Modeling of SAW Delay Lines

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Introduction

Integrated Vehicle Health Monitoring (IVHM) of aerospace vehicles requires rugged sensors having reduced volume, mass, and power that can be used to measure a variety of phenomena. Wireless systems are preferred when retro-fitting sensors onto existing vehicles [1]. Surface Acoustic Wave (SAW) devices are capable of sensing: temperature, pressure, strain, chemical species, mass loading, acceleration, and shear stress. SAW technology is low cost, rugged, lightweight, and extremely low power. To aid in the development of SAW sensors for IVHM applications, a first order model of a SAW Delay line has been created.

Background

The SAW Delay line structure is composed of two sets of InterDigitated (IDT) metal fingers on a piezoelectric substrate (Figure 1). Sensors can be developed by adding a sensing medium between the two IDTs. When one of the IDTs is excited by a sinusoidal electrical signal the piezoelectric effect generates mechanical waves. The waves that propagate to the other IDT generate an electrical signal through the inverse piezoelectric effect. The lack of support for transcendentals within the Differential Algebraic Equation framework prevents the use of the frequency domain analysis in analog extended Hardware Description Languages (HDLs). Although frequency domain analysis exists in VHDL-AMS it is rudimentary and is not sufficient for some general frequency domain modeling [3], that is why we have chosen to use a mathematical modeling tool.

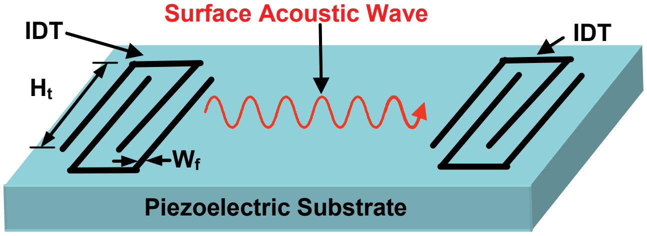


Figure 1. Basic SAW Delay line Device.

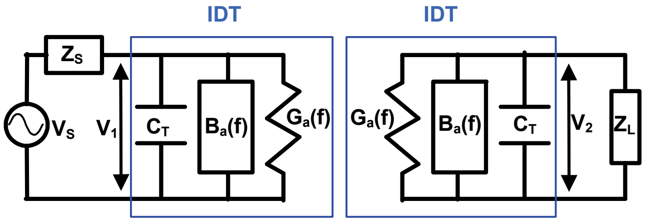


Figure 2. Mason equivalent circuit model.

Model Implementation

The Impulse Response method was used to model the SAW delay line[2]. This is a first order model only; however, it does model the piezoelectric, mechanical and electrical behaviors of the SAW device. The Impulse Response method uses the Mason equivalent circuit shown in Figure 2. The figure shows the source voltage and both the source and load impedances which are not part of the model. For each IDT the model is comprised of the radiation conductance $G_a(f)$, the acoustic susceptance $B_a(f)$, and the total capacitance C_T.

Saw Delay Line Example

A simple SAW delay line that consists of two identical IDTs will be used to demonstrate the model. The synchronous frequency is 52.633 MHz. The substrate is ST cut Quartz. The Null Bandwidth (NBW) is 1.5 MHz. The delay length between the two IDTs is 5 wavelengths. Both the source and load resistances are assumed to be 50 ohms. The selection of a substrate material determines the capacitance, Cs = 0.503385 pf/cm, the piezoelectric coefficient, k = 0.04, and the acoustic velocity, v = 3158 m/s for the SAW device [3].

Inputs:

$f_0 := 52.633 \cdot \text{MHz}$	Synchronous Frequency
Substrate := "ST Cut Quartz"	Substrate Material
$\text{NBW} := 1.5 \cdot \text{MHz}$	Null Bandwidth
$\text{Delay} := 5.0$	Delay Length in Wavelengths

Material Properties for ST-Cut Crystal Quartz:

$v := 3158 \cdot \frac{\text{m}}{\text{s}}$	Acoustic Velocity
$k := \sqrt{0.0016}$	Piezoelectric Coefficient
$C_s := 0.503385 \times 10^{-12} \cdot \frac{\text{F}}{\text{cm}}$	Capacitance/ finger pair/ m

From the Impulse Response model one can calculate wavelength (λ), and the number of finger pairs (Np), and the electrode widths, using the following equations, where v is the acoustic velocity in the media, f_0 is the center or synchronous frequency, and NBW is the null bandwidth or fractional frequency.

$\lambda := \frac{v}{f_0}$	$\lambda = 60\,\mu\text{m}$	Wavelength
$W_f := \frac{\lambda}{4}$	$W_f = 15\,\mu\text{m}$	Width of electrode fingers and spaces.
$N_p := \text{round}\left(\frac{2}{\text{NBW}} \cdot f_0\right)$	$N_p = 70$	Number of Finger pairs

Aperture Optimization

An optimal design must match the IDT resistance (real impedance) to the source resistance. The IDT finger overlap or aperture height (H_a) is often adjusted so that the IDT design achieves the correct IDT resistance. The following equation was used to optimize the aperture in terms of the input resistance (50Ω for most cases). Using these values above yields an optimized aperture of 2399.0 μm.

$$H_a := \frac{1}{50\cdot\Omega}\cdot\left(\frac{1}{f_0 2\cdot C_s\cdot N_p}\right)\cdot\mathrm{Re}\left(\frac{1}{4\cdot k^2\cdot N_p + j\cdot\pi}\right) \quad . \qquad H_a = 2399\,\mu\mathrm{m}$$

Finger Aperture

Frequency Response

The frequency response (transfer function) of a SAW device (Fig. 2) is the ratio of V_2 over V_1, and is shaped by the sinc function. The variable x is used to simplify the frequency response equation and is defined as [2]:

$$x(f) := \mathrm{if}\left[f = f_0, 0, N_p\cdot\pi\cdot\left(\frac{f - f_0}{f_0}\right)\right] \quad .$$

x is used in the equation for the frequency response [2]:

$$H_n(f) := 20\cdot\log\left[\left|4\cdot k^2\cdot C_s\cdot\frac{m}{F}\cdot f_0\cdot s\cdot N_p^{\,2}\cdot\left(\frac{\sin(x(f))}{x(f)}\right)^2\cdot e^{\frac{-j\cdot\left(N_p+5\right)}{f_0\cdot s}}\right|\right] \quad .$$

Frequency Response

where f is the frequency, k is the piezoelectric coupling coefficient, D is the delay between IDTs in wavelengths, and C_s is the capacitance for a finger pair per unit length. Often the frequency response is normalized using the log equation.

Radiation Conductance

The real part of the IDT admittance is called the radiation conductance. The radiation conductance is also shaped by the sinc function [2], and is normalized by dividing by the radiation conductance at the synchronous frequency:

$$G_a(f) := 8\cdot k^2\cdot C_s\cdot H_a\cdot f_0\cdot N_p^{\,2}\cdot\left(\frac{\sin(x(f))}{x(f)}\right)^2 \quad ,$$

Radiation Conductance

$$G_n(f) := \frac{G_a(f)}{G_a\left(f_0\right)} \quad .$$

Normalized Radiation Conductance

Acoustic Susceptance

The second element of the model is the imaginary part of the IDT admittance and is called the acoustic susceptance. It is the acoustic wave phenomena modeled as an electrical parameter. Notice that the acoustic susceptance is normalized using the radiation conductance since the acoustic susceptance at the synchronous frequency is zero. The Hilbert transform of the radiation conductance gives the acoustic susceptance [2]:

$$B_a(f) := G_a\left(f_0\right)\cdot\frac{(\sin(2\cdot x(f))) - 2\cdot x(f)}{2\cdot x(f)^2} \quad ,$$

Acoustic Susceptance

$$B_n(f) := \frac{B_a(f)}{G_a\left(f_0\right)} \quad .$$

Normalized Acoustic Susceptance

Total Capacitance

The third element of the model is the total static capacitance (C_T) for the IDT, and is found by multiplying the capacitance per unit length for a pair of fingers (C_s) times the finger overlap or aperture (H_a) times the number of finger pairs (N_p).

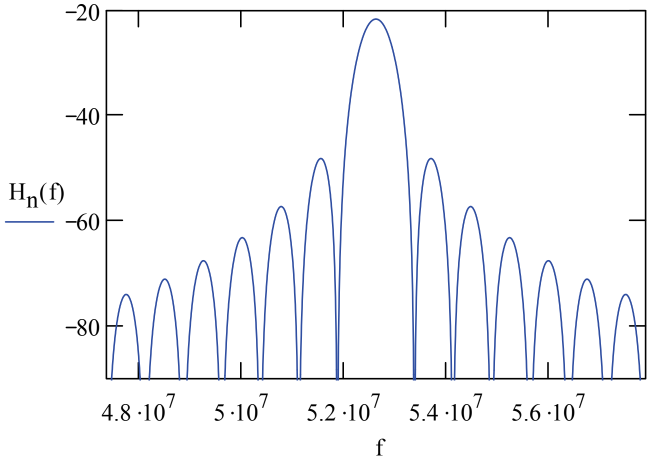
$$C_T := N_p\cdot C_s\cdot H_a \quad .$$

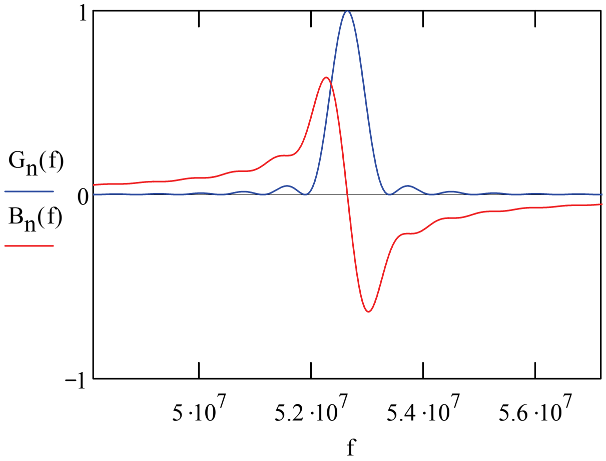
Total IDT Capacitance

Analysis

The plots below are the frequency response, radiation conductance, and the acoustic susceptance of the example design. These plots allow complete analysis of the SAW device.

Values for Plotting: $f_{min} := 0.5 \cdot f_0$ $f_{max} := 1.5 \cdot f_0$ $f := f_{min}, f_{min} + \frac{f_{max} - f_{min}}{10000} .. f_{max}$

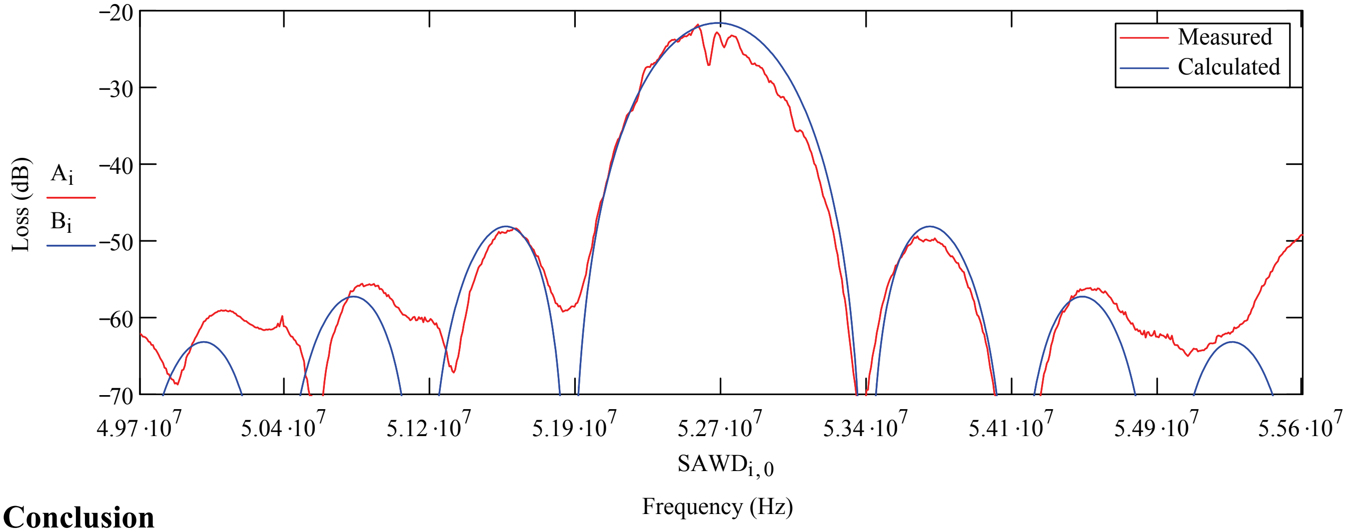




Results

The calculated and the measured frequency response of the SAW delay line is plotted below. The model captures the main characteristics the central lobe and the first side lobes. However, it does not do a good job on the second or subsequent side lobes.
It also does not handle any second order effects such as the ripples seen on the main lobe caused by triple transit echoes.

SAWD := READFILE("D3Q3_Mod1.TXT" ,"delimited") $i := 0 .. 800$ $A_i := SAWD_{i,1}$ $B_i := H_n(SAWD_{i,0} \cdot Hz)$



Conclusion

An implementation of the Impulse Response Model for a SAW delay line device has been presented. The model has been presented, along with the results from an example SAW delay line. The model first order model captures the main characteristics of the central lobe and the first side lobes.

Further Information

See "1st Order Modeling of a SAW Delay Line using MathCAD", by W. C. Wilson, and G. M. Atkinson, NASA Technical Report Server **(http://ntrs.nasa.gov)**, NASA Center: Langley Research Center, Document ID: 20070016024, May 9, 2007.

References

[1] W. H. Prosser, "Development of Structural Health Management Technology for Aerospace Vehicles", NASA LaRC, JANNAF 39th CS/27th APS/21st PSHS/3rd MSS Joint Subcom. Meeting, Dec. 1-5, 2003; 20031216.

[2] C.S. Hartmann, D. T., Bell, R. C. Rosenfeld, "Impulse Model Design of Acoustic Surface-Wave Filters", Microwave Theory and Techniques, IEEE Trans. on, Vol. 21, Iss. 4, Apr. 1973, pp. 162 ~ 175.

[3] E. Christen, K. Bakalar, "VHDL-AMS-a Hardware Description Language for Analog and Mixed-Signal Applications", Circuits and Systems II: Analog and Digital Signal Processing, IEEE Trans. on, Vol. 46, Iss. 10, Oct. 1999, pp. 1263 ~ 1272.

[4] A. J. Slobodnik Jr., "Surface acoustic waves and SAW materials", Proc. of the IEEE, Vol. 64, Iss. 5, May 1976, pp. 581 ~ 595.