NASA/TP—2007–214962

# Topology Synthesis of Structures Using Parameter Relaxation and Geometric Refinement

*P.V. Hull and M.L. Tinker*
*Marshall Space Flight Center, Marshall Space Flight Center, Alabama*

***June 2007***

## The NASA STI Program…in Profile

Since its founding, NASA has been dedicated to the advancement of aeronautics and space science. The NASA Scientific and Technical Information (STI) Program Office plays a key part in helping NASA maintain this important role.

The NASA STI program operates under the auspices of the Agency Chief Information Officer. It collects, organizes, provides for archiving, and disseminates NASA's STI. The NASA STI program provides access to the NASA Aeronautics and Space Database and its public interface, the NASA Technical Report Server, thus providing one of the largest collections of aeronautical and space science STI in the world. Results are published in both non-NASA channels and by NASA in the NASA STI Report Series, which includes the following report types:

- TECHNICAL PUBLICATION. Reports of completed research or a major significant phase of research that present the results of NASA programs and include extensive data or theoretical analysis. Includes compilations of significant scientific and technical data and information deemed to be of continuing reference value. NASA's counterpart of peer-reviewed formal professional papers but has less stringent limitations on manuscript length and extent of graphic presentations.
- TECHNICAL MEMORANDUM. Scientific and technical findings that are preliminary or of specialized interest, e.g., quick release reports, working papers, and bibliographies that contain minimal annotation. Does not contain extensive analysis.
- CONTRACTOR REPORT. Scientific and technical findings by NASA-sponsored contractors and grantees.
- CONFERENCE PUBLICATION. Collected papers from scientific and technical conferences, symposia, seminars, or other meetings sponsored or cosponsored by NASA.
- SPECIAL PUBLICATION. Scientific, technical, or historical information from NASA programs, projects, and missions, often concerned with subjects having substantial public interest.
- TECHNICAL TRANSLATION. English-language translations of foreign scientific and technical material pertinent to NASA's mission.

Specialized services also include creating custom thesauri, building customized databases, and organizing and publishing research results.

For more information about the NASA STI program, see the following:

- Access the NASA STI program home page at <http://www.sti.nasa.gov>
- E-mail your question via the Internet to <help@sti.nasa.gov>
- Fax your question to the NASA STI Help Desk at 301–621–0134
- Phone the NASA STI Help Desk at 301–621–0390
- Write to:
  NASA STI Help Desk
  NASA Center for AeroSpace Information
  7115 Standard Drive
  Hanover, MD  21076–1320

NASA/TP—2007–214962

# Topology Synthesis of Structures Using Parameter Relaxation and Geometric Refinement

*P.V. Hull and M.L. Tinker*
*Marshall Space Flight Center, Marshall Space Flight Center, Alabama*

National Aeronautics and
Space Administration

Marshall Space Flight Center • MSFC, Alabama 35812

***June 2007***

Available from:

NASA Center for AeroSpace Information
7115 Standard Drive
Hanover, MD 21076–1320
301–621–0390

This report is also available in electronic form at
<https://www2.sti.nasa.gov>

## TABLE OF CONTENTS

## LIST OF FIGURES

## LIST OF ACRONYMS

| | |
|---|---|
| CAD | computer-aided design |
| CM | compliant mechanism |
| CNC | computer numerical control |
| FEA | finite element analysis |
| FIN | input force |
| GA | genetic algorithm |
| TP | Technical Publication |

## NOMENCLATURE

| | |
|---|---|
| $cp$ | control point |
| $cp^j$ | initial control set |
| $cp^{j+1}$ | new set of control points |
| $cp_{\text{thickness}}$ | thickness of control point |
| $E$ | modulus of elasticity |
| $f$ | fitness or object function |
| $k_s$ | spring stiffness |
| $\mathbf{K}_{\text{tot}}$ | stiffness matrix representing both mechanism and the external springs |
| $M$ | mass |
| $\mathbf{S}$ | subdivision matrix |
| $t$ | thickness |
| $\mathbf{u}$ | vector of nodal displacement |
| $u_o$ | scalar output displacement in the direction of the spring |
| $W_i$ | strain energy in the system |
| $W_o$ | work out of the system |
| $\delta$ | deflection |
| $\sigma$ | stress |

TECHNICAL PUBLICATION

# TOPOLOGY SYNTHESIS OF STRUCTURES USING PARAMETER RELAXATION AND GEOMETRIC REFINEMENT

## 1. INTRODUCTION

Traditionally, structural synthesis problems are formulated by shape, size, and/or topology. Shape formulations change the boundary of the device or specific members to achieve a desired effect. Size optimization varies the thickness or width of specific elements to acquire a better solution. Topology formulations optimally distribute solid and void material over the fixed design space. This is the typical way of parameterizing the topology optimization problems.[1–5] The majority of the optimization tools commonly used share the same general process. They define the design parameters as a discretization of the design space, and then the discretized areas are characteristically assigned density parameter values corresponding to no material or material state at a point in the continuum (or apply relaxation by defining a range of values for the discrete density parameters to exist). A gradient-based or genetic algorithm (GA) optimization problem is then constructed, based on an objective function with associated constraints that attempt to combine in an optimal method, minimal mass, deflection, and stress. The objective function is evaluated using finite element analysis (FEA) on the candidate topology within the discretized design space. The sensitivity derivative calculation is expedited with the use of a constant-size stiffness matrix.

Current research efforts have focused their attention within this procedural framework, modifying components of this process, such as the objective function formulation, the type of objective function evaluation used, or the optimization techniques employed. Many researchers choose to employ the method of parameter relaxation at this point in the design process by replacing the discrete valued parameters with defined parameters over a range. Parameter relaxation is used to find a more optimum solution to the problem; it is commonly applied to density values or spatial parameters over the discrete areas. For the density relaxation, a penalty scheme is then frequently exploited to suppress the intermediate density parameters by removing low, unused densities to produce a design that is more concrete.

Proposed here is a thickness relaxation scheme applied to a control point parameterization using subdivision. This control point parameterization, first demonstrated by Hull and Canfield,[1] defines the design domain in terms of control meshes. Using this parameterization, relaxation is implemented and parameter suppression-focused penalty schemes are avoided. A brief review of relaxation pertaining to compliant mechanism (CM) design problems is presented next.

## 2. RELAXATION

Relaxation of the design parameters in structural problems is used to improve the optimality of a solution when compared to the traditional constant thickness problems. This relaxation of the constant thickness design parameter transfers the problem from a discrete to a continuous mathematical programming problem. In addition to changing the nature of the optimization problem, relaxation of the variables also demonstrates the ability to use alternate materials or material properties over the design space to achieve added functionality. The literature demonstrates that the 0-1 (void-full) discrete topology optimization problems, absent of parameter relaxation, lack optimum solutions in general.[6] This is due to the radical change in the efficiency measure with the introduction of relaxed intermediate variables. Often the relaxation principle is applied to composites to allow for the anisotropic properties of the design material. The relaxation process applied generally for structural design problems uses a $\rho$-type method (relax on a single parameter) or homogenization method (relax on multiple parameters and then find average constitutive parameters of the more complex material description). The foremost motivation for using relaxation of the design parameters here is to produce a greater convergence of the solutions.

Many researchers apply relaxation techniques to CM or structural design problems. Saxena uses a multiple material approach by relaxing the discrete density variable, giving a proportional value.[7] He uses this approach for multimaterial design without increasing the number of design parameters. Borvall and Petersson and Jog and Haber use density relaxation for structural design.[8,9] This technique is demonstrated on discrete elements in figure 1. Poulson relaxes density parameters and constraint values for design of CMs.[10] Although many researchers are using density relaxation, it is well known that the relaxed density parameter CM design problems do not lead to useful designs.[6] Based on that assumption, several researchers have developed specific techniques to suppress intermediary density variables either during or after optimization. Lau et al. applied such a penalty scheme and Zhou and Rozvany used the SIMP technique.[11,12]

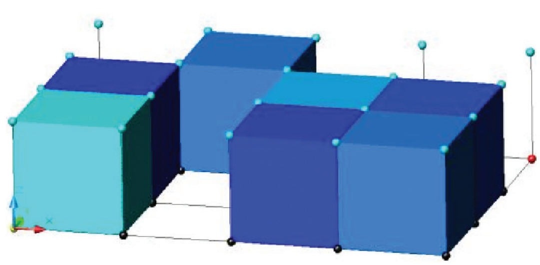

Figure 1. Density relaxed values for structural design problem—variable densities represented by variable shades.

Another method for relaxing design variables in CM design problems is to hold the material properties constant but change the sizes or shapes of the discrete elements on the macro scale.[4,7–9] A method called restriction is used to keep any design variable changes in a discrete range. Similar to density relaxation, researchers use this method to expand the range of optimum solutions through the introduction of a geometric optimal variable. One advantage of this method is that it is relatively simple to solve. The technique offered in this Technical Publication (TP) relaxes the thickness variable on the discretized control points, subdivides the model, and produces a manufacturable design with isotropic material without the need for any penalization scheme or modification of the final design. Node wandering is another topic for relaxation of design variables, which will not be reviewed in this TP.

This TP is based largely on a previous paper by the author; a brief review of that work is provided here. Hull and Canfield proposed approaching the CM design problem with an alternate design space parameterization through control meshes.[1] Then, the geometric refinement technique of subdivision was applied over the design space that created a solid model representation. This control point discretization scheme easily converts the given set of design parameters to a solid model for direct analysis and manufacture without user translation. Hull and Canfield use the geometric subdivision technique to define a smooth curve or surface as the limit of a sequence of successive refinements while also removing singularity points, high stress anomalies from checkerboard patterns, and distinctive high stress concentrations from "block" discretizations.[1] This same parameterization will be used here, but the two-dimensional subdivision method will be extended to three-dimensional space.

In this TP, topology synthesis of structures is performed using thickness relaxation and three-dimensional geometric subdivision with GA optimization. This problem is formulated as follows: The design space is discretized with a control mesh parameterization, followed by thickness parameter relaxation, then objective function definition, three-dimensional subdivision implementation, objective function evaluation using a commercial FEA program, and last, optimization through GAs. This procedure is applied, focusing on the alternate designs created by relaxing the spatial parameter thickness. In addition, it will be shown that relaxation of certain parameters may extend the range of problems that can be addressed; e.g., in permitting limited out-of-plane motion for a path generation problem for the nonsymmetric subdivision problem.

## 3. PROBLEM FORMULATION

The traditional design space parameterization for structural design problems is posed in a void-full form, such that density parameters assume discrete values, indicating a material or no material state.[9] Relaxation is often applied to the discrete density parameters during optimization, resulting in a more optimum topology that possesses discretized regions with variable densities. In the essence and purpose of density relaxation, a thickness relaxation scheme with subdivision is presented here. The thickness parameter is relaxed over the discretized control points in the design space; then, geometric smoothing is performed over the variable control point thicknesses.

The classical structural design problem is optimized using a fixed stiffness matrix. The designer defines the constant stiffness matrix prior to optimization that is used throughout the search for an optimum design. Parameter relaxation is easily implemented by varying global stiffness values, such as density and modulus, on a discrete level. Proposed here is a solid model representation of the structural design, which results in a specific stiffness matrix with each design throughout the optimization process. Relaxation of the thickness parameter is applied over each design. This solid model representation is achieved through the control point discretization and subdivision refinement.

### 3.1 Control Point Introduction and Parameterization Scheme

There are many different discretization schemes used to formulate the structural design problems. Parsons and Canfield offer a frame element discretization,[13] while Hull and Canfield, Yin and Ananthasuresh, Fanjoy and Crossley, and Poulson offer a discretized block element topology.[1,4,5,10] A reference-based discretization is offered by Zhou and Rozvany,[12] and Saxena and Saxena present a honeycomb discretization area.[14] Presented here is a control mesh parameterization.

The control mesh parameterization used in this research facilitates conversion of the design parameters (information about the control points) to a solid model in a standard form for later analysis or manufacture. This transition provides a high-level definition of the solid model ready for prototype fabrication with a limited number of design parameters. Each structural design throughout the optimization process is presented as a solid model description with a high level of resolution and definition by use of subdivision. The control point design parameter space provides a description of the topology, shape, size, and material properties of a potential structural design. The solid model description, translated from control mesh to solid model through subdivision, is accepted by commercially available FEA, CAD, and CNC software. The solid model implementation to commercial FEA programs in batch mode facilitates an efficient modeling of the material elastic deformation, while a representation of the design in this form permits direct manufacture through numerical-controlled machining. The solid model definition also eliminates the numerical problems that frequently occur in structural design problems, including FEA checkerboard anomalies, high stress regions, and mesh dependency.

The control points in this parameterization scheme are equally spaced over the design space in the x, y, and z coordinates according to the divisions specified in the problem definition. This discretization of the structural design space is unique because the control meshes that assemble this parameterization contain specific material properties, position, and boundary information. This information is used in the transition to a solid model description. The material properties potentially include modulus, Poisson's ratio, thermal conductivity, etc. The control mesh parameterization applied is specifically a single-layer, three-dimensional parameterization of the design space. The boundary information found at each control mesh quantifies the existence of surrounding discrete areas with uniform thickness (40 control points characterized into six node types, shown in fig. 2).

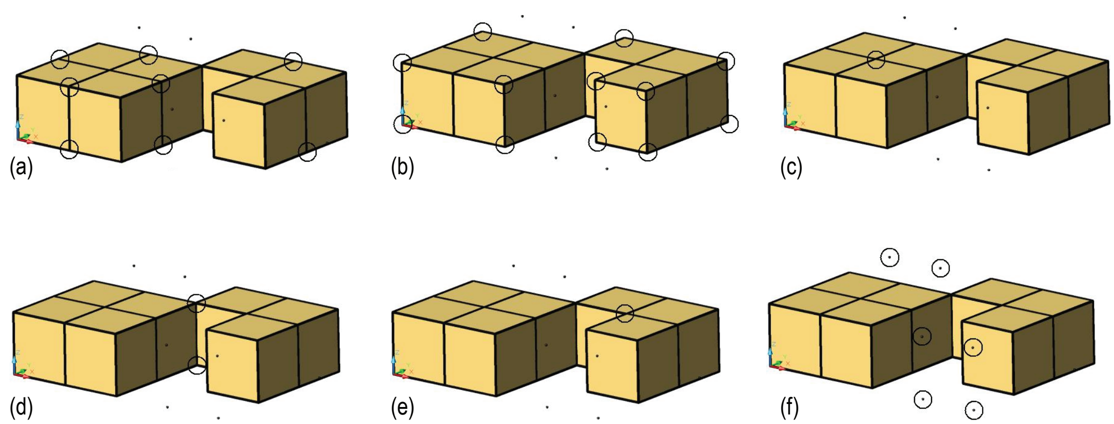


Figure 2. Control points characterized into six node types: (a) edge node, (b) exterior corner node, (c) interior node, (d) singularity node, (e) interior corner node, and (f) void nodes.

The chosen control mesh design space parameterization and subsequent solid model definition through three-dimensional geometric subdivision has many implications on the structural design problem it is used to facilitate. The subdivision step in the design process is discussed in section 3.4. Next is the discussion of thickness relaxation at the control points.

### 3.2 Thickness Parameter Relaxation

The method of relaxation is commonly applied to the initial design parameters of a structural design problem to transfer the problem from a discrete to a continuous mathematical programming problem. Also, by changing the disposition of the synthesis problem, relaxation displays the ability to model alternate materials and geometric properties within the design parameters. Relaxation of the design parameters is often performed on the discrete element density parameters because it is well known that the binary {0-1} topology optimization problems lack optimum solutions.[6]

The control mesh discretized CM problem described by Hull and Canfield[1] constrains the thickness variable to two parameters, either zero or one as shown below, where the optimization problem is given as follows:

Minimize: $f\left(cp\right)$

Subject to: $cp_{\text{thickness}} = \begin{cases} 1 \\ 0 \end{cases}$ .

Hull and Canfield demonstrate an optimal solution using control meshes;[1] however, as shown in figure 3(a), each design is held to a constant thickness. A benefit to this method is that the design is readily manufactured with a two-axis CNC; conversely, a significantly improved solution is available using a variable thickness. Relaxation of the thickness parameter enables the optimal CM synthesis problem to reach a greater level of optimal designs unavailable to the unrelaxed problem. Relaxation of the thickness parameter at the control points is formulated as follows:

Minimize: $f\left(cp\right)$

Subject to: $0 \le cp_{\text{thickness}} \le 1$ .

A comparison of the unrelaxed thickness problem to the relaxed thickness problems at the control points is given in figure 3.

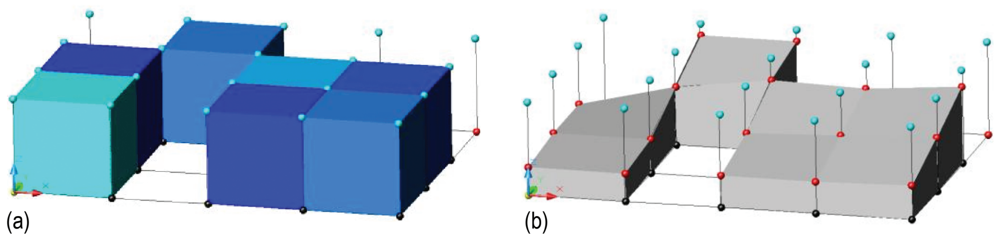


Figure 3. Control meshes: (a) traditional binary discretized design space with varying density values and (b) control point averaged discretized design space with varying thickness.

A vector of 20 thickness values at the control points creates the relaxed thickness design shown in figure 3(b). Likewise, figure 1 demonstrates the binary parameterization by either material or no material states with relaxed density values, while this traditional design space is given as a set 12 of density values. In addition to the thickness relaxation problem, it will be shown that relaxation of certain parameters also extends the range of problems that can be addressed; e.g., in permitting limited out-of-plane motion. This is discussed further in the subdivision section.

## 3.3 Objective Function Definition

### 3.3.1 Structures

There are many fitness functions available for topology optimization problems. Two common objective function types will be described in this TP—minimal mass with stress and deflection constraints—for structures, and minimal strain energy for CMs.

**3.3.1.1 Structural Objective Functions**. A common fitness or objective function for structural problems follows the precedent of designing a structure for minimal mass ($M$) while maintaining certain deflection ($\delta$) and stress ($\sigma$) constraints. The objective function formulation is given as follows:

$$\text{Objective} = M \ \text{kg} \times \delta_{(\text{constraint})} \times \sigma_{(\text{constraint})} \quad . \tag{1}$$

Each design problem has an allowable deflection and stress.

**3.3.1.2 Compliant Mechanism Objective—Strain Energy**. The fitness or objective function for this problem will follow the precedent of designing for flexibility as well as stiffness. While many objective functions have been defined and could be implemented, a measure defined in Parsons and Canfield is applied here.[13] For this function, a measure of flexibility is defined as the total work in the output spring ($W_o$):

$$W_o = \text{sign}(u_o) \times \frac{1}{2} \times k_s \times {u_o}^2 \quad , \tag{2}$$

where $u_o$ is the scalar output displacement in the direction of the spring and $k_s$ is the spring stiffness. The measure of stiffness will be defined as the strain energy in the system ($W_i$):

$$W_i = \frac{1}{2} \times \mathbf{u}^T \times \mathbf{K}_{\text{tot}} \times \mathbf{u} \quad , \tag{3}$$

where $\mathbf{u}$ is the vector of nodal displacements and $\mathbf{K}_{\text{tot}}$ is the stiffness matrix representing both the mechanism and the external springs. The objective function that maximizes the output energy while minimizing total strain energy in the mechanism is then given as

$$f = \frac{\text{sign}(u_o) \times \frac{1}{2} \times k_s \times {u_o}^2}{\frac{1}{2} \times \mathbf{u}^T \times \mathbf{K}_{\text{tot}} \times \mathbf{u}} \quad , \tag{4}$$

where $f$ is the fitness or objective function.

## 3.4 Formulation of the Three-Dimensional Subdivision Scheme

Subdivision is a "corner cutting" technique used to describe smooth curves or surfaces; the initial idea is traceable to the early 1940s. However, the concrete formulation for surface modeling using subdivision was offered in two papers by Doo and Sabin and Catmull and Clark, each offering their respective subdivision schemes.[15,16] All subdivided surfaces begin with some type of polygonal surface, referred to as the initial surface. This surface of polygonal faces is subdivided into supplementary polygons. In this TP, all polygons used are quadrilaterals. This method of geometric surface refinement is widely used in animated features such as "A Bug's Life" and "Toy Story." This surface refinement method offers a modeling approach for structural topology problems that holds several benefits, including efficiency, compact support, local definition, affined invariance, simplicity, and definable continuity of the surface.[17]

There are many subdivision schemes available. The method used here is fundamentally based on the Doo and Sabin scheme.[15] This scheme is a stationary subdivision method (constant weightings) generating a $C^1$ continuous surface from an arbitrary mesh. The process starts with an initial control mesh, given as a set of equally spaced control points (**cp**), defined by the specified division of the design space. The three-dimensional subdivision scheme classifies the initial control points into several categories as shown in figure 2. The six classes (a to f) are given by the vector of control points ($\mathbf{cp^j}$), shown in equation (5):

$$cp_i = \begin{cases} \text{type a} \\ \text{type b} \\ \text{type c} \\ \text{type d} \\ \text{type e} \\ \text{type f} \end{cases} , \quad \forall i, \quad i = 1 \quad n \ . \qquad (5)$$

These classified control points contain averaged or subdivided information, including position coordinates, material properties, and boundary classification. This control point classification scheme is directly dependent upon the orientation and number of vertices, as shown in figure 2. The control points ($\mathbf{cp^j}$) are modified at successive steps to create a new set of control points ($\mathbf{cp^{j+1}}$). This numerical refinement is performed by multiplying $\mathbf{cp^j}$ by a subdivision matrix (S) as shown here:

$$\begin{pmatrix} cp_0^{j+1} \\ cp_1^{j+1} \\ \\ cp_n^{j+1} \end{pmatrix} = \mathbf{S} \begin{pmatrix} cp_0^{j} \\ cp_1^{j} \\ \\ cp_n^{j} \end{pmatrix} . \qquad (6)$$

The initial control set ($\mathbf{cp^j}$) describes the design parameter space, while the final control set ($\mathbf{cp^{j+1}}$) describes the solid model space. The subdivision matrix (**S**) is a matrix of weightings that define the creation of $\mathbf{cp^{j+1}}$. It is the relationship between the initial control points and the subdivided control points. The subdivision matrix used in this TP is defined here. This matrix is determined by the authors' preference for subdivision weightings:

$$S = \begin{bmatrix} \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\ \frac{3}{8} & \frac{3}{8} & \frac{1}{16} & \frac{1}{16} & 0 & \frac{1}{16} & \frac{1}{16} & 0 & 0 \\ \frac{3}{8} & \frac{1}{16} & \frac{3}{8} & 0 & \frac{1}{16} & \frac{1}{16} & 0 & \frac{1}{16} & 0 \\ \frac{3}{8} & \frac{1}{16} & 0 & \frac{3}{8} & \frac{1}{16} & 0 & \frac{1}{16} & 0 & \frac{1}{16} \\ \frac{3}{8} & 0 & \frac{1}{16} & \frac{1}{16} & \frac{3}{8} & 0 & 0 & \frac{1}{16} & \frac{1}{16} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \end{bmatrix} . \qquad (7)$$

The total volume charted by the initial control points is larger then the shape defined by the subdivided control points. This is called the approximating subdivision method. The case where the subdivided shape is larger then the unsubdivided shape is the interpolating method. The approximating scheme is demonstrated in figure 4.

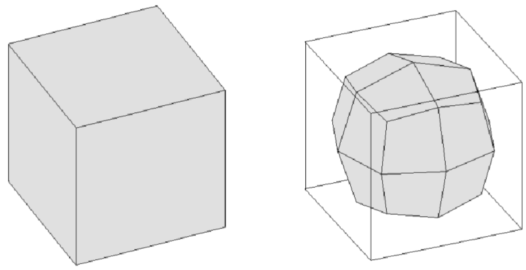

Figure 4. Single subdivided block.

Figure 5(a) displays a thickness relaxed control point discretization of the design space while (b) demonstrates a three-dimensional, subdivided solid model of the topology.

The foundation for the subdivision method is spline curves; however, subdivision differs in that it produces information of the definite surface as a sequence of control points. Requirements on **S** include eigenvectors to form a basis when the first eigenvalue is equal to 1 and the remainder less than 1.

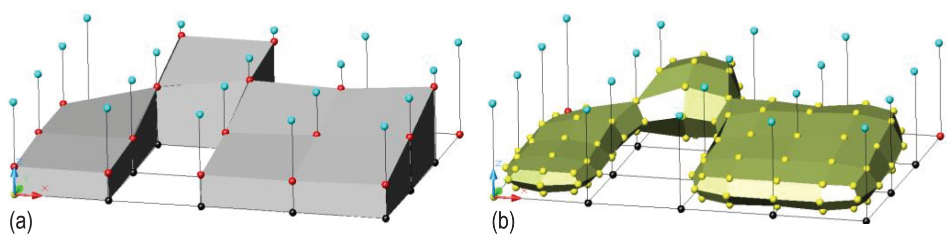


Figure 5. Design space and topology: (a) control point averaged discretized design space with varying thickness and (b) a three-dimensional, subdivided solid model of the topology.

The original set of control points are defined as the design parameters. The subdivision matrix for a three-dimensional surface mapped volume, similar to the two-dimensional method defined by Hull and Canfield,[1] creates a new point for every control point, line, and face. For example, given the block in figure 4, there are 8 control points, 12 lines, and 6 control meshes. Totaling these parameters gives the sum of 26 new subdivided control points.

### 3.5 Control Point Orientation Alternatives

The geometric refinement of the thickness relaxed structural problem is described in section 3.3. The subdivision method uses a weighting matrix, when applied on the control points, creates a new set of control points representing a solid model. Discussed here are the initial control point orientation alternatives and the effect they have on final solid model shape, specifically the nonsymmetric and symmetric problems as shown in figure 6.

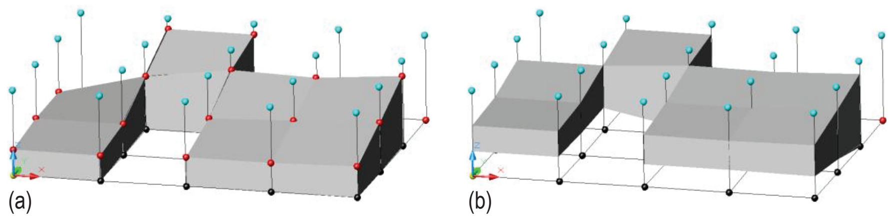


Figure 6. Control point orientation: (a) nonsymmetric and (b) symmetric.

Three solid model orientations are available for optimization and can be evaluated from a postsubdivision model—the nonsymmetric, flat-sided, and symmetric solid model. The nonsymmetric model permits out-of-plane motion of the structural design. This may be used in a path generation problem. Using this control point orientation allows the user to objectively design a structure or CM that moves in a third-dimensional direction. This is due to the nonsymmetric mass placements as shown in figure 7(a). The flat-side control point orientation is a full subdivision problem with one side flattened for designing a structure for application on a surface, shown in figure 7(b). Finally, the symmetric orientation problem constrains motions to a path similar to the nonrelaxed problem, where symmetry exists about an xy-plane, demonstrated in figure 7(c).

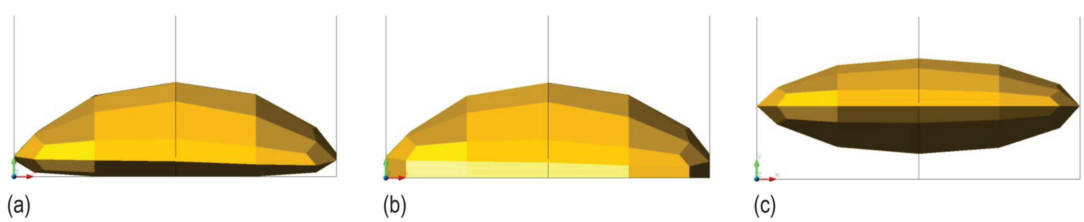


Figure 7. Postsubdivision model orientation: (a) nonsymmetric, (b) flat-sided, and (c) symmetric.

Presented in this TP are several examples following the symmetric problem and a specific application to the flat-side and nonsymmetric problem.

## 3.6 Genetic Algorithm Search Method

Genetic algorithm optimization is a highly suitable method for the optimal design of structures. Algorithms of this nature are guided random searches and therefore remove the requirement for gradient derivations. This permits a decidedly diverse choice of possible objective functions. Genetic algorithms possess many advantageous qualities, such as the ability to handle both convex and nonconvex objective functions and finding an optimal family of solutions. These benefits give the designer greater freedom to select the final design. A disadvantage is that genetic algorithms are costly to use due to high computation time and convergence performance that is complicated to predict.

In this work, a combination of large population, sufficient mutation rate, and stochastically-selected initial states were used to help guarantee a solution near the global optimum. If a more globally optimal solution is desired beyond that selected by the GA, a hybrid GA could be used. Further discussion of GAs as applied to topology optimization can be found in Hull and Canfield.[1]

## 3.7 Example Designs

Section 3.7 demonstrates the use of subdivision as part of the structural/compliant mechanisms design optimization tool. To perform this demonstration, an example that is common to the CM design literature is selected for analysis here—compliant inverter. This example is a common design problem in CM optimization. It is chosen to demonstrate the design tool implementation over CM topologies that possess compliant flexural joints.

### 3.7.1 Compliant Inverter

The design domain for the compliant inverter is shown in figure 8. Pin boundary conditions are utilized at the upper and lower lefthand corners. This problem is discretized with finite blocks as shown. The design parameters given for this problem are as follows: Design space size is 4 in × 2 in; the thickness ($t$) equals 0.1 in; the design parameter mesh size is 0.25 in × 0.25 in; the modulus of elasticity ($E$) is 83,000 psi; the FEA element type is eight-node structural solid; input force (FIN) is 50 lb; and the spring stiffness (k) is 5 in-lb. The results from this problem are shown in figure 9 (half of the symmetric gripper shown). Figure 10 shows the subdivided topology and strain plot of subdivided topology.

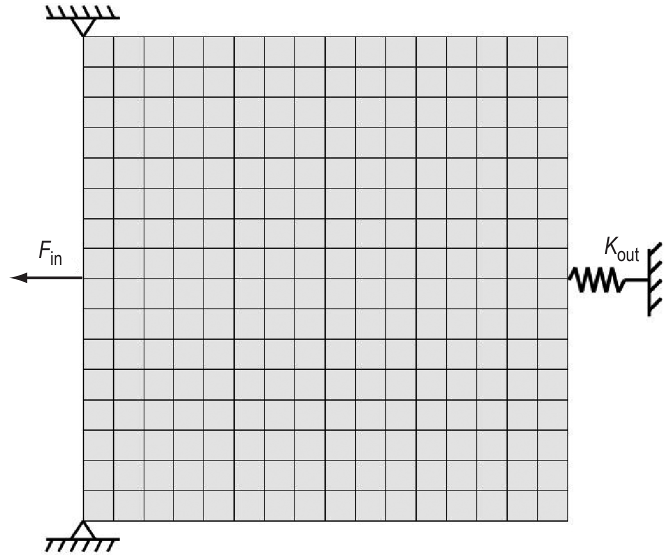


Figure 8. Symmetric inverter problem.

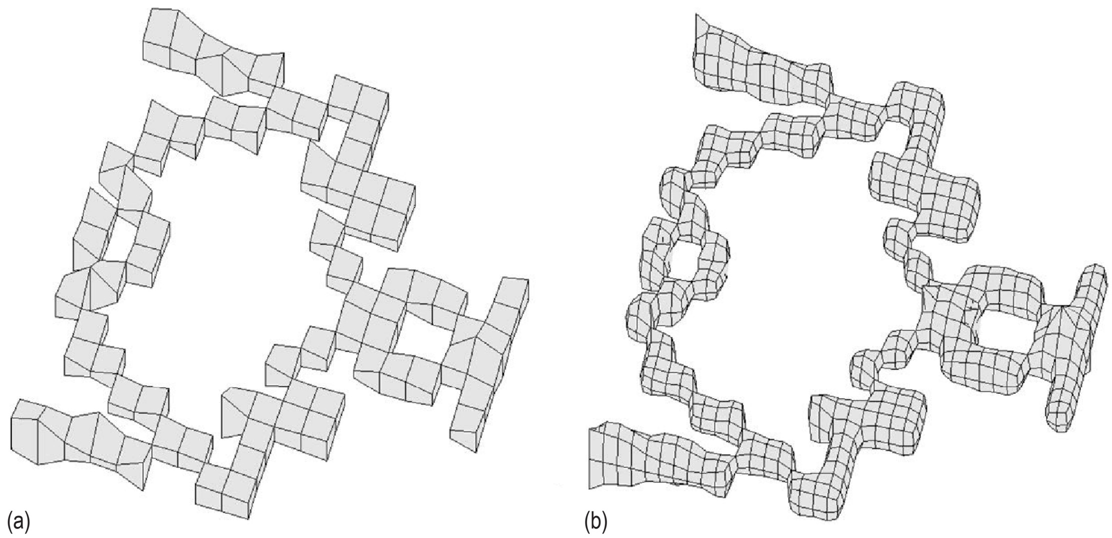


Figure 9. Isometric view of compliant inverter: (a) optimal block topology and (b) subdivided topology.

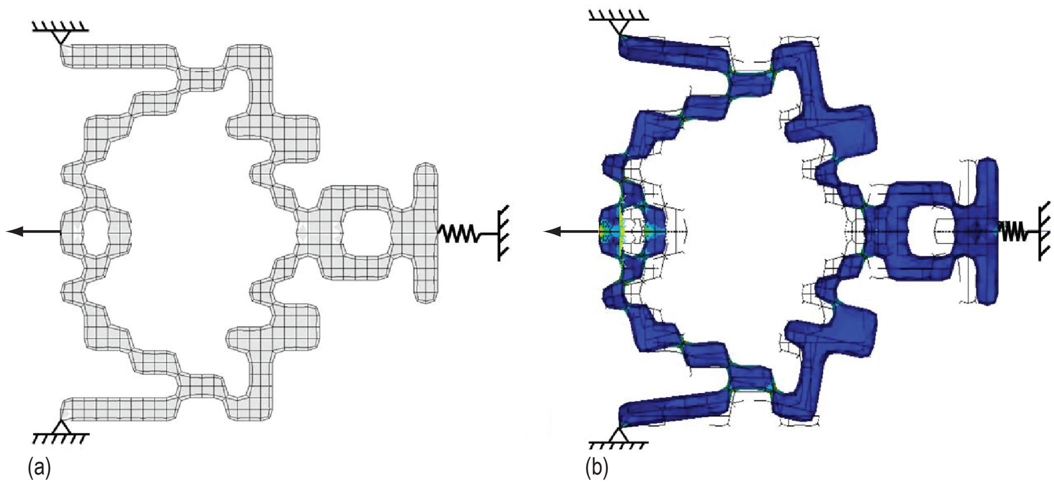


Figure 10. Compliant inverter: (a) subdivided topology and (b) strain plot of subdivided topology.

### 3.8 Truss

The design domain for a truss is shown in figure 11; pin boundary conditions are found on the left side of the design. This problem is discretized with finite blocks as shown. The design parameters given for this problem are the design space size = 720 in×360 in; the thickness, $t$ = 0.1 in; the design parameter mesh size = 15.6 in×15.6 in; $E$ = 83,000 psi; FEA element type = 8 node structural solid; and input force, $P_1 = P_2$ = 1,000 lb. The results from this problem are shown in figure 12.

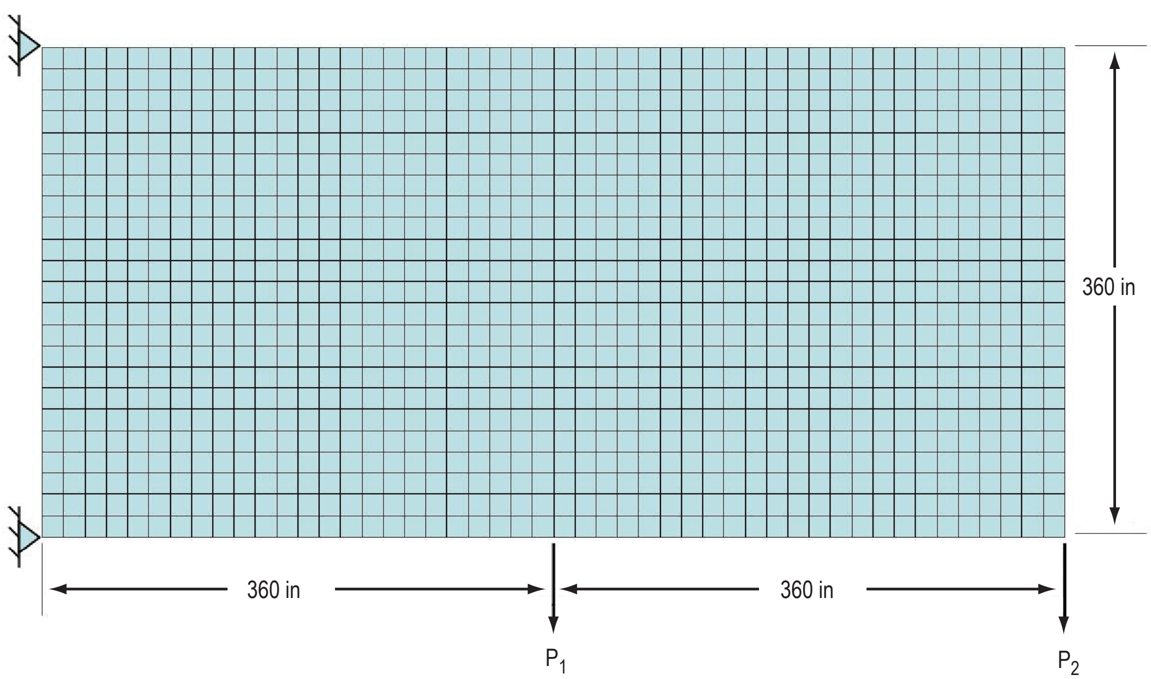


Figure 11. Symmetric inverter problem.

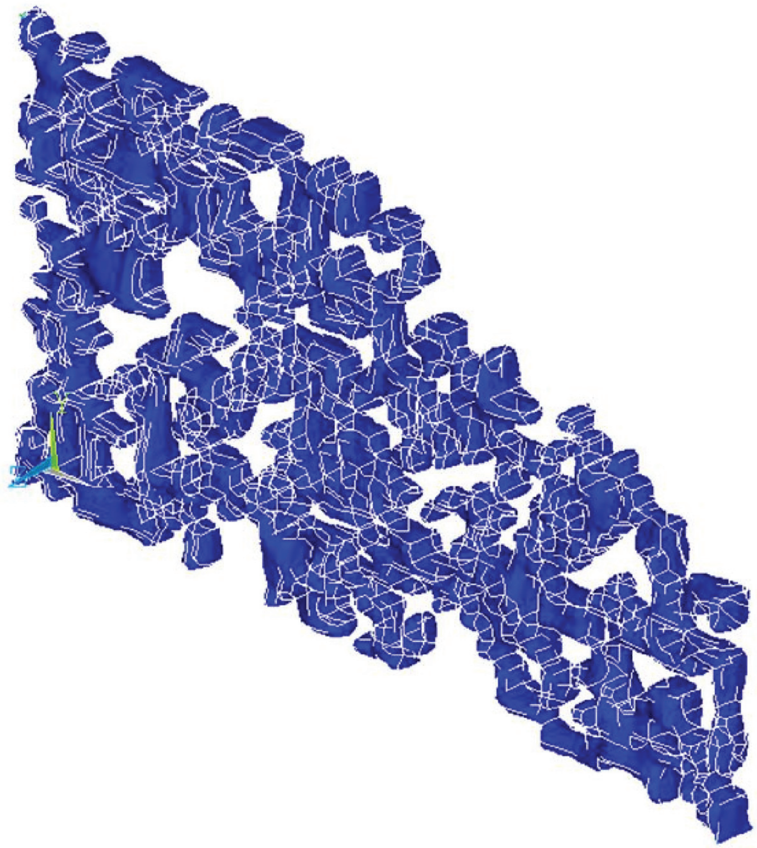

Figure 12. FEA strain model of a relaxed thickness truss.

The truss problem demonstrated above is a difficult problem to solve due to the immense size of the design region. The design region for this problem is as follows: topology design region: $246\times23=2^{2,186}$, relaxed thickness design region yields (applied on the control points with an 8-bit number defining the range) $47\times24^{256}$. A design region of such an immense size creates a problem very difficult to solve. Each design FEA evaluation takes ≈7 min using a Linux dual 3.8 GHz, 8 Gb ram, scsi computer. The above solution was arrived at through 5 mo of continuous calculations.

## 4. CONCLUSIONS

One of the significant results of the relaxation process offered in this TP is that direct manufacturability of the optimized design will be maintained without the need for designer intervention or translation. While the relaxed problems are not readily manufactured on a CNC, they are in a numeric form that is readily acceptable by a rapid prototyping, ultrasonic object consolidation, stereo lithography, or any other three-dimensional fabrication machine.

The future holds great promise for solving problems with such immense design regions and evaluation times as the truss example. Hopefully, this design tool will be fully utilized when the computing power is developed.

## REFERENCES


1. Hull, P.V.; and Canfield, S.: "Optimal Synthesis of Compliant Mechanisms Using Subdivision and Commercial FEA," *J. Mech. Design*, Submitted April 2004.

2. Diaz, A.; and Sigmund, O.: "Checkerboard Patterns in Layout Optimization," *Struc. Optim.*, Vol. 10, pp. 40–45, 2001.

3. Sigmund, O.; and Petersson, J.: "Numerical Instabilities in Topology Optimization: A Survey on Procedures Dealing With Checkerboards, Mesh Dependencies and Local Minima," *Struc. Optim.*, Vol. 16, pp. 68–75, 1998.

4. Yin, L.; and Ananthsuresh, G.K.: "Topology Optimization of Compliant Mechanisms With Multiple Materials Using a Peak Function Material Interpolation Scheme," *Struc. Multidisc. Optim*, Vol. 23, pp. 49–62, 2001.

5. Fanjoy, D.W.; and Crossley, W.A.: "Topology Design of Planar Cross-Sections With a Genetic Algorithm: Part 2—Bending, Torsion and Combined Loading Applications," *Eng. Optim.*, Vol. 34, No. 1, pp. 49–64, 2002.

6. Allaire, G.; and Kohn, R.V.: "Topology Optimization and Optimal Shape Design Using Homogenization," in *Topology Design of Structures*, M.P. Bendsoe and C.A. Mota Soares (eds.), Kluwer, Dordrecht, pp. 207–218, 1993.

7. Saxena, A.: "On Multiple-Material Optimal Compliant Topologies: Discrete Variable Parameterization Using Genetic Algorithm," *Proc. of The 2002 Design Engineering Technical Conferences*, Montreal, Canada, DETC2002/MECH-34209, 2002.

8. Borvall, T.; and Petersson, J.: "Topology Optimization Using Regularized Intermediate Density Control," *Comput. Meth. Appl. Mech. Eng.*, Vol. 190, pp. 4911–4928, 2001.

9. Jog, C.S.; and Haber, R.B.: "Stability of Finite Element Models for Distributed-Parameter Optimization and Topology Design," *Comput. Meth. Appl. Mech. Eng.*, Vol. 130, pp. 203–226, 1996.

10. Poulson, T.A.: "A New Scheme for Imposing a Minimum Length Scale in Topology Optimization," *Int. J. Num. Meth. Eng.*, Vol. 57, pp. 741–760, 2003.

11. Lau, G.K.; Du, H.; and Lim, M.K.: "Use of Functional Specifications as Objective Functions in Topological Optimization of Compliant Mechanism," *Comput. Meth. Appl. Mech. Eng.*, Vol. 190, pp. 4421–4433, 2001.

12. Zhou, M.; and Rozvany, G.I.N.: "The COC Algorithm, Part II: Topological, Geometrical and Generalized Shape Optimization," *Comput. Meth. Appl. Mech. Eng.*, Vol. 89, pp. 309–336, 1991.

13. Parsons, R.; and Canfield, S.: "Developing Genetic Programming Techniques for the Design of Compliant Mechanisms," *J. Int. Soc. Struc. Multidisc. Opt.*, Vol. 24, No. 1, pp. 78–86, 2002.

14. Saxena, R.; and Saxena, A.: "On Honeycomb Parameterization for Topology Optimization of Compliant Mechanisms," *Proceedings of the ASME Design Engineering Technical Conference*, Vol. 2 B, pp. 975–985, 2003.

15. Doo, D.; and Sabin, M.: "1978 Behaviour of Recursive Division Surfaces Near Extraordinary Points," *Computer Aided Design*, Vol. 10, No. 6, pp. 356–360, 1978.

16. Catmull, E.; and Clark, J.: "Recursively Generated B-spline Surfaces on Arbitrary Topological Meshes," *Computer Aided Design*, Vol. 10, pp. 350–355, 1978.

17. Sorin, D.; and Schroder, P.: Organizers of the "Subdivision for Modeling and Animation," SIGRAPH 2000 Course Notes, 2000.

| **REPORT DOCUMENTATION PAGE** | | | *Form Approved OMB No. 0704-0188* |
|---|---|---|---|



| 1. AGENCY USE ONLY *(Leave Blank)* | 2. REPORT DATE | 3. REPORT TYPE AND DATES COVERED |
|---|---|---|
| | June 2007 | Technical Publication |

**4. TITLE AND SUBTITLE**

Topology Synthesis of Structures Using Parameter Relaxation and Geometric Refinement

**5. FUNDING NUMBERS**

**6. AUTHORS**

P.V. Hull and M.L. Tinker

**7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)**

George C. Marshall Space Flight Center
Marshall Space Flight Center, AL 35812

**8. PERFORMING ORGANIZATION REPORT NUMBER**

M–1191

**9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)**

National Aeronautics and Space Administration
Washington, DC 20546–0001

**10. SPONSORING/MONITORING AGENCY REPORT NUMBER**

NASA/TP—2007–214962

**11. SUPPLEMENTARY NOTES**

Prepared by the Spacecraft and Vehicle Systems Department, Engineering Directorate

**12a. DISTRIBUTION/AVAILABILITY STATEMENT**

Unclassified-Unlimited
Subject Category 39
Availability: NASA CASI 301–621–0390

**12b. DISTRIBUTION CODE**

**13. ABSTRACT** *(Maximum 200 words)*

Typically, structural topology optimization problems undergo relaxation of certain design parameters to allow the existence of intermediate variable optimum topologies. Relaxation permits the use of a variety of gradient-based search techniques and has been shown to guarantee the existence of optimal solutions and eliminate mesh dependencies. This Technical Publication (TP) will demonstrate the application of relaxation to a control point discretization of the design workspace for the structural topology optimization process. The control point parameterization with subdivision has been offered as an alternative to the traditional method of discretized finite element design domain. The principle of relaxation demonstrates the increased utility of the control point parameterization. One of the significant results of the relaxation process offered in this TP is that direct manufacturability of the optimized design will be maintained without the need for designer intervention or translation. In addition, it will be shown that relaxation of certain parameters may extend the range of problems that can be addressed; e.g., in permitting limited out-of-plane motion to be included in a path generation problem.

**14. SUBJECT TERMS**

topology, optimization, finite element analysis, genetic algorithms, stress analysis

**15. NUMBER OF PAGES**

28

**16. PRICE CODE**

| 17. SECURITY CLASSIFICATION OF REPORT | 18. SECURITY CLASSIFICATION OF THIS PAGE | 19. SECURITY CLASSIFICATION OF ABSTRACT | 20. LIMITATION OF ABSTRACT |
|---|---|---|---|
| Unclassified | Unclassified | Unclassified | Unlimited |

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-18
298-102

National Aeronautics and
Space Administration
IS20
**George C. Marshall Space Flight Center**
Marshall Space Flight Center, Alabama
35812