.6 | 92 | LLAM | $18 /-1400$ |
| Insertion | $00: 00: 05: 40$ | 812 | -3.9 | 90 | LLAM | $101 / 15$ |
| Height Maneuver | $00: 01: 01: 44$ | 17 | -10.2 | -1 | LLAM | $101 / 90$ |
| Phasing Maneuver | $00: 02: 02: 05$ | 24 | -11.2 | 15 | LLAM | $213 / 90$ |
| Coelliptic Maneuver | $00: 04: 04: 40$ | 28 | 3.3 | 15 | LLAM | $90 / 84$ |
| Terminal Phase Initiation | $00: 05: 07: 14$ | 4 | 1.0 | 15 | LLAM | $105 / 83$ |
| Teminal Phase Finalization | $00: 05: 56: 24$ | 4 | 0 | 0 | LLAM | $106 / 98$ |
| Docking | $00: 07: 00: 00$ | - |  |  |  | $106 / 98$ |
|  |  | 889 |  |  |  |  |

Note: At the time of rendezvous initiation,
the CEV is in a $106 \times 98 \mathrm{~km}$ orbit.


Figure 15: Relative Motion between the LLAM and the CEV for the PDI +120 Seconds Case

Case 3a: Abort at the time of PDI plus 258 seconds / Insertion target apolune equal to $100 \mathbf{k m}$
For Case 3a, the PDI abort occurs 258 seconds after burn initiation. In this case, as in cases 1 and 2, the insertion apolune target altitude is 100 km . Following the abort call, the LLAM separates from the LLDM and then performs an insertion burn for approximately 691 seconds, after which the LLAM has returned to the post-DOI orbit (approximately 15 by 100 km ).

At the first apolune following insertion, the LLAM performs a height maneuver. The height maneuver is targeted to create a desired differential altitude between the LLAM and the CEV at the next perilune.

One-half rev later, at perilune, the LLAM will perform the phasing burn. In this rendezvous scheme, the phasing burn is designed to create a specified phase angle at the time of the coelliptic maneuver.

The LLAM will remain in the phasing orbit for 1 rev. Thus, at the next perilune, approximately two hours later, the coelliptic maneuver will occur. This maneuver is designed to place the LLAM in an orbit that is coelliptic to the CEV orbit. After the burn, the LLAM will have an apolune and a perilune that
have a constant $\Delta \mathrm{h}$ relative to the CEV apolune and perilune, and the LLAM line of apsides will be collinear with the CEV line of apsides.

At a specified time later, the TPI burn occurs. TPI is designed to place the LLAM in the vicinity of the CEV, after a designated orbital transfer angle.

If necessary, the LLAM will perform mid-course correction burns during the transfer time. Near the end of the transfer time, the LLAM will perform small braking maneuvers to slow its approach rate to the CEV, finally arriving at a stable orbit condition with the CEV.

The Case 3a mission sequence is presented in Table 6. Note that the total $\Delta \mathrm{V}$ for the rendezvous sequence is approximately $1883 \mathrm{~m} / \mathrm{s}$. The relative motion between the LLAM and the CEV, starting with insertion, is shown in Figure 16.

Table 6: Case $3 a-$ Abort at PDI + 258 Seconds / Insertion target apolune equal to 100 km

| Trajectory Event | Phase Elapsed <br> Time | AV of <br> Burn | Phase <br> Angle | Delta <br> Altitude | Active <br> Vehicle | Resultant <br> ha/ $\mathbf{h p}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | dd:hh:mm:ss | $\mathrm{m} / \mathrm{s}$ | deg | km |  | km |
|  |  |  |  |  |  |  |
| PDI | $00: 00: 00: 00$ |  | -6.6 | 89 | LLDM | $101 / 15$ |
| PDI Abort | $00: 00: 04: 18$ |  | -0.2 | 105 | LLAM | $5 /-1700$ |
| Insertion | $00: 00: 11: 31$ | 1848 | 10.5 | 90 | LLAM | $101 / 15$ |
| Height Maneuver | $00: 01: 07: 46$ | 17 | 4.3 | -1 | LLAM | $100 / 90$ |
| Phasing Maneuver | $00: 02: 08: 06$ | 3 | 3.2 | 15 | LLAM | $116 / 90$ |
| Coelliptic Maneuver | $00: 04: 07: 53$ | 7 | 3.3 | 15 | LLAM | $91 / 84$ |
| Teminal Phase Initiation | $00: 05: 10: 33$ | 4 | 1.0 | 15 | LLAM | $106 / 84$ |
| Teminal Phase Finalization | $00: 05: 59: 44$ | 4 | 0 | 0 | LLAM | $106 / 98$ |
| Docking | $00: 07: 00: 00$ |  |  |  |  | $106 / 98$ |
|  |  | 1883 |  |  |  |  |

Note: At the time of rendezvous initiation,
the CEV is in a $106 \times 98 \mathrm{~km}$ orbit.
RELATIVE MOTION BETWEEN LLAM AND CEV


Figure 16: Relative Motion between the LLAM and the CEV for the PDI +258 Seconds Case, where the Insertion Apolune is equal to 100 km

## Case 3b: Abort at the time of PDI plus 258 seconds / Target apolune equal to $\mathbf{1 2 0} \mathbf{~ k m}$

For Case 3b, as for case 3a, the PDI abort occurs 258 seconds after burn initiation. However, in this case, the insertion apolune target altitude is 120 km . Following the abort call, the LLAM separates
from the LLDM and then performs an insertion burn for approximately 692 seconds. At insertion, the LLAM is in an orbit where perilune is approximately 15 km and apolune is approximately 120 km .

At the first apolune following insertion, the LLAM performs a height maneuver. The height maneuver is targeted to create a desired differential altitude between the LLAM and the CEV at the next perilune.

One-half rev later, at perilune, the LLAM will perform the phasing burn. In this rendezvous scheme, the phasing burn is designed to create a specified phase angle at the time of the coelliptic maneuver.

The LLAM will remain in the phasing orbit for 1 rev. Thus, at the next perilune, approximately two hours later, the coelliptic maneuver will occur. This maneuver is designed to place the LLAM in an orbit that is coelliptic to the CEV orbit. After the burn, the LLAM will have an apolune and a perilune that have a constant $\Delta \mathrm{h}$ relative to the CEV apolune and perilune, and the LLAM line of apsides will be collinear with the CEV line of apsides.

At a specified time later, the TPI burn occurs. TPI is designed to place the LLAM in the vicinity of the CEV, after a designated orbital transfer angle.

If necessary, the LLAM will perform mid-course correction burns during the transfer time. Near the end of the transfer time, the LLAM will perform small braking maneuvers to slow its approach rate to the CEV, finally arriving at a stable orbit condition with the CEV.

The Case 3 b mission sequence is presented in Table 7. Note that the total $\Delta \mathrm{V}$ for the rendezvous sequence is approximately $1886 \mathrm{~m} / \mathrm{s}$. The relative motion between the LLAM and the CEV, starting with insertion, is shown in Figure 17.

Table 7: Case 3b - Abort at PDI + 258 Seconds / Insertion target apolune equal to 120 km

| Trajectory Event | Phase Elapsed <br> Time | $\Delta V$ of <br> Burn | Phase <br> Angle | Delta <br> Altitude | Active <br> Vehicle | Resultant <br> ha/ hp |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | dd:hh:mm:ss | $\mathrm{m} / \mathrm{s}$ | deg | km |  | km |
|  |  |  |  |  |  |  |
| PDI | $00: 00: 00: 00$ |  | -6.6 | 89 | LLDM | $101 / 15$ |
| PDI Abort | $00: 00: 04: 18$ |  | -0.2 | 105 | LLAM | $5 /-1700$ |
| Insertion | $00: 00: 11: 32$ | 1852 | 10.5 | 90 | LLAM | $121 / 15$ |
| Height Maneuver | $00: 01: 08: 07$ | 17 | 5.8 | -21 | LLAM | $120 / 90$ |
| Phasing Maneuver | $00: 02: 08: 56$ | 6 | 6.1 | 15 | LLAM | $96 / 90$ |
| Coelliptic Maneuver | $00: 04: 07: 44$ | 3 | 3.3 | 15 | LLAM | $91 / 84$ |
| Terminal Phase Initiation | $00: 05: 10: 24$ | 4 | 1.0 | 15 | LLAM | $106 / 84$ |
| Terminal Phase Finalization | $00: 05: 59: 35$ | 4 | 0 | 0 | LLAM | $106 / 98$ |
| Docking | $00: 07: 00: 00$ |  |  |  |  | $106 / 98$ |
|  |  | 1886 |  |  |  |  |

Note: At the time of rendezvous initiation,
the CEV is in a $106 \times 98 \mathrm{~km}$ orbit.


Figure 17: Relative Motion between the LLAM and the CEV for the PDI +258 Seconds Case, where the Insertion Apolune is equal to 120 km

These four cases indicate the following.

1. As more of PDI is completed before the abort, the insertion delta-V dramatically increases. Since PDI creates a large negative perilune very quickly into the burn, this is to be expected.
2. The phase angle at insertion will help dictate the length of the rendezvous. Based on the cases presented in this study, a $4-r e v$ rendezvous (approximately 7 hours) is feasible for the LLAM rendezvous following either an early or a late PDI abort. However, most likely, the mid-PDI abort cases will require more than 4 revs for rendezvous. In those cases, the phase angle at insertion is small enough that the relative motion distance between the LLAM and the CEV will too small to be safe during the phasing orbit. An increase in the amount of time that the LLAM spends in the phasing orbit will allow the LLAM to get far enough ahead to allow it to phase safely above the CEV.
3. Since there is a likelihood the LLAM will have to phase above the CEV orbit altitude, it is important to study the relative motion of each rendezvous maneuver profile. For a particular rendezvous plan, the total time required to complete the rendezvous may be operationally feasible, and the total rendezvous $\Delta \mathrm{V}$ may be within the LLAM budget. However, if, during the rendezvous, before the final approach, the LLAM relative range to the CEV is small, then that particular plan should be regarded as unfeasible.

## CONCLUSIONS

A nominal lunar ascent and rendezvous trajectory has been developed, using a short rendezvous approach. A single coelliptic, 2-rev rendezvous can also be completed for the same nominal delta-V costs, taking approximately 100 min longer. Though the shortest possible rendezvous sequence is preferred at this time, a dispersion analysis should be completed to better understand the impacts to navigation performance.

For the nominal trajectory design, there is minimal difference in the total delta-V for the $1-\mathrm{rev}$ rendezvous maneuver sequence and the $2-r e v$ rendezvous maneuver sequence. However, operational constraints (e.g., navigational dispersions) will increase the LL rendezvous propellant requirements and, ultimately, determine if one of the two rendezvous maneuver sequences (or, for that matter, another sequence) is preferred over the other.

The number of LL launch opportunities is directly related to the amount of delta-V that is available for planar (yaw) steering above the nominal amount that is needed for ascent into lunar orbit. For
a 6 -hour launch window (plus or minus 3 hours around the in-plane point), the maximum amount of additional delta-V needed to steer into the CEV plane is about $10 \mathrm{~m} / \mathrm{s}(33 \mathrm{ft} / \mathrm{s})$. During that 6 -hour period, since the orbital period of the CEV is almost 2 hours, 4 LL launch opportunities occur.

Generally, for anytime abort from the lunar surface, there is a lower delta-V cost associated with targeting ascent to an inclination/node combination that minimizes the wedge angle between the LL and CEV and then performing a plane change maneuver, as compared to the LL performing yaw steering to the CEV plane during ascent.

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| NOTATION |  |
| :--- | :--- |
| AM | Ascent Module |
| CEV | Crew Exploration Vehicle |
| Delta-V | Change in velocity |
| DOF | Degrees of Freedom |
| DM | Descent Module |
| FAS | Flight Analysis System |
| JSC | Johnson Space Center |
| LAN | Longitude of the Ascending Node |
| LL | Lunar Lander (previously known as LSAM) |
| LLO | Low Lunar Orbit |
| LSAM | Lunar Surface Access Module (previous name for Lunar Lander) |
| NASA | National Aeronautics and Space Administration |
| NCC | Number, Corrective Combination |
| NPSOL | Nonlinear Programming Solver from Stanford's Systems Optimization Laboratory |
| PEG | Powered Explicit Guidance |
| SAR | Single Axis Rotation |
| SORT | Simulation and Optimization of Rocket Trajectories |
| TPI | Terminal Phase Initiation |
| TPF | Terminal Phase Finalization |
| T/W | Thrust to Weight |

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