JET Noise Prediction

by

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Abstract:

Aerodynamic noise prediction has been an important and challenging research area since James Lighthill first introduced his Acoustic Analogy Approach over fifty years ago. This talk attempts to provide a unified framework for the subsequent theoretical developments in this field. It assumes that there is no single approach that is optimal in all situations and uses the framework as a basis for discussing the strengths weaknesses of the various approaches to this topic. But the emphasis here will be on the important problem of predicting the noise from high speed air jets. Specific results will presented for round jets in the 0.5 to 1.4 Mach number range and compared with experimental data taken on the Glenn SHAR rig. It is demonstrated that non-parallel mean flow effects play an important role in predicting the noise at the supersonic Mach numbers. The results explain the failure of previous attempts based on the parallel flow Lilley model (which has served as the foundation for most jet noise analyses during past two decades).

Generalized Acoustic Analogy

$$\rho = \overline{\rho} + \rho', \quad p = \overline{p} + p', \quad h = \tilde{h} + h' \quad v_i = \tilde{v}_i + v_i'$$

- $\overline{\rho}, \overline{p}, \tilde{v}_i, \tilde{h}$ satisfy the usual hyperbolic conservation laws (HCL)
- Euler, N-S, RANS, etc.

- Residual variables satisfy formally linear HCL

$$L_{\mu\nu} u_\nu = D^{\mu}_{\lambda j} e''_{\lambda j} \qquad \text{for j=1,2,3} \quad \lambda\text{=1,2,..4} \qquad \mu, \nu = 1, 2, \ldots, 5$$

$$\{u_\nu\} \equiv \{\rho v_1', \rho v_2', \rho v_3', p_e', \rho'\}$$

$$e''_{\lambda i} \equiv e'_{\lambda i} - \tilde{e}_{\lambda i} \qquad \lambda = 1, 2, 3, 4$$

Far Field Pressure Autocovariance

- $$\overline{p^2}(\boldsymbol{x},t) = \int_{-\infty}^{\infty}\iint_V \overline{\gamma}_{\kappa j\lambda l}(\boldsymbol{x}|\boldsymbol{y};\boldsymbol{\eta},t+\tau)\mathcal{R}_{\kappa j\lambda l}(\boldsymbol{y};\boldsymbol{\eta},\tau)\,d\boldsymbol{y}\,d\boldsymbol{\eta}\,d\tau$$

$$\mathcal{R}_{\kappa j\lambda l}(\boldsymbol{y};\boldsymbol{\eta},\tau) \equiv \frac{1}{2T}\int_{-T}^{T} e''_{\kappa j}(\boldsymbol{y},\tau_0)e''_{\lambda l}(\boldsymbol{y}+\boldsymbol{\eta},\tau_0+\tau)\,d\tau_0$$

For Base Flow=Mean Flow

$$R_{\kappa j\lambda l}(\boldsymbol{y};\boldsymbol{\eta},\tau) \equiv \frac{1}{2T}\int_{-T}^{T}\left[\rho v'_\kappa v'_j - \overline{\rho v'_\kappa v'_j}\right](\boldsymbol{y},\tau_0)\left[\rho v'_\lambda v'_l - \overline{\rho v'_\lambda v'_l}\right](\boldsymbol{y}+\boldsymbol{\eta},\tau_0+\tau)\,d\tau_0$$

$$\mathcal{R}_{\kappa j\lambda l} = R_{\kappa j\lambda l} - \frac{\gamma-1}{2}\left(\delta_{\kappa j}R_{kk\lambda l} + \delta_{\lambda l}R_{\kappa jkk}\right) + \left(\frac{\gamma-1}{2}\right)^2 \delta_{\kappa j}\delta_{\lambda l}R_{iikk}\,,$$

Uni-directional Mean Flow Analogy

- Advantage
- Simple expression for the propagator $\overline{\gamma}_{\nu j \mu l}$
- Drawbacks
- Only applies to nearly parallel shear flows
- Mean flow interaction effects not completely eliminated from the source function
- Source function still has a mean flow component
- Rayleigh operator can support linear instability waves that grow without bound in this model
- Propagator develops a strong non-integrable singularity at the critical layer when the observation point is in the far field--- **sound is infinitely loud!**

Asymptotic Green's function for Slowly Diverging Mean Flow

- $Y \equiv \varepsilon y_1$

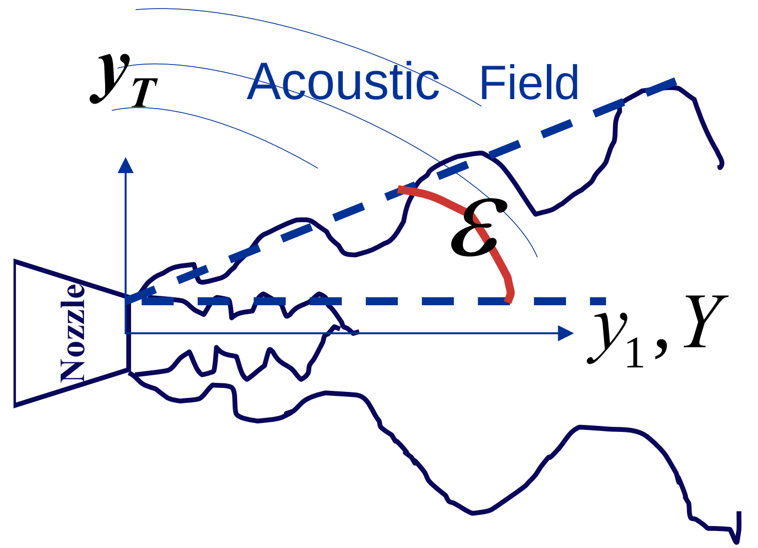
$$\tilde{v}_1 = U\left(Y, \boldsymbol{y}_T\right) + \varepsilon U^{(1)}\left(Y, \boldsymbol{y}_T\right) + \ldots$$

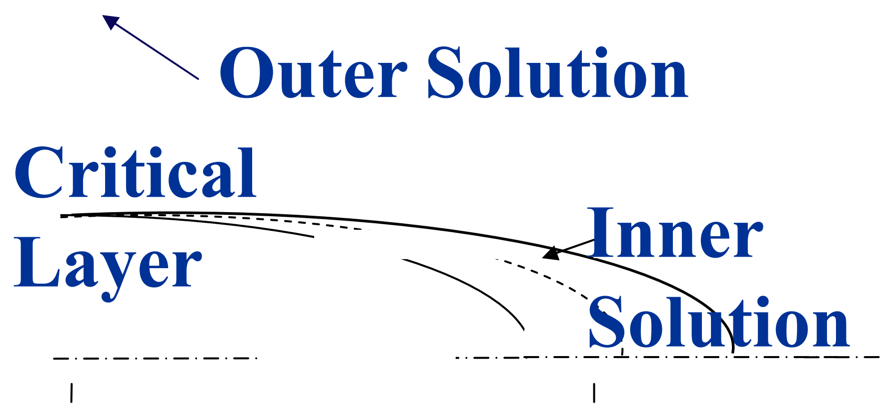
$$\tilde{\boldsymbol{v}}_T = \varepsilon \boldsymbol{V}\left(Y, \boldsymbol{y}_T\right) + \varepsilon^2 \boldsymbol{V}^{(1)}\left(Y, \boldsymbol{y}_T\right) + \ldots$$

$$\boldsymbol{y}_T = \left\{y_{2,} y_3\right\} \qquad \tilde{\boldsymbol{v}}_T = \left\{\tilde{v}_2, \tilde{v}_3\right\}$$

Outer Solution

$$g_{\nu\mu}^{a} = g_{\nu\mu}^{a,0} + \varepsilon g_{\nu\mu}^{a,1} + \ldots$$

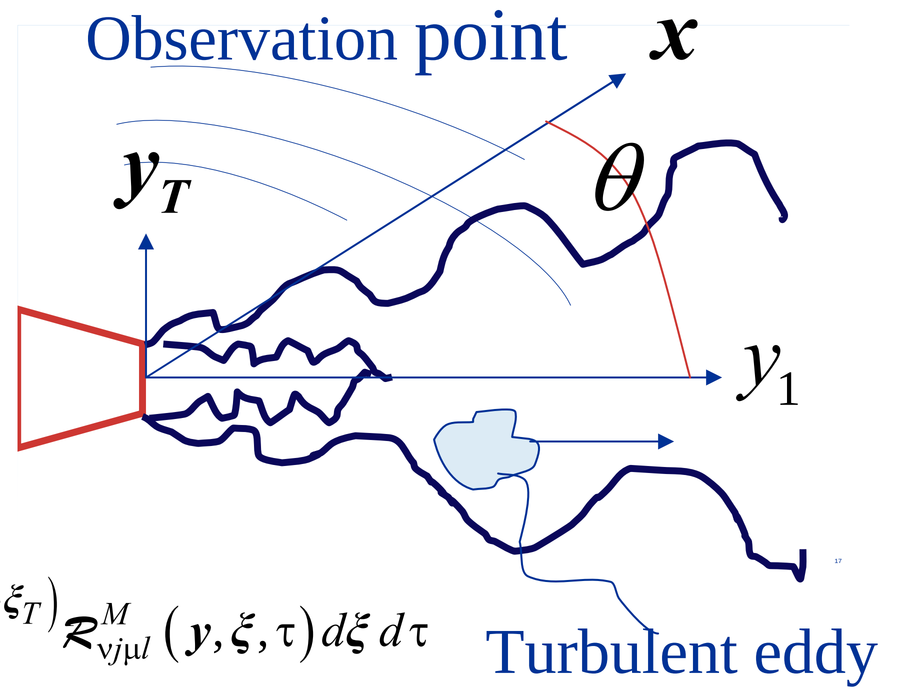




Far Field Spectrum
(non-causal)

Speed of sound c_∞

Neglect contribution from instability



$$I_\omega(\boldsymbol{x}) = \int_V I_\omega(\boldsymbol{x}|\boldsymbol{y})\,d\boldsymbol{y}$$

$$\Phi^*_{\nu j\mu l}(\boldsymbol{y};k_1,\boldsymbol{k}_T,\omega) \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\omega\tau}\int_V e^{i(k_1\xi_1+\boldsymbol{k}_T\bullet\boldsymbol{\xi}_T)}\mathcal{R}^M_{\nu j\mu l}(\boldsymbol{y},\xi,\tau)\,d\xi\,d\tau$$

$$I_\omega(\boldsymbol{x}|\boldsymbol{y}) \to \left(\frac{2\pi}{x}\right)^2 \underbrace{\overline{\Gamma}_{\kappa j\lambda l}(\boldsymbol{x}|\boldsymbol{y}_T,Y;\omega)}_{\text{propagator}} \underbrace{\Phi^*_{\kappa j\lambda l}\left(\boldsymbol{y};\frac{\omega}{c_\infty}\cos\theta,\frac{\omega}{c_\infty}\nabla S,\omega(1-M_c\cos\theta)\right)}_{\text{spectral tensor}} \quad as \quad x\to\infty$$

Source Modeling

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$$\Psi_{ijkl}\left(\boldsymbol{y};k_1,\boldsymbol{k}_T,\omega\right)\equiv\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-i\omega\tau}\int_V e^{i\left(k_1\xi_1+\boldsymbol{k}_T\bullet\boldsymbol{\xi}_T\right)}R_{ijkl}^{M}\left(\boldsymbol{y},\boldsymbol{\xi},\tau\right)d\boldsymbol{\xi}\,d\tau,$$

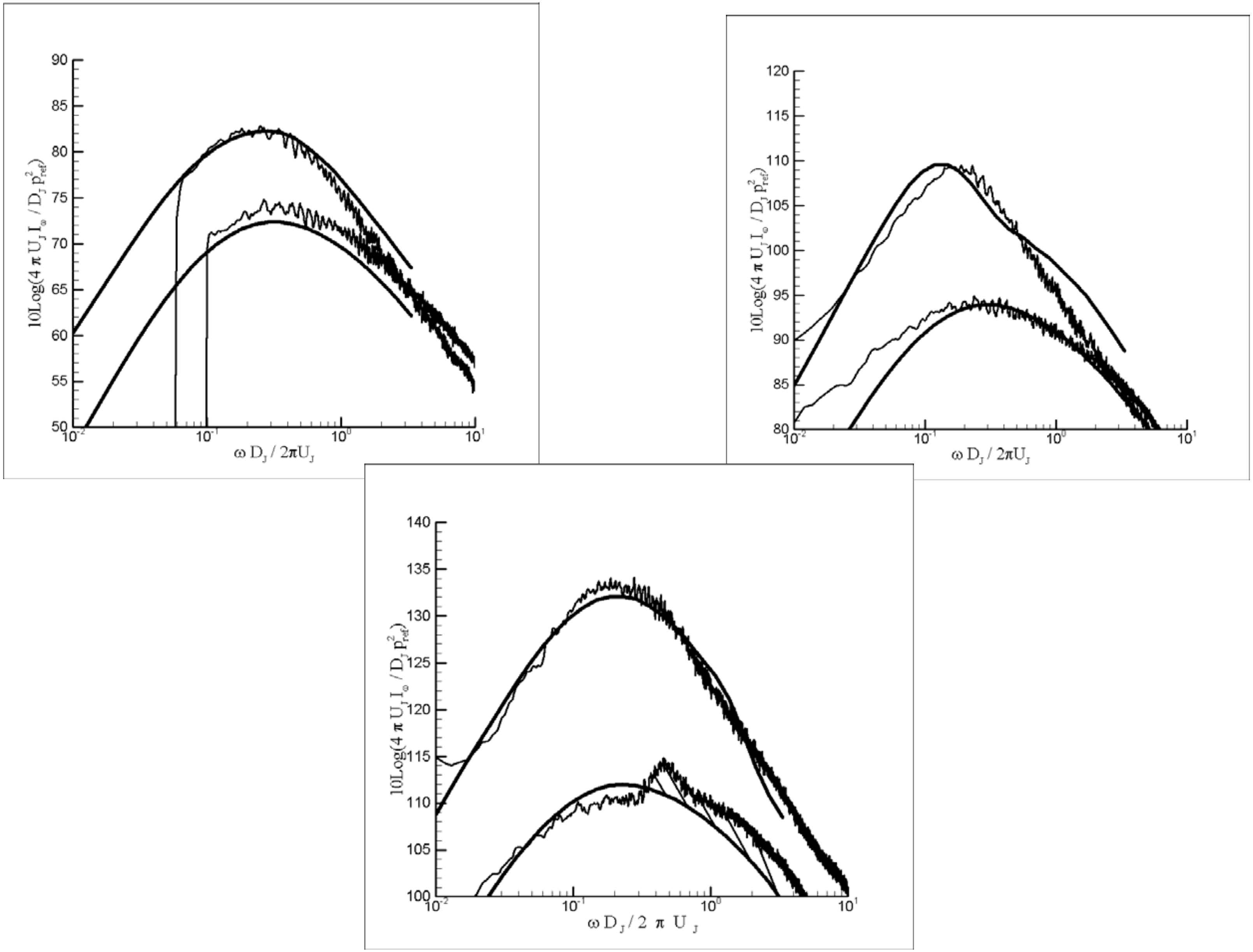
Requirements

1) Reduce number of independent components
2) Remaining components of spectral tensor inexpensive to compute

Must capture details of turbulence structure!

$$R_{ijkl}^{M}\left(\boldsymbol{\xi},\tau\right)=\sum_{m=0}^{\infty}\sum_{l=0}^{\infty}a_{m,l}D_1^m D_\tau^l e^{-X\left(\tilde{\boldsymbol{\xi}},\tilde{\tau}\right)},\quad D_1\equiv\tilde{\xi}_1\frac{\partial}{\partial\tilde{\xi}_1},\quad D_\tau\equiv\tilde{\tau}\frac{\partial}{\partial\tilde{\tau}}$$

Comparison with Data



Comparison with JeNo results for M=1.4 unheated jet at two emission angles